

# Apêndice A

## Tabelas

### Identidades Trigonométricas

$$(1) \quad \sin^2 x + \cos^2 x = 1$$

$$(2) \quad 1 + \tan^2 x = \sec^2 x$$

$$(3) \quad 1 + \cot^2 x = \operatorname{cosec}^2 x$$

$$(4) \quad \sin^2 x = 1/2(1 - \cos 2x)$$

$$(5) \quad \cos^2 x = 1/2(1 + \cos 2x)$$

$$(6) \quad \sin 2x = 2 \sin x \cos x$$

$$(7) \quad \sin x \cos y = 1/2[\sin(x - y) + \sin(x + y)]$$

$$(8) \quad \sin x \sin y = 1/2[\cos(x - y) - \cos(x + y)]$$

$$(9) \quad \cos x \cos y = 1/2[\cos(x - y) + \cos(x + y)]$$

### Tabela de Derivadas

Nesta tabela  $u$  e  $v$  são funções deriváveis de  $x$  e  $c$ ,  $\alpha$  e  $a$  são constantes.

$$(1) \quad y = c \Rightarrow y' = 0$$

$$(2) \quad y = x \Rightarrow y' = 1$$

$$(3) \quad y = c \cdot u \Rightarrow y' = c \cdot u'$$

$$(4) \quad y = u + v \Rightarrow y' = u' + v'$$

$$(5) \quad y = u \cdot v \Rightarrow y' = u' \cdot v' + v \cdot u'$$

$$(6) \quad y = \frac{u}{v} \Rightarrow y' = \frac{v \cdot u' - u \cdot v'}{v^2}$$

$$(7) \quad y = u^\alpha (\alpha \neq 0) \Rightarrow y' = \alpha \cdot u^{\alpha-1} \cdot u'$$

$$(8) \quad y = a^u (a > 0, a \neq 1) \Rightarrow y' = a^u \cdot \ln a \cdot u'$$

$$(9) \quad y = e^u \Rightarrow y' = e^u \cdot u'$$

$$(10) \quad y = \log_a u \Rightarrow y' = \frac{u'}{u} \log_a e$$

$$(11) \quad y = \ln u \Rightarrow y' = \frac{u'}{u}$$

$$(12) \quad y = u^v \Rightarrow y' = v \cdot u^{v-1} \cdot u' + u^v \cdot \ln u \cdot v' (u > 0)$$

$$(13) \quad y = \sin u \Rightarrow y' = \cos u \cdot u'$$

$$(14) \quad y = \cos u \Rightarrow y' = -\sin u \cdot u'$$

(15)  $y = \operatorname{tg} u \Rightarrow y' = -\sec^2 u \cdot u'$

(16)  $y = \operatorname{cotg} u \Rightarrow y' = -\operatorname{cosec}^2 u \cdot u'$

(17)  $y = \sec u \Rightarrow y' = \sec u \cdot \operatorname{tg} u \cdot u'$

(18)  $y = \operatorname{cosec} u \Rightarrow y' = -\operatorname{cosec} u \cdot \operatorname{cotg} u \cdot u'$

(19)  $y = \operatorname{arc sen} u \Rightarrow y' = \frac{u'}{\sqrt{1-u^2}}$

(20)  $y = \operatorname{arc cos} u \Rightarrow y' = \frac{-u'}{\sqrt{1-u^2}}$

(21)  $y = \operatorname{arc tg} u \Rightarrow y' = \frac{u'}{1+u^2}$

(22)  $y = \operatorname{arc cotg} u \Rightarrow y' = \frac{-u'}{1+u^2}$

(23)  $y = \operatorname{arc sec} u, |u| \geq 1 \Rightarrow y' = \frac{u'}{|u|\sqrt{u^2-1}}, |u| > 1$

(24)  $y = \operatorname{arc cosec} u, |u| \geq 1 \Rightarrow y' = \frac{-u'}{|u|\sqrt{u^2-1}}, |u| > 1$

(25)  $y = \operatorname{senh} u \Rightarrow y' = \cosh u \cdot u'$

(26)  $y = \cosh u \Rightarrow y' = \operatorname{senh} u \cdot u'$

(27)  $y = \operatorname{tgh} u \Rightarrow y' = \operatorname{sech}^2 u \cdot u'$

(28)  $y = \operatorname{cotgh} u \Rightarrow y' = -\operatorname{cosech}^2 u \cdot u'$

(29)  $y = \operatorname{sech} u \Rightarrow y' = -\operatorname{sech} u \cdot \operatorname{tgh} u \cdot u'$

(30)  $y = \operatorname{cosech} u \Rightarrow y' = -\operatorname{cosech} u \cdot \operatorname{cotgh} u \cdot u'$

(31)  $y = \operatorname{argsenh} u \Rightarrow y' = \frac{u'}{\sqrt{u^2+1}}$

(32)  $y = \operatorname{arg cosh} u \Rightarrow y' = \frac{u'}{\sqrt{u^2-1}}, u > 1$

(33)  $y = \operatorname{arg tgh} u \Rightarrow y' = \frac{u'}{1-u^2}, |u| < 1$

(34)  $y = \operatorname{arg cotgh} u \Rightarrow y' = \frac{u'}{1-u^2}, |u| > 1$

(35)  $y = \operatorname{arg sech} u \Rightarrow y' = \frac{-u'}{u\sqrt{1-u^2}}, 0 < u < 1$

(36)  $y = \operatorname{arg cosech} u \Rightarrow y' = \frac{-u'}{|u|\sqrt{1+u^2}}, u \neq 0.$

## Tabela de Integrais

(1)  $\int du = u + C$

(2)  $\int \frac{du}{u} = \ln |u| + C$

(3)  $\int u^\alpha du = \frac{u^{\alpha+1}}{\alpha+1} + C (\alpha \text{ é constante } \neq -1)$

(4)  $\int a^u du = \frac{a^u}{\ln a} + C$

(5)  $\int e^u du = e^u + C$

(6)  $\int \operatorname{sen} u du = -\cos u + C$

(7)  $\int \cos u du = \operatorname{sen} u + C$

(8)  $\int \operatorname{tg} u du = \ln |\sec u| + C$

$$(9) \quad \int \cotg u \, du = \ln |\sen u| + C$$

$$(11) \quad \int \sec u \, du = \ln |\sec u + \tg u| + C$$

$$(13) \quad \int \cosec^2 u \, du = -\cotg u + C$$

$$(15) \quad \int \cosec u \cdot \cotg u \, du = -\cosec u + C$$

$$(17) \quad \int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctg \frac{u}{a} + C$$

$$(19) \quad \int \senh u \, du = \cosh u + C$$

$$(21) \quad \int \sech^2 u \, du = \tgh u + C$$

$$(23) \quad \int \sech u \cdot \tgh u \, du = -\sech u + C$$

$$(25) \quad \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$(27) \quad \int \frac{du}{u \sqrt{a^2 \pm u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 \pm u^2}}{u} \right| + C$$

$$(10) \quad \int \cosec u \, du = \ln |\cosec u - \cotg u| + C$$

$$(12) \quad \int \sec^2 u \, du = \tg u + C$$

$$(14) \quad \int \sec u \cdot \tg u \, du = \sec u + C$$

$$(16) \quad \int \frac{du}{\sqrt{a^2 - u^2}} = \arcsen \frac{u}{a} + C$$

$$(18) \quad \int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arc sec} \left| \frac{u}{a} \right| + C$$

$$(20) \quad \int \cosh u \, du = \senh u + C$$

$$(22) \quad \int \cosech^2 u \, du = -\cotgh u + C$$

$$(24) \quad \int \cosech u \cdot \cotgh u \, du = -\cosech u + C$$

$$(26) \quad \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u + a}{u - a} \right| + C$$

## Fórmulas de Recorrência

$$(1) \quad \int \sen^n u \, du = -\frac{1}{n} \sen^{n-1} u \cos u + \frac{n-1}{n} \int \sen^{n-2} u \, du$$

$$(2) \quad \int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sen u + \frac{n-1}{n} \int \cos^{n-2} u \, du$$

$$(3) \quad \int \tg^n u \, du = \frac{1}{n-1} \tg^{n-1} u - \int \tg^{n-2} u \, du$$

$$(4) \quad \int \cotg^n u \, du = -\frac{1}{n-1} \cotg^{n-1} u - \int \cotg^{n-2} u \, du$$

$$(5) \quad \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tg u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$$