On Denoting 14

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By a 'denoting phrase' I mean a phrase such as any one of the following: a man, some man, any man, every man, all men, the present King of England, the present King of France, the center of mass in the solar system at the first instant of the twentieth century, the revolution of the earth round the sun, the revolution of the sun round the earth. Thus a phrase is denoting solely in virtue of its *form*. We may distinghish three cases: (1) A phrase may be denoting, and yet not denote anything; e.g., 'the present King of France'. (2) A phrase may denote one definite object, e.g., 'the present King of England' denotes a certain man. (3) A phrase may denote ambiguously; e.g., 'a man' denotes not many men, but an ambiguous man. The interpretation of such phrases is a matter of considerable difficulty; indeed, it is very hard to frame any theory not susceptible of formal refutation. All the difficulties with which I am acquainted are met, so far as I can discover, by the theory which I am about to explain.

The subject of denoting is of very great importance, not only in logic and mathematics, but also in theory of knowledge. For example, we know that the center of mass of the solar system at a definite instant is some definite point, and we can affirm a number of propositions about it; but we have no immediate *acquaintance* with this point, which is only known to us by description. The distinction between *acquaintance* and *knowledge about* is the distinction between the things we have presentations of, and the things we only reach by denoting phrases. It often happens that we know that a certain phrase denotes unambiguously, although we have no acquaintance with what it denotes; this occurs in the above case of the center of mass. In perception we have acquaintance with the objects of perception, and in thought we have acquaintance with objects of a more abstract logical character; but we do not necessarily have acquaintance with the objects denoted by phrases composed of words with whose meanings we are acquainted. To take a very important instance: there seems no reason to believe that we are ever acquainted with other people's minds, seeing that these are not directly perceived; hence what we know about them is obtained through denoting. All thinking has to start from acquaintance, but it succeeds in thinking *about* many things with which we have no acquaintance.

The course of my argument will be as follows. I shall begin by stating the theory I intend to advocate;[1] I shall then discuss the theories of Frege and Meinong, showing why neither of them satisfies me; then I shall give

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the grounds in favor of my theory; and finally I shall briefly indicate the philosophical consequences of my theory.

My theory, briefly, is as follows. I take the notion of the *variable* as fundamental; I use '$C(x)$' to mean a proposition[2] in which x is a constituent, where x, the variable, is essentially and wholly undetermined. Then we can consider the two notions '$C(x)$ is always true' and '$C(x)$ is sometimes true.'[3] Then *everything* and *nothing* and *something* (which are the most primitive of denoting phrases) are to be interpreted as follows:

> C (everything) means '$C(x)$ is always true';
>
> C (nothing) means ' "$C(x)$ is false" is always true';
>
> C (something) means 'It is false that "$C(x)$ is false" is always true'.[4]

Here the notion '$C(x)$ is always true' is taken as ultimate and indefinable, and the others are defined by means of it. *Everything, nothing,* and *something* are not assumed to have any meaning in isolation, but a meaning is assigned to *every* proposition in which they occur. This is the principle of the theory of denoting I wish to advocate: that denoting phrases never have any meaning in themselves, but that every proposition in whose verbal expression they occur has a meaning. The difficulties concerning denoting are, I believe, all the result of a wrong analysis of propositions whose verbal expressions contain denoting phrases. The proper analysis, if I am not mistaken, may be further set forth as follows.

Suppose now we wish to interpret the proposition, 'I met a man'. If this is true, I met some definite man; but that is not what I affirm. What I affirm is, according to the theory I advocate:

> ' "I met x, and x is human" is not always false'.

Generally, defining the class of men as the class of objects having the predicate *human,* we say that:

> 'C (a man)' means ' "C (x) and x is human" is not always false'.

This leaves 'a man', by itself, wholly destitute of meaning, but gives a meaning to every proposition in whose verbal expression 'a man' occurs.

Consider next the proposition 'all men are mortal'. This proposition[5] is really hypothetical and states that *if* anything is a man, it is mortal. That is, it states that if x is a man, x is mortal, whatever x may be. Hence, substituting 'x is human' for 'x is a man', we find:

> 'All men are mortal' means ' "If x is human, x is mortal" is always true'.

This is what is expressed in symbolic logic by saying that 'all men are mortal' means ' "x is human" implies "x is mortal" for all values of x'. More generally, we say:

> 'C (all men)' means ' "If x is human, then C (x) is true" is always true'.

Similarly

> 'C (no men)' means ' "If x is human, then C (x) is false" is always true'.
>
> 'C (some men)' will mean the same as 'C (a man)',[6] and
>
> 'C (a man)' means 'It is false that "C (x) and x is human" is always false'.
>
> 'C (every man)' will mean the same as 'C (all men)'.

It remains to interpret phrases containing *the.* These are by far the most interesting and difficult of denoting phrases. Take as an example 'the father of Charles II was executed'. This asserts that there was an x who was the father of Charles II and was executed. Now, *the,* when it is strictly used, involves uniqueness; we do, it is true, speak of '*the son* of So-and-so' even when So-and-so has several sons, but it would be more correct to say '*a* son of So-and-so'. Thus for our purposes we take *the* as involving uniqueness. Thus when we say 'x was *the* father of Charles II' we not only assert that x had a certain relation to Charles II, but also that nothing else had this relation. The relation in question, without the assumption of uniqueness, and without any denoting phrases, is expressed by 'x begat Charles II'. To get an equivalent of 'x was the father of Charles II', we must add, 'If y is other than x, y did not beget Charles II', or what is equivalent, 'If y begat Charles II, y is identical with x'. Hence, 'x is the father of

Charles II' becomes: '*x* begat Charles II; and "if *y* begat Charles II, *y* is identical with *x*" is always true of *y*'.

Thus 'the father of Charles II was executed' becomes:

> 'It is not always false of *x* that *x* begat Charles II and that *x* was executed and that "if *y* begat Charles II, *y* is identical with *x*" is always true of *y*'.

This may seem a somewhat incredible interpretation; but I am not at present giving reasons, I am merely *stating* the theory.

To interpret '*C* (the father of Charles II)', where *C* stands for any statement about him, we have only to substitute *C* (*x*) for '*x* was executed' in the above. Observe that, according to the above interpretation *C* may be, '*C* (the father of Charles II)' implies:

> 'It is not always false of *x* that "if *y* begat Charles II, *y* is identical with *x*" is always true of *y*',

which is what is expressed in common language by 'Charles II had one father and no more'. Consequently if this condition fails, *every* proposition of the form '*C* (the father of Charles II)' is false. Thus e.g. every proposition of the form '*C* (the present King of France)' is false. This is a great advantage in the present theory. I shall show later that it is not contrary to the law of contradiction, as might be at first supposed.

The above gives a reduction of all propositions in which denoting phrases occur to forms in which no such phrases occur. Why it is imperative to effect such a reduction, the subsequent discussion will endeavor to show.

The evidence for the above theory is derived from the difficulties which seem unavoidable if we regard denoting phrases as standing for genuine constituents of the propositions in whose verbal expressions they occur. Of the possible theories which admit such constituents the simplest is that of Meinong.[7] This theory regards any grammatically correct denoting phrase as standing for an *object.* Thus 'the present King of France', 'the round square', etc., are supposed to be genuine objects. It is admitted that such objects do not *subsist,* but nevertheless they are supposed to be objects. This is in itself a difficult view; but the chief objection is that such objects, admittedly, are apt to infringe the law of contradiction. It is contended, for example, that the existent present King of France exists, and also does not exist; that the round square is round, and also not round, etc. But this is intolerable; and if any theory can be found to avoid this result, it is surely to be preferred.

The above breach of the law of contradiction is avoided by Frege's theory. He distinguishes, in a denoting phrase, two elements, which we may call the *meaning* and the *denotation.*[8] Thus 'the center of mass of the solar system at the beginning of the twentieth century' is highly complex in *meaning,* but its *denotation* is a certain point, which is simple. The solar system, the twentieth century, etc., are constituents of the *meaning;* but the *denotation* has no constituents at all.[9] One advantage of this distinction is that it shows why it is often worthwhile to assert identity. If we say 'Scott is the author of *Waverley*', we assert an identity of denotation with a difference of meaning. I shall, however, not repeat the grounds in favor of this theory, as I have urged its claims elsewhere (loc. cit.), and am now concerned to dispute those claims.

One of the first difficulties that confronts us, when we adopt the view that denoting phrases *express* a meaning and *denote* a denotation,[10] concerns the cases in which the denotation appears to be absent. If we say 'the King of England is bald', that is, it would seem, not a statement about the complex *meaning* 'the King of England', but about the actual man denoted by the meaning. But now consider 'the King of France is bald'. By parity of form, this also ought to be about the denotation of the phrase 'the King of France'. But this phrase, though it has a *meaning* provided 'the King of England' has a meaning, has certainly no denotation, at least in any obvious sense. Hence one would suppose that 'the King of France is bald' ought to be nonsense; but it is not nonsense, since it is plainly false. Or again consider such a proposition as the following: 'If *u* is a class which has only one member, then that one member is a member of *u*', or, as we may state it, 'If *u* is a unit class, the *u* is a *u*'. This proposition ought to be *always* true, since the conclusion is true whenever the hypothesis is true. But 'the *u*' is a denoting phrase, and it is the denotation, not the meaning, that is said to be a *u*. Now if *u* is *not* a unit class, "the *u*" seems to denote nothing; hence our proposition would seem to become nonsense as soon as *u* is not a unit class.

Now it is plain that such propositions do *not* become nonsense merely because their hypotheses are false. The king in *The Tempest* might say, 'If Ferdinand is not drowned, Ferdinand is my only son'. Now 'my only son' is a denoting phrase, which, on the face of it, has a denotation when, and only when, I have exactly one son. But the above statement would nevertheless have remained true if Ferdinand had been in fact drowned. Thus we must either provide a denotation in cases in which it is at first sight absent, or we must abandon the view that the denotation is what is concerned in propositions which contain denotating phrases. The latter is the course that I advocate. The former course may be taken, as by Meinong, by admitting objects which do not subsist, and denying that they obey the law of contradiction; this, however, is to be avoided if possible. Another way of taking the same course (so far as our present alternative is concerned) is adopted by Frege, who provides by definition some purely conventional denotations for the cases in which otherwise there would be none. Thus 'the King of France', is to denote the null-class; 'the only son of Mr. So-and-so' (who has a fine family of ten), is to denote the class of all his sons; and so on. But this procedure, though it may not lead to actual logical error, is plainly artificial, and does not give an exact analysis of the matter. Thus if we allow that denoting phrases, in general, have the two sides of meaning and denotation, the cases where there seems to be no denotation cause difficulties both on the assumption that there really is a denotation and on the assumption that there really is none.

A logical theory may be tested by its capacity for dealing with puzzles, and it is a wholesome plan, in thinking about logic, to stock the mind with as many puzzles as possible, since these serve much the same purpose as is served by experiments in physical science. I shall therefore state three puzzles which a theory as to denoting ought to be able to solve; and I shall show later that my theory solves them.

(1) If *a* is identical with *b*, whatever is true of the one is true of the other, and either may be substituted for the other in any proposition without altering the truth or falsehood of that proposition. Now George IV wished to know whether Scott was the author of *Waverley;* and in fact Scott *was* the author of *Waverley*. Hence we may substitute *Scott* for *the author of 'Waverley'*, and thereby prove that George IV wished to know whether Scott was Scott. Yet an interest in the law of identity can hardly be attributed to the first gentleman of Europe.

(2) By the law of excluded middle, either '*A* is *B*' or '*A* is not *B*' must be true. Hence either 'the present King of France is bald' or 'the present King of France is not bald' must be true. Yet if we enumerated the things that are bald, and then the things that are not bald, we should not find the present King of France in either list. Hegelians, who love a synthesis, will probably conclude that he wears a wig.

(3) Consider the proposition '*A* differs from *B*'. If this is true, there is a difference between *A* and *B*, which fact may be expressed in the form 'the difference between *A* and *B* subsists'. But if it is false that *A* differs from *B*, then there is no difference between *A* and *B*, which fact may be expresssed in the form 'the difference between *A* and *B* does not subsist'. But how can a non-entity be the subject of a proposition? 'I think, therefore I am' is no more evident than 'I am the subject of a proposition, therefore I am', provided 'I am' is taken to assert subsistence or being,[11] not existence. Hence, it would appear, it must always be self-contradictory to deny the being of anything; but we have seen, in connection with Meinong, that to admit being also sometimes leads to contradictions. Thus, if *A* and *B* do not differ, to suppose either that there is, or that there is not, such an object as 'the difference between *A* and *B*' seems equally impossible.

The relation of the meaning to the denotation involves certain rather curious difficulties, which seem in themselves sufficient to prove that the theory which leads to such difficulties must be wrong.

When we wish to speak about the *meaning* of a denoting phrase, as opposed to its *denotation,* the natural mode of doing so is by inverted commas. Thus we say:

> The center of mass of the solar system is a point, not a denoting complex;
>
> 'The center of mass of the solar system' is a denoting complex, not a point.

Or again,

> The first line of Gray's Elegy states a proposition.

"The first line of Gray's Elegy" does not state a proposition. Thus taking any denoting phrase, say *C,* we wish to consider the relation between *C* and '*C*', where the difference of the two is the kind exemplified in the above two instances.

We say, to begin with, that when *C* occurs it is the *denotation* that we are speaking about; but when '*C*' occurs, it is the *meaning.* Now the relation of meaning and denotation is not merely linguistic through the phrase: there must be a logical relation involved, which we express by saying that the meaning denotes the denotation. But the difficulty which confronts us is that we cannot succeed in *both* preserving the connection of meaning and denotation *and* preventing them from being one and the same; also that the meaning cannot be got at except by means of denoting phrases. This happens as follows.

The one phrase *C* was to have both meaning and denotation. But if we speak of 'the meaning of *C*', that gives us the meaning (if any) of the denotation. 'The meaning of the first line of Gray's Elegy' is the same as 'The meaning of "The curfew tolls the knell of parting day",' and is not the same as 'The meaning of "the first line of Gray's Elegy".' Thus in order to get the meaning we want, we must speak not of 'the meaning of *C*', but of 'the meaning of "*C*",' which is the same as '*C*' by itself. Similarly 'the denotation of *C*' does not mean the denotation we want, but means something which, if it denotes at all, denotes what is denoted by the denotation we want. For example, let '*C*' be 'the denoting complex occurring in the second of the above instances'. Then *C* = 'the first line of Gray's Elegy', and the denotation of *C* = The curfew tolls the knell of parting day. But what we *meant* to have as the denotation was 'the first line of Gray's Elegy'. Thus we have failed to get what we wanted.

The difficulty in speaking of the meaning of a denoting complex may be stated thus: The moment we put the complex in a proposition, the proposition is about the denotation; and if we make a proposition in which the subject is 'the meaning of *C*', then the subject is the meaning (if any) of the denotation, which was not intended. This leads us to say that, when we distinguish meaning and denotation, we must be dealing with the meaning: the meaning has denotation and is a complex, and there is not something other than the meaning, which can be called the complex, and be said to *have* both meaning and denotation. The right phrase, on the view in question, is that some meanings have denotations.

But this only makes our difficulty in speaking of meanings more evident. For suppose *C* is our complex; then we are to say that *C is* the meaning of the complex. Nevertheless, whenever *C* occurs without inverted commas, what is said is not true of the meaning, but only of the denotation, as when we say: The center of mass of the solar system is a point. Thus to speak of *C* itself, i.e., to make a proposition about the meaning, our subject must not be *C,* but something which denotes *C.* Thus '*C*', which is what we use when we want to speak of the meaning, must be not the meaning, but something which denotes the meaning. And *C* must not be a constituent of this complex (as it is of 'the meaning of *C*'); for if *C* occurs in the complex, it will be its denotation, not its meaning, that will occur, and there is no backward road from denotations to meanings, because every object can be denoted by an infinite number of different denoting phrases.

Thus it would seem that '*C*' and *C* are different entities, such that '*C*' denotes *C;* but this cannot be an explanation, because the relation of '*C*' to *C* remains wholly mysterious; and where we are to find the denoting complex '*C*' which is to denote *C*? Moreover, when *C* occurs in a proposition, it is not *only* the denotation that occurs (as we shall see in the next paragraph); yet, on the view in question, *C* is only the denotation, the meaning being wholly relegated to '*C*'. This is an inextricable tangle and seems to prove that the whole distinction of meaning and denotation has been wrongly conceived.

That the meaning is relevant when a denoting phrase occurs in a proposition is formally proved by the puzzle about the author of *Waverley*. The proposition 'Scott was the author of *Waverley*' has a property not possessed by 'Scott was Scott', namely the property that George IV wished to know whether it was true. Thus the two are not identical propositions; hence the meaning of 'the author of *Waverley*' must be relevant as well as the denotation, if we adhere to the point of view to which this distinction belongs. Yet, as we have just seen, so long as we adhere to this point of view, we are compelled to hold that only the denotation can be relevant. Thus the point in question must be abandoned.

It remains to show how all the puzzles we have been considering are solved by the theory explained at the beginning of this article.

According to the view which I advocate, a denoting phrase is essentially *part* of a sentence, and does not, like most single words, have any significance on its own account. If I say 'Scott was a man', that is a statement of the form '*x* was a man', and it has 'Scott' for its subject. But if I say 'the author of *Waverley* was a man', that is not a statement of the form '*x* was a man', and does not have 'the author of *Waverley*' for its subject. Abbreviating the statement made at the beginning of this article, we may put, in place of 'the author of *Waverley* was a man', the following: 'One and only one entity wrote *Waverley,* and that one was a man'. (This is not so strictly what is meant as what was said earlier; but it is easier to follow.) And speaking generally, suppose we wish to say that the author of *Waverley* had the property ϕ, what we wish to say is equivalent to 'One and only one entity wrote *Waverley,* and that one had the property ϕ'.

The explanation of *denotation* is now as follows. Every proposition in which 'the author of *Waverley*' occurs being explained as above, the proposition 'Scott was the author of *Waverley*' (i.e., 'Scott was identical with the author of *Waverley*') becomes 'One and only one entity wrote *Waverley,* and Scott was identical with that one'; or, reverting to the wholly explicit form: 'It is not always false of *x* that *x* wrote *Waverley,* that it is always true of *y* that if *y* wrote *Waverley, y* is identical with *x,* and that Scott is identical with *x*'. Thus if '*C*' is a denoting phrase, it may happen that there is one entity *x* (there cannot be more than one) for which the proposition '*x* is identical with *C*' is true, this proposition being interpreted as above. We may then say that the entity *x* is the denotation of the phrase '*C*'. Thus Scott is the denotation of 'the author of *Waverley*'. The '*C*' in inverted commas will be merely the *phrase,* not anything that can be called the *meaning.* The phrase *per se* has no meaning, because in any proposition in which it occurs the proposition, fully expressed, does not contain the phrase, which has been broken up.

The puzzle about George IV's curiosity is now seen to have a very simple solution. The proposition 'Scott was the author of *Waverley*', which was written out in its unabbreviated form in the preceding paragraph, does not contain any constituent 'the author of *Waverley*' for which we could substitute 'Scott'. This does not interfere with the truth of inferences resulting from making what is *verbally* the substitution of 'Scott' for 'the author of *Waverley*', so long as 'the author of *Waverley*' has what I call a *primary* occurrence in the proposition considered. The difference of primary and secondary occurrences of denoting phrases is as follows:

When we say: 'George IV wished to know whether so-and-so', or when we say 'So-and-so is surprising' or 'So-and-so is true', etc., the 'so-and-so' must be a proposition. Suppose now that 'so-and-so' contains a denoting phrase. We may either eliminate this denoting phrase from the subordinate proposition 'so-and-so', or from the whole proposition in which 'so-and-so' is a mere constituent. Different propositions result according to which we do. I have heard of a touchy owner of a yacht to whom a guest, on first seeing it, remarked, 'I thought your yacht was larger than it is'; and the owner replied, 'No, my yacht is not larger than it is'. What the guest meant was, 'The size that I thought your yacht was is greater than the size your yacht is'; the meaning attributed to him is, 'I thought the size of your yacht was greater than the size of your yacht'. To return to George IV and *Waverley,* when we say 'George IV wished to know whether Scott was the author of *Waverley*', we normally mean 'George IV wished to know whether one and only one man wrote *Waverley* and Scott was that man'; but we *may* also mean: 'One and only one man wrote *Waverley,* and George IV wished to know whether Scott was that man'. In the latter, 'the author of *Waverley*' has a *primary* occurrence; in the former, a *secondary.* The latter might be expressed by 'George IV wished to know, concerning the man who in fact wrote *Waverley,* whether he was Scott'. This would be true, for example, if George IV had seen Scott at a distance, and had asked 'Is that Scott?'. A *secondary* occurrence of a denoting phrase may be defined as one in which the phrase occurs in a proposition p which is a mere constituent of the proposition we are considering, and the substitution for the denoting phrase is to be effected in p, not in the whole proposition concerned. The ambiguity as between primary and secondary occurrences is hard to avoid in language; but it does no harm if we are on our guard against it. In symbolic logic it is of course easily avoided.

The distinction of primary and secondary occurrences also enables us to deal with the question whether the present King of France is bald or not bald, and generally with the logical status of denoting phrases that denote nothing. If 'C' is a denoting phrase, say 'the term having the property F', then

> 'C has the property ϕ' means 'one and only one term has the property F, and that one has the property ϕ'.[12]

If now the property F belongs to no terms, or to several, it follows that 'C has the property ϕ' is false for *all* values of ϕ. Thus 'the present King of France is bald' is certainly false; and 'the present King of France is not bald' is false if it means

> 'There is an entity which is now King of France and is not bald'.

but is true if it means

> 'It is false that there is an entity which is now King of France and is bald'.

That is, 'the King of France is not bald' is false if the occurrence of 'the King of France' is *primary,* and true if it is *secondary.* Thus all propositions in which 'the King of France' has a primary occurrence are false; the denials of such propositions are true, but in them 'the King of France' has a secondary occurrence. Thus we escape the conclusion that the King of France has a wig.

We can now see also how to deny that there is such an object as the difference between A and B in the case when A and B do not differ. If A and B do differ, there is one and only one entity x such that 'x is the difference between A and B' is a true proposition; if A and B do not differ, there is no such entity x. Thus according to the meaning of denotation lately explained, 'the difference between A and B' has a denotation when A and B differ, but not otherwise. This difference applies to true and false propositions generally. If '$a\ R\ b$' stands for 'a has the relation R to b', then when $a\ R\ b$ is true, there is such an entity as the relation R between a and b; when $a\ R\ b$ is false, there is no such entity. Thus out of any proposition we can make a denoting phrase, which denotes an entity if the proposition is true, but does not denote an entity if the proposition is false. E.g., it is true (at least we will suppose so) that the earth revolves around the sun, and false that the sun revolves around the earth; hence 'the revolution of the earth round the sun' denotes an entity, while 'the revolution of the sun round the earth' does not denote an entity.[13]

The whole realm of non-entities, such as 'the round square', 'the even prime other than

2', 'Apollo', 'Hamlet', etc., can now be satisfactorily dealt with. All these are denoting phrases which do not denote anything. A proposition about Apollo means what we get by substituting what the classical dictionary tells us is meant by Apollo, say 'the sun-god'. All propositions in which Apollo occurs are to be interpreted by the above rules for denoting phrases. If 'Apollo' has a primary occurrence, the proposition containing the occurrence is false; if the occurrence is secondary, the proposition may be true. So again 'the round square is round' means 'there is one and only one entity x which is round and square, and that entity is round', which is a false proposition, not, as Meinong maintains, a true one. 'The most perfect Being has all perfections; existence is a perfection; therefore the most perfect Being exists' becomes:

'There is one and only one entity x which is most perfect; that one has all perfections; existence is a perfection; therefore that one exists'. As a proof, this fails for want of a proof of the premise 'there is one and only one entity x which is most perfect'.[14]

Mr. MacColl (*Mind,* N.S., No. 54, and again No. 55, page 401) regards individuals as of two sorts, real and unreal; hence he defines the null-class as the class consisting of all unreal individuals. This assumes that such phrases as 'the present King of France', which do not denote a real individual, do, nevertheless, denote an individual, but an unreal one. This is essentially Meinong's theory, which have seen reason to reject because it conflicts with the law of contradiction. With our theory of denoting, we are able to hold that there are no unreal individuals; so that the null-class is the class containing no members, not the class containing as members all unreal individuals.

It is important to observe the effect of our theory on the interpretation of definitions which proceed by means of denoting phrases. Most mathematical definitions are of this sort: for example '$m - n$ means the number which, added to n, gives m'. Thus $m - n$ is defined as meaning the same as a certain denoting phrase; but we agreed that denoting phrases have no meaning in isolation. Thus what the definition really ought to be is: 'Any proposition containing $m-n$ is to mean the proposition which results from substituting for "$m - n$" "the number which, added to n, gives m".' The resulting proposition is interpreted according to the rules already given for interpreting propositions whose verbal expression contains a denoting phrase. In the case where m and n are such that there is one and only one number x which, added to n, gives m, there is a number x which can be substituted for $m - n$ in any proposition containing $m - n$ without altering the truth or falsehood of the proposition. But in other cases, all propositions in which '$m - n$' has a primary occurrence are false.

The usefulness of *identity* is explained by the above theory. No one outside a logic-book ever wishes to say 'x is x', and yet assertions of identity are often made in such forms as 'Scott was the author of *Waverley*' or 'thou art the man'. The meaning of such propositions cannot be stated without the notion of identity, although they are not simply statements that Scott is identical with another term, the author of *Waverley,* or that thou art identical with another term, the man. The shortest statement of 'Scott is the author of *Waverley*' seems to be 'Scott wrote *Waverley;* and it is always true of y that if y wrote *Waverley, y* is identical with Scott'. It is in this way that identity enters into 'Scott is the author of *Waverley*'; and it is owing to such uses that identity is worth affirming.

One interesting result of the above theory of denoting is this: when there is anything with which we do not have immediate acquaintance, but only definition by denoting phrases, then the propositions in which this thing is introduced by means of a denoting phrase do not really contain this thing as a constituent, but contain instead the constituents expressed by the several words of the denoting phrase. Thus in every proposition that we can apprehend (i.e. not only in those whose truth or falsehood we can judge of, but in all that we can think about), all the constituents are really entities with which we have immediate acquaintance. Now such things as matter (in the sense in which matter occurs in physics) and the minds of other people are known to us only by denoting phrases, i.e. we are not *acquainted* with them, but we know them as what has such and such properties. Hence, although we can form propositional functions $C(x)$ which must hold of such and such a material particle, or of So-and-so's mind, yet we are not acquainted with the propositions which affirm these things that we know must be true, because we cannot apprehend the actual entities concerned. What we know is 'So-and-so has a mind which has such and such properties', where A is the mind in question. In such a case, we know the properties of a thing without having acquaintance with the thing itself, and without, consequently, knowing any single proposition of which the thing itself is a constituent.

Of the many other consequences of the view I have been advocating, I will say nothing. I will only beg the reader not to make up his mind against the view—as he might be tempted to do, on account of its apparently excessive complication—until he has attempted to construct a theory of his own on the subject of denotation. This attempt, I believe, will convince him that, whatever the true theory may be, it cannot have such a simplicity as one might have expected beforehand.

NOTES

1. I have discussed this subject in *Principles of Mathematics,* Chap. V, and § 476. The theory there advocated is very nearly the same as Frege's, and is quite different from the theory to be advocated in what follows.
2. More exactly, a propositional function.
3. The second of these can be defined by means of the first, if we take it to mean, 'It is not true that "$C(x)$ is false" is always true'.
4. I shall sometimes use, instead of this complicated phrase, the phrase '$C(x)$ is not always false', or '$C(x)$ is sometimes true', supposed *defined* to mean the same as the complicated phrase.
5. As has been ably argued in Mr. Bradley's *Logic,* Book I, Chap. II.
6. Psychologically 'C (a man)' has a suggestion of *only* one, and 'C (some men)' has a suggestion of *more than one;* but we may neglect these suggestions in a preliminary sketch.
7. See *Untersuchungen zur Gegenstandstheorie und Psychologie* (Leipzig, 1904) the first three articles (by Meinong, Ameseder, and Mally respectively).
8. See his "Ueber Sinn und Bedeutung," *Zeitschrift für Phil. und Phil. Kritik,* 100. [*Reprinted in this volume.*]
9. Frege distinguishes the two elements of meaning and denotation everywhere, and not only in complex denoting phrases. Thus it is the *meanings* of the constituents of a denoting complex that enter into its *meaning,* not their *denotation.* In the proposition 'Mont Blanc is over 1,000 metres high', it is, according to him the *meaning* of 'Mont Blanc', not the actual mountain, that is a constituent of the *meaning* of the proposition.
10. In this theory, we shall say that the denoting phrase *expresses* a meaning; and we shall say both of the phrase and of the meaning that they *denote* a denotation. In the other theory, which I advocate, there is no *meaning,* and only sometimes a *denotation.*
11. I use these as synonyms.
12. This is the abbreviated, not the stricter, interpretation.
13. The propositions from which such suppositions are derived are not identical either with these entities or with the propositions that these entities have being.
14. The argument can be made to prove validly that all members of the class of most perfect Beings exist; it can also be proved formally that this class cannot have *more* than one member; but, taking the definition of perfection as possession of all positive predicates, it can be proved almost equally formally that the class does not have even one member.

Descriptions 15

BERTRAND RUSSELL

We dealt in the preceding chapter with the words *all* and *some;* in this chapter we shall consider the word *the* in the singular, and in the next chapter we shall consider the word *the* in the plural. It may be thought excessive to devote two chapters to one word, but to the philosophical mathematician it is a word of very great importance: like Browning's Grammarian with the enclitic δε, I would give the doctrine of this word if I were "dead from the waist down" and not merely in a prison.

We have already had occasion to mention "descriptive functions," i.e. such expressions as "the father of x" or "the sine of x." These are to be defined by first defining "descriptions."

A "description" may be of two sorts, definite and indefinite (or ambiguous). An indefinite description is a phrase of the form "a so-and-so," and a definite description is a phrase of the form "the so-and-so" (in the singular). Let us begin with the former.

"Who did you meet?" "I met a man." "That is a very indefinite description." We are therefore not departing from usage in our terminology. Our question is: What do I really assert when I assert "I met a man"? Let us assume, for the moment, that my assertion is true, and that in fact I met Jones. It is clear that what I assert is *not* "I met Jones." I may say "I met a man, but it was not Jones"; in that case, though I lie, I do not contradict myself, as I should do if when I say I met a man I really mean that I met Jones. It is clear also that the person to whom I am speaking can understand what I say, even if he is a foreigner and has never heard of Jones.

But we may go further: not only Jones, but no actual man, enters into my statement. This becomes obvious when the statement is false, since then there is no more reason why Jones should be supposed to enter into the proposition than why anyone else should. Indeed the statement would remain significant, though it could not possibly be true, even if there were no man at all. "I met a unicorn" or "I met a sea-serpent" is a perfectly significant assertion, if we know what it would be to be a unicorn or a sea-serpent, i.e. what is the definition of these fabulous monsters. Thus it is only what we may call the *concept* that enters into the proposition. In the case of "unicorn," for example, there is only the concept: there is not also, somewhere among the shades, something unreal which may be called "a unicorn." Therefore, since it is significant (though false) to say "I met a unicorn," it is clear that this proposition, rightly analyzed, does not contain a constituent "a unicorn," though it does contain the concept "unicorn."

The question of "unreality," which con-

From *Introduction to Mathematical Philosophy* (London: George Allen and Unwin Ltd., 1919), pp. 167–180.

fronts us at this point, is a very important one. Misled by grammar, the great majority of those logicians who have dealt with this question have dealt with it on mistaken lines. They have regarded grammatical form as a surer guide in analysis than, in fact, it is. And they have not known what differences in grammatical form are important. "I met Jones" and "I met a man" would count traditionally as propositions of the same form, but in actual fact they are of quite different forms: the first names an actual person, Jones; while the second involves a propositional function, and becomes, when made explicit: "The function 'I met x and x is human' is sometimes true." (It will be remembered that we adopted the convention of using "sometimes" as not implying more than once.) This proposition is obviously not of the form "I met x," which accounts for the existence of the proposition "I met a unicorn" in spite of the fact that there is no such thing as "a unicorn."

For want of the apparatus of propositional functions, many logicians have been driven to the conclusion that there are unreal objects. It is argued, e.g. by Meinong,[1] that we can speak about "the golden mountain," "the round square," and so on; we can make true propositions of which these are the subjects; hence they must have some kind of logical being, since otherwise the propositions in which they occur would be meaningless. In such theories, it seems to me, there is a failure of that feeling for reality which ought to be preserved even in the most abstract studies. Logic, I should maintain, must no more admit a unicorn than zoology can; for logic is concerned with the real world just as truly as zoology, though with its more abstract and general features. To say that unicorns have an existence in heraldry, or in literature, or in imagination, is a most pitiful and paltry evasion. What exists in heraldry is not an animal, made of flesh and blood, moving and breathing of its own initiative. What exists is a picture, or a description in words. Similarly, to maintain that Hamlet, for example, exists in his own world, namely, in the world of Shakespeare's imagination, just as truly as (say) Napoleon existed in the ordinary world, is to say something deliberately confusing, or else confused to a degree which is scarcely credible. There is only one world, the "real" world: Shakespeare's imagination is part of it, and the thoughts that he had in writing Hamlet are real. So are the thoughts that we have in reading the play. But it is of the very essence of fiction that only the thoughts, feelings, etc., in Shakespeare and his readers are real, and that there is not, in addition to them, an objective Hamlet. When you have taken account of all the feelings roused by Napoleon in writers and readers of history, you have not touched the actual man; but in the case of Hamlet you have come to the end of him. If no one thought about Hamlet, there would be nothing left of him; if no one had thought about Napoleon, he would have soon seen to it that some one did. The sense of reality is vital in logic, and whoever juggles with it by pretending that Hamlet has another kind of reality is doing a disservice to thought. A robust sense of reality is very necessary in framing a correct analysis of propositions about unicorns, golden mountains, round squares, and other such pseudo-objects.

In obedience to the feeling of reality, we shall insist that, in the analysis of propositions, nothing "unreal" is to be admitted. But, after all, if there *is* nothing unreal, how, it may be asked, *could* we admit anything unreal? The reply is that, in dealing with propositions, we are dealing in the first instance with symbols, and if we attribute significance to groups of symbols which have no significance, we shall fall into the error of admitting unrealities, in the only sense in which this is possible, namely, as objects described. In the proposition "I met a unicorn," the whole four words together make a significant proposition, and the word "unicorn" by itself is significant, in just the same sense as the word "man." But the *two* words "a unicorn" do not form a subordinate group having a meaning of its own. Thus if we falsely attribute meaning to these two words, we find ourselves saddled with "a unicorn," and with the problem how there can be such a thing in a world where there are no unicorns. "A unicorn" is an indefinite description which describes nothing. It is not an indefinite description which describes something unreal. Such a proposition as "x is unreal" only has meaning when "x" is a description, definite or indefinite; in that case the proposition will be true if "x" is a description which describes nothing. But whether the description "x" describes something or describes nothing, it is in any case not a constituent of the proposition in which it occurs; like "a unicorn" just now, it is not a subordinate group having a meaning of its own. All this results from the fact that, when "x" is a description, "x is unreal" or "x does not exist" is not nonsense, but is always significant and sometimes true.

We may now proceed to define generally the meaning of propositions which contain ambiguous descriptions. Suppose we wish to make some statement about "a so-and-so," where "so-and-so's" are those objects that have a certain property ϕ, i.e. those objects x for which the propositional function ϕx is true. (E.g. if we take "a man" as our instance of "a so-and-so," ϕx will be "x is human.") Let us now wish to assert the property ψ of "a so-and-so," i.e. we wish to assert that "a so-and-so" has that property which x has when ψx is true. (E.g. in the case of "I met a man," ψx will be "I met x.") Now the proposition that "a so-and-so" has the property ψ is *not* a proposition of the form "ψx." If it were, "a so-and-so" would have to be identical with x for a suitable x; and although (in a sense) this may be true in some cases, it is certainly not true in such a case as "a unicorn." It is just this fact, that the statement that a so-and-so has the property ψ is not of the form ψx, which makes it possible for "a so-and-so" to be, in a certain clearly definable sense, "unreal." The definition is as follows:

> The statement that "an object having the property ϕ has the property ψ"

means:

> "The joint assertion of ϕx and ψx is not always false."

So far as logic goes, this is the same proposition as might be expressed by "some ϕ's are ψ's"; but rhetorically there is a difference, because in the one case there is a suggestion of singularity, and in the other case of plurality. This, however, is not the important point. The important point is that, when rightly analyzed, propositions verbally about "a so-and-so" are found to contain no constituent represented by this phrase. And that is why such propositions can be significant even when there is no such thing as a so-and-so.

The definition of *existence,* as applied to ambiguous descriptions, results from what was said at the end of the preceding chapter [chapter 15 of *Introduction to Mathematical Philosophy*]. We say that "men exist" or "a man exists" if the propositional function "x is human" is sometimes true; and generally "a so-and-so" exists if "x is so-and-so" is sometimes true. We may put this in other language. The proposition "Socrates is a man" is no doubt *equivalent* to "Socrates is human," but it is not the very same proposition. The *is* of "Socrates is human" expresses the relation of subject and predicate; the *is* of "Socrates is a man" expresses identity. It is a disgrace to the human race that it has chosen to employ the same word "is" for these two entirely different ideas—a disgrace which a symbolic logical language of course remedies. The identity in "Socrates is a man" is identity between an object named (accepting "Socrates" as a name, subject to qualifications explained later) and an object ambiguously described. An object ambiguously described will "exist" when at least one such proposition is true, i.e. when there is at least one true proposition of the form "x is a so-and-so," where "x" is a name. It is characteristic of ambiguous (as opposed to definite) descriptions that there may be any number of true propositions of the above form—Socrates is a man, Plato is a man, etc. Thus "a man exists" follows from Socrates, or Plato, or anyone else. With definite descriptions, on the other hand, the corresponding form of proposition, namely, "x is the so-and-so" (where "x" is a name), can only be true for one value of x at most. This brings us to the subject of definite descriptions, which are to be defined in a way analogous to that employed for ambiguous descriptions, but rather more complicated.

We come now to the main subject of the present chapter, namely, the definition of the word *the* (in the singular). One very important point about the definition of "a so-and-so" applies equally to "the so-and-so"; the definition to be sought is a definition of propositions in which this phrase occurs, not a definition of the phrase itself in isolation. In the case of "a so-and-so," this is fairly obvious: no one could suppose that "a man" was a definite object, which could be defined by itself. Socrates is a man, Plato is a man, Aristotle is a man, but we cannot infer that "a man" means the same as "Socrates" means and also the same as "Plato" means and also the same as "Aristotle" means, since these three names have different meanings. Nevertheless, when we have enumerated all the men in the world, there is nothing left of which we can say, "This is a man, and not only so, but it is *the* 'a man,' the quintessential entity that is just an indefinite man without being anybody in particular." It is of course quite clear that whatever there is in the world is definite: if it is a man it is one definite man and not any other. Thus there cannot be such an entity as "a man" to be found in the world, as opposed to specific men. And accordingly it is natural that we do not define "a man" itself, but only the propositions in which it occurs.

In the case of "the so-and-so" this is equally true, though at first sight less obvious. We may demonstrate that this must be the case, by a consideration of the difference between a *name* and a *definite description.* Take the proposition, "Scott is the author of *Waverley.*" We have here a name, "Scott," and a description, "the author of *Waverley,*" which are asserted to apply to the same person. The distinction between a name and all other symbols may be explained as follows:

A name is a simple symbol whose meaning is something that can only occur as subject, i.e. something of the kind that we defined as an "individual" or a "particular." And a "simple" symbol is one which has no parts that are symbols. Thus "Scott" is a simple symbol, because, though it has parts (namely, separate letters), these parts are not symbols. On the other hand, "the author of *Waverley*" is not a simple symbol, because the separate words that compose the phrase are parts which are symbols. If, as may be the case, whatever *seems* to be an "individual" is really capable of further analysis, we shall have to content ourselves with what may be called "relative individuals," which will be terms that, throughout the context in question, are never analyzed and never occur otherwise than as subjects. And in that case we shall have correspondingly to content ourselves with "relative names." From the standpoint of our present problem, namely, the definition of descriptions, this problem, whether these are absolute names or only relative names, may be ignored, since it concerns different stages in the hierarchy of "types," whereas we have to compare such couples as "Scott" and "the author of *Waverley,*" which both apply to the same object, and do not raise the problem of types. We may, therefore, for the moment, treat names as capable of being absolute; nothing that we shall have to say will depend upon this assumption, but the wording may be a little shortened by it.

We have, then, two things to compare: (1) a *name,* which is a simple symbol, directly designating an individual which is its meaning, and having this meaning in its own right, independently of the meanings of all other words; (2) a *description,* which consists of several words, whose meanings are already fixed, and from which results whatever is to be taken as the "meaning" of the description.

A proposition containing a description is not identical with what that proposition becomes when a name is substituted, even if the name names the same object as the description describes. "Scott is the author of *Waverley*" is obviously a different proposition from "Scott is Scott": the first is a fact in literary history, the second a trivial truism. And if we put anyone other than Scott in place of "the author of *Waverley,*" our proposition would become false, and would therefore certainly no longer be the same proposition. But, it may be said, our proposition is essentially of the same form as (say) "Scott is Sir Walter," in which two names are said to apply to the same person. The reply is that, if "Scott is Sir

Walter" really means "the person named 'Scott' is the person named 'Sir Walter,' " then the names are being used as descriptions: i.e. the individual, instead of being named, is being described as the person having that name. This is a way in which names are frequently used in practice, and there will, as a rule, be nothing in the phraseology to show whether they are being used in this way or *as* names. When a name is used directly, merely to indicate what we are speaking about, it is no part of the *fact* asserted, or of the falsehood if our assertion happens to be false: it is merely part of the symbolism by which we express our thought. What we want to express is something which might (for example) be translated into a foreign language; it is something for which the actual words are a vehicle, but of which they are no part. On the other hand, when we make a proposition about "the person called 'Scott,' " the actual name "Scott" enters into what we are asserting, and not merely into the language used in making the assertion. Our proposition will now be a different one if we substitute "the person called 'Sir Walter.' " But so long as we are using names *as* names, whether we say "Scott" or whether we say "Sir Walter" is as irrelevant to what we are asserting as whether we speak English or French. Thus so long as names are used *as* names, "Scott is Sir Walter" is the same trivial proposition as "Scott is Scott." This completes the proof that "Scott is the author of *Waverley*" is not the same proposition as results from substituting a name for "the author of *Waverley*," no matter what name may be substituted.

When we use a variable, and speak of a propositional function, ϕx say, the process of applying general statements about x to particular cases will consist in substituting a name for the letter "x," assuming that ϕ is a function which has individuals for its arguments. Suppose, for example, that ϕx is "always true"; let it be, say, the "law of identity," $x=x$. Then we may substitute for "x" any name we choose, and we shall obtain a true proposition. Assuming for the moment that "Socrates," "Plato," and "Aristotle" are names (a very rash assumption), we can infer from the law of identity that Socrates is Socrates, Plato is Plato, and Aristotle is Aristotle. But we shall commit a fallacy if we attempt to infer, without further premisses, that the author of *Waverley* is the author of *Waverley*. This results from what we have just proved, that, if we substitute a name for "the author of *Waverley*" in a proposition, the proposition we obtain is a different one. That is to say, applying the result to our present case: If "x" is a name, "$x=x$" is not the same proposition as "the author of *Waverley* is the author of *Waverley*," no matter what name "x" may be. Thus from the fact that all propositions of the form "$x=x$" are true we cannot infer, without more ado, that the author of *Waverley* is the author of *Waverley*. In fact, propositions of the form "the so-and-so is the so-and-so" are not always true: it is necessary that the so-and-so should *exist* (a term which will be explained shortly). It is false that the present King of France is the present King of France, or that the round square is the round square. When we substitute a description for a name, propositional functions which are "always true" may become false, if the description describes nothing. There is no mystery in this as soon as we realize (what was proved in the preceding paragraph) that when we substitute a description the result is not a value of the propositional function in question.

We are now in a position to define propositions in which a definite description occurs. The only thing that distinguishes "the so-and-so" from "a so-and-so" is the implication of uniqueness. We cannot speak of "*the* inhabitant of London," because inhabiting London is an attribute which is not unique. We cannot speak about "the present King of France," because there is none; but we can speak about "the present King of England." Thus propositions about "the so-and-so" always imply the corresponding propositions about "a so-and-so," with the addendum that there is not more than one so-and-so. Such a proposition as "Scott is the author of Waverley" could not be true if *Waverley* had never been written, or if several people had written it; and no more could any other proposition resulting from a propositional function x by the substitution of "the author of *Waverley*" for "x." We may say that "the author of *Waverley*" means "the value of x for which 'x wrote *Waverley*' is true." Thus the proposition "the author of *Waverley* was Scotch," for example, involves:

(1) "x wrote *Waverley*" is not always false
(2) "if x and y wrote *Waverley*, x and y are identical" is always true
(3) "if x wrote *Waverley*, x was Scotch" is always true

These three propositions, translated into ordinary language, state:

(1) at least one person wrote *Waverley*
(2) at most one person wrote *Waverley*
(3) whoever wrote *Waverley* was Scotch

All these three are implied by "the author of *Waverley* was Scotch." Conversely, the three together (but no two of them) imply that the author of *Waverley* was Scotch. Hence the three together may be taken as defining what is meant by the proposition "the author of *Waverley* was Scotch."

We may somewhat simplify these three propositions. The first and second together are equivalent to: "There is a term c such that 'x wrote *Waverley*' is true when x is c and is false when x is not c." In other words, "There is a term c such that 'x wrote *Waverley*' is always equivalent to 'x is c.' " (Two propositions are "equivalent" when both are true or both are false.) We have here, to begin with, two functions of x, "x wrote *Waverley*" and "x is c," and we form a function of c by considering the equivalence of these two functions of x for all values of x; we then proceed to assert that the resulting function of c is "sometimes true," i.e. that it is true for at least one value of c. (It obviously cannot be true for more than one value of c.) These two conditions together are defined as giving the meaning of "the author of *Waverley* exists."

We may now define "the term satisfying the function ϕx exists." This is the general form of which the above is a particular case. "The author of *Waverley*" is "the term satisfying the function 'x wrote *Waverley*.' " And "the so-and-so" will always involve reference to some propositional function, namely, that which defines the property that makes a thing a so-and-so. Our definition is as follows:

> "The term satisfying the function ϕx exists" means:
>
> "There is a term c such that ϕx is always equivalent to 'x is c.' "

In order to define "the author of *Waverley* was Scotch," we have still to take account of the third of our three propositions, namely, "Whoever wrote *Waverley* was Scotch." This will be satisfied by merely adding that the c in question is to be Scotch. Thus "the author of *Waverley* was Scotch" is:

> "There is a term c such that (1) 'x wrote *Waverley*' is always equivalent to 'x is c,' (2) c is Scotch."

And generally: "the term satisfying ϕx satisfies ψx" is defined as meaning:

> "There is a term c such that (1) ϕx is always equivalent to 'x is c,' (2) ψx is true."

This is the definition of propositions in which descriptions occur.

It is possible to have much knowledge concerning a term described, i.e. to know many propositions concerning "the so-and-so," without actually knowing what the so-and-so is, i.e. without knowing any proposition of the form "x is the so-and-so," where "x" is a name. In a detective story propositions about "the man who did the deed" are accumulated, in the hope that ultimately they will suffice to demonstrate that it was A who did the deed. We may even go so far as to say that, in all such knowledge as can be expressed in words—with the exception of "this" and "that" and a few other words of which the meaning varies on different occasions—no names, in the strict sense, occur, but what seem like names are really descriptions. We may inquire significantly whether Homer existed, which we could not do if "Homer" were a name. The proposition "the so-and-so exists" is significant, whether true or false; but if a is the so-and-so (where "a" is a name), the words "a exists" are meaningless. It is only of descriptions—definite or indefinite—that existence can be significantly asserted; for, if "a" is a name, it *must* name something: what does not name anything is not a name, and therefore, if intended to be a name, is a symbol devoid of meaning, whereas a description, like "the present King of France," does not become incapable of occurring significantly merely on the ground that it describes nothing, the reason being that it is a *complex* symbol, of which the meaning is derived from that of its constituent symbols. And so, when we ask whether Homer existed, we are using the word "Homer" as an abbreviated description: we may replace it by (say) "the author of the *Iliad* and the *Odyssey.*" The same considerations apply to almost all uses of what look like proper names.

When descriptions occur in propositions, it is necessary to distinguish what may be called "primary" and "secondary" occurrences. The abstract distinction is as follows. A description has a "primary" occurrence when the proposition in which it occurs results from substituting the description for "x" in some propositional function ϕx; a description has a "secondary" occurrence when the result of substituting the description for x in ϕx gives only *part* of the proposition concerned. An instance will make this clearer. Consider "the present King of France is bald." Here "the present King of France" has a primary occurrence, and the proposition is false. Every proposition in which a description which describes nothing has a primary occurrence is false. But now consider "the present King of France is not bald." This is ambiguous. If we are first to take "x is bald," then substitute "the present King of France" for "x," and then deny the result, the occurrence of "the present King of France" is secondary and our proposition is true; but if we are to take "x is not bald" and substitute "the present King of France" for "x," then "the present King of France" has a primary occurrence and the proposition is false. Confusion of primary and secondary occurrences is a ready source of fallacies where descriptions are concerned.

Descriptions occur in mathematics chiefly in the form of *descriptive functions,* i.e. "the term having the relation R to y," or "the R of y" as we may say, on the analogy of "the father of y" and similar phrases. To say "the father of y is rich," for example, is to say that the following propositional function of c: "c is rich, and 'x begat y' is always equivalent to 'x is c,' " is "sometimes true," i.e. is true for at least one value of c. It obviously cannot be true for more than one value.

The theory of descriptions, briefly outlined in the present chapter, is of the utmost importance both in logic and in theory of knowledge. But for purposes of mathematics, the more philosophical parts of the theory are not essential, and have therefore been omitted in the above account, which has confined itself to the barest mathematical requisites.

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