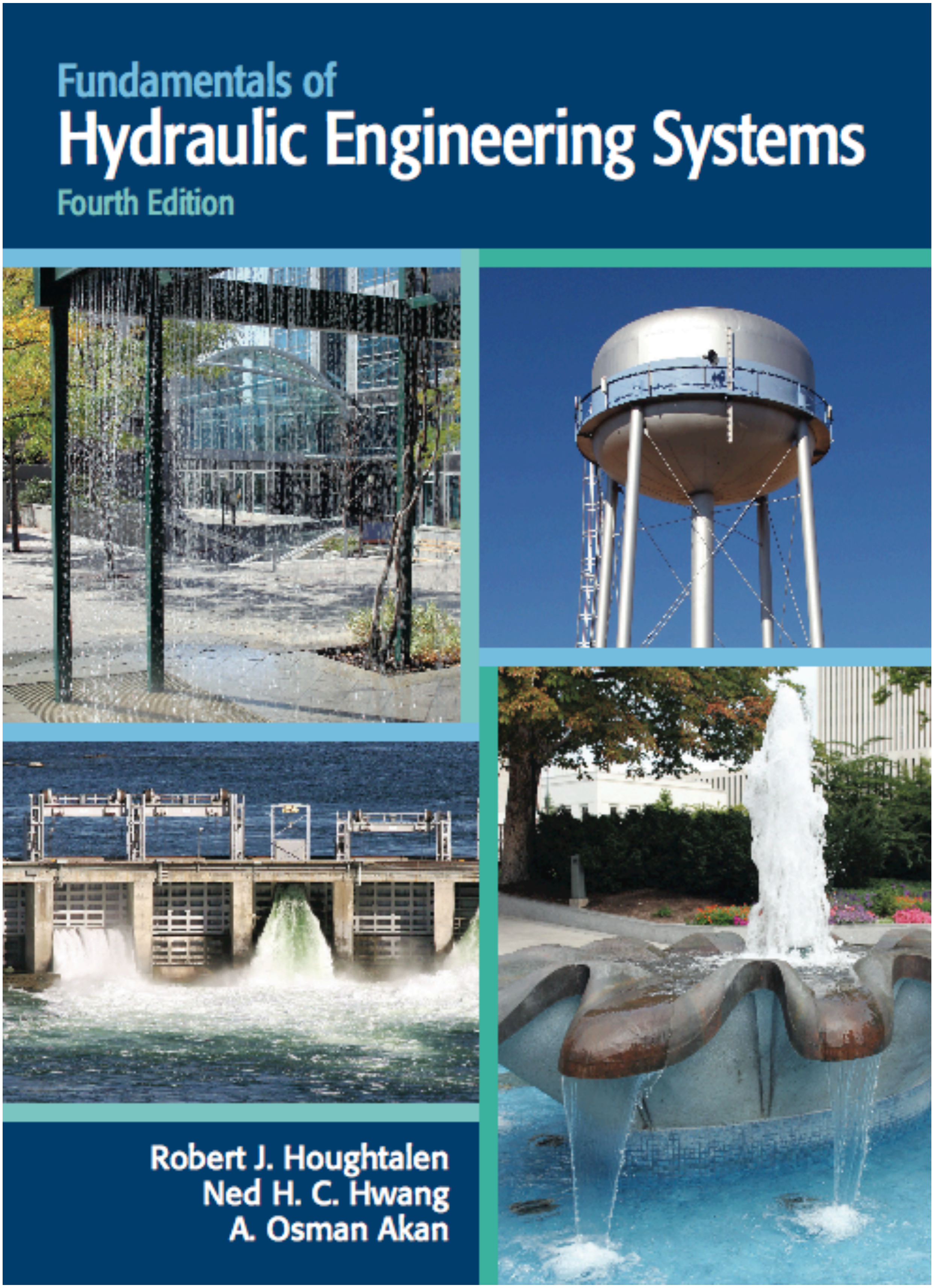
– Solutions Manual –

for



Contents

Chapter 1 – Problem Solutions

1.2.1

E_1 = energy req'd to bring ice temperature to 0°C

E_1 = (250 L)(1000 g/L)(20°C)(0.465 cal/g·°C)

$E_1 = 2.33 \times 10^6$ cal

E_2 = energy required to melt ice

E_2 = (250 L)(1000 g/L)(79.7 cal/g·°C)

$E_2 = 1.99 \times 10^7$ cal

E_3 = energy required to raise the water temperature to 20°C

E_3 = (250 L)(1000 g/L)(20°C)(1 cal/g·°C)

$E_3 = 5.00 \times 10^6$ cal

$\mathbf{E_{total}} = E_1 + E_2 + E_3 = \mathbf{2.72 \times 10^7}$ **cal**

1.2.2

At 0.9 bar (ambient pressure), the boiling temperature of water is 97°C (see Table 1.1).

E_1 = energy required to bring the water temperature to 97°C

E_1 = (1200 g)(97°C - 45°C)(1 cal/g·°C)

$E_1 = 6.24 \times 10^4$ cal

E_2 = energy required to vaporize the water

E_2 = (1200 g)(597 cal/g)

$E_2 = 7.16 \times 10^5$ cal

$\mathbf{E_{total}} = E_1 + E_2 = \mathbf{7.79 \times 10^5}$ **cal**

1.2.3

E_1 = energy required to change water to ice

E_1 = (100 g)(79.7 cal/g)

$E_1 = 7.97 \times 10^3$ cal

E_2 = energy required to change vapor to ice

E_2 = (100 g)(597 cal/g) + (100 g)(79.7 cal/g)

$E_2 = 6.77 \times 10^4$ cal

Total energy removed to freeze water and vapor.

$\mathbf{E_{total} = E_1 + E_2 = 7.57 \times 10^4}$ **cal**

1.2.4

E_1 = energy needed to vaporize the water

E_1 = (100 L)(1000 g/L)(597 cal/g)

$E_1 = 5.97 \times 10^7$ cal

The energy remaining (E_2) is:

$E_2 = E - E_1$

$E_2 = 6.80 \times 10^7$ cal – 5.97×10^7 cal

$E_2 = 8.30 \times 10^6$ cal

The temperature change possible with the remaining energy is:

8.30×10^6 cal = (100 L)(1000 g/L)(1 cal/g·°C)(ΔT)

ΔT = 83°C, making the temperature

T = 93°C when it evaporates.

Therefore, based on Table 1.1,

$\therefore$ **P = 0.777 atm**

1.2.5

E_1 = energy required to raise the temperature to 100°C

$E_1 = (5000\ g)(100°C – 25°C)(1\ cal/g·°C)$

$E_2 = 3.75x10^5$ cal

E_2 = energy required to vaporize 2.5 kg of water

$E_2 = (2500\ g)(597\ cal/g)$

$E_2 = 1.49x10^6$ cal

$E_{total} = E_1 + E_2 = 1.87x10^6$ cal

Time required = $(1.87x10^6\ cal)/(500\ cal/s)$ =
3740 sec = 62.3 min

1.2.6

E_1 = energy required to melt ice

E_1 = (5 slugs)(32.2 lbm/slug)(32°F - 20°F)(0.46 BTU/lbm·°F) + (5 slugs)(32.2 lbm/slug)(144 BTU/lbm)

$E_1 = 2.41 \times 10^4$ BTU

To melt the ice, the temperature of the water will decrease to:

2.41×10^4 BTU = (10 slugs)(32.2 lbm/slug)(120°F – T_1)(1 BTU/lbm·°F)

T_1 = 45.2°F

The energy lost by the water (to lower its temp. to 45.2°F) is that required to melt the ice. Now you have 5 slugs of water at 32°F and 10 slugs at 45.2°F.

Therefore, the final temperature of the water is:

[(10 slugs)(32.2 lbm/slug)(45.2°F – T_2)(1 BTU/lbm·°F)]

= [(5 slugs)(32.2 lbm/slug)(T_2 - 32°F)(1 BTU/lbm·°F)]

T_2 = 40.8°F

1.3.1

The weight of water in the container is 814 N.

m = W/g = (814 N)/(9.81 m/sec^2) = 83.0 kg

At 20˚C, 998 kg = 1 m^3

Therefore, the volume can be determined by

Vol = (83.0 kg)(1 m^3/998 kg)

Vol = 8.32 x 10^{-2} m^3

1.3.2

F = m·a   Letting a = g results in Equation 1.1

W = m·g, dividing both sides of the equation by volume yields

$\gamma = \rho \cdot g$

1.3.3

$\gamma = \rho \cdot g = (13{,}600\ kg/m^3)(9.81\ m/s^2)$

γ = 133 kN/m^3

S.G. = $\gamma_{liguid}/\gamma_{water\ at\ 4°C}$

S.G. = (133,000 N/m^3)/(9810 N/m^3)

S.G. = 13.6 (mercury)

1.3.4

The force exerted on the tank bottom is equal to the weight of the water body.

F = W = m · g = [ρ · (Vol)] (g)

F = [1.94 slugs/ft^3 (π · (5 ft)^2 · 3 ft)] (32.2 ft/sec^2)

F = 1.47 x 10^4 lbs

(Note: 1 slug = 1 lb·sec^2/ft)

1.3.5

Weight of water on earth = 7.85 kN

m = W/g = (7,850 N)/(9.81 m/s^2)

m = 800 kg

Note: mass on moon is the same as mass on earth

W (moon) = mg = (800 kg)[(9.81 m/s^2)/(6)]

W(moon) = 1310 N

1.3.6

W = mg = (0.258 slug)(32.2 ft/s^2)

W = 8.31 lb

Note: a slug has units of (lb·s^2)/(ft)

Volume of 1 gal = 0.134 ft^3

S.G. = $\gamma_{\text{liguid}}/\gamma_{\text{water at 4°C}}$

γ = (8.31 lb)/(0.134 ft^3) = 62.0 lb/ft^3

S.G. = (62.0 lb/ft^3)/(62.4 lb/ft^3)

S.G. = 0.994

1.3.7

Density can be expressed as:

ρ = m/Vol

and even though volume changes with temperature, mass does not. Thus,

(ρ_1)(Vol$_1$) = (ρ_2)(Vol$_2$) = constant; or

Vol$_2$ = (ρ_1)(Vol$_1$)/(ρ_2)

Vol$_2$ = (1000 kg/m^3)(100 m^3)/(958 kg/m^3)

Vol$_2$ = 104.4 m^3 (or a 4.4% change in volume)

1.3.8

(1 N)[(1 lb)/(4.448 N)] = **0.2248 lb**

1.3.9

(1 N·m)[(3.281 ft)/(1 m)][(0.2248 lb)/(1 N))]

= **7.376 x 10^{-1} ft·lb**

1.4.1

$[\mu(\text{air})/\mu(H_2O)]_{20°C}$ = (1.817x10^{-5})/(1.002x10^{-3})

$[\mu(\text{air})/\mu(H_2O)]_{20°C}$ = 1.813x10^{-2}

$[\mu(\text{air})/\mu(H_2O)]_{80°C}$ = (2.088x10^{-5})/(0.354x10^{-3})

$[\mu(\text{air})/\mu(H_2O)]_{80°C}$ = 5.90x10^{-2}

$[\nu(\text{air})/\nu(H_2O)]_{20°C}$ = (1.509x10^{-5})/(1.003x10^{-6})

$[\nu(\text{air})/\nu(H_2O)]_{20°C}$ = 15.04

$[\nu(\text{air})/\nu(H_2O)]_{80°C}$ = (2.087x10^{-5})/(0.364x10^{-6})

$[\nu(\text{air})/\nu(H_2O)]_{80°C}$ = 57.3

Note: The ratio of absolute and kinematic viscosities of air and water increases with temperature because the viscosity of air increases with temperature, but that of water decreases with temperature. Also, the values of kinematic viscosity (ν) for air and water are much closer than those of absolute viscosity. Why?

1.4.2

$\mu(\text{water})_{20°C}$ = 1.002x10^{-3} N·sec/m^2

$\nu(\text{water})_{20°C}$ = 1.003x10^{-6} m^2/s

(1.002x10^{-3} N·sec/m^2)·[(0.2248 lb)/(1 N)]·
[(1 m)^2/(3.281 ft)^2] = **2.092x10^{-5} lb·sec/ft^2**

(1.003x10^{-6} m^2/s)[(3.281 ft)^2/(1 m)^2] = **1.080x10^{-5} ft^2/s**

1.4.3

(a) 1 poise = 0.1 N·sec/m^2

(0.1 N·sec/m^2)[(0.2248 lb)/(1 N)][(1 m)^2/(3.281 ft)^2] =
2.088x10^{-3} lb·sec/ft^2

alternatively,

1 lb·sec/ft^2 = 478.9 poise

(b) 1 stoke = 1 cm^2/sec

(1 cm^2/s)[(0.3937 in)^2/(1 cm)^2][(1 ft)^2/(12 in)^2] =
1.076x10^{-3} ft^2/sec

alternatively,

1 ft^2/sec = 929.4 stoke

1.4.4

Assuming a Newtonian relationship:

$\tau = \mu(dv/dy) = \mu(\Delta v/\Delta y)$

τ = (2.09x10^{-5} lb·sec/ft^2)[(5 ft/sec)/(0.25 ft)]

τ = (4.18x10^{-4} lb/ft^2)

F = τ·A = (4.18x10^{-4} lb/ft^2)(10 ft)(30 ft)

F = 0.125 lbs

1.4.5

v = y^2 – 2y, where y is in inches and v is in ft/s

Making units consistent yields

v = 144y^2 – 24y, where y is in ft and v is in ft/s

Taking the first derivative w/respect to y:

dv/dy = 288y – 24 sec^{-1}

$\tau = \mu(dv/dy)$

τ = (0.375 N·sec/m^2)(288y – 24 sec^{-1})

1.4.5 (cont.)

Solutions:

y = 0 ft, τ = - 9.00 N/m^2

y = 1/12 ft, τ = 0 N/m^2

y = 1/6 ft, τ = 9.00 N/m^2

y = 1/4 ft, τ = 18.0 N/m^2

y = 1/3 ft, τ = 27.0 N/m^2

1.4.6

Based on the geometry of the incline

$T_{shear\ force}$ = W(sin15°) = τ·A = μ(dv/dy)A

Δy = [(μ)(Δv)(A)] / [(W)(sin15°)]

Δy = [(1.29 N·sec/m^2)(0.025 m/sec)
(0.50m)(0.75m)]/[(220 N)(sin15°)]

Δy = 2.12 x 10^{-4} m = 2.12 x 10^{-2} cm

1.4.7

$\sum F_y$ = 0 (constant velocity motion)

W = $T_{shear\ force}$ = τ·A;   where A is the surface area

(of the cylinder) in contact with the oil film:

A = (π)[(5.48/12)ft][(9.5/12)ft] = 1.14 ft^2

Now, τ = W/A = (0.5 lb)/(1.14 ft^2) = 0.439 lb/ft^2

$\tau = \mu(dv/dy) = \mu(\Delta v/\Delta y)$, where

Δv = v (the velocity of the cylinder). Thus,

v = (τ)(Δy)/μ

v = [(0.439 lb/ft^2){(0.002/12)ft}] / (0.016 lb·s/ft^2)

v = 4.57 x 10^{-3} ft/sec

1.4.8

$\tau = \mu(\Delta v/\Delta y)$

$\tau = (0.0065 \text{ lb·sec/ft}^2)[(1 \text{ ft/s})/(0.5/12 \text{ ft})$

$\tau = 0.156 \text{ lb/ft}^2$

$F = (\tau)(A) = (2 \text{ sides})(0.156 \text{ lb/ft}^2)(2 \text{ ft}^2)$

F = 0.624 lb

1.4.9

$\mu = \tau/(dv/dy) = (F/A)/(\Delta v/\Delta y)$;

Torque = Force·distance = F·R; R = radius

Thus; $\mu = (\text{Torque}/R)/[(A)(\Delta v/\Delta y)]$

$$\mu = \frac{Torque/R}{(2\pi)(R)(h)(\omega \cdot R/\Delta y)} = \frac{Torque \cdot \Delta y}{(2\pi)(R^3)(h)(\omega)}$$

$$\mu = \frac{(1.50 N \cdot m)(0.0002m)}{(2\pi)(0.025m)^3(0.04m)(2000rpm)\left(\frac{2\pi rad/\sec}{60rpm}\right)}$$

$\mu = 3.65\text{x}10^{-1}$ N·sec/m^2

1.4.10

$\mu = (16)(1.002\text{x}10^{-3} \text{ N·sec/m}^2)$

$\mu = 1.603\text{x}10^{-2} \text{ N·sec/m}^2$

$$\text{Torque} = \int_0^R (r)dF = \int_0^R r \cdot \tau \cdot dA$$

$$\text{Torque} = \int_0^R (r)(\mu)(\frac{\Delta v}{\Delta y})dA$$

$$\text{Torque} = \int_0^R (r)(\mu)(\frac{(\omega)(r) - 0}{\Delta y})(2\pi r)dr$$

$$\text{Torque} = \frac{(2\pi)(\mu)(\omega)}{\Delta y}\int_0^R (r^3)dr$$

$$\text{Torque} = \frac{(2\pi)(1.603 \cdot 10^{-2} N \cdot \sec/m^2)(0.65 rad/\sec)}{0.0005m}\left[\frac{(1m)^4}{4}\right]$$

Torque = 32.7 N·m

1.5.1

$h = [(4)(\sigma)(\sin\theta)] / [(\gamma)(D)]$

But sin 90˚ = 0, $\sigma = 7.132\text{x}10^{-2}$ N/m

and $\gamma = 9790 \text{ N/m}^3$ (at 20˚C)

thus, $D = [(4)(\sigma)] / [(\gamma)(h)]$; for h = 3.0 cm

$D = [(4)(7.132\text{x}10^{-2} \text{ N/m})] / [(9790 \text{ N/m}^3)(0.03\text{m})]$

$D = 9.71 \text{ x } 10^{-4} \text{ m} = 9.71 \text{ x } 10^{-2}$ cm; thus,

for h = 3.0 cm, D = 0.0971 cm

for h = 2.0 cm, D = 0.146 cm

for h = 1.0 cm, D = 0.291 cm

1.5.2

The concept of a line force is logical for two reasons:

1) The surface tension acts along the perimeter of the tube pulling the column of water upwards due to adhesion between the water and the tube.
2) The surface tension is be multiplied by the tube perimeter, a length, to obtain the upward force used in force balance development of the equation for capillary rise.

1.5.3

$\sigma = [(h)(\gamma)(D)] / [(4)(\sin\theta)]$

$\sigma = [(0.6/12)\text{ft}(1.94\text{slug/ft}^3)(32.2 \text{ lb/ft}^3)(0.02/12)\text{ft}]/$
$[(4)(\sin 54°)]$

$\sigma = 1.61 \text{ x } 10^{-3}$ lb/ft

1.5.4

Capillary rise in the 0.25 cm. tube is found using:

$h = [(4)(\sigma)(\sin\theta)] / [(\gamma)(D)]$

where $\sigma = (6.90 \times 10^{-2})(1.2) = 8.28 \times 10^{-2}$ N/m

and $\gamma = (9752)(1.03) = 1.00 \times 10^4$ N/m^3

$$h = \frac{4(8.28\cdot 10^{-2}\, N/m)(\sin 30)}{(1.00\cdot 10^{4}\, N/m^{3})(0.0025m)}$$

$h = 6.62 \times 10^{-3}$ m = 0.662 cm

1.5.5

Condition 1: $h_1 = [(4)(\sigma_1)(\sin\theta_1)] / [(\gamma)(D)]$

$h_1 = [(4)(\sigma_1)(\sin 30°)] / [(\gamma)(0.7 \text{ mm})]$

Condition 2: $h_2 = [(4)(\sigma_2)(\sin\theta_2)] / [(\gamma)(D)]$

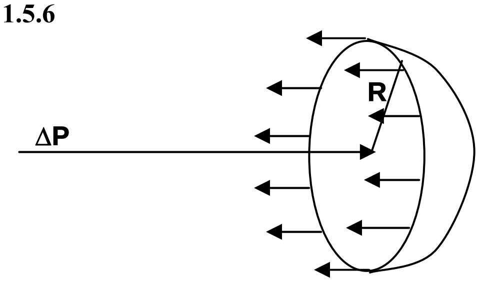
$h_2 = [(4)(0.9\sigma_1)(\sin 42°)] / [(\gamma)(0.7 \text{ mm})]$

h_2/h_1 $= [(0.9)(\sin 42°)] / (\sin 30°) =$ **1.204**

alternatively,

$h_2 = 1.204(h_1)$, about a 20% increase!

1.5.6



$\Delta P = P_i - P_e$ (internal pressure minus external pressure)

$\sum F_x = 0;\quad 2\pi(R)(\sigma) - \Delta P(\pi)(R^2) = 0$

$\Delta P = 2\sigma/R$

1.6.1

E_b $= -\Delta P/(\Delta Vol/Vol)$ **$= 9.09 \times 10^9$ N/m^2**

1.6.2

$P_1 = 25$ bar $= 25 \times 10^5$ N/m^2 $= 2.50 \times 10^6$ N/m^2

$\Delta Vol/Vol = -\Delta P/E_b$

$\Delta Vol/Vol = -(4.5 \times 10^5 \text{ N/m}^2 - 2.5 \times 10^6 \text{ N/m}^2)/(2.2\times 10^9 \text{ N/m}^2)$

$\Delta Vol/Vol = 9.3 \times 10^{-4} = 0.093\%$ (volume increase)

$\Delta\rho/\rho = -\Delta Vol/Vol = -0.093\%$ (density decreases)

1.6.3

$\rho_o = 1.94$ slugs/ft^3 (based on temp. & pressure)

$m = \rho_o \cdot Vol_o = (1.94 \text{ slug/ft}^3)(120 \text{ ft}^3) = 233$ slugs

W = mg = (233 slugs)(32.2 ft/sec^2) = **7,500 lb**

$\rho = \rho_o/[1+(\Delta Vol/Vol)]$; see example 1.3

ρ = 1.94 slug/ft^3/[1+(-0.545/120)] = **1.95 slug/ft^3**

1.6.4

$P_i = 30$ N/cm^2 = 300,000 N/m^2 = 3 bar

$\Delta P = 30$ bar – 3 bar = 27 bar = 27×10^5 N/m^2

Amount of water that enters pipe = ΔVol

$Vol_{pipe} = [(\pi)(1.50 \text{ m})^2/(4)]\cdot(2000 \text{ m}) = 3530 \text{ m}^3$

$\Delta Vol = (-\Delta P/E_b)(Vol) = [(-27\times 10^5 \text{ N/m}^2)/(2.2\times 10^9 \text{ N/m}^2)]*(3530 \text{ m}^3)$

$\Delta Vol = -4.33$ m^3

Water in the pipe is compressed by this amount.

∴ **The volume of H_2O that enters the pipe is 4.33 m^3**

Chapter 2 – Problem Solutions

2.2.1

$P = \gamma \cdot h$; where $\gamma = (1.03)(9810\ N/m^3) = 1.01\text{x}10^4\ N/m^3$

(using the specific weight of water at standard conditions since water gets very cold at great depths)

$P = \gamma \cdot h = (1.01\text{x}10^4\ N/m^3)(730\ m)$

$\mathbf{P = 7.37\text{x}10^6\ N/m^2 = 1{,}070\ psi}$

The pressure given is gage pressure. To get absolute pressure, atmospheric pressure must be added.

2.2.2

a) The force exerted on the tank bottom is equal to the weight of the water body.

$F = W = m \cdot g = [\rho \cdot (Vol)]\ (g)$

$F = [1.94\ slugs/ft^3\ (\pi \cdot (5\ ft)^2 \cdot 3\ ft)]\ (32.2\ ft/sec^2)$

$\mathbf{F = 1.47\ x\ 10^4\ lbs}$

(Note: 1 slug = 1 $lb \cdot sec^2/ft$)

b) The force exerted on the tank bottom is equal to the pressure on the bottom times the area of the bottom.

$P = \gamma \cdot h = (62.3\ lb/ft^3)(3\ ft) = 187\ lb/ft^2$

$F = P \cdot A = (187\ lb/ft^2)(\pi \cdot (5\ ft)^2)$

$\mathbf{F = 1.47\ x\ 10^4\ lbs}$

2.2.3

γ_{water} at 30°C = 9.77 kN/m^3

P_{vapor} at 30°C = 4.24 kN/m^2

$P_{atm} = P_{column} + P_{vapor}$

$P_{atm} = (9.8\ m)(9.77\ kN/m^3) + (4.24\ kN/m^2)$

$\mathbf{P_{atm} = 95.7\ kN/m^2 + 4.24\ kN/m^2 = 99.9\ kN/m^2}$

2.2.3 (cont.)

The percentage error if the direct reading is used and the vapor pressure is ignored is:

$Error = (P_{atm} - P_{column})/(P_{atm})$

$Error = (99.9\ kN/m^2 – 95.7\ kN/m^2)/(99.9\ kN/m^2)$

Error = 0.0420 = **4.20%**

2.2.4

The atm. pressure found in problem 2.2.3 is 99.9 kN/m^2

$P_{atm} = (\gamma_{Hg})(h)$

$h = P_{atm}/\gamma_{Hg} = (99.9\ kN/m^2) / [(13.6)(9.77\ kN/m^3)]$

h = 0.752 m = 75.2 cm = 2.47 ft

2.2.5

The force exerted on the tank bottom is equal to the pressure on the bottom times the area of the bottom.

$P = \gamma \cdot h = (9.79\ kN/m^3)(6\ m) = 58.7\ kN/m^2$

$F = P \cdot A = (58.7\ kN/m^2)(36\ m^2)$

F = 2,110 N

The force exerted on the sides of the tank may be found in like manner (pressure times the area). However, the pressure is not uniform on the tank sides since $P = \gamma \cdot h$. Therefore, the average pressure is required. Since the pressure is a linear relationship, the average pressure occurs at half the depth. Now,

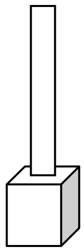
$P_{avg} = \gamma \cdot h_{avg} = (9.79\ kN/m^3)(3\ m) = 29.4\ kN/m^2$

$F = P_{avg} \cdot A = (29.4\ kN/m^2)(36\ m^2)$

F = 1,060 N

Obviously, the force on the bottom is greater than the force on the sides by a factor of two.

2.2.6



$W_{total} = (Vol_{container})(\gamma_{water}) + (Vol_{pipe})(\gamma_{water})$

$W_{total} = (3\text{ ft})^3(62.3\text{ lb/ft}^3)$
$\quad + [(\pi)(0.50\text{ ft})^2(30\text{ ft})](62.3\text{ lb/ft}^3)$

$\mathbf{W_{total}} = 1680\text{ lb} + 1470\text{ lb} = \mathbf{3{,}150\text{ lb}}$

$P_{bottom} = \gamma h = (62.3\text{ lb/ft}^3)(33\text{ ft}) = 2060\text{ lb/ft}^2$

$\mathbf{F_{bottom}} = (2060\text{ lb/ft}^2)(9\text{ ft}^2) = \mathbf{18{,}500\text{ lb}}$

Note: The weight of the water is not equal to the force on the bottom. Why? (Hint: Draw a free body diagram of the 3 ft x 3 ft x 3 ft water body labeling all forces (vertical) acting on it. Don't forget the pressure from the container top. Now, to determine the side force:

$P_{avg} = \gamma \cdot h_{avg} = (62.3\text{ lb/ft}^3)(31.5\text{ ft}) = 1960\text{ lb/ft}^2$

$F = P_{avg} \cdot A = (1960\text{ lb/ft}^2)(9\text{ ft}^2)$

$\mathbf{F = 17{,}600\text{ lb}}$

2.2.7

$P_{bottom} = P_{gage} + (\gamma_{liquid})(1.4\text{ m})$; and

$\gamma_{liquid} = (SG)(\gamma_{water}) = (0.80)(9790\text{ N/m}^3) = 7830\text{ N/m}^3$

$\therefore P_{bottom} = 4.50\text{x}10^4\text{ N/m}^2 + (7830\text{ N/m}^3)(1.4\text{ m})$

$\mathbf{P_{bottom} = 5.60 \text{ x } 10^4\text{ N/m}^2}$

The pressure at the bottom of the liquid column can be determined two different ways which must be equal. Hence,

$(h)(\gamma_{liquid}) = P_{gage} + (\gamma_{liquid})(1\text{ m})$

$h = (P_{gage})/(\gamma_{liquid}) + 1\text{m}$

$\mathbf{h} = (4.50\text{x}10^4\text{ N/m}^2)/7830\text{ N/m}^3 + 1\text{ m} = \mathbf{6.75\text{ m}}$

2.2.8

$\gamma_{seawater} = (SG)(\gamma_{water}) = (1.03)(9790\text{ N/m}^3)$

$\gamma_{seawater} = 1.01 \text{ x } 10^4\text{ N/m}^3$

$P_{tank} = (\gamma_{water})(\Delta h) = (1.01 \text{ x } 10^4\text{ N/m}^3)(6\text{ m})$

$\mathbf{P_{tank} = 6.06 \text{ x } 10^4\text{ N/m}^2}$ **(Pascals) = 8.79 psi**

2.2.9

$\gamma_{oil} = (SG)(\gamma_{water}) = (0.85)(62.3\text{ lb/ft}^3) = 53.0\text{ lb/ft}^3$

$P_{10ft} = P_{air} + (\gamma_{oil})(10\text{ ft})$

$P_{air} = P_{10ft} - (\gamma_{oil})(10\text{ ft})$

$P_{air} = 23.7\text{ psi }(144\text{ in}^2/\text{ft}^2) - (53.0\text{ lb/ft}^3)(10\text{ ft})$

$\mathbf{P_{air} = 2.88 \text{ x } 10^4\text{ lb/ft}^2}$ **(20.0 psi); Gage pressure**

$P_{abs} = P_{gage} + P_{atm} = 20.0\text{ psi} + 14.7\text{ psi}$

$\mathbf{P_{abs} = 34.7\text{ psi }(5.00 \text{ x } 10^4\text{ lb/ft}^2)}$**; Absolute pressure**

2.2.10

The mechanical advantage in the lever increases the input force delivered to the hydraulic jack. Thus,

$F_{input} = (9)(50\text{ N}) = 450\text{ N}$

The pressure developed in the system is:

$P_{input} = F/A = (450\text{ N})/(25\text{ cm}^2) = 18\text{ N/cm}^2$

$\mathbf{P_{input} = 180\text{ kN/m}^2}$

From Pascal's law, the pressure at the input piston should equal the pressure at the two output pistons.

$\therefore$ The force exerted on each output piston is:

$P_{input} = P_{output}$ equates to: $18\text{ N/cm}^2 = F_{output}/250\text{ cm}^2$

$F_{output} = (18\text{ N/cm}^2)(250\text{ cm}^2)$

$\mathbf{F_{output} = 4.50\text{ kN}}$

2.4.1

Since the line passing through points 7 and 8 represents an equal pressure surface;

$P_7 = P_8$ or $(h_{water})(\gamma_{water}) = (h_{oil})(\gamma_{oil})$

However; $(h_{oil})(\gamma_{oil}) = (h_{oil})(\gamma_{water})(SG_{oil})$, thus

$\mathbf{h_{oil}} = (h_{water})/(SG_{oil}) = (52.3\text{ cm})/(0.85) = \mathbf{61.5\text{ cm}}$

2.4.2

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$(3\text{ ft})(\gamma_{water}) = (h)(\gamma_{Hg})$

$h = (3\text{ ft})(\gamma_{water}/\gamma_{Hg}) = (3\text{ ft}) / (SG_{Hg}) = (3\text{ ft}) / (13.6)$

h = 0.221 ft = 2.65 in.

2.4.3

The pressure at the bottom registered by the gage is equal to the pressure due to the liquid heights. Thus,

$(h_{Hg})(SG_{Hg})(\gamma_{water}) = (4h)(\gamma_{water}) + (h)(SG_{oil})(\gamma_{water})$

$h = (h_{Hg})(SG_{Hg})/(4 + SG_{oil})$

$\mathbf{h} = (26.3\text{ cm})(13.6)/(4 + 0.82) = \mathbf{74.2\text{ cm}}$

2.4.4

A surface of equal pressure can be drawn at the mercury-water meniscus. Therefore,

$P_A + (y)(\gamma_{water}) = (h)(\gamma_{Hg})$

$P_A + (0.034\text{ m})(\gamma_{water}) = (0.026\text{ m})(\gamma_{Hg})$

$P_A = (0.026\text{ m})(13.6)(9{,}790\text{ N/m}^3) - (0.034\text{ m})(9{,}790\text{ N/m}^3)$

$\mathbf{P_A = 3{,}130\ N/m^2\ (Pascals) = 3.13\ kN/m^2}$

2.4.5

A surface of equal pressure can be drawn at the mercury-water meniscus. Therefore,

$P_{pipe} + (2\text{ ft})(\gamma_{water}) = (h)(\gamma_{Hg})$

$(16.8\text{ lb/in}^2)(144\text{ in}^2/\text{ft}^2) + (2\text{ ft})(\gamma_{water}) = (h)(\gamma_{Hg})$

$(2.42 \times 10^3\text{ lb/ft}^2) + (2\text{ ft})(62.3\text{ lb/ft}^3) = (h)(13.6)(62.3\text{ lb/ft}^3)$

h = 3.00 ft (manometer is correct)

2.4.6

Using the "swim through" technique, start at the end of the manometer which is open to the atmosphere and thus equal to zero gage pressure. Then "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach the pipe. The computations are below:

$$0 - (0.66\text{ m})(\gamma_{CT}) + [(0.66 + y + 0.58)\text{m}](\gamma_{air}) - (0.58\text{ m})(\gamma_{oil}) = P_{pipe}$$

The specific weight of air is negligible when compared to fluids, so that term in the equation can be dropped.

$P_{pipe} = 0 - (0.66\text{ m})(SG_{CT})(\gamma) - (0.58\text{ m})(SG_{oil})(\gamma)$

$P_{pipe} = 0 - (0.66\text{ m})(1.60)(9790\text{ N/m}^3) - (0.58\text{ m})(0.82)(9790\text{ N/m}^3)$

$P_{pipe} = -15.0\text{ kN/m}^2$ Pressure can be converted to height (head) of any liquid through $P = \gamma \cdot h$. Thus,

$\mathbf{h_{pipe}} = (-15{,}000\text{ N/m}^2)/(9790\text{ N/m}^3) = \mathbf{-1.53\ m\ of\ water}$

2.4.7

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$P + (h_1)(\gamma) = (h_2)(\gamma_{Hg}) = (h_2)(SG_{Hg})(\gamma)$

$P + (0.575\text{ ft})(62.3\text{ lb/ft}^3) = (2.00\text{ ft})(13.6)(62.3\text{ lb/ft}^3)$

$\mathbf{P = 1{,}660\ lb/ft^2 = 11.5\ psi}$

2.4.8

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$P_{pipe} + (h_1)(\gamma) = (h_2)(\gamma_{Hg}) = (h_2)(SG_{Hg})(\gamma)$

$P_{pipe} + (0.20\text{ m})(9790\text{ N/m}^3) = (0.67\text{ m})(13.6)(9790\text{ N/m}^3)$

$\mathbf{P_{pipe} = 8.72 \times 10^4\ N/m^2}$ **(Pascals) = 87.2 KPa**

When the manometer reading rises or falls, mass balance must be preserve in the system. Therefore,

$Vol_{res} = Vol_{tube}$ or $A_{res}{\cdot}h_1 = A_{tube}{\cdot}h_2$

$h_1 = h_2\,(A_{tube}/A_{res}) = h_2\,[(D_{tube})^2/(D_{res})^2]$

$\mathbf{h_1} = (10\text{ cm})[(0.5\text{ cm})^2/(5\text{ cm})^2] = \mathbf{0.1\ cm}$

2.4.9

Using the "swim through" technique, start at pipe *A* and "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach pipe B. The computations are:

$P_A + (5.33\text{ ft})(\gamma) - (1.67\text{ft})(\gamma_{Hg}) - (1.0\text{ ft})(\gamma_{oil}) = P_B$

$P_A - P_B = (62.3\text{ lb/ft}^3)\,[(1.0\text{ ft})(0.82) - (5.33\text{ ft}) + (1.67\text{ ft})(13.6)]$

$\mathbf{P_A - P_B = 1{,}130\ lb/ft^2 = 7.85\ psi}$

2.4.10

Using the "swim through" technique, start at P_2 and "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach P_1. The computations are:

$P_2 + (\Delta h)(\rho_1{\cdot}g) + (y)(\rho_1{\cdot}g) + (h)(\rho_2{\cdot}g) - (h)(\rho_1{\cdot}g) - (y)(\rho_1{\cdot}g) = P_1$

where y is the vertical elevation difference between the fluid surface in the left hand reservoir and the interface between the two fluids on the right side of the U-tube.

$P_1 - P_2 = (\Delta h)(\rho_1{\cdot}g) + (h)(\rho_2{\cdot}g) - (h)(\rho_1{\cdot}g)$

2.4.10 – cont.

When the manometer reading (h) rises or falls, mass balance must be preserve in the system. Therefore,

$Vol_{res} = Vol_{tube}$ or $A_{res}{\cdot}(\Delta h) = A_{tube}{\cdot}\,h$

$\Delta h = h\,(A_{tube}/A_{res}) = h\,[(d_2)^2/(d_1)^2]$; substituting yields

$\mathbf{P_1 - P_2 = h\,[(d_2)^2/(d_1)^2]\,(\rho_1{\cdot}g) + (h)(\rho_2{\cdot}g) - (h)(\rho_1{\cdot}g)}$

$\mathbf{P_1 - P_2 = h{\cdot}g\,[\rho_2 - \rho_1 + \rho_1\,\{(d_2)^2/(d_1)^2\}]}$

$\mathbf{P_1 - P_2 = h{\cdot}g\,[\rho_2 - \rho_1\,\{1 - (d_2)^2/(d_1)^2\}]}$

2.4.11

Using the "swim through" technique, start at both ends of the manometers which are open to the atmosphere and thus equal to zero gage pressure. Then "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach the pipes in order to determine P_A and P_B. The computations are below:

$0 + (23)(13.6)(\gamma) - (44)(\gamma) = P_A$; $P_A = 269{\cdot}\gamma$

$0 + (46)(0.8)(\gamma) + (20)(13.6)(\gamma) - (40)(\gamma) = P_B$

$P_B = 269{\cdot}\gamma$; Therefore, $P_A = P_B$ and $\mathbf{h = 0}$

2.4.12

Using the "swim through" technique, start at the sealed right tank where the pressure is known. Then "swim through" the tanks and pipes, adding pressure when "swimming" down and subtracting when "swimming" up until you reach the left tank where the pressure is not known. The computations are as follows:

$20\text{ kN/m}^2 + (4.5\text{ m})(9.79\text{ kN/m}^3) - (2.5\text{ m})(1.6)(9.79\text{ kN/m}^3) - (5\text{ m})(0.8)(9.79\text{ kN/m}^3) = P_{left}$

$\mathbf{P_{left} = -14.3\ kN/m^2}$ **(or -14.3 kPa)**

$P_B = (-14.3\text{ kN/m}^2) / [(SG_{Hg})(\gamma)]$

$P_B = (-14.3\text{ kN/m}^2) / [(13.6)(9.79\text{ kN/m}^3)]$

$\mathbf{P_B = -0.107\ m = 10.7\ cm\ (Hg)}$

2.5.1

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)[(3m)/(3)]·[6 m^2]

F = 5.87 x 10^4 N = 58.7 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{\left[(4m)(3m)^3/36\right]}{\left[(4m)(3m)/2\right](1.00m)} + 1.00m$$

y_p = 1.50 m (depth to center of pressure)

2.5.2

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(30 ft)/2]·[(30 ft)(1 ft)]

F = 2.80 x 10^4 lbs per foot of length

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{\left[(1ft)(30ft)^3/12\right]}{\left[(30ft)(1ft)\right](15ft)} + 15ft$$

y_p = 20.0 ft (depth to the center of pressure)

In summing moments about the toe of the dam ($\sum M_A$), the weight acts to stabilize the dam (called a righting moment) and the hydrostatic force tends to tip it over (overturning moment).

M = (Wt.)[(2/3)(10)] - F(10ft) =

[1/2 (10 ft)(30 ft)·(1 ft)](2.67)(62.3 lb/ft^3)·(6.67 ft) –

(2.80 x 10^4 lbs)·(10 ft) = -1.14 x 10^5 ft-lbs

M = 1.14 x 10^5 ft-lbs (overturning; dam is unsafe)

2.5.3

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(1 m)(π)(0.5 m)^2

F = 7.69 x 10^3 N = 7.69 kN ; $\bar{y} = \bar{h}/\sin 45°$;

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{\left[\pi(1m)^4/64\right]}{\left[\pi(1m)^2/4\right](1.414m)} + 1.414m$$

y_p = 1.46 m (distance from water surface to the center of pressure along the incline).

2.5.4

$F_{square} = \gamma \cdot \bar{h} \cdot A$ = γ(L/2)(L^2) = (γ/2)·L^3

$F_{tri} = \gamma \cdot \bar{h} \cdot A$ = γ(L+H/3)(LH/2) = (γ/2)[L^{2}H + LH^2/3]

Setting the two forces equal: $F_{square} = Y_{tri}$;

(γ/2)·L^3 = (γ/2)[L^{2}H + LH^2/3]

L^2 - HL - H^2/3 = 0; divide by H^2 and solve quadratic

(L/H)^2 – (L/H) - 1/3 = 0; L/H = 1.26 or **H/L = 0.791**

2.5.5

$F_{left} = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(0.5 m)[(1.41m)(3m)]

F_{left} = 20.7 kN (where A is "wet" surface area)

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{\left[(3m)(1.41m)^3/12\right]}{\left[(3m)(1.41m)\right](0.705m)} + 0.705m$$

y_p = 0.940 m (inclined distance to center of pressure)

Location of this force from the hinge (moment arm):

Y' = 2 m – 1.41 m + 0.940 m = 1.53 m

$F_{right} = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(h/2 m)[(h/cos45°)(3m)]

F_{right} = 20.8·h^2 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{\left[(3)(1.41\cdot h)^3/12\right]}{\left[(3)(1.41\cdot h)\right](0.705\cdot h)} + 0.705\cdot h$$

y_p = (0.940·h)m; Moment arm of force from hinge:

Y" = 2 m – (h/sin 45)m + (0.940·h)m = 2m – (0.474·h)m

The force due to the gate weight: W = 20.0 kN
Moment arm of this force from hinge: X = 0.707 m

Summing moments about the hinge yields: $\sum M_{hinge}$= 0

(20.8·h^2)[2m – (0.474·h)] – 20.7(1.53) – 20(0.707) = 0

h = 1.25 m (gate opens when depth exceeds 1.25 m)

2.5.6

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(7 ft)]·[π(6 ft)^2/4]

F = 1.23 x 10^4 lbs

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(6ft)^4/64]}{[\pi(6ft)^2/4](7ft)} + 7ft$$

y_p = 7.32 ft (depth to the center of pressure)

Thus, summing moments: $\sum M_{hinge} = 0$

P(3 ft) – (1.23 x 10^4 lbs)(0.32 ft) = 0

P = 1.31 x 10^3 lbs

2.5.7

In order for the balance to be maintained at h = 4 feet, the center of pressure should be at the pivot point (i.e., the force at the bottom check block is zero). As the water rises above h = 4 feet, the center of pressure will rise above the pivot point and open the gate. Below h = 4 feet, the center of pressure will be lower than the pivot point and the gate will remain closed. Thus, for a unit width of gate, the center of pressure is

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1ft)(10ft)^3/12]}{[(1ft)(10ft)](9ft)} + 9ft$$

y_p = 9.93 ft (vertical distance from water surface to the center of pressure)

Thus, the horizontal axis of rotation (0-0') should be 14 ft – 9.93 ft = 4.07 ft above the bottom of the gate.

2.5.8

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(2.5 m)[(π){(1.5)^2–(0.5)^2}m^2]

F = 1.54 x 10^5 N = 154 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(3m)^4/64 - \pi(1m)^4/64)]}{[\pi(3m)^2/4 - \pi(1m)^2/4](2.5m)} + 2.5m$$

y_p = 2.75 m (below the water surface)

2.5.9

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(2.5 m)[(π)(1.5 m)^2–(1.0 m)^2]

F = 1.49 x 10^5 N = 149 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(3m)^4/64 - (1m)(1m)^3/12)]}{[\pi(1.5m)^2 - (1m)^2](2.5m)} + 2.5m$$

y_p = 2.76 m (below the water surface)

2.5.10

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(2.5m)[(5/cos 30˚)(3 m)]

F = 4.24 x 10^5 N = 424 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(3m)(5.77m)^3/12]}{[(3m)(5.77m)](2.89m)} + 2.89m$$

y_p = 3.85 m (inclined depth to center of pressure)

Summing moments about the base of the dam; $\sum M = 0$

(424 kN)(5.77 m – 3.85 m) – (F_{AB})(5.77m/2) = 0

F_{AB} = 282 kN

2.5.11

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(d/2)ft]·[{(d /cos30)ft}(8 ft)]

F = 288·d^2 lbs

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(8)(d/\cos 30°)^3/12]}{[(8)(d/\cos 30°)](d/2\cos 30°)} + (d/2\cos 30)$$

y_p = [(0.192·d) + 0.577·d)] ft = 0.769·d (inclined depth)

Thus, summing moments: $\sum M_{hinge} = 0$

(288·d^2)[(d/cos30)-0.769d] – (5,000)(15) = 0

d = 8.77 ft A depth greater than this will make the gate open, and anything less will make it close.

2.5.12

The force on the side of the gate can be found as:

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)[(h/2)m][(h m)(1 m)]
F = (4.90 x 10^3)h^2 N (per meter of gate width)

$$Y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1)(h)^3/12]}{[(1)(h)](h/2)} + h/2 = (2/3)h \text{ m}$$

The force on the bottom of the gate can be found as:

F = p·A = (9790 N/m^3)(h)[(1m)(1m)]
F = (9.79 x 10^3)h N (per meter of gate width)

This force is located 0.50 m from the hinge.
Summing moments about the hinge; $\sum M_h = 0$

[(4.90 x 10^3)h^2][h - (2/3)h] – [(9.79 x 10^3)h](0.5) = 0

h = 1.73 m

2.5.13

The total force from fluids A and B can be found as:

$F_A = \gamma \cdot \bar{h} \cdot A$ = (γ_A)(h_A)·[π(d)^2/4]
$F_B = \gamma \cdot \bar{h} \cdot A$ = (γ_B)(h_B)·[π(d)^2/4]

For equilibrium, forces must be equal, opposite, and collinear.

$F_A = F_B$; (γ_A)(h_A)·[π(d)^2/4] = (γ_B)(h_B)·[π(d)^2/4]

h_A = [(γ_B)/(γ_A)](h_B)

2.5.14

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(30 ft)]·[(10 ft)(6 ft)]

F = 1.12 x 10^5 lbs (Horizontal force on gate)

Thus, summing vertical forces: $\sum F_y = 0$
T_{up} – W – F($C_{friction}$) = 0

T = 3 tons (2000 lbs/1 ton) + (1.12 x 10^5 lbs)(0.2)

T = 2.84 x 10^4 lbs (lifting force required)

2.6.1

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved gate.

$F_H = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(5m)[(10 m)(2 m)]

F_H = 9.79 x 10^5 N = 979 kN

Now obtain the vertical component of the total hydrostatic pressure force by determining the weight of the water column above the curved gate.

$F_V = \gamma \cdot Vol$ = (9790 N/m^3)[(4m)(2m)+π/4(2m)^2](10 m)

F_V = 1.09 x 10^6 N = 1090 kN; The total force is

F = [(979 kN)^2 + (1090 kN)^2]$^{1/2}$ = **1470 kN**

θ = tan^{-1} (F_V/F_H) = **48.1°**

Since all hydrostatic pressures pass through point A (i.e., they are all normal to the surface upon which they act), then the resultant must also pass through point A.

2.6.2

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the viewing port.

$F_H = \gamma \cdot \bar{h} \cdot A$ = (1.03·9790N/m^3)(4m)[π(1m)^2]=127 kN

Now obtain the net vertical component of the total hydrostatic pressure force by combining the weight of the water column above the top of the viewing port (which produces a downward force) and the upward force on the bottom of the viewing port (equivalent to the weight of the water above it). The difference in the two columns of water is the weight of water in a hemi-spherical volume (the viewing port) acting upwards.

$F_V = \gamma \cdot Vol$ =(1.03·9790 N/m^3)[(1/2)(4/3)π(1m)^3] = 21.1 kN

F = [(127 kN)^2 + (21.1 kN)^2]$^{1/2}$ = **129 kN**

θ = tan^{-1} (F_V/F_H) = **9.43°**

The resultant force will pass through the center of the hemisphere since all pressures pass through this point.

2.6.3

The vertical component of the total hydrostatic pressure force is equal to the weight of the water column above it to the free surface. In this case, it is the imaginary or displaced weight of water above the shell since the pressure is from below).

$F_V = \gamma \cdot Vol$ = (62.3 lb/ft^3)[(1/2)(4/3)(π)(3.0 ft)^3] = 3,520 lb

The weight must be equal to this; thus **W = 3,520 lb**

2.6.4

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of curved surface AB (both sides).

$$F_H = (\gamma \cdot \bar{h} \cdot A)_{right} - (\gamma \cdot \bar{h} \cdot A)_{left} = (\gamma \cdot A)(\bar{h}_{right} - \bar{h}_{left})$$
$$= (9790 \text{ N/m}^3)\,[(1.75 \text{ m})(1 \text{ m})]\,(3.875 \text{ m} - 0.875 \text{ m})$$

F_H = 5.14 x 10^4 N **= 51.4 kN (towards the barge)**

Now obtain the resultant vertical component of the total hydrostatic pressure force subtracting the weight of the water column above the curved surface (the water in the barge) from the displaced weight for the case of the water on the outside of the barge.

$$F_V = (\gamma \cdot Vol)_{displaced} - (\gamma \cdot Vol)_{leaked}$$
$$= (9790 \text{ N/m}^3)[(1.75 \text{ m})(1 \text{ m})(3 \text{ m})]$$

F_V = 5.14 x 10^4 N **= 51.4 kN (upwards)**

2.6.5

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved gate. The height of the vertical projection is (R)(sin 45˚) = 8.49 m. Thus,

$F_H = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(4.25m)[(10 m)(8.49 m)]

F_H = 3.53 x 10^6 N = 3,530 kN

Now obtain the vertical component of the total hydrostatic pressure force by determining the weight of the water column above the curved gate.

2.6.5 (continued)

The volume of water above the gate is:

Vol = ($A_{rectangle}$ - $A_{triangle}$ - A_{arc})(length)
Vol =[(12m)(8.49m)-(1/2)(8.49m)(8.49m)-(π/8)(12m)^2](10 m)
Vol = 92.9 m^3

$F_V = \gamma \cdot Vol$ = (9790 N/m^3)(92.9 m^3)

F_V = 9.09 x 10^5 N = 909 kN; The total force is

F = [(3,530 kN)^2 + (909 kN)^2]$^{1/2}$ = **3650 kN**

θ = tan^{-1} (F_V/F_H) **= 14.4**˚

Since all hydrostatic pressures pass through point O (i.e., they are all normal to the surface upon which they act), then the resultant must also pass through point O.

2.6.6

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved gate. Thus,

$F_H = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)(7.0 ft)[(8.0 ft)(1.0 ft)]

F_H = 3.49 x 10^3 lb = 3,490 lb (per unit length of gate)

Obtain the vertical component of the total hydrostatic pressure force by determining the imaginary (displaced) weight of the water column above the curved gate. The volume of displaced water above the gate is:

Vol = ($A_{rectangle}$ + A_{arc} - $A_{triangle}$)(length)

Vol = [(4 ft)(3 ft)+(53.1˚/360˚)π(10 ft)^2 - (1/2)(8 ft)(6 ft)](1 ft)

Vol = 34.3 ft^3

$F_V = \gamma \cdot Vol$ = (62.3 lb/ft^3)(34.3 ft^3)

F_V = 2.14 x 10^3 lb = 2,140 lb; The total force is

F = [(3,490 lb)^2 + (2,140 lb)^2]$^{1/2}$ = **4,090 lb**

θ = tan^{-1} (F_V/F_H) **= 31.5**˚

The resultant force will pass through the center of the gate radius since all pressures pass through this point.

2.6.7

The force on the end of the cylinder is:

$F = \gamma \cdot \bar{h} \cdot A$ = (0.9)(62.3 lb/ft^3)(10 ft)[π(2 ft)^2]

F = 7,050 lb

The force on the side of the cylinder is:

$F_H = \gamma \cdot \bar{h} \cdot A$ = (0.9)(62.3 lb/ft^3)(10.0 ft)[(10 ft)(4 ft)]

F_H = 22,400 lb

Based on the same theory as Example 2.6, the vertical force is downward and equal to the weight of the water in half of the tank. Thus,

$F_V = \gamma \cdot Vol$ = (0.9)(62.3 lb/ft^3)[π(2ft)^2/2](10 ft)

F_V = 3,520 lb (acting downward); The total force is

F = [(22,400 lb)^2 + (3,520 lb)^2]$^{1/2}$ = **22,700 lb**

θ = tan^{-1} (F_V/F_H) = **8.93°** ↘

The resultant force will pass through the center of the tank since all pressures pass are normal to the tank wall and thus pass through this point.

2.6.8

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the curved surface ABC.

$F_H = \gamma \cdot \bar{h} \cdot A$ = (γ)(R)[(2R)(1)] = **2(γ)(R)^2**

Now obtain the vertical component of the total hydrostatic pressure force by determining the weight of the water above the curved surface ABC. The volume of water above the curved surface is:

Vol = ($A_{\text{quaarter circle}} + A_{\text{rectangle}} - A_{\text{quarter circle}}$)(unit length)

Vol = ($A_{\text{rectangle}}$)(unit length) = (2R)(R)(1) = 2(R)^2

$F_V = \gamma \cdot Vol$ = γ[2(R)^2] = **2(γ)(R)^2**

2.6.9

Obtain the horizontal component of the total hydrostatic pressure force by determining the total pressure on the vertical projection of the projecting surface. Thus,

$F_H = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)(8.0 ft)[(12.0 ft)(1.0 ft)]

F_H = 5,980 lb (per unit length of surface)

$$Y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{\left[(1ft)(12ft)^3/12\right]}{\left[(1ft)(12ft)\right](8ft)} + 8ft \ \mathbf{= 9.50\ ft}$$

The vertical component of the total hydrostatic pressure force is equal to the weight of the water displaced by the quadrant and the triangle. In parts (and using Table 2.1 to locate the forces), we have

$F_{VTriangle} = \gamma \cdot Vol$ = (62.3 lb/ft^3)[(1/2)(8 ft)(4 ft)](1 ft)

$F_{VTriangle}$ = 1000 lb upwards 1.33 ft from wall

$F_{Vquadrant} = \gamma \cdot Vol$ = (62.3 lb/ft^3)[(π/4)(4 ft)^2](1 ft)

$F_{Vquadrant}$ = 780 lb upwards 1.70 ft from wall

2.6.10

There are four vertical forces at work on the cone plug. The cone unplugs when $\sum F_y = 0$.

The pressure force of fluid A on top of the plug (down):

$F_{ATop} = \gamma \cdot Vol$ = (9790 N/m^3)[π(0.15m)^2(0.3m)] = 208 N

The pressure force of fluid A on the cone sides (up):

$F_{ASides} = \gamma \cdot Vol$ = [π(0.15m)^2(0.3m) + (π/3) (0.15m)^2(0.3m) - (π/3) (0.05m)^2(0.1m)](9790 N/m^3) = 274 N

The pressure force of fluid B on the cone bottom (up):

$F_{Bbottom} = \gamma \cdot Vol$ = (0.8)(9790 N/m^3) [π(0.05m)^2(1.5m) + (π/3) (0.05m)^2(0.1m)] = 94.3 N

$\sum F_y$ = 208 – 274 – 94.3 + (γ_{cone})(π/3) (0.15m)^2(0.3m) = 0;

γ_{cone} = 22,700 N/m^3; S.G. = 2.32

2.6.11

Everything is the same as in problem 2.6.10 except:

The equivalent depth of oil based on the air pressure is:

$h = P/\gamma$ = (8,500 N/m^2)/[(0.8)(9790 N/m^3)] = 1.09 m

The pressure force of fluid B on the cone bottom (up):

$F_{Bbottom} = \gamma \cdot Vol =$ (0.8)(9790 N/m^3) [π(0.05m)^2(1.09m) + (π/3) (0.05m)^2(0.1m)] = 69.1 N

$\sum F_y$ = 208 – 274 – 69.1 + (γ_{cone})(π/3) (0.15m)^2(0.3m) = 0;

γ_{cone} = 19,100 N/m^3; S.G. = 1.95

2.6.12

The horizontal component of the hydrostatic pressure force due to fluids A and B are found as follows:

$F_{HALeft} = \gamma \cdot \bar{h} \cdot A$ = (0.8)(9790 N/m^3)(6 m)[(1 m)(1.41 m)]

F_{HALeft} = 66.3 kN

$F_{HARight} = \gamma \cdot \bar{h} \cdot A$ = (0.8)(9790 N/m^3)(5.65 m)[(1m)(0.707m)]

$F_{HARight}$ = 31.3 kN

$F_{HBRight} = \gamma \cdot \bar{h} \cdot A$ = (1.5)(9790 N/m^3)(5.35 m)[(1 m)(0.707 m)]

$F_{HBRight}$ = 55.5 kN ; Thus, **F_H = 20.5 kN** $\leftarrow$

The vertical component of the total hydrostatic pressure force due to fluids A and B are found as follows:

F_{VATop} = (0.8)(9790N/m^3)[(1.41m)(1m)(6m) - π/2(0.707m)^2(1.0m)]

F_{VATop} = 60.1 kN

$F_{VABottom}$ = (0.8)(9790N/m^3)[(0.707m)(1m)(6m)+π/4(0.707m)^2(1.0m)]

$F_{VABottom}$ = 36.3 kN

$F_{VBBottom}$ = (1.5)(9790N/m^3)[(0.707m)(1m)(5m)+π/4(0.707m)^2(1.0m)]

$F_{VBBottom}$ = 57.7 kN

$W_{Cylinder}$ = (2.0)(9790N/m^3)[π(0.707m)^2(1.0m)] = 30.7 kN

Thus, F_V = 3.2 kN $\uparrow$

2.8.1

The buoyant force equals the weight reduction. Thus,

B = 301 N – 253 N = 48.0 N In addition,

B = wt. of water displaced = $\gamma \cdot Vol$ = (9790 N/m^3)(Vol)

Thus, Vol = 4.90 x 10^{-3} m^3 and

γ_{metal} = W/Vol = **6.14 x 10^4 N/m^3**

S.G. = (6.14 x 10^4 N/m^3)/9.79 x 10^3 N/m^3) **= 6.27**

2.8.2

For floating bodies, weight equals the buoyant force.

W = B; and using w & L for width & length of blocks

γ_A(H)(w)(L) + γ_B(1.5·H)(w)(L) = γ (2·H)(w)(L)

γ_A + (1.5γ_A)(1.5) = γ (2); γ_A (1 + 2.25) = γ (2);

γ_A = 0.615γ; and since **γ_B** = 1.5·γ_A = **0.923γ**

2.8.3

When the sphere is lifted off the bottom, equilibrium in the y-direction occurs with W = B. Therefore,

W = γ_{sphere} [(4/3)π(0.15m)^3] + γ_{buoy}[π(0.25m)^2(2m)]

W = (13.5γ)[0.0141m^3] + (0.45γ)[0.393m^3] = 0.367 γ

B = γ_{sea}[(4/3)π(0.15m)^3] + γ_{sea}[π(0.25m)^2(0.30m + h)]

B = 0.0145 γ + 0.0607 γ + 0.202·h· γ

Equating; **h = 1.45 m**

2.8.4

Theoretically, the lake level will fall. When the anchor is in the boat, it is displacing a volume of water equal to its weight. When the anchor is thrown in the water, it is only displacing its volume. Since it has a specific gravity greater than 1.0, it will displace more water by weight than by volume.

2.8.5

When the anchor is lifted off the bottom, equilibrium in the y-direction occurs ($\sum F_y = 0$). Therefore,

T (sin60°) + B = W; where T = anchor line tension

B = buoyancy force, and W= anchor weight

B = (62.3 lb/ft^3)π(0.75 ft)^2(1.2 ft) = 132 lb

W = (2.7)(62.3 lb/ft^3)π(0.75 ft)^2(1.2 ft) = 357 lb

Substituting, **T = 260 lb**

2.8.6

Two forces act on the gate, the hydrostatic pressure and the buoy force. The hydrostatic pressure is

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(1.5m)[(1m/sin45°)^2]

F = 29.4 kN; acting normal to the gate surface.

The location of the force is

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y}$$

$$y_P = \frac{\left[(1m/\sin 45°)(1m/\sin 45°)^3/12\right]}{\left[(1m/\sin 45°)(1m/\sin 45°)\right](1.5/\sin 45°)} + (1.5/\sin 45°)$$

y_p = 2.20 m; This is the distance down the incline from the water surface.

The distance up the incline from the hinge is

y' = (2m/sin45°) – 2.20 m = 0.628 m

The buoyant force (on half the sphere) is

B =γ·Vol=(9790 N/m^3)(1/2)(4/3)π(R)^3 = 20.5(R)^3 kN

$\sum M_{hinge}$ = 0, ignoring the weights (gate and buoy)

(29.4 kN)(0.628 m) – [20.5(R)^3 kN](1 m) = 0

R = 0.966 m

2.8.7

Three forces act on the rod; the weight, buoyant force, and the hinge force. The buoyant force is

B = (62.3 lb/ft^3)(0.5 ft)(0.5 ft)(7 ft/sin θ) = 109 lb/sin θ;

B = 109 lb/sin θ; ↑ The buoyant force acts at the center of the submerged portion. W = 150 lb

$\sum M_{hinge}$ = 0, and assuming the rod is homogeneous,

(109 lb/sin θ)(3.5 ft/tan θ) – (150 lb)[(6 ft)(cos θ)] = 0

Noting that tan θ = (sin θ/cos θ) and dividing by cos θ

(sin θ)^2 = 0.424; sin θ = 0.651; **θ = 40.6°**

2.8.8

The center of gravity (G) is given as 1 m up from the bottom of the barge. The center of buoyancy (B) is 0.75 m up from the bottom since the draft is 1.5 m. Therefore GB = 0.25 m, and GM is found using

$\overline{GM} = \overline{MB} \pm \overline{GB} = \frac{I_0}{\text{Vol}} \pm \overline{GB}$; where Io is the waterline moment of inertial about the tilting axis. Chopping off the barge at the waterline and looking down we have a rectangle which is 14 m by 6 m. Thus,

$$\overline{GM} = \frac{I_0}{\text{Vol}} \pm \overline{GB} = \frac{\left[(14m)(6m)^3/12\right]}{(14m)(6m)(1.5m)} - 0.25m = \mathbf{1.75\ m};$$

Note: Vol is the submerged volume and a negative sign is used since G is located above the center of buoyancy.

$M = W \cdot \overline{GM} \cdot \sin\theta$

M = [(1.03)(9790 N/m^3)(14 m)(6m)(1.5m)](1.75m)(sin 4°)

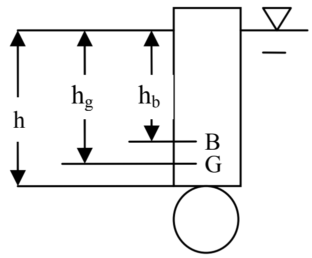
M = 155 kN·m (for a heel angle of 4°)

M = 309 kN·m (for a heel angle of 8°)

M = 462 kN·m (for a heel angle of 12°)

2.8.9

First determine how much the wooden pole is in the water. Summing forces in the y-direction, W = B



$(Vol_s)(SG_s)(\gamma) + (Vol_p)(SG_p)(\gamma) = (Vol_s)(\gamma) + (Vol_{p'})(\gamma)$

$[(4/3)\pi(0.25m)^3](1.4)+[\pi(0.125m)^2(2m)](0.62) =$

$[(4/3)\pi(0.25m)^3]+[\pi(0.125m)^2(h)]$;

$0.0916m^3 + 0.0609m^3 = 0.0654m^3 + (0.0491m^2)h$

h = 1.77 m; Find "B" using the principle of moments.

$[(Vol_s)(\gamma)+(Vol_{p'})(\gamma)](h_b)=(Vol_s)(\gamma)(h+0.25m)+(Vol_{p'})(\gamma)(h/2)$

$[(0.0654m^3 + (0.0491m^3)(1.77m)](h_b) =$
$(0.0654m^3)(1.77m + 0.25m) + [(0.0491m^3)(1.77m)](1.77m/2)$

h_b = 1.37 m; Find "G" using the principle of moments.

$(W)(h_g+0.23m) = (W_s)(2.0\text{ m} + 0.25\text{ m}) + (W_p)(2.0m/2)$

$[(0.0916m^3 + 0.0609m^3)](h_g +0.23m) =$
$(0.0916m^3)(2.25m) + (0.0609m^3)(1.00m)$

h_g = 1.52 m; GB = $h_g - h_b$ = 0.15 m

MB = I_o/Vol

MB = $[(1/64)\pi(0.25m)^4] / [(0.0654m^3 + (0.0491m^3)(1.77m)]$

MB = 1.26×10^{-3} m

GM = MB + GB = 1.26×10^{-3} m + 0.15 m **= 0.151 m**

2.8.10

If the metacenter is at the same position as the center of gravity, then GM = 0 and the righting moment is $M = W \cdot \overline{GM} \cdot \sin\theta = 0$. **With no righting moment, the block will not be stable.**

2.8.11

The center of gravity (G) is estimated as 17 ft up from the bottom of the tube based on the depth of the water inside it. The center of buoyancy (B) is 21 feet from the bottom since 42 feet is in the water.
Therefore GB = 4.0 ft, and GM is found using

$\overline{GM} = \overline{MB} \pm \overline{GB} = \frac{I_0}{\text{Vol}} \pm \overline{GB}$; where Io is the waterline moment of inertial about the tilting axis. Chopping off the tube at the waterline and looking down we have a circle with a 36 ft diameter. Thus,

$$\overline{GM} = \frac{I_0}{\text{Vol}} \pm \overline{GB} = \frac{[\pi(36ft)^4/64]}{(\pi/4)(36ft)^2(42ft)} + 4.0ft = \mathbf{5.93\ ft;}$$

Note: Vol is the submerged volume and a positive sign is used since G is located below the center of buoyancy.

$M = W \cdot \overline{GM} \cdot \sin\theta$

M = $[(1.02)(62.3\text{ lb/ft}^3)(\pi/4)(36\text{ ft})^2(42\text{ ft})](5.93\text{ ft})(\sin 4°)$

M = 1.12×10^6 ft·lb (for a heel angle of 4°)

2.8.12

The center of gravity (G) is roughly 1.7 m up from the bottom if the load is equally distributed. The center of buoyancy (B) is 1.4 m from the bottom since the draft is 2.0 m. Therefore GB = 0.3 m, and GM is found using

$$\overline{GM} = \frac{I_0}{\text{Vol}} \pm \overline{GB} = \frac{[(12m)(4.8m)^3/12]}{(12m)(4.8m)(2m)} - 0.3m = 0.660\text{ m};$$

Note: Vol is the submerged volume and a negative sign is used since G is located above the center of buoyancy.

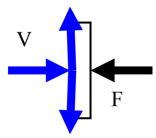
$M = W \cdot \overline{GM} \cdot \sin\theta$

M = $[(1.03)(9790\text{ N/m}^3)(12m)(4.8m)(2m)](0.660m)(\sin 15°)$

M = 198 kN·m; The distance G can be moved is

d = GM (sin θ) = (0.66 m)(sin 15°) **= 0.171 m**

3.3.1



From Equation (3.7a), $\sum F_x = \rho Q(V_{x,out} - V_{x,in})$; there are no pressure forces (water is exposed to the atmosphere), $V_{x,out} = 0$; $V_{x,in} = 3.44$ m/sec

$Q = V \cdot A = (3.44 \text{ m/sec})[(\pi/4)\{(0.20)\text{m}\}^2] = 0.108 \text{ m}^3/\text{sec}$

$-F_x = (998 \text{ kg/m}^3)(0.108 \text{ m}^3/\text{sec})[0 - (3.44 \text{ m/sec})] = -371$ N

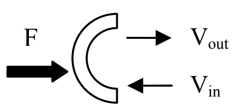
Thus, $F_x = 331$ N ⟵ $F_y = 0$ N; $F_z = 0$ N; since all flow into the control volume is x-directed and the outflow is zero in the y and z direction since the spray is equal in all directions. Note: This is the force exerted on the water (control volume) by the plate. The force exerted on the plate by the water is equal and opposite

$F_x = 371$ N ⟶

3.3.2

The force exerted on the hemispherical lid is double the force on the flat lid. Note that with the flat lid, the hydrodynamic force results from redirecting the water though an angle of 90˚, while the hemispherical lid redirects the flow through 180˚.

3.3.3



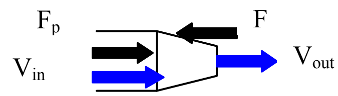
Since the flow is turned 180˚, $V_{x,in} = -V_{x,out} = -V$

$\sum F_x = \rho Q(V_{x,out} - V_{x,in}) = \rho Q(2V) = \rho(V \cdot A)(2V)$

$233 \text{ lb} = 2(1.94 \text{ slug/ft}^3)[(\pi/4)\{(1/12)\text{ft}\}^2](V)^2$

V = 105 ft/sec

3.3.4



No exit pressure force exists since the water is exposed to the atmosphere, and the entrance pressure force is

$F_p = P \cdot A = (270{,}000 \text{ N/m}^2)[\pi/4(0.60\text{m})^2] = 76.3$ kN

$V_{x,out} = Q/A = (1.1 \text{ m}^3/\text{s}) / [(\pi/4)(0.30\text{m})^2] = 15.6$ m/sec

$V_{x,in} = Q/A = (1.1 \text{ m}^3/\text{s}) / [(\pi/4)(0.60\text{m})^2] = 3.89$ m/sec

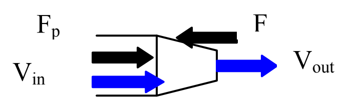
Using Equation (3.7a); $\sum F_x = \rho Q(V_{x,out} - V_{x,in})$

$76.3 \text{ kN} - F = (998 \text{ kg/m}^3)(1.1 \text{ m}^3/\text{sec})[(15.6 – 3.89) \text{ m/sec}]$

F = 63.4 kN⟵ (Force on water; control volume)

F = 63.4 kN ⟶ (Force on the connection)

3.3.5



No exit pressure force exists since the water is exposed to the atmosphere, and the entrance pressure force is

$F_p = P \cdot A = 0.196 \cdot P$;

The force on the control volume from the nozzle is:

F (nozzle) = F = -43.2 kN

$V_{x,out} = Q/A = (0.9 \text{ m}^3/\text{s})/[(\pi/4)(0.25\text{m})^2] = 18.3$ m/s

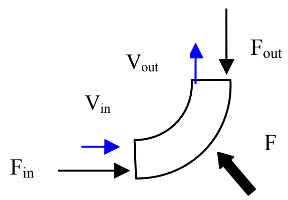
$V_{x,in} = Q/A = (0.9 \text{ m}^3/\text{s})/[(\pi/4)(0.50\text{m})^2] = 4.58$ m/s

Using Equation (3.7a); $\sum F_x = \rho Q(V_{x,out} - V_{x,in})$

$0.196 \cdot P – 43{,}200 \text{ N} = (998 \text{ kg/m}^3)(0.9 \text{ m}^3/\text{s})[(18.3 - 4.58) \text{ m/s}]$

$P = 2.83 \times 10^5$ N/m² (Pascals)

3.3.6



$F_{in} = P \cdot A = (15.1\ lbs/in^2)\ [(\pi/4)(0.5ft)^2] = 2.96\ lbs$

$F_{out} = P \cdot A = (14.8\ lbs/in^2)\ [(\pi/4)(0.5ft)^2] = -\ 2.91\ lbs$

$V = Q/A = (3.05\ ft^3/s)/[(\pi/4)(0.5ft)^2] = 15.5\ ft/s$

Using component equations (3.7a) and (3.7b);

$\sum F_x = \rho Q(V_{x,out} - V_{x,in})$; ($\longrightarrow$ +), Assume F_x is negative

$2.96\ lbs - F_x = (1.94\ slug/ft^3)(3.05\ ft^3/s)[(0 - 15.5)\ ft/s]$

$F_x = 94.7\ lb \leftarrow$

$\sum F_y = \rho Q(V_{y,out} - V_{y,in})$; ($\uparrow$ +); Assume F_y is positive

$F_y\ - 2.91\ lb = (1.94\ slug/ft^3)(3.05\ ft^3/s)[(15.5 - 0)\ ft/s]$

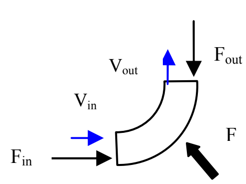
$F_y = 94.6\ lb \uparrow$

F $= [(94.7\ lb)^2 + (94.6\ lb)^2]^{1/2} =$ **134 lb**

$\boldsymbol{\theta} = \tan^{-1}(F_y/F_x) =$ **45.0°**

Note: The signs on the reaction force are assumed in the equations. If the assumed directions are correct, as in this case, the result will have a positive sign. If the assumed direction is wrong, the result will have a negative sign and the actual direction is opposite.

3.3.7



$F_{in} = P \cdot A = (10\ m)(9.79\ kN/m^3)[(\pi/4)(0.6m)^2] = 27.7\ kN$

$F_{out} = P \cdot A = (9.8m)(9.79\ kN/m^3)[(\pi/4)(0.6m)^2] = 27.1\ kN$

$Q/A = \dot{m}/\rho = (985\ kg/s)/(998\ kg/m^3) = 0.987\ m^3/sec$

3.3.7 (continued)

$V = Q/A = (0.987\ m^3/sec) / [\pi/4(0.6m)^2] = 3.49\ m/s$

Using component equations (3.7a) and (3.7b);

$\sum F_x = \rho Q(V_{x,out} - V_{x,in})$; ($\longrightarrow$ +), Assume F_x is negative

$27.7\ kN - F_x = (985\ kg/s)[(0 - 3.49)\ m/s]\ (1\ kN/1000\ N)$

$F_x = 31.1\ kN \leftarrow$

$\sum F_y = \rho Q(V_{y,out} - V_{y,in})$; ($\uparrow$ +); Assume F_y is positive

$F_y\ - 27.1\ kN = (985\ kg/s)[(3.49 - 0)\ m/s]\ (1\ kN/1000\ N)$

$F_y = 30.5\ kN \uparrow$

F $= [(31.1\ kN)^2 + (30.5\ kN)^2]^{1/2} =$ **43.6 kN**

$\boldsymbol{\theta} = \tan^{-1}(F_y/F_x) =$ **44.4°**

3.3.8

$F_{x,in} = P \cdot A = (250{,}000\ N/m^2)[(\pi/4)(0.15\ m)^2] = +4{,}420\ N$

$F_{y,in} = 0$ (all flow x-directed);

$F_{x,out} = (130{,}000\ N/m^2)[(\pi/4)(0.075m)^2]\cos 30° = -497\ N$

$F_{y,out} = (130{,}000\ N/m^2)[(\pi/4)(0.075m)^2]\sin 30° = -287\ N$

$V_{in} = 4\ m/sec = V_{x,in}$, $V_{y,in} = 0$; $Q = VA = 0.0707\ m^3/sec$

$V_{out} = Q/A = (0.0707\ m^3/sec) / [(\pi/4)(0.075m)^2] = 16.0\ m/sec$

$V_{x,out} = (16.0\ m/sec)(\cos 30°) = 13.9\ m/sec$; $V_{y,out} = 8.00\ m/sec$

$\Sigma F_x = \rho Q(V_{x_{out}} - V_{x_{in}})$; ($\longrightarrow$ +) assume F_x negative

$4{,}420N - 497N - F_x = (998\ kg/m^3)(0.0707\ m^3/s)[(13.9 - 4)\ m/s]$

$\Sigma F_y = \rho Q(V_{y_{out}} - V_{y_{in}})$; ($\uparrow$ +), assume F_y is positive

$-287N + F_y = (998\ kg/m^3)(0.0707\ m^3/s)[(8 - 0)\ m/s]$;

$F_x = 3220\ N \leftarrow$ $\quad F_y = 851\ N \uparrow$ The resultant is

F $= [(3220\ N)^2 + (851\ N)^2]^{1/2} =$ **3330 N**

and its direction is $\boldsymbol{\theta} = \tan^{-1}(F_y/F_x) =$ **14.8°**

3.5.1

To determine the friction factor, we must solve for the relative roughness and the Reynolds number.

e/D = (0.045 mm)/(1500 mm) = 0.00003

V = Q/A = (3.5 m^3/s)/[(π/4)(1.5 m)^2] = 1.98 m/sec

N_R = DV/ν = [(1.5m)(1.98 m/s)]/(1.00 x 10^{-6} m^2/s)

N_R = 2.97 x 10^6 From Moody diagram; **f = 0.011**

Thus, the flow is **turbulent – transitional zone**.

3.5.2

To determine the friction factor, we must solve for the relative roughness and the Reynolds number.

e/D = (0.15 ft)/(10.0 ft) = 0.015

V = Q/A = (628 ft^3/s)/[(π/4)(10.0 ft)^2] = 8.00 ft/sec

N_R = DV/ν = [(10.0 ft)(8.00 ft/s)]/(1.08 x 10^{-5} ft^2/s)

N_R = 7.41 x 10^6 From Moody diagram; **f = 0.044**

Thus, the flow is **turbulent – rough pipe.**

3.5.3

e/D = (0.0015 mm)/(15 mm) = 0.0001

V = Q/A = (0.01m^3/60s)/[(π/4)(0.015 m)^2] = 0.943 m/s

N_R = DV/ν = [(0.015m)(0.943 m/s)]/(1.00 x 10^{-6} m^2/s)

N_R = 1.41 x 10^4 From Moody diagram; **f = 0.028**

Determine the friction head loss and convert to ΔP

h_f = f (L/D)(V^2/2g); for a 1000 m length of pipe

h_f = (0.028)(30m/0.015m)[(0.943 m/s)^2/(2·9.81 m/s^2)]

h_f = 2.54 m; and from Eq'n (3:15a)

ΔP = (9,790 N/m^3)(2.54 m) **= 24.9 kN/m^2**

3.5.4

The pressure drop is computed from the energy eq'n:

$$\frac{V_1^2}{2g}+\frac{P_1}{\gamma}+h_1=\frac{V_2^2}{2g}+\frac{P_2}{\gamma}+h_2+h_L \text{ ; where } h_L = h_f$$

e/D = (0.0005 ft)/(1.25 ft) = 0.0004

V = Q/A = (18 ft^3/s)/[(π/4)(1.25 ft)^2] = 14.7 ft/sec

N_R = DV/ν = [(1.25ft)(14.7 ft/s)]/(1.08 x 10^{-5} ft^2/s)

N_R = 1.70 x 10^6 From Moody diagram; **f = 0.0165**

h_f = f (L/D)(V^2/2g); for a 65 ft length of pipe

h_f = (0.0165)(65ft/1.25ft)[(14.7 ft/s)^2/(2·32.2 ft/s^2)]

h_f = 2.88 ft; and from the energy equation (v_1 = v_2);

$$\frac{P_1-P_2}{\gamma}=h_2-h_1+h_f = (65\text{ ft})(1/50)+2.88\text{ ft} = 4.18\text{ ft}$$

ΔP = (62.3 lb/ft^3)(4.18 ft) **=** 260 lb/ft^2 (**1.81 lb/in^2**)

3.5.5

The tower height can be found from energy eq'n:

$$\frac{V_1^2}{2g}+\frac{P_1}{\gamma}+h_1=\frac{V_2^2}{2g}+\frac{P_2}{\gamma}+h_2+h_L \text{ ; where } h_L = h_f$$

e/D = (0.004 mm)/(400 mm) = 0.00001; V = 7.95 m/sec

N_R = DV/ν = [(0.4m)(7.95 m/s)]/(1.57 x 10^{-6} m^2/sec)

N_R = 2.02 x 10^6 From Moody diagram; **f = 0.011**

h_f = f (L/D)(V^2/2g); for a 50 m length of pipe

h_f = (0.011)(100m/0.4m)[(7.95 m/s)^2/(2·9.81 m/s^2)]

h_f = 8.86 m; from the energy eq'n (V_1 = P_1 = P_2 = 0);

h = $(V_2)^2$/2g + h_2 + h_f ; w/datum at the ground elev.

h = (7.95m/s)^2/(2·9.81 m/s^2) + (-2 m) + 8.86 m

h = 10.1 m

3.5.6

Use the Darcy-Weisbach equation:

$h_f = f\,(L/D)(V^2/2g)$;

$4.6\text{ m} = f\,(2000\text{m}/0.30\text{m})(V^2/2\cdot9.81\text{m/s}^2)$

The Moody diagram is required to find f. However, V is not available so N_R can not be solved. Use (e/D) and the Moody diagram to obtain a trial f value by assuming flow is in the complete turbulence regime.

$e/D = (0.26\text{ mm})/(300\text{ mm}) = 0.000867$; $f \approx 0.02$, and

$4.6\text{ m} = (0.02)(2000\text{m}/0.30\text{m})(V^2/2\cdot9.81\text{m/s}^2)$

$V = 0.823$ m/sec; Now with $\nu = 1.31 \times 10^{-6}\text{ m}^2/\text{sec}$

$N_R = 1.88 \times 10^5$ From Moody; $f = 0.0205$, ok - close

$4.6\text{ m} = (0.0205)(2000\text{m}/0.30\text{m})(V^2/2\cdot9.81\text{m/s}^2)$

$V = 0.813$ m/sec; **Q** = AV **= 0.0575 m^3/sec**

3.5.7

First use the energy equation to determine h_f;

$$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L\ ;$$

where $h_L = h_f$, $V_A = V_B$, and use B as a datum elev.

$8.3\text{m} + 100\text{m} = 76.7\text{m} + 0\text{m} + h_f$; $h_f = 31.6$ m

$e/D = (0.9\text{ mm})/(4000\text{ mm}) = 0.000225$;

V is not available so N_R can not be solved. Use e/D and the Moody diagram to obtain a trial f value by assuming complete turbulence. Thus $f = 0.014$, and solve $h_f = f\,(L/D)(V^2/2g)$; to obtain a trial V. Hence,

$31.6\text{m} = (0.014)(4500\text{m}/4\text{m})[V^2/(2\cdot9.81\text{ m/s}^2)]$

$V = 6.27$ m/sec; Now with $\nu = 1.00 \times 10^{-6}\text{ m}^2/\text{sec}$

$N_R = 2.51 \times 10^7$ From Moody; $f = 0.014$ ok

Q = AV = $[(\pi/4)(4\text{m})^2](6.27\text{ m/s})$ = **78.8 m^3/sec**

3.5.8

Apply the energy equation to determine V;

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L\ ;\ \text{where } h_L = h_f$$

"1" is at the surface of the reservoir; thus $V_1 = 0 = P_1$

"2" is at the pipe outfall, therefore $V_2 = V$ and $P_2 = 0$.

With the datum at "2", and substituting Darcy-Weisbach into the energy equation, we have

$0 + 0 + (3 + 52.8)\text{ft} = V^2/2g + 0 + 0 + f(5280/1.5)V^2/2g$

$55.8\text{ ft} = [V^2/(2\cdot32.2\text{ft/s}^2)](1 + 3520\cdot f)$

$e/D = (0.0006\text{ ft})/(1.5\text{ ft}) = 0.0004$;

V is not available so N_R can not be solved. Use e/D and the Moody diagram to obtain a trial f value by assuming complete turbulence. Thus $f = 0.016$, and solve for V

$55.8\text{ ft} = [V^2/(2\cdot32.2\text{ft/s}^2)](1 + 3520\cdot0.016)$

$V = 7.92$ ft/sec; Now with $\nu = 1.69 \times 10^{-5}\text{ ft}^2/\text{sec}$

$N_R = 7.03 \times 10^5$ From Moody; $f = 0.017$ ok - close

$55.8\text{ ft} = [V^2/(2\cdot32.2\text{ft/s}^2)](1 + 3520\cdot0.017)$; $V = 7.69$ ft/s

Q = AV = $[(\pi/4)(1.5\text{ft})^2](7.69\text{ ft/s})$ = **13.6 ft^3/sec**

3.5.9

From Eq'n (3.15a) and noting 16.3 psi = 2350 lb/ft^2:

$(P_1 - P_2)/\gamma = h_L = h_f$; $(2350\text{ lb/ft}^2)/(62.3\text{ lb/ft}^3) = h_f$

$h_f = 37.7$ ft; Substituting this into Darcy-Weisbach

$e/D = (0.0005\text{ ft})/(0.5\text{ ft}) = 0.001$

$V = Q/A = (1.34\text{ ft}^3/\text{s})/[(\pi/4)(0.5\text{ ft})^2] = 6.82$ ft/sec

$N_R = DV/\nu = [(0.5\text{ ft})(6.82\text{ ft/s})]/(1.08 \times 10^{-5}\text{ ft}^2/\text{s})$

$N_R = 3.16 \times 10^5$ From Moody diagram; **f = 0.0195**

$h_f = f\,(L/D)(V^2/2g)$; for a 6-in. diameter pipe

$37.7\text{ ft} = (0.0195)(L/0.5\text{ft})[(6.82\text{ ft/s})^2/(2\cdot32.2\text{ ft/s}^2)]$

L = 1340 ft

3.5.10

Apply the Darcy-Weisbach eq'n: $h_f = f\,(L/D)(V^2/2g)$

$9.8\text{ m} = f\cdot(200/D)[V^2/(2\cdot9.81\text{m/s}^2)]$; but $V = Q/A = 4Q/\pi D^2$

Thus; $V = 4(0.010\text{ m}^3/\text{sec})/[\pi(D^2)] = 0.0127/D^2$ and

$9.8\text{ m} = f\cdot(200/D)[(0.0127/D^2)^2/(2\cdot9.81\text{m/s}^2)]$; yielding

$D^5 = 0.000168\cdot f$; Neither D nor V is available so e/D and N_R can not be determined. Iterate with f = 0.02 as a first trial, which is near midrange of typical f values.

Solving for D: $D^5 = 0.000168\cdot(0.02)$; D = 0.0804m

Now, e/D = 0.045mm/80.4mm = 0.000560

$V = 0.0127/D^2 = 1.96$ m/s; and w/ν = 1.00×10^{-6} m²/s

$N_R = 1.58 \times 10^5$ From Moody, f = 0.019; the new D:

$D^5 = 0.000168\cdot(0.019)$; **D = 0.0796m ≈ 0.080 m**

3.5.11

From Eq'n (3.15a) and noting 43 psi = 6190 lb/ft²:

$(P_1 - P_2)/\gamma = h_L = h_f$; $(6190\text{ lb/ft}^2)/(62.3\text{ lb/ft}^3) = h_f$

$h_f = 99.4$ ft; Substituting this into the Darcy-Weisbach equation: $h_f = f\,(L/D)(V^2/2g)$, we have

$99.4\text{ ft} = f\cdot(5{,}280/D)[V^2/(2\cdot32.2\text{ft/s}^2)]$; $V = Q/A = 4Q/\pi D^2$

Thus; $V = 4(16.5\text{ ft}^3/\text{sec})/[\pi(D^2)] = 21.0/D^2$ and

$99.4\text{ ft} = f\cdot(5{,}280/D)[(21.0/D^2)^2/(2\cdot32.2\text{ ft/s}^2)]$; yielding

$D^5 = 364\cdot f$; Neither D nor V is available so e/D and N_R can not be determined. Iterate with f = 0.02 as a first trial, which is near midrange of typical f values.

Solving for D: $D^5 = 364\cdot(0.02)$; D = 1.49 ft

Now, e/D = 0.002ft/1.49ft = 0.00134

$V = 21.0/D^2 = 9.46$ ft/s; and w/ν = 1.08×10^{-5} ft²/s

$N_R = 1.31 \times 10^6$ From Moody, f = 0.022; the new D:

$D^5 = 364\cdot(0.022)$; **D = 1.52 ft ≈ 1.5 ft**

3.5.12

First, apply the energy equation to the pipeline;

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L \text{ ; where } h_L = h_f$$

"1" is at the surface of the reservoir; thus $V_1 = 0 = P_1$

"2" is at the surface of the tank, therefore $V_2 = 0 = P_2$.

With the datum at "2", and substituting Darcy-Weisbach into the energy equation, we have

$0 + 0 + (-1 + 8000/500 + 6)\text{m} = 0 + 0 + 0 + f(8000/D)V^2/2g$

$21\text{ m} = f\cdot(8000/D)[V^2/(2\cdot9.81\text{m/s}^2)]$; but $V = Q/A = 4Q/\pi D^2$

$Q = (1800\text{m}^3/\text{day})(1\text{day}/24\text{hr})(1\text{hr}/3600\text{sec}) = 0.0208\text{ m}^3/\text{sec}$

Thus; $V = 4(0.0208\text{ m}^3/\text{sec})/[\pi(D^2)] = 0.0265/D^2$ and

$21\text{ m} = f\cdot(8000/D)[(0.0265/D^2)^2/(2\cdot9.81\text{m/s}^2)]$; yielding

$D^5 = 0.0136\cdot f$; Neither D nor V is available so e/D and N_R can not be determined. Iterate with f = 0.02 as a first trial, which is near midrange of typical f values.

Solving for D: $D^5 = 0.0136\cdot(0.02)$; D = 0.194m

Now, e/D = 0.36mm/194mm = 0.00186

$V = 0.0265/D^2 = 0.704$ m/s; and w/ν = 1.57×10^{-6} m²/s

The controlling diameter will be dictated by 4°C water.

$N_R = 8.70 \times 10^4$ From Moody, f = 0.025, the new D:

$D^5 = 0.0136\cdot(0.025)$; **D = 0.202 m ≈ 0.2 m**

3.5.13

Substituting equation (3.19) into (3.20) for one of the V's in the V² term yields:

$$\frac{P_1 - P_2}{\gamma} = f\left(\frac{L}{D}\right)\left[\frac{(P_1 - P_2)D^2}{32\mu L}\right]\frac{V}{2g}$$

$$f = \frac{64}{\gamma}\frac{\mu g}{VD}$$

3.5.14

First, apply the energy equation to the pipeline;

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L \text{ ; where } h_L = h_f$$

$V_1 = V_2$; $P_1 = P_2$; Thus, $h_1 - h_2 = h_f$

Now use the Darcy Weisbach equation to find h_f .

e/D = (0.045 mm)/(200 mm) = 0.000225

V = Q/A = $(0.08\text{m}^3/\text{s})/[(\pi/4)(0.20 \text{ m})^2]$ = 2.55 m/s

$N_R = DV/\nu = [(0.20\text{m})(2.55 \text{ m/s})]/(1.00 \times 10^{-6} \text{ m}^2/\text{s})$

$N_R = 5.10 \times 10^5$ From Moody diagram; **f = 0.017**

Determine the friction head loss and convert to slope

$h_f = f\,(L/D)(V^2/2g)$; for a 100 m length of pipe

$h_f = (0.017)(100\text{m}/0.20\text{m})[(2.55 \text{ m/s})^2/(2\cdot 9.81 \text{ m/s}^2)]$

h_f = 2.82 m; h_f/L = 2.82/100 = 0.0282 or **S = 2.82%**

3.5.15

The pressure drop from each of the gage pairs represents the friction loss for horizontal, uniform pipes based on Eq'n (3.15a). The flow rate can then be determined from the Darcy-Weisbach equation upstream and downstream. The difference in flow rates, if any, represents the amount leaking. Thus, the upstream flowrate is determined as

$(P_1 - P_2)/\gamma = h_L = h_f$; $(23{,}000 \text{ N/m}^2)/(9{,}790 \text{ N/m}^3) = h_f$

h_f = 2.35 m; Substituting this into Darcy-Weisbach

$h_f = f\,(L/D)(V^2/2g)$;

$2.35 \text{ m} = f\,(100\text{m}/0.30\text{m})(V^2/2\cdot 9.81\text{m/s}^2)$

The Moody diagram is required to find f. However, V is not available so N_R can not be solved. Use (e/D) and the Moody diagram to obtain a trial f value by assuming

3.5.15 (continued)

flow is in the complete turbulence regime.

e/D = (0.26 mm)/(300 mm) = 0.000867; $f \approx 0.02$, and

$2.35 \text{ m} = (0.02)(100\text{m}/0.30\text{m})(V^2/2\cdot 9.81\text{m/s}^2)$

V = 2.63 m/sec; Now with $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{sec}$

$N_R = 7.89 \times 10^5$ From Moody; f = 0.0195, ok - close

$2.35 \text{ m} = (0.0195)(100\text{m}/0.30\text{m})(V^2/2\cdot 9.81\text{m/s}^2)$

V = 2.66 m/sec; **Q** = AV **= 0.188 m³/sec (188 L/sec)**

The downstream flowrate is determined as

$(P_1 - P_2)/\gamma = h_L = h_f$; $(20{,}900 \text{ N/m}^2)/(9{,}790 \text{ N/m}^3) = h_f$

h_f = 2.13 m; Substituting this into Darcy-Weisbach

$h_f = f\,(L/D)(V^2/2g)$;

$2.13 \text{ m} = f\,(100\text{m}/0.30\text{m})(V^2/2\cdot 9.81\text{m/s}^2)$

The Moody diagram is required to find f. However, V is not available so N_R can not be solved. Use (e/D) and the Moody diagram to obtain a trial f value by assuming flow is in the complete turbulence regime.

e/D = (0.26 mm)/(300 mm) = 0.000867; $f \approx 0.02$, and

$2.13 \text{ m} = (0.02)(100\text{m}/0.30\text{m})(V^2/2\cdot 9.81\text{m/s}^2)$

V = 2.50 m/sec; Now with $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{sec}$

$N_R = 7.50 \times 10^5$ From Moody; f = 0.0195, ok - close

$2.13 \text{ m} = (0.0195)(100\text{m}/0.30\text{m})(V^2/2\cdot 9.81\text{m/s}^2)$

V = 2.54 m/sec; **Q** = AV **= 0.180 m³/sec (180 L/sec)**

Therefore, the leak is:

$\mathbf{Q_{up} - Q_{down}}$ = (188 – 180)L/sec = **8 L/sec**

Even though this does not seem like much of a leak, it amounts to almost 5% of the pipe flow. Further investigation is required to prevent the loss of water.

3.7.1

All the equations can be written in the form: $h_f = KQ^m$

a) **Darcy-Weisbach**: $K = (0.0826fL)/D^5$, $m = 2$

$e/D = 0.26/300 = 0.000867$; $\nu = 0.800 \times 10^{-6}\ m^2/sec$

$V = Q/A = (0.320\ m^3/s)/[\pi(0.15m)^2] = 4.53\ m/sec$

$N_R = VD/\nu = [(4.53)(0.30)]/0.80 \times 10^{-6} = 1.70 \times 10^6$

From the Moody diagram, f = 0.0195

$K = (0.0826 \cdot 0.0195 \cdot 6000)/(0.30)^5 = 3980$

$\mathbf{h_f} = KQ^m = (3980)(0.320)^2 = \mathbf{408\ m}$

b) **Hazen-Williams**: $K = (10.67L)/(D^{4.87} \cdot C^{1.85})$; $m = 1.85$

$K = (10.67 \cdot 6000)/(0.3^{4.87} \cdot 130^{1.85}) = 2770$

$\mathbf{h_f} = KQ^m = (2770)(0.320)^{1.85} = \mathbf{337\ m}$

c) **Manning**: $K = (10.3 \cdot n^2 \cdot L)/(D^{5.33})$; $m = 2$

$K = (10.3 \cdot 0.011^2 \cdot 6000)/(0.30^{5.33}) = 4580$

$\mathbf{h_f} = KQ^m = (4580)(0.320)^2 = \mathbf{469\ m}$

3.7.2

First, apply the energy equation to the pipeline;

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$; where $h_L = h_f$

$V_1 = V_2 = 0$; $P_1 = P_2 = 0$; Thus, $h_1 = h_2 + h_f = 5280 + h_f$

Both equations can be written in the form: $h_f = KQ^m$

a) **Hazen-Williams**: $K = (4.73L)/(D^{4.87} \cdot C^{1.85})$; $m = 1.85$

$K = (4.73 \cdot 2 \cdot 5280)/(2.5^{4.87} \cdot 110^{1.85}) = 0.0964$

$h_f = KQ^m = (0.0964)(77.6)^{1.85} = 302$ ft; $\mathbf{h_1 = 5582}$ **ft (ok)**

b) **Manning**: $K = (4.64 \cdot n^2 \cdot L)/(D^{5.33})$; $m = 2$

$K = (4.64 \cdot 0.017^2 \cdot 2 \cdot 5280)/(2.5^{5.33}) = 0.107$

$h_f = KQ^m = (0.107)(77.6)^2 = 644$ ft; $\mathbf{h_1 = 5924}$ **ft (Not ok)**

3.7.3

First, apply the energy equation to the pipeline;

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$; where $h_L = h_f$

$V_1 = V_2 = 0$; $P_1 = P_2 = 0$; Thus, $h_1 - h_2 = h_f = 5$ m

All the equations can be written in the form: $h_f = KQ^m$

a) **Darcy-Weisbach**: $K = (0.0826fL)/D^5$, $m = 2$

$e/D = 0.18/500 = 0.000360$; From Moody, try f = 0.016

based on the complete turbulence assumption.

$K = (0.0826 \cdot 0.016 \cdot 1200)/(0.50)^5 = 50.7$

$h_f = 5 = KQ^m = (50.7)(Q)^2$; $Q = 0.314\ m^3/sec$

Now $V = Q/A = (0.314\ m^3/s)/[\pi(0.25m)^2] = 1.60\ m/sec$

$N_R = VD/\nu = [(1.60)(0.50)]/1.0 \times 10^{-6} = 8.00 \times 10^5$

From the Moody diagram, f = 0.0165

$K = (0.0826 \cdot 0.0165 \cdot 1200)/(0.50)^5 = 52.3$

$h_f = 5 = KQ^m = (52.3)(Q)^2$; $\mathbf{Q = 0.309\ m^3/sec}$

b) **Hazen-Williams**: $K = (10.67L)/(D^{4.87} \cdot C^{1.85})$; $m = 1.85$

$K = (10.67 \cdot 1200)/(0.5^{4.87} \cdot 140^{1.85}) = 40.1$

$h_f = 5 = KQ^m = (40.1)(Q)^{1.85}$; $\mathbf{Q = 0.325\ m^3/sec}$

b) **Manning**: $K = (10.3 \cdot n^2 \cdot L)/(D^{5.33})$; $m = 2$

$K = (10.3 \cdot 0.011^2 \cdot 1200)/(0.50^{5.33}) = 60.2$

$h_f = 5 = KQ^m = (60.2)(Q)^2$; $\mathbf{Q = 0.288\ m^3/sec}$

3.7.4

There are some additional empirical equations available in the literature (some just to estimate the "f" value for Darcy-Wiesbach), but none are as widely used or accepted as the three covered in your textbook (Darcy-Weisbach, Hazen-Williams, and Manning)

3.7.5

First use the energy equation to determine h_f;

$$\frac{V_A^2}{2g}+\frac{P_A}{\gamma}+h_A=\frac{V_B^2}{2g}+\frac{P_B}{\gamma}+h_B+h_L;$$

where $h_L = h_f$, $V_A = V_B$, and use B as a datum elev.

8.3m + 100m = 76.7m + 0m + h_f ; h_f = 31.6 m

Both equations can be written in the form: $h_f = KQ^m$

a) **Hazen-Williams**: $K = (10.67L)/(D^{4.87}\cdot C^{1.85})$; m = 1.85

$K = (10.67\cdot 4500)/(4.0^{4.87}\cdot 110^{1.85}) = 0.00939$

$h_f = 31.6 = KQ^m = (0.00939)(Q)^{1.85}$; **Q = 80.6 m^3/sec**

b) **Manning**: $K = (10.3\cdot n^2\cdot L)/(D^{5.33})$; m = 2

$K = (10.3\cdot 0.017^2\cdot 4500)/(4.0^{5.33}) = 0.00828$

$h_f = 31.6 = KQ^m = (0.00828)(Q)^2$; **Q = 61.8 m^3/sec**

The Hazen-Williams solution compares favorably to Darcy-Weisbach; but the Manning solution does not!

3.7.6

First, apply the energy equation to the pipeline;

$$\frac{V_1^2}{2g}+\frac{P_1}{\gamma}+h_1=\frac{V_2^2}{2g}+\frac{P_2}{\gamma}+h_2+h_L;$$ where $h_L = h_f$

$V_1 = V_2$; $h_1 = h_2$; Thus, $(P_1 - P_2)/\gamma = h_f$ = 29.9 ft

The Manning equation can be written as : $h_f = KQ^m$

Manning (n = 0.012): $K = (4.64\cdot n^2\cdot L)/(D^{5.33})$; m = 2

$K = (4.64\cdot 0.012^2\cdot L)/(2.0^{5.33}) = 0.0000166\cdot L$

$h_f = 29.9 = KQ^m = (0.0000166\cdot L)(30)^2$; **L = 2000 ft**

Manning (n = 0.013): $K = (4.64\cdot n^2\cdot L)/(D^{5.33})$; m = 2

$K = (4.64\cdot 0.013^2\cdot L)/(2.0^{5.33}) = 0.0000195\cdot L$

$h_f = 29.9 = KQ^m = (0.0000195\cdot L)(30)^2$; **L = 1,700 ft**

The length changes by **15%!**

3.7.7

The energy equation yields, $h_1 - h_2 = h_f$ = 20 m

The Hazen-Williams eq'n can be written as: $h_f = KQ^m$

a) **One 30-cm pipe**: $K = (10.67L)/(D^{4.87}\cdot C^{1.85})$; m = 1.85

$K = (10.67\cdot 2000)/(0.30^{4.87}\cdot 140^{1.85}) = 804$

$h_f = 20 = KQ^m = (804)(Q)^{1.85}$; **$Q_{30}$ = 0.136 m^3/sec**

b) **Two 20-cm pipes**: $K = (10.67L)/(D^{4.87}\cdot C^{1.85})$; m = 1.85

$K = (10.67\cdot 2000)/(0.20^{4.87}\cdot 140^{1.85}) = 5790$

$h_f = 20 = KQ^m = (5790)(Q)^{1.85}$; $Q_{20} = 0.0467\ m^3/sec$

Q_{20s} = 2(Q) = **0.0934 m^3/sec**; Not as much flow!!

3.7.8

Need to use the Manning eq'n in its original form:

$Q = VA = (1.486/n)AR_h^{2/3}S^{1/2}$

$A = \pi(r)^2/2 = \pi(1.0ft)^2/2 = 1.57\ ft^2$

$R_h = A/P$; and $P = \pi r + 2r = \pi(1\ ft) + 2(1\ ft) = 5.14$ ft

Therefore, $R_h = A/P = (1.57\ ft^2)/(5.14\ ft) = 0.305$ ft

$S = h_f/L = h_f/(1200\ ft)$; $Q = (1.486/n)AR_h^{2/3}S^{1/2}$

$15\ ft^3/sec = (1.486/0.013)(1.57\ ft^2)(0.305\ ft)^{2/3}(h_f/1200ft)^{1/2}$;

h_f = 40.8 ft

3.7.9

The energy equation yields, $h_f = (P_1 - P_2)/\gamma$

$h_f = (366{,}000\ N/m^2)/(9790\ N/m^3) = 37.4$ m

$h_f = 37.4\ m = KQ^m = (K)(0.136)^{1.85}$; K = 1500

$K = 1500 = (10.67L)/(D^{4.87}\cdot C^{1.85}) =$

$(10.67\cdot 2000)/(0.30^{4.87}\cdot C^{1.85})$; **$C_{HW}$ = 99.9**

3.11.1

Contraction: $h_c = K_c[(V_2)^2/2g]$; V_2 is the small pipe

$V_2 = Q/A = (0.106\ m^3/s)/[\pi(0.075m)^2] = 6.00$ m/sec

$\mathbf{h_c} = (0.33)[(6.00\ m/s)^2/2\cdot9.81\ m/s^2]$ **= 0.606 m**

Expansion: $h_E = [(V_1 - V_2)^2/2g]$; V_2 is now large pipe

$V_2 = Q/A = (0.106\ m^3/s)/[\pi(0.15m)^2] = 1.50$ m/sec

$\mathbf{h_E} = [(6.00\ m/s – 1.50\ m/s)^2/2\cdot9.81\ m/s^2]$ **= 1.03 m**

Enlargement loss is much greater (70%).

3.11.2

Confusor: $h_c' = K_c'[(V_2)^2/2g]$; V_2 is the small pipe

$V_2 = Q/A = (0.106\ m^3/s)/[\pi(0.075m)^2] = 6.00$ m/sec

With $A_2/A_1 = 0.25$, extrapolate Fig 3.11; $K_c' = 0.01$

$\mathbf{h_c'} = (0.01)[(6.00\ m/s)^2/2\cdot9.81\ m/s^2]$ **= 0.018 m**

Diffusor: $h_E' = K_E'[(V_1^2 - V_2^2)/2g]$; V_2 is now large pipe

$V_2 = Q/A = (0.106\ m^3/s)/[\pi(0.15m)^2] = 1.50$ m/sec

From “α” table for diffusors: With α = 15°, $K_E' = 0.194$

$\mathbf{h_E} = (0.194)[(6.00)^2 – (1.50)^2]/2\cdot9.81]$ **= 0.334 m**

The diffusor loss is much greater, but both are reduced greatly from the abrupt contraction and expansion.

3.11.3

The headloss is expressed as: $h_L = [K_v]\ (V)^2/2g$

$\Delta P/\gamma = (100{,}00\ N/m^2)/(9{,}790\ N/m^3) = 10.2\ m = h_L$

$V = Q/A = (0.04\ m^3/s)/[\pi(0.04\ m)^2] = 7.96$ m/sec; thus

$10.2\ m = [K_v]\ (7.96\ m/s)^2/2\cdot9.81\ m/s^2$

$\mathbf{K_v = 3.16}$

3.11.4

The headloss is expressed as: $h_L = [\sum K_v]\ (V)^2/2g$

$\Delta P/\gamma = (5.19\ lb/in.^2)(144\ in.^2/ft^2)/(62.3\ lb/ft^3) = 12.0\ ft = h_L$

$12.0\ ft = [2.5 + 10]\ (V)^2/2\cdot32.2\ ft/s^2$, $V = 7.86$ ft/sec

$\mathbf{Q} = V\cdot A = (7.86\ ft/s)[\pi\{(1/3)ft\}^2]$ = $\mathbf{2.74\ ft^3/sec}$

3.11.5

First, apply the energy equation to the pipeline;

$$\frac{V_1^2}{2g}+\frac{P_1}{\gamma}+h_1=\frac{V_2^2}{2g}+\frac{P_2}{\gamma}+h_2+h_L$$; where $h_L = h_c$

$h_c = K_c(V_2^2/2g)$; $h_1 = h_2$, $V_1 = (A_2/A_1)V_2 = 0.25\cdot V_2$

Since K_c depends on V_2, assume $V_2 \approx 6$ m/s and thus

$K_c = 0.33$ with $D_2/D_1 = 0.5$ (Table 3.5), Thus

$(0.25\cdot V_2^2/2g)+(285/9.79)=[1+0.33](V_2^2/2g)+(265/9.79)$

V_2.= 6.09 m/sec (OK – no more iterations needed)

$\mathbf{Q} = V\cdot A = (6.09\ m/s)[\pi(0.15m)^2]$ = $\mathbf{0.430\ m^3/sec}$

3.11.6

First, apply the energy equation to the pipeline;

$$\frac{V_A^2}{2g}+\frac{P_A}{\gamma}+h_A=\frac{V_B^2}{2g}+\frac{P_B}{\gamma}+h_B+h_L$$; where

$h_L = h_f + [\sum K](V)^2/2g$; $V_A = V_B$, and $h_A = h_B$. Thus

$P_A/\gamma = P_B/\gamma + [f(L/D) + \sum K](V^2/2g)$

$V = Q/A = (0.006\ m^3/s)/[\pi(0.02m)^2] = 4.77$ m/sec

$N_R = DV/\nu = [(0.04m)(4.77\ m/s)]/(1.00 \times 10^{-6}\ m^2/s)$

$N_R = 1.91 \times 10^5$; e/D = 0.045mm/40 mm = 0.00113

From Moody; **f = 0.022**; Thus, the energy e’qn. is

$P_A/(9{,}790) = (192{,}000/9{,}790) + [0.022(50/0.04) +$

$(0.15) + 2(0.17)][(4.77)^2/2\cdot9.81]$

$\mathbf{P_A = 510{,}000\ N/m^2}$ **(Pascals) = 510 kPa**

3.11.7

Applying the energy equation from the surface of the storage tank (1) to the outlet of the pipe (2) yields;

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$; where $P_1 = P_2 = 0$;

$h_L = h_f + [\sum K](V^2/2g)$; $V_1 = 0$ and $h_2 = 0$. Thus

$h_1 = [1 + f(L/D) + \sum K](V^2/2g)$; where $V_2 = V$ (pipe V)

and $K_e = 0.5$; $K_b = 0.19$ (R/D = 2); $K_v = 0.15$; and

assuming complete turbulence for the first trial:

e/D = 0.00085ft/0.5ft = 0.00170; thus; **f = 0.022,** and

$h_1 = 60.2 = [1+0.022(500/0.5)+0.5+2(0.19)+0.15](V^2/2g)$;

V = 12.7 ft/sec; $N_R = DV/\nu = [(0.5)(12.7)]/(1.08\text{x}10^{-5})$

$N_R = 5.88 \text{ x } 10^5$; From Moody; new **f = 0.022**; (ok)

Q = V·A = (12.7 ft/s)[π(0.25ft)^2] = **2.49 ft^3/sec**

Theoretically, the pipe entrance coefficient of 0.5 should be adjusted according to the pipe velocity and the first column of Table 3.5. However, this usually does not affect the final result and a loss coefficient of 0.5 is often used for square-edged entrances.

3.11.8

Apply the energy equation from the surface of the supply tank (1) to the surface of the receiving tank (2):

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$; where $P_1 = P_2 = 0$;

$h_L = h_f + [\sum K](V^2/2g)$; and $V_1 = V_2 = 0$. Therefore

$h_1 - h_2 = [f(L/D) + \sum K_v](V^2/2g)$; where V is pipe V;

$K_e = 0.5$; $K_b = 0.35$ (R/D = 1); $K_d = 1.0$ (exit loss);

and assuming complete turbulence for the first trial:

e/D = 0.26mm/150mm = 0.00173; thus; **f = 0.022,** and

3.11.8 (continued)

$h_1 - h_2 = 5 = [0.022(75/0.15) + 0.5 + 0.35 + 1.0](V^2/2g)$

V = 2.76 m/sec; $N_R = DV/\nu = [(0.15)(2.76)]/(1.0\text{x}10^{-6})$

$N_R = 4.14 \text{ x } 10^5$; From Moody; new **f = 0.0225**; Now

solving the energy eq'n again yields V = 2.74 m/s and

Q = V·A = (2.74 m/s)[π(0.075m)^2] = **0.0484 m^3/sec**

Note: Theoretically, the pipe entrance coefficient of 0.5 should be adjusted according to the pipe velocity and the first column of Table 3.5. However, this usually does not affect the final result and a loss coefficient of 0.5 is often used for square-edged entrances.

3.11.9

Applying the energy equation from the surface of the storage tank (1) to the outlet of the pipe (2) yields;

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$; where $P_1 = P_2 = 0$;

$h_L = [\sum K](V^2/2g)$; $h_f = 0$ (short pipe), $V_1 = h_2 = 0$. Thus

$h_1 = [1 + \sum K](V^2/2g)$; where $V_2 = V$ (pipe V);

$K_e = 0.5$; & $K_v = 10.0$. Rearranging the energy eq'n

$V = [2g(h_1)/(1 + \sum K)]^{1/2}$; Also,

$Q = d(Vol)/dt = (\pi D^2/4)dh/dt$; and

$Q = AV = (\pi d^2/4)\,[2g(h)/(1 + \sum K)]^{1/2}$;
where d = pipe diameter; D = tank diameter. Now,

$(\pi D^2/4)dh/dt = (\pi d^2/4)\,[2g(h)/(1 + \sum K)]^{1/2}$

$dt = [(D^2/d^2)dh] / [2g(h)/(1 + \sum K)]^{1/2}$

$t = [(1 + \sum K)/2g]^{1/2}\,(D^2/d^2)\int_{h_1=1.5}^{h_2=3} h^{-1/2}dh$

$t = [(11.5)/2\cdot 9.81]^{1/2}\,(5^2/0.2^2)\,2[(3)^{1/2}-(1.5)^{1/2}]$

t = 485 sec

3.11.10

Determine the difference between the head losses under existing conditions and the proposed conditions.

Existing Pipeline: Friction losses (Darcy-Weisbach):

$h_f = f\,(L/D)(V^2/2g)$; e/D = 0.045/200 = 0.000225

$V = Q/A = (0.10\ m^3/s)/[\pi(0.10m)^2] = 3.18$ m/sec

$N_R = DV/\nu = [(0.20m)(3.18\ m/s)]/(1.00 \times 10^{-6}\ m^2/s)$

$N_R = 6.36 \times 10^5$; From Moody; **f = 0.0155**;

$h_f = (0.0155)(800/0.20)(3.18^2/2\cdot9.81) = 32.0$ m

Note: The friction losses, apart from any minor losses, use up almost all of the available head (34 m).

New Pipeline: Friction losses (30 cm pipe):

$h_f = f\,(L/D)(V^2/2g)$; e/D = 0.045/300 = 0.00015

$V = Q/A = (0.10\ m^3/s)/[\pi(0.15m)^2] = 1.41$ m/sec

$N_R = DV/\nu = [(0.30m)(1.41\ m/s)]/(1.00 \times 10^{-6}\ m^2/s)$

$N_R = 4.23 \times 10^5$; From Moody; **f = 0.0155**;

$h_f = (0.0155)(0.94\cdot800/0.30)(1.41^2/2\cdot9.81) = 3.94$ m

Friction losses(20 cm pipe); 6% of existing pipe losses:

$h_f = (0.06)(32.0\ m) = 1.92$ m

Confusor: $h_c' = K_c'[(V_2)^2/2g]$; V_2 is the small pipe

$V_2 = Q/A = (0.10\ m^3/s)/[\pi(0.10m)^2] = 3.18$ m/sec

With A_2/A_1=0.444, extrapolate Fig 3.11; $K_c' = 0.025$

$h_c' = (0.025)[(3.18\ m/s)^2/2\cdot9.81\ m/s^2] = 0.0129$ m

Total Head Loss = 3.94 + 1.92 + 0.0129 = 5.87 m

Pressure Head Gain = 32.0 – 5.87 = 26.1 m

Note: Theoretically, the pipe entrance loss may change slightly with a bigger pipe. However, this will be minor in comparison to the friction losses.

3.11.11

The contraction headloss is expressed as:

$h_c = K_c[(V_S)^2/2g]$; where V_S is the velocity in the smaller pipe. Based on Table 3.5, the value of K_c ranges from 0.29 to 0.38 for $D_S/D_L = 0.5$.

The expansion headloss is expressed as:

$h_E = [(V_S - V_L)^2/2g]$; where V_L is the velocity in the larger pipe and V_S is the smaller pipe velocity. But,

$h_E = [(V_S - V_L)^2/2g] = (1 - V_L/V_S)^2\,[(V_S)^2/2g]$

which is in the same form as h_c above. Since

$V_L = (A_S/A_L)V_S = 0.25\cdot V_S$; for $D_S/D_L = 0.5$

$h_E = [1 - (0.25V_S)/V_S]^2\,[(V_S)^2/2g] = 0.563\,[(V_S)^2/2g]$

The expansion coefficient (0.563) is always larger than the contraction coefficient (0.29 to 0.38 range).

3.12.1

For the parallel pipe system (Fig 3.19);

$h_{f1} = h_{f2} = h_{fE}$ and $Q_E = Q_1 + Q_2$ From Table 3.4:

$h_f = [(10.3n^2L)/D^{5.33}]Q^2$ for Manning equation (SI) or

$Q = [(h_f\,D^{5.33})/(10.3n^2L)]^{1/2}$; Substituting into the flow eq'n above for pipes 1, 2, and E and simplifying yields:

$[(D_E^{5.33})/(n_E^2L_E)]^{1/2} = [(D_1^{5.33})/(n_1^2L_1)]^{1/2} + [(D_2^{5.33})/(n_2^2L_2)]^{1/2}$

$\mathbf{[(D_E^{5.33})/(n_E^2L_E)]^{1/2} = \sum [(D_i^{5.33})/(n_i^2L_i)]^{1/2} \Rightarrow i = 1\ to\ N}$

and this is appropriate for both systems of units.

3.12.2

For the parallel pipe system (Fig 3.19);

$h_{f1} = h_{f2} = h_{fE}$ and $Q_E = Q_1 + Q_2$ From Table 3.4:

$h_f = [(10.7\cdot L)/(D^{4.87}C^{1.85})]Q^{1.85}$: Hazen-Williams (SI) or

$Q = [(h_fD^{4.87}C^{1.85})/(10.7\cdot L)]^{1/1.85}$; Substituting into the flow eq'n for pipes 1, 2, and E and simplifying yields:

$[(D_E^{4.87}C_E^{1.85})/L_E]^{1/1.85} = [(D_1^{4.87}C_1^{1.85})/L_1]^{1/1.85} + [(D_2^{4.87}C_2^{1.85})/L_2)]^{1/1.85}$

$\mathbf{[(D_E^{4.87}C_E^{1.85})/L_E]^{1/1.85} = \sum [(D_i^{4.87}C_i^{1.85})/L_i]^{1/1.85} \Rightarrow i = 1\ to\ N}$

and this is appropriate for both systems of units.

3.12.3

Reworking Example 3.10 using the Manning eq'n and arbitrarily letting D = 4 ft and n = 0.013 yields

$[(D_E^{5.33})/(n_E^2L_E)]^{1/2} = [(D_1^{5.33})/(n_1^2L_1)]^{1/2} + [(D_2^{5.33})/(n_2^2L_2)]^{1/2}$

$[(4^{5.33})/(0.013^2 \cdot L_E)]^{1/2} = [(3^{5.33})/(1800 \cdot 0.013^2)]^{1/2} + [(2^{5.33})/(1500 \cdot 0.013^2)]^{1/2}$

L_E = 4430 ft. Then $h_{fAF} = h_{fAB} + h_{fBC} + h_{fCF}$

$h_{fAF} = [(4.64 \cdot 1800 \cdot 0.013^2)/4^{5.33}]120^2 + [(4.64 \cdot 4430 \cdot 0.013^2) \div 4^{5.33}]120^2 + [(4.64 \cdot 1500 \cdot 0.013^2)/4^{5.33}]120^2$

$\mathbf{h_{fAF}}$ = 12.6 + 30.9 + 10.5 = **54.0 ft**; Flow in pipe branches,

h_{fBC} = 30.9 ft = $[(4.64 \cdot 1800 \cdot 0.013^2)/3^{5.33}]Q_1^2$; $\mathbf{Q_1}$ **= 87.5 cfs**

h_{fBC} = 30.9 ft = $[(4.64 \cdot 1500 \cdot 0.013^2)/2^{5.33}]Q_2^2$; $\mathbf{Q_2}$ **= 32.5 cfs**

3.12.4

Reworking Example 3.10 using Hazen-Williams and arbitrarily letting D = 4 ft and C_{HW} = 100 yields

$[(D_E^{4.87}C_E^{1.85})/L_E]^{1/1.85} = [(D_1^{4.87}C_1^{1.85})/L_1]^{1/1.85} + [(D_2^{4.87}C_2^{1.85})/L_2)]^{1/1.85}$

$[(4^{4.87} \cdot 100^{1.85})/L_E)]^{1/1.85} = [(3^{4.87} \cdot 100^{1.85})/1800]^{1/1.85} + [(2^{4.87} \cdot 100^{1.85})/1500]^{1/1.85}$

L_E = 4030 ft. Then $h_{fAF} = h_{fAB} + h_{fBC} + h_{fCF}$

$h_{fAF} = [(4.73 \cdot 1800)/(4^{4.87} \cdot 100^{1.85})]120^{1.85} + [(4.73 \cdot 4030) \div (4^{4.87} \cdot 100^{1.85})]120^{1.85} + [(4.73 \cdot 1500)/ (4^{4.87} \cdot 100^{1.85})]120^{1.85}$

$\mathbf{h_{fAF}}$ = 14.0 + 31.2 + 11.6 = **56.8 ft**; Flow in pipe branches,

h_{fBC}=31.2ft=$[(4.73 \cdot 1800)/(3^{4.87} \cdot 100^{1.85})]Q_1^{1.85}$; $\mathbf{Q_1}$ **= 87.0 cfs**

h_{fBC}=31.2ft=$[(4.73 \cdot 1500)/(2^{4.87} \cdot 100^{1.85})]Q_2^{1.85}$; $\mathbf{Q_2}$ **= 33.0 cfs**

3.12.5

Find the equivalent pipe to replace Branches 1 and 2, arbitrarily letting D = 3 m and f = 0.02 yields

$[(D_E^5)/(f_E \cdot L_E)]^{1/2} = [(D_1^5)/(f_1 \cdot L_1)]^{1/2} + [(D_2^5)/(f_2 \cdot L_2)]^{1/2}$

$[(3^5)/(0.02 \cdot L_E)]^{1/2} = [(2^5)/(0.018 \cdot 1000)]^{1/2} + [(3^5)/(0.02 \cdot 800)]^{1/2}$

3.12.5 (continued)

L_E = 444 m. Then $h_{fAF} = h_{fAB} + h_{fBC} + h_{fCF}$

$h_{fAF} = [(0.0826 \cdot 0.02 \cdot 1000)/3^5]60^2 + [(0.0826 \cdot 0.02 \cdot 444) \div 3^5]80^2 + [(0.0826 \cdot 0.02 \cdot 900)/3^5]70^2$

$\mathbf{h_{fAF}}$ = 24.5 + 19.3 + 30.0 = **73.8 m**; Flow in pipe branches,

h_{fBC}= 19.3 m =$[(0.0826 \cdot 0.018 \cdot 1000)/2^5]Q_1^2$; $\mathbf{Q_1 = 20.4\ m^3/s}$

h_{fBC}= 19.3 m = $[(0.0826 \cdot 0.020 \cdot 800)/3^5]Q_2^2$; $\mathbf{Q_2 = 59.6\ m^3/s}$

3.12.6

Yes. In Example 3.10, an equivalent pipe was found for the parallel pipes resulting in 3 pipes in series. Now you can replace these 3 pipes with an equivalent pipe. Letting D = 4 ft and f = 0.02 for the pipe

$[(f_E \cdot L_E)/(D_E^5)] = [(f_1 \cdot L_1)/(D_1^5)]+[(f_2 \cdot L_2)/(D_2^5)]+[(f_3 \cdot L_3)/(D_3^5)]$

$[(0.02 \cdot L_E)/(4^5)] = [(0.02 \cdot 1800)/(4^5)] + [(0.02 \cdot 3310)/(4^5)] + [(0.02 \cdot 1500)/(4^5)]$

$\mathbf{L_E}$ **= 6610 ft**. (Note: The individual pipe lengths could be added directly since all of them have the same D and f.

3.12.7

Yes. In Problem 3.12.5, an equivalent pipe was found for the parallel pipes resulting in 3 pipes in series. Now you can replace these 3 pipes with an equivalent pipe. Letting D = 3 m and f = 0.02 for the pipe

$[(f_E \cdot L_E)/(D_E^5)] = [(f_1 \cdot L_1)/(D_1^5)]+[(f_2 \cdot L_2)/(D_2^5)]+[(f_3 \cdot L_3)/(D_3^5)]$

$[(0.02 \cdot L_E)/(3^5)] = [(0.02 \cdot 1000)/(3^5)] + [(0.02 \cdot 444)/(3^5)] + [(0.02 \cdot 900)/(3^5)]$

$\mathbf{L_E}$ **= 2344 m**. (Note: The individual pipe lengths could be added directly since all of them have the same D and f.

Chapter 4 – Problem Solutions

4.1.1 Learning to understand and construct energy grade lines (EGLs) and hydraulic grade lines (HGLs) provides tremendous insight into pipe flow and open channel flow problems. Each of the questions presented has a logical explanation based on the theory and practical pipe flow problems covered in Chapter 3.

a) The location of the EGL at the reservoirs. (The EGL is at the water surface; all position head and no velocity or pressure head. The HGL is located at the water surface too since there is no velocity head.)

b) The drop in the EGL moving from reservoir A into pipe 1 (This accounts for the entrance loss.)

c) The slope of the EGL in pipe 1. (This accounts for the friction loss; $h_f = f\,(L/D)(\,V^2/2g)$)

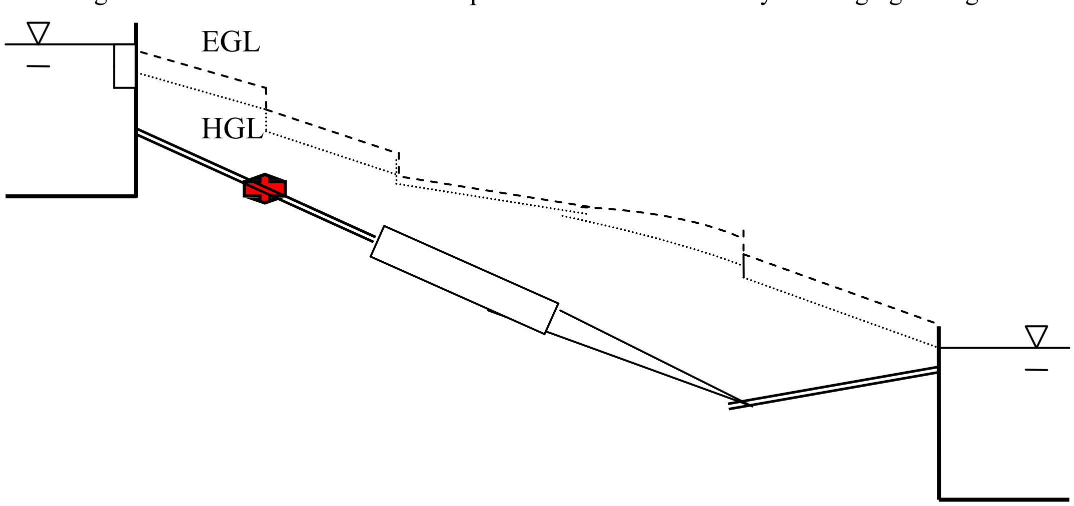
d) The separation distance between the EG L and the HGL. (This represents the velocity head, $V^2/2g$.)

e) The drop in the EGL moving from pipe 1 to pipe 2. (This accounts for the contraction loss.)

f) The steeper slope of the EGL in pipe 2 (steeper than pipe 1). (This accounts for the friction loss. Since pipe 2 is smaller than pipe 1, the loss of energy to friction is greater over distance; $h_f = f\,(L/D)(\,V^2/2g)$ or $h_f/L = f\,(1/D)(\,V^2/2g)$ where D is smaller and V is greater than in pipe 1.)

g) The drop in the EGL moving from pipe 2 to reservoir B. (This accounts for the exit loss, which is one full velocity head, $V^2/2g$. Thus, the HGL meets the water surface at reservoir B.)

4.1.2 Start drawing your EGL and HGL from the reservoir surface (on the left) and move to the right accounting for head losses along the way. Note that a) the EGL and HGL both start and end at the reservoir surfaces, b) the minor losses (entrance, valve, expansion, bend, and exit) are accounted for with an abrupt drop in the EGL and HGL; c) the separation distance between the EGL and HGL (which accounts for the velocity head, $V^2/2g$, is less with the large middle pipe than the smaller pipes on the end, d) the slope of the EGL, which represents the friction loss over length, is less for the large middle pipe than the smaller end pipes, and e) the pipe confusor going from the large middle pipe to the end pipe is the only location on the drawing where the EGL and HGL are not parallel because the velocity is changing throughout.



4.1.3 From the Moody Diagram, any value of Reynold's number greater than about 1.0×10^6 is in the complete turbulence regime (f = 0.02). Since $N_R \geq 1.0 \times 10^6$; $VD/\nu = VD/(1.0 \times 10^{-6}\ m^2/s) \geq 1.0 \times 10^6$. Therefore, $V{\cdot}D \geq 1.00\ m^2/s$; and **for V = 1 m/s, D = 1 m; for V = 2 m/s, D = 0.5 m; for V = 4.0 m/s, D = 0.25 m; etc.** (Note: Velocities of 1 m/sec to 4 m/sec are not unusual in water transmission systems.)

4.1.4 Applying the energy equation; $\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$; where $V_A = V_B = P_A = P_B = 0$,

$h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$, $K_b = 0.19$, $K_v = 0.15$, and $K_d = 1.0$ (exit coefficient). Thus,

$h_A - h_B = [f(L/D) + \sum K](V^2/2g) = [f(200/0.25) + 0.5 + 2(0.19) + 0.15 + 1.0](V^2/2g)$

$V = Q/A = (0.50\ ft^3/s)/[(\pi/4)(0.25\ ft)^2] = 10.2$ ft/sec; $e/D = 0.00085ft/0.25ft = 0.0034$

$N_R = DV/\nu = [(0.25\ ft)(10.2\ ft/s)]/(1.08 \times 10^{-5}\ ft^2/s) = 2.36 \times 10^5$; From Moody; **f = 0.027**; Thus,

$\mathbf{h_A - h_B} = [0.027(200/0.25) + 0.5 + 2(0.19) + 0.15 + 1.0]{\cdot}[(10.2)^2/2{\cdot}32.2]$ **= 38.2 ft**

4.1.5 Applying the energy equation; $\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$; where $V_A = V_B = P_A = P_B = 0$,

$h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$, $K_v = 10$, $K'_C = 0.3$, $K_b = 0.2$, $K_d = 1.0$ (exit), and $h_E = (V_S - V_L)^2/2g$. Thus,

$h_A = h_B + [f(L/D)_S + \sum K](V_S^2/2g) + h_E + [f(L/D)_L](V_L^2/2g)$; V_S & V_L are the small & large pipe velocities

$h_A = 750 + [f_S(200/0.5)+12](V_S^2/2g) + (V_S - V_L)^2/2g + f_L(100/1)(V_L^2/2g)$

$V_S = Q/A = (1.2\ m^3/s)/[(\pi/4)(0.5\ m)^2] = 6.11$ m/sec; $e/D_S = 0.18mm/500mm = 0.00036$

$N_R = DV_S/\nu = [(0.5\ m)(6.11\ m/s)]/(1.00 \times 10^{-6}\ m^2/s) = 3.06 \times 10^6$; From Moody; $\mathbf{f_S = 0.0155}$;

$V_L = Q/A = (1.2\ m^3/s)/[(\pi/4)(1.0\ m)^2] = 1.53$ m/sec; $e/D_L = 0.18mm/1000mm = 0.00018$

$N_R = DV_L/\nu = [(1.0\ m)(1.53\ m/s)]/(1.00 \times 10^{-6}\ m^2/s) = 1.53 \times 10^6$; From Moody; $\mathbf{f_L = 0.0145}$;

$\mathbf{h_A} = 750 + [0.0155(200/0.5)+12][(6.11)^2/2g] + (6.11 - 1.53)^2/2g + [0.0145(100/1)][(1.53)^2/2g]$ **= 786 m**

4.1.6 Applying the energy equation; $\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 + h_L$; where $V_A = 0$, $V_1 = V$ (pipeline V),

$h_L = h_f + [\sum K](V)^2/2g$; $h_A = 10$ m, $h_1 = 7$ m, and $P_A/\gamma = P_0/\gamma = (9.79\ kN/m^2)/(9.79\ kN/m^3) = 1$ m. Thus,

$P_1/\gamma = P_0/\gamma + h_A - h_1 - [1 + f(L/D) + \sum K](V^2/2g) = 4\ m - [1 + f(L/D) + \sum K](V^2/2g)$

$V = Q/A = (0.0101\ m^3/s)/[(\pi/4)(0.102\ m)^2] = 1.24$ m/sec; $e/D = 0.045mm/102mm = 0.000441$

$N_R = DV/\nu = [(0.102m)(1.24\ m/s)]/(1.00 \times 10^{-6}\ m^2/s) = 1.26 \times 10^5$; From Moody; **f = 0.02**; Thus,

$P_1/\gamma = 4\ m - [1 + 0.02(10m/0.102m) + 0.5 + 12.0][(1.24\ m/s)^2/2{\cdot}9.81\ m/s^2] = 2.79$ m; $\mathbf{P_1 = 27.3\ kPa}$

For P_2, the energy equation yields: $P_2/\gamma = P_1/\gamma + h_1 - h_2 - h_L$, where $h_L = h_f = f(L/D)(V^2/2g)$. Thus,

$P_2/\gamma = 2.79\ m + 7\ m - 2\ m - [0.02(5m/0.102m)[(1.24\ m/s)^2/2{\cdot}9.81\ m/s^2] = 7.71$ m; $\mathbf{P_2 = 75.5\ kPa}$

For P_3, the energy equation yields: $P_3/\gamma = P_2/\gamma - h_L$, where $h_L = h_f = f(L/D)(V^2/2g)$. Thus,

$P_3/\gamma = 7.71\ m - [0.02(5m/0.102m)[(1.24\ m/s)^2/2{\cdot}9.81\ m/s^2] = 7.63$ m; $\mathbf{P_3 = 74.7\ kPa}$

(Problem 4.1.6 – continued)

For P_4, the energy equation yields: $P_4/\gamma = P_3/\gamma + h_3 - h_4 - h_L$, where $h_L = h_f = f(L/D)(V^2/2g)$. Thus,

$P_4/\gamma = 7.63\text{ m} + 2\text{ m} - 9.5\text{ m} - [0.02(7.5\text{m}/0.102\text{m})[(1.24\text{ m/s})^2/2\cdot9.81\text{ m/s}^2] = 0.0148\text{ m}$; $\mathbf{P_4 = 0.145\ kPa}$

For P_5, the energy equation yields: $P_5/\gamma = P_4/\gamma - h_L$, where $h_L = h_f = f(L/D)(V^2/2g)$. Thus,

$P_5/\gamma = 0.0148\text{ m} - [0.02(5\text{m}/0.102\text{m})[(1.24\text{ m/s})^2/2\cdot9.81\text{ m/s}^2] = -0.0620\text{ m}$; $\mathbf{P_5 = -0.607\ kPa}$

For P_6, the energy equation yields: $P_6/\gamma = P_5/\gamma + h_5 - h_6 - h_L$, where $h_L = h_f = f(L/D)(V^2/2g)$. Thus,

$P_6/\gamma = -0.0620\text{ m} + 9.5\text{ m} - 7\text{ m} - [0.02(2.5\text{m}/0.102\text{m})[(1.24\text{ m/s})^2/2\cdot9.81\text{ m/s}^2] = 2.40\text{ m}$; $\mathbf{P_5 = 23.5\ kPa}$

4.1.7 Applying the energy equation; $\dfrac{V_A^2}{2g} + \dfrac{P_A}{\gamma} + h_A = \dfrac{V_B^2}{2g} + \dfrac{P_B}{\gamma} + h_B + h_L$; where $V_A = V_B = P_A = P_B = 0$,

$h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$, $K_{v1} = 12$, $K_{v2} = 10$, $K_d = 1.0$ (exit coef)., and $h_A - h_B = 2.5$ m. Thus,

2.5 m = $[f(40\text{m}/0.102\text{m}) + 23.5](V^2/2g)$; where V is pipe V. Assume complete turbulence for the first trial:

For commercial steel: e/D = 0.045mm/102mm = 0.000441; thus; **f = 0.0165** and solving energy eq'n

2.5 m = $[0.0165(40\text{m}/0.102\text{m}) + 23.5](V^2/2g)$; V = 1.28 m/sec. Now we can solve for Reynolds number,

$N_R = DV/\nu = [(0.102)(1.28)]/(1.00\text{x}10^{-6}) = 1.31 \text{ x } 10^5$; From Moody; new **f = 0.02**; and the new V is

2.5 m = $[0.02(40\text{m}/0.102\text{m}) + 23.5](V^2/2g)$; V = 1.25 m/sec. $N_R = DV/\nu = 9.73 \text{ x } 10^4$; **f = 0.02**; OK

Thus, **Q** = V·A = (1.25 m/s)[(π/4)(0.102m)^2] = 0.0102 m^3/sec = **10.2 L/sec**

4.1.8 Applying the energy equation; $\dfrac{V_A^2}{2g} + \dfrac{P_A}{\gamma} + h_A = \dfrac{V_B^2}{2g} + \dfrac{P_B}{\gamma} + h_B + h_L$; where $V_A = V_B = P_A = P_B = 0$,

$h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$, $K_d = 1.0$ (exit coef.), and $h_A - h_B = 9$ m. Thus,

9 m = $[f(700\text{m}/0.4\text{m}) + 1.5](V^2/2g)$; where V is pipe V. Assume complete turbulence for the first trial:

a) For commercial steel: e/D = 0.045mm/400mm = 0.000113; thus; **f = 0.012** and solving energy eq'n

9 m = $[0.012(700\text{m}/0.4\text{m}) + 1.5](V^2/2g)$; V = 2.80 m/sec. Now we can solve for Reynolds number,

$N_R = DV/\nu = [(0.40)(2.80)]/(1.0\text{x}10^{-6}) = 1.12 \text{ x } 10^6$; From Moody; new **f = 0.014**; and the new V is

9 m = $[0.014(700\text{m}/0.4\text{m}) + 1.5](V^2/2g)$; V = 2.61 m/sec. New N_R = $1.04 \text{ x } 10^6$; & new **f = 0.014** ok

Thus, **Q** = V·A = (2.61 m/s)[π(0.2m)^2] = **0.328 m^3/sec**

b) For cast iron: e/D = 0.26mm/400mm = 0.00065; thus; **f = 0.018** and solving energy eq'n

9 m = $[0.018(700\text{m}/0.4\text{m}) + 1.5](V^2/2g)$; V = 2.31 m/sec. Now we can solve for Reynolds number,

$N_R = DV/\nu = [(0.40)(2.31)]/(1.0\text{x}10^{-6}) = 9.24 \text{ x } 10^5$; From Moody; new **f = 0.018**; ck

Thus, **Q** = V·A = (2.31 m/s)[π(0.2m)^2] = **0.290 m^3/sec**

c) For smooth concrete: e/D = 0.18mm/400mm = 0.00045; thus; **f = 0.0165** and solving energy eq'n

9 m = $[0.0165(700\text{m}/0.4\text{m}) + 1.5](V^2/2g)$; V = 2.41 m/sec. Now we can solve for Reynolds number,

$N_R = DV/\nu = [(0.40)(2.41)]/(1.0\text{x}10^{-6}) = 9.64 \text{ x } 10^5$; From Moody; new **f = 0.017**; and the new V is

9 m = $[0.017(700\text{m}/0.4\text{m}) + 1.5](V^2/2g)$; V = 2.38 m/sec. New N_R = $9.52 \text{ x } 10^6$; & new **f = 0.017** ok

Thus, **Q** = V·A = (2.38 m/s)[π(0.2m)^2] = **0.299 m^3/sec**

Lowest capacity: cast iron. Highest capacity: commercial steel. **% Gain** = (0.328-0.290)/0.290 **= 13.1%**

4.1.9 Applying the energy equation; $\frac{V_A^2}{2g}+\frac{P_A}{\gamma}+h_A=\frac{V_B^2}{2g}+\frac{P_B}{\gamma}+h_B+h_L$; where $V_A = V_B = P_A = P_B = 0$,

$h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$, $K_b = 0.17$, $K_d = 1.0$ (exit coef.), and $h_E = (V_S - V_L)^2/2g$ where V_S & V_L are the small & large pipe velocities. Thus, $h_A - h_B = [f(L/D)_S + \sum K](V_S^2/2g) + h_E + [f(L/D)_L + \sum K](V_L^2/2g)$;

60 ft = $[f_S(1000/0.667)+0.5](V_S^2/2g) + (V_S - V_L)^2/2g + [f_L(1000/1.33) + 4(0.17) + 1.0](V_L^2/2g)$

Based on the continuity equation: $(A_S)(V_S) = (A_L)(V_L)$; $V_S = [(D_L)^2/(D_S)^2]\cdot(V_L) = 4V_L$; and now

60 ft = $[f_S(1000/0.667)+0.5][(4V_L)^2/2g] + (3V_L)^2/2g + [f_L(1000/1.33) + 4(0.17) + 1.0](V_L^2/2g)$

$e/D_S = 0.00085/0.667 = 0.00127$; $e/D_L = 0.00085/1.33 = 0.000639$; Assuming complete turbulence:

from Moody; $\mathbf{f_S = 0.021}$ and $\mathbf{f_L = 0.018}$; and substituting into the energy equation yields;

60 ft = $[0.021(1000/0.667)+0.5][(4V_L)^2/2g] + (3V_L)^2/2g + [0.018(1000/1.33) + 4(0.17) + 1.0](V_L^2/2g)$

$\mathbf{V_L}$ = 2.69 ft/sec, and $V_S = 4V_L$ = 10.8 ft/sec. Now determine the Reynolds numbers and check f;

$N_R = DV_S/\nu = [(0.667 \text{ ft})(10.8 \text{ ft/s})]/(1.08 \times 10^{-5} \text{ ft}^2/\text{s}) = 6.67 \times 10^5$; From Moody; $\mathbf{f_S = 0.021}$;

$N_R = DV_L/\nu = [(1.33 \text{ ft})(2.69 \text{ ft/s})]/(1.08 \times 10^{-5} \text{ ft}^2/\text{s}) = 3.31 \times 10^5$; From Moody; $\mathbf{f_L = 0.0185}$; thus

60 ft = $[0.021(1000/0.667)+0.5][(4V_L)^2/2g] + (3V_L)^2/2g + [0.0185(1000/1.33) + 4(0.17) + 1.0](V_L^2/2g)$

$\mathbf{V_L}$ = 2.68 ft/sec, $V_S = 4V_L$ = 10.7 ft/sec, and $\mathbf{Q} = (V_L)(A_L) = (2.68)[\pi\cdot(0.667)^2]$ **= 3.75 cfs**

For a 16-inch line throughout: 60 ft = $[f_L(2000/1.33) + 0.5 + 4(0.17) + 1.0](V_L^2/2g)$; Try $\mathbf{f_L = 0.018}$

60 ft = $[0.018(2000/1.33) + 0.5 + 4(0.17) + 1.0](V_L^2/2g)$; $\mathbf{V_L}$ = 11.5 ft/sec; $N_R = 1.42 \times 10^6$; f is ok

New $\mathbf{Q} = (V_L)(A_L) = (11.5)[\pi\cdot(0.667)^2]$ **= 16.1 cfs**; the **capacity increases by 429%.**

4.1.10 Applying the energy equation; $\frac{V_A^2}{2g}+\frac{P_A}{\gamma}+h_A=\frac{V_B^2}{2g}+\frac{P_B}{\gamma}+h_B+h_L$; where $V_A = V_B = P_A = P_B = 0$,

$h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$, $K_d = 1.0$ (exit coef.), $h_A - h_B$ = 18.4 m, and $V = Q/A = 0.0727/D^2$. Thus, 18.4 m = $[f(600\text{m}/D) + 1.5]\cdot[(0.0727/D^2)^2/2g]$. Assume D = 0.2 m, thus V = 1.82 m/s, e/D = 0.36/200 = 0.0018; $N_R = DV/\nu = [(0.2)(1.82)]/(1.00\times10^{-6}) = 3.64 \times 10^5$; and $\mathbf{f = 0.023}$; solving energy e'qn for new D;

18.4 m = $[0.023(600\text{m}/D) + 1.5]\cdot[(0.0727/D^2)^2/2g]$; D = 0.183 m which differs from the trial size, thus V = 2.17 m/s, e/D = 0.00197; $N_R = [(0.183)(2.17)]/(1.00\times10^{-6}) = 3.97 \times 10^5$; and new $\mathbf{f = 0.0235}$; and

18.4 m = $[0.0235(600\text{m}/D) + 1.5]\cdot[(0.0727/D^2)^2/2g]$; D = 0.184 m; close enough so **D = 0.184 m**

Without minor losses; 18.4 m = $[0.0235(600\text{m}/D)]\cdot[(0.0727/D^2)^2/2g]$; **D = 0.183 m**; essentially the same.

Note: The solution process is a little easier without minor losses; solve this first and add minor losses later.

4.1.11 Applying the energy equation; $\frac{V_A^2}{2g}+\frac{P_A}{\gamma}+h_A=\frac{V_B^2}{2g}+\frac{P_B}{\gamma}+h_B+h_L$; where $V_A = V_B = P_A = P_B = 0$,

$h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$, $K_v = 10$, $K_d = 1.0$ (exit coef), $h_A - h_B$ = 4.6 ft, $V = Q/A = 3.18/D^2$. Thus, 4.6 ft = $[f(75 \text{ ft}/D) + 11.5]\cdot[(3.18/D^2)^2/2g]$. Assume D = 1.0 ft, thus V = 3.18 ft/s, e/D = 0.00015/1 = 0.00015; $N_R = DV/\nu = [(1.0)(3.18)]/(1.69\times10^{-5}) = 1.88 \times 10^5$; and $\mathbf{f = 0.018}$; solve energy e'qn for new D;

4.6 ft = $[0.018(75 \text{ ft}/D) + 11.5]\cdot[(3.18/D^2)^2/2g]$. D = 0.82 ft; differs from trial size, so V = 4.73 ft/s, e/D = 0.000183; $N_R = [(0.82)(4.73)]/(1.69\times10^{-5}) = 2.30 \times 10^5$; new $\mathbf{f = 0.0165}$; solve energy eq'n. **D = 0.82 ft.** ok

4.1.12 Apply the energy eq'n; $\frac{V_A^2}{2g}+\frac{P_A}{\gamma}+h_A=\frac{V_B^2}{2g}+\frac{P_B}{\gamma}+h_B+h_L$; where $V_A = V_B$; $h_A = h_B$; and $h_L = h_f$.

Also, $(P_A – P_B)/\gamma$ = (0.05 m of Hg)(13.6/1.1) = 0.618 m (pressure head for the water/glycerin solution). In addition, Q = 0.0000833 m^3/s; and V = Q/A = $0.000106/D^2$. Substituting into the energy eq'n, 0.618 m = $[f(2.5/D)]\cdot[(0.000106/D^2)^2/2g]$. Assume D = 10mm (0.01m), thus V = 1.06 m/s, e/D = 0.003/10 = 0.0003; $N_R = DV/\nu = [(0.01)(1.06)]/(1.03\text{x}10^{-5})$ = 1,030; laminar flow f = 64/1030 **= 0.0621**; solving energy e'qn for new D; 0.618 m = $[0.0621(2.5/D)]\cdot[(0.000106/D^2)^2/2g]$. D = 0.0108 m which differs from the trial size, thus V = 0.909 m/s, $N_R = [(0.0108)(0.909)]/(1.03\text{x}10^{-5})$ = 953; new f = 64/953 **= 0.0672**; and 0.618 m = $[0.0672(2.5/D)]\cdot[(0.000106/D^2)^2/2g]$. **D = 0.0109 m (10.9 mm)**

4.1.13 Apply the energy eq'n; $\frac{V_A^2}{2g}+\frac{P_A}{\gamma}+h_A=\frac{V_1^2}{2g}+\frac{P_1}{\gamma}+h_1+h_L$; where $V_A = 0$; $P_A/\gamma = P_0/\gamma$; $V_1 = V$ (pipe V);

P_1/γ = (39,300 N/m^2)/(9790 N/m^3) = 4.01 m; $h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$, $K_v = 12.0$. Therefore, P_0/γ + 10m = 4.01m + $[1 + f(10/0.102\text{m}) + 12.5]\cdot(V^2/2g)$ + 7m. Assuming complete turbulence; e/D = 0.045mm/102mm = 0.000441; From Moody; **f = 0.0165**; and substituting into the energy eq'n, $P_0/\gamma = 1.01 + 15.1(V^2/2g)$ Since there are too many unknowns, apply the energy eq'n for A to B:

$\frac{V_A^2}{2g}+\frac{P_A}{\gamma}+h_A=\frac{V_B^2}{2g}+\frac{P_B}{\gamma}+h_B+h_L$; where $V_A = V_B = 0$; $P_A/\gamma = P_0/\gamma$; $P_B/\gamma = 0$; h_A = 10 m; h_B = 7.5 m;

$h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$, $K_v = 12.0$, $K_{v'} = 10.0$, $K_d = 1.0$. Again substitute into the energy eq'n, P_0/γ + 10m = 7.5 m + $[f(40/0.102\text{m}) + 23.5]\cdot(V^2/2g)$; and assuming complete turbulence with **f = 0.0165**; $P_0/\gamma = -2.5\text{m} + 30.0(V^2/2g)$; Equating the values of P_0/γ from the two energy expressions yields V = 2.15 m/sec. Now, $N_R = DV/\nu = [(0.102)(2.15)]/(1.00\text{x}10^{-6}) = 2.19 \text{ x } 10^5$; and new **f = 0.0185**; thus $P_0/\gamma = 1.01 + 15.3(V^2/2g)$ and $P_0/\gamma = -2.5\text{m} + 30.8(V^2/2g)$; equating and solving yields V = 2.11 m/s Substituting this into either P_0/γ expression; P_0/γ = 4.48 m; and $\mathbf{P_0}$ = (4.48 m)(9790 N/m2) **= 43.9 kPa**

4.1.14 Replace the parallel pipes with an equivalent pipe letting D = 2 ft (to match AB & CD); C_{HW} = 100. Thus, $[(D_E^{4.87}C_E^{1.85})/L_E]^{1/1.85} = [(D_1^{4.87}C_1^{1.85})/L_1]^{1/1.85} + [(D_2^{4.87}C_2^{1.85})/L_2)]^{1/1.85}$ Therefore, $[(2^{4.87}\cdot 100^{1.85})/L_E)]^{1/1.85} = [(1^{4.87}\cdot 100^{1.85})/2800]^{1/1.85} + [(1.5^{4.87}\cdot 100^{1.85})/3000]^{1/1.85}$**;** thus $L_{E(BC)}$ = 6920 ft. From the energy eq'n; $h_{fAD} = h_A - h_D$ = 130 ft; from equivalent pipe theory; $h_{fAD} = h_{fAB} + h_{fBC} + h_{fCD}$; thus $h_{fAD} = 130 = [(4.73\cdot 3000)/(2^{4.87}\cdot 100^{1.85})]Q^{1.85} + [(4.73\cdot 6920)/(2^{4.87}\cdot 100^{1.85})]Q^{1.85} + [(4.73\cdot 2500)/(2^{4.87}\cdot 100^{1.85})]Q^{1.85}$

Q = 22.8 cfs, which is the discharge between the reservoirs and **also the discharge in pipes AB and CD**. To determine the total head at B and C we again use the expression: $h_f = [(4.73\cdot L)/(D^{4.87}\cdot C^{1.85})]Q^{1.85}$; thus $h_{fAB} = [(4.73\cdot 3000)/(2^{4.87}\cdot 100^{1.85})](22.8)^{1.85}$ = 31.5 ft; therefore, $\mathbf{H_B} = h_A - h_{fAB}$= 230 ft – 31.5 ft **= 198.5 ft** $h_{fBC} = [(4.73\cdot 6920)/(2^{4.87}\cdot 100^{1.85})](22.8)^{1.85}$ = 72.6 ft; thus, $\mathbf{H_C} = h_B - h_{fBC}$= 198.5ft – 72.6ft **= 125.9 ft** Finally, the discharge in pipes BC1 and BC2 can be found using the headloss from B to C (h_{fBC}); $h_{fBC1} = 72.6 = [(4.73\cdot 2800)/(1^{4.87}\cdot 100^{1.85})]Q^{1.85}$; $\mathbf{Q_{BC1}}$ **= 6.00 cfs**; and $\mathbf{Q_{BC2}} = Q_{BC} - Q_{BC1}$ = 22.8 – 6.00 **= 16.8 cfs**

4.1.15 Replace the parallel pipes with an equivalent pipe letting D = 2 ft (to match AB & CD); C_{HW} = 100. Thus, $[(D_E^{4.87}C_E^{1.85})/L_E]^{1/1.85} = [(D_1^{4.87}C_1^{1.85})/L_1]^{1/1.85} + [(D_2^{4.87}C_2^{1.85})/L_2)]^{1/1.85}$ Therefore, $[(2^{4.87}\cdot 100^{1.85})/L_E)]^{1/1.85} = [(1^{4.87}\cdot 100^{1.85})/2800]^{1/1.85} + [(1.5^{4.87}\cdot 100^{1.85})/3000]^{1/1.85}$**;** thus $L_{E(BC)}$ = 6920 ft.

From the energy eq'n; $h_{fAD} = h_A - h_D$ = 130 ft; from equivalent pipe theory; $h_{fAD} = h_{fAB} + h_{fBC} + h_{fCD}$; thus $h_{fAD} = 130 = [(4.73\cdot 3000)/(2^{4.87}\cdot 100^{1.85})]Q^{1.85} + [(4.73\cdot 6920)/(2^{4.87}\cdot 100^{1.85})](Q+8)^{1.85} + [(4.73\cdot 2500)/(2^{4.87}\cdot 100^{1.85})]Q^{1.85}$

Q = 18.0 cfs, which is the discharge between the reservoirs and **also the discharge in pipes AB and CD.**

To determine the total head at B and C we again use the expression: $h_f = [(4.73\cdot L)/(D^{4.87}\cdot C^{1.85})]Q^{1.85}$; thus

$h_{fAB} = [(4.73\cdot 3000)/(2^{4.87}\cdot 100^{1.85})](18.0)^{1.85}$ = 20.3 ft; therefore, $\mathbf{H_B} = h_A - h_{fAB}$ = 230 ft - 20.3 ft **= 209.7 ft**

$h_{fBC} = [(4.73\cdot 6920)/(2^{4.87}\cdot 100^{1.85})](26.0)^{1.85}$ = 92.6 ft; thus, $\mathbf{H_C} = h_B - h_{fBC}$ = 209.7ft - 92.6ft = **117.1 ft**

Finally, the discharge in pipes BC1 and BC2 can be found using the headloss from B to C (h_{fBC});

$h_{fBC1} = 92.6 = [(4.73\cdot 2800)/(1^{4.87}\cdot 100^{1.85})]Q^{1.85}$; $\mathbf{Q_{BC1}}$ = **6.84 cfs**; and $\mathbf{Q_{BC2}} = Q_{BC} - Q_{BC1}$ = 26.0 – 6.84 **= 19.2 cfs**

4.2.1 Balancing energy between the upstream reservoir and the summit, where cavitation is most likely to occur, would yield two unknowns, the pipe velocity and the pressure at the summit. Therefore, balance energy between the upstream reservoir (A) and the downstream reservoir (B) to determine the velocity. Thus,

$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$; where $V_A = V_B = 0$; $P_A = P_B = 0$; $h_A - h_B$ = 25 m; and head losses are

$h_L = h_f + [\sum K](V)^2/2g$; where $K_e = 0.5$, $K_{v1} = 0.15$, $K_d = 1.0$ (exit coef.). Therefore,

25 m = [f(300m/0.20m) + 1.65](V^2/2g); where V is pipe V. Assume complete turbulence for the first trial:

For ductile iron: e/D = 0.12mm/200mm = 0.0006; thus; **f ≈ 0.018** and solving energy eq'n

25 m = [0.018(300m/0.20m) + 1.65](V^2/2g); V = 4.14 m/sec. Now we can solve for Reynolds number,

$N_R = DV/\nu = [(0.20)(4.14)]/(1.00\times10^{-6}) = 8.28 \times 10^5$; From Moody; new **f = 0.018**; OK

Now balancing energy from A to the summit; $\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_S^2}{2g} + \frac{P_S}{\gamma} + h_S + h_L$; $V_A = P_A = 0$; $V_S = V$;

$h_A = \Delta s$ = 7.0 m (datum at h_s); and thus 7.0 m = $(4.14)^2/2g + P_S/\gamma$ + [0.5 + 0.018(150/0.20)]$(4.14)^2/2g$;

$\mathbf{P_S/\gamma}$ **= -6.10 m (> -10.1 m; no cavitation concerns; see paragraph prior to Example 4.4 in book)**

4.2.2 Applying the energy equation from the reservoir surface (A) to the outlet of the siphon (B) yields;

$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$; where $V_A = 0$; $P_A = P_B = 0$; V_B = V (siphon V); and

h_L = 0.8 m + 1.8 m = 2.6 m. Also, $h_A - h_B$ = 5.0 m, thus 5m = V^2/2g + 2.6m; V = 6.86 m/s

Thus, **Q** = V·A = (6.86 m/s)[π(0.06m)^2] = **0.0776 m^3/sec (77.6 L/s)**

Balancing energy between the upstream reservoir and the summit yields;

$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_S^2}{2g} + \frac{P_S}{\gamma} + h_S + h_L$; where $V_A = 0$; $P_A/\gamma = 0$; h_L = 0.8 m; $h_A - h_S$ = -2.0 m; and thus

-2 m = $(6.86 \text{ m/s})^2/2g + P_S/\gamma$ + 0.8 m; P_S/γ = -5.20 m. Thus $\mathbf{P_S}$ = (-5.20 m)(9,790 N/m^2) **= -50.9 kPa**

4.2.3 Balancing energy between the upstream reservoir and the summit, where cavitation is most likely to occur, would yield two unknowns, the velocity and the pressure at the summit. Therefore, balance energy between the upstream reservoir (A) and the downstream reservoir (B) to determine the velocity. Thus,

$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$; where $V_A = V_B = 0$; $P_A = P_B = 0$; $h_A - h_B = 50$ ft; & head losses are

$h_L = h_f + [\sum K](V)^2/2g$; where $K_e = 0.5$, $K_d = 1.0$ (exit coef.). Therefore,

50 ft = $[f(200/2.0) + 1.5](V^2/2g)$; where V is pipe V. Assume complete turbulence for the first trial:

For rough concrete: e/D = 0.002ft/2.0ft = 0.001; thus; **f ≈ 0.02** and solving energy eq'n

50ft = $[0.02(200/2.0) + 1.5](V^2/2g)$; V = 30.3 ft/sec. Now we can solve for Reynolds number,

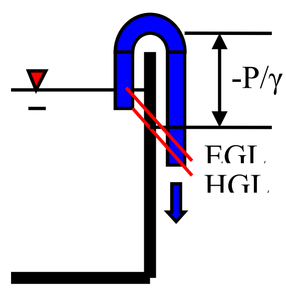
$N_R = DV/\nu = [(2.0)(30.3)]/(1.08\times10^{-5}) = 5.61 \times 10^6$; From Moody; new **f = 0.02**; OK

Now balancing energy from A to the summit yields;

$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_S^2}{2g} + \frac{P_S}{\gamma} + h_S + h_L$; where $V_A = P_A = 0$; $V_S = V$; $h_A - h_s = -5.0$ ft $K_e = 0.5$; and thus

-5.0 ft = $(30.3)^2/2g + P_S/\gamma + [0.5 + 0.02(60/2.0)](30.3)^2/2g$; solving yields $\mathbf{P_S/\gamma = -34.9}$ **ft** which is below the vapor pressure of water of 0.344 lb/in^2 (at 68°F, found in front jacket of book and Table 1.1) which equates to a gage pressure of -14.4 lb/in^2 ($P_v - P_{atm}$ = 0.344 lb/in^2 - 14.7 lb/in^2) and converts to a pressure head (gage) of -33.3 ft of water $[(P_v - P_{atm})/\gamma$ = (-14.4 lb/in^2 x 144 in^2/ft^2)/62.3 lb/ft^3], **cavitation will occur.**

4.2.4 Yes, all siphons encounter negative pressure at their summits. By definition, negative pressure will occur in any pipeline where the center line of the pipe rises above the HGL. Since a siphon carries water to a higher elevation than the supply reservoir, negative pressure is inevitable as can be seen in the EGL and the HGL sketches below.



4.2.5 Applying the energy equation just upstream (A) and just downstream (B) of the confusor yields;

$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$; where $h_A = h_B$; P_A = 84 kPa; P_B/γ = - 8 m; h_L = 0 (smooth); and

$V_A = Q/A$ = (0.440 m^3/s)/[(π/4)(0.40 m)^2] = 3.50 m/sec; thus

(3.50m/s^2)^2/2g + [(84 kN/m^2)/(9.79 kN/m^2)] = $V_B^2/2g$ - 8m; $V_B^2/2g$ = 17.2 m; V_B = 18.4 m/s

V_B = 18.4 m/s = Q/A_B = (0.440 m^3/s)/[(π/4)$(D_B)^2$]; $\mathbf{D_B}$ **= 0.174 m = 17.4 cm**

4.2.6 Balancing energy between the supply reservoir (A) and reservoir (B) yields

$H_A + H_p = H_B + h_L$; where $H_B - H_A$ = 50 m; and $h_L = h_f + [\sum K](V)^2/2g = 11[(V)^2/2g]$; Therefore,

$H_p = 50 \text{ m} + 11[(V)^2/2g]$ ➔ Equation (a)

Balancing energy between the reservoir (A) and the inlet of the pump (C) yields

$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_C^2}{2g} + \frac{P_C}{\gamma} + h_C + h_L$; where V_A= 0; V_C = V (pipe velocity); P_A = 0; P_C/γ = -6 m; and

$h_L = 4[(V)^2/2g]$; Therefore, $4 \text{ m} = V^2/2g - 6 \text{ m} + 4[(V)^2/2g]$; therefore, $V^2/2g$ = 2 m; Substituting into Equation (a) yields: $\mathbf{H_p}$ = 50 m + 11[2 m] **= 72 m** Note: The pressure head the pump delivers must overcome the elevation difference between the two reservoirs (50 m) and the losses (22 m).

Sketch the EGL and HGL such that a) both start & end at the reservoir surfaces, b) the minor losses (entrance, bends, exit) are accounted for with an abrupt drop in the EGL and HGL; c) the EGL and HGL are parallel lines, and d) the pump adds a significant boost (abrupt rise) to the EGL.

4.2.7 Balancing energy from reservoir A to the suction side of the pump yields;

$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_S^2}{2g} + \frac{P_S}{\gamma} + h_S + h_L$; where $V_A = P_A = 0$; V_S = V; $h_A - h_s$ = 4.0 m; and

$h_L = h_f + [\sum K](V)^2/2g$; where K_e = 0.5; and V = Q/A = $5/[\pi/4(0.8)^2]$ = 9.95 m/s. Therefore,

$4.0\text{m} = (9.95)^2/2g + P_S/\gamma + [f(L/0.8) + 0.5](9.95)^2/2g$; To determine f; e/D = 0.60mm/800mm = 0.00075

$N_R = DV/\nu = [(0.80)(9.95)]/(1.00\text{x}10^{-6}) = 7.96 \text{ x } 10^6$; From Moody; **f = 0.0185**; and the vapor pressure of water at 20˚C is 2,335 N/m^2 (Table 1.1). This is an absolute pressure; gage pressure is found by subtracting atmospheric pressure ($1.014 \text{ x } 10^5$ N/m^2) or in terms of pressure head,

$(P_v - P_{atm})/\gamma$ = (2,335 – 101,400)/9790 = -10.1 m. Substituting back into the energy equation;

$4.0\text{m} = (9.95)^2/2g - 10.1\text{m} + [0.0185(L/0.8) + 0.5](9.95)^2/2g$; thus **L = 56.0 m**

4.2.8 Balancing energy between the upstream side of the pump (2) and the receiving tank (3) yields

$\frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 = \frac{V_3^2}{2g} + \frac{P_3}{\gamma} + h_3 + h_L$; where V_3= 0; $P_3/\gamma = [(32.3 \text{ lb/in}^2)(144 \text{ in}^2/\text{ft}^2)]/(62.3\text{lb/ft}^2)$ = 74.7 ft;

h_3 = 20 ft; h_2 = 10 ft; $V_2 = Q/A = (8 \text{ ft}^3/\text{s})/[(\pi/4)(1.0\text{ft})^2]$ = 10.2 ft/sec; and head losses are

$h_L = h_f + [\sum K](V)^2/2g$; where K_d = 1.0 (exit coef.). Thus, $h_L = [f(130/1.0) + 1.0](V^2/2g)$; & V is pipe V.

For ductile iron: e/D = 0.0004ft/1.0ft = 0.0004; $N_R = DV/\nu = [(1.0)(10.2)]/(1.08\text{x}10^{-5}) = 9.44 \text{ x } 10^5$;

From Moody; **f = 0.017**; and $h_L = [0.017(130/1.0) + 1.0]\cdot[(10.2)^2/2g)$ = 5.18 ft; now from the energy eq'n;

$(10.2)^2/2g + P_2/\gamma + 10$ ft = 74.7 ft + 20 ft + 5.18 ft; therefore, $\mathbf{P_2/\gamma}$ **= 88.3 ft and** $\mathbf{P_2}$ **= 38.2 psi**

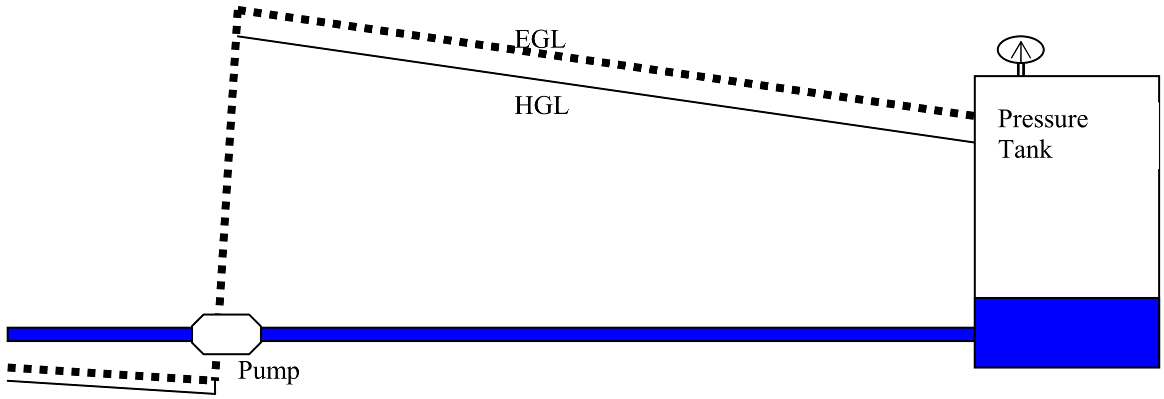
Now balancing energy from the suction side of the pump (1) to the discharge side (2) yields;

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + H_P = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2$; where V_2 = 10.2 ft/s; $V_1 = Q/A = (8 \text{ ft}^3/\text{s})/[(\pi/4)(1.33\text{ft})^2]$ = 5.76 ft/sec;

(Problem 4.2.8 – continued)

P_2/γ = 88.3 ft; $h_1 = h_2$; H_P = 111 ft; and thus $(5.76)^2/2g + P_1/\gamma$ + 111 ft = $(10.2)^2/2g$ + 88.3 ft; yielding

P_S/γ = -21.6 ft which is above the vapor pressure of water of 0.344 lb/in^2 (at 68˚F, found in front jacket of the book and Table 1.1) or a gage pressure of -14.4 lb/in^2 ($P_v - P_{atm}$ = 0.344 lb/in^2 - 14.7 lb/in^2). This converts to a pressure head (gage) of -33.3 ft of H_2O [$(P_v - P_{atm})/\gamma$ = (-14.4 lb/in^2 x 144 in^2/ft^2)/62.3 lb/ft^3], therefore **cavitation will not occur.** The EGL and HGL sketch appears below.



Sketch the EGL and HGL such that a) the EGL and HGL start below the pump, b) they are separated by the velocity head, c) the pump adds a significant boost (abrupt rise) to the EGL, d) the EGL slopes toward the tank based on the friction loss, e) the HGL runs parallel to and below the EGL separated by the velocity head, and f) the HGL ends at the tank a distance above the water surface due to the pressure head.

4.2.9 Balancing energy between the upstream reservoir (A) and the downstream reservoir (B) yields

$H_A + H_p = H_B + h_L$; where $H_A - H_B$ = 30 m; $h_L = h_f + [\sum K](V)^2/2g$; K_e = 0.5, K_d = 1.0 (exit coef). Thus,

30 m + H_P = [f(2000m/0.40m) + 1.5](V^2/2g); where V is pipe V. If the flow is to be doubled;

V = 2(3.09 m/s) = 6.18 m/sec (based on doubling the velocity from Example 4.4). Now

$N_R = DV/\nu$ = [(0.40)(6.18)]/(1.31x10^{-6}) = 1.89 x 10^6. From Moody; **f = 0.0105**; and solving energy eq'n

30 m + H_P = [0.0105(2000m/0.40m) + 1.5]·[(6.18)^2/2g); **H_P = 75.1 m**

4.2.10 Balancing energy between the summit and the downstream reservoir (B), yields,

$\frac{V_S^2}{2g} + \frac{P_S}{\gamma} + h_S = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$; where $V_B = P_B$ = 0; $h_S - h_B$ = 22 m; P_S/γ = -6m; and head losses are

$h_L = h_f + [\sum K](V)^2/2g$; w/$K_v$ = 0.15; K_d = 1.0 (exit). Thus, 22m + (-6m) = [f(150m/0.20m)+1.15](V^2/2g);

Assume complete turbulence: e/D = 0.12mm/200mm = 0.0006; thus; **f ≈ 0.018** and solving energy eq'n

16m = [0.018(150m/0.20m) + 1.15](V^2/2g); V = 4.63 m/sec. Now we can solve for Reynolds number,

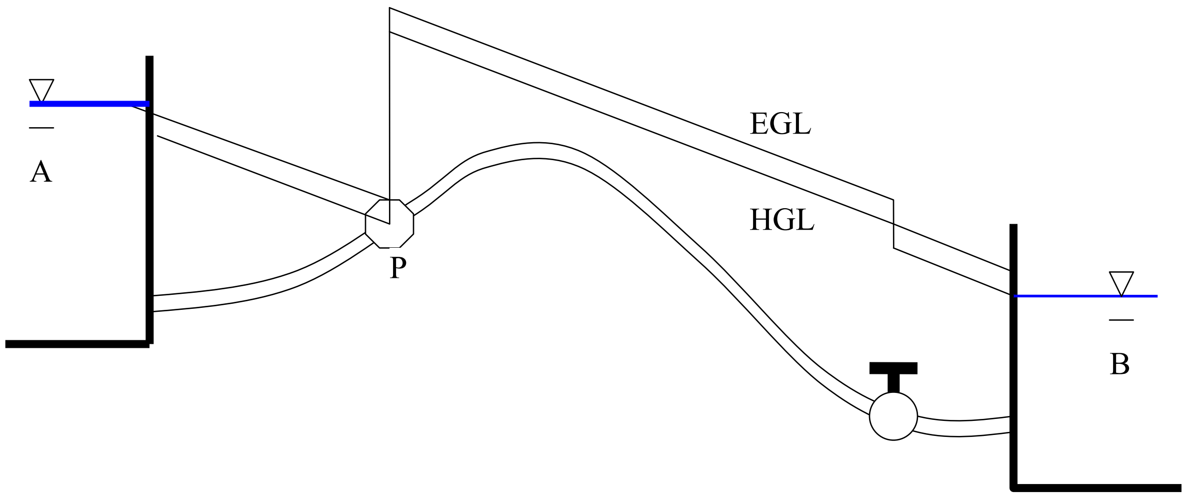
$N_R = DV/\nu$ = [(0.20)(4.63)]/(1.00x10^{-6}) = 9.26 x 10^5; From Moody; new **f = 0.018**; OK

Now balancing energy from A to B; $H_A + H_p = H_B + h_L$; $H_A - H_B$ = 25m; K_e = 0.5; K_v = 0.15; K_d = 1.0

25 m + H_P = [0.018(300m/0.20m) + 1.65]·[(4.63)^2/2g); **H_P = 6.30 m**

(Problem 4.2.10 – continued)

Start drawing your EGL and HGL from the reservoir surface (on the left) and move to the right accounting for head losses along the way. Note that a) the EGL and HGL both start and end at the reservoir surfaces, b) the minor losses (entrance, valve, and exit) are accounted for with an abrupt drop in the EGL and HGL; c) the separation distance between the EGL and HGL accounts for the velocity head, $V^2/2g$, so they are parallel lines, d) the slope of the EGL represents the friction loss over length, and e) the pump adds energy shown as an abrupt rise in the EGL and HGL.



4.2.11 The most likely location for negative pressure in a pipeline is at the highest point in the system. Since points 4 and 5 are at the same height, choose point 5 since it is further from reservoir A and therefore more head losses have accrued. Now applying the energy equation between reservoir A and point 5 yields

$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + h_A = \frac{V_5^2}{2g} + \frac{P_5}{\gamma} + h_5 + h_L$; where $V_A = 0$, $V_5 = V$ (pipeline V), $h_A = 10$ m, and $h_5 = 9.5$ m.

Also, $P_A/\gamma = P_0/\gamma$ and $P_5/\gamma = 0$ (to meet the condition of the problem), and $h_L = h_f + [\sum K](V)^2/2g$; where $K_e = 0.5$, $K_V = 12.0$, e/D = 0.045mm/102mm = 0.000441, assume f = 0.02 (not likely full turbulence) thus,

$P_0/\gamma + 10\text{m} = V^2/2g + 9.5\text{ m} + [0.5 + 0.02(32.5/0.102)+12]\cdot[V^2/2g]$ and $P_0/\gamma = 19.9[V^2/2g] - 0.5$ (eq'n 1)

Balance energy from A to B to get V; $\frac{V_A^2}{2g} + \frac{P_0}{\gamma} + h_A = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + h_B + h_L$ where $V_A = V_B = P_B = 0$, and

$P_0/\gamma + 10\text{m} = 7.5\text{ m} + [0.5 + 0.02(40/0.102) + 12 + 10 + 1.0]\cdot[V^2/2g]$; $P_0/\gamma = 31.3[V^2/2g] - 2.5$ (eq'n 2)

Solving equations (1) and (2) simultaneously yields; V = 1.86 m/sec; thus

$N_R = DV/\nu = [(0.102)(1.86)]/(1.00\times10^{-6}) = 1.90 \times 10^5$; From Moody; new **f = 0.0185**; thus

$P_0/\gamma = 19.4[V^2/2g] - 0.5$ (eq'n 1) and $P_0/\gamma = 30.8[V^2/2g] - 2.5$ (eq'n 2) and new V = 1.86 (and f is OK)

Therefore; $P_0/\gamma = 19.4[(1.86)^2/2g] - 0.5 = 2.92$ m; and $\mathbf{P_0} = (2.92\text{ m})(9{,}790\text{ N/m}^3) =$ **28.6 kPa**

4.3.1 The middle reservoir's water surface elevation is a good first estimate for the total energy level at the junction (P). It reduces the number of computations in the first pass through the problem, and the result will indicate whether a higher or lower energy level is needed for the next iteration. Thus, the direction of flow (to or from) the middle reservoir is established. The following spreadsheet solves the 3-reservoir problem.

Three Reservoir Problem (Example 4.6)

Reservoir Water Surface Elevations			Total Head at Junction (m)	Pipe Diameters		
WS1 =	120	M	P = 99	D1 =	0.30	m
WS2 =	100	M	Trial until ∑Qs	D2 =	0.50	m
WS3 =	80	M	Balance	D3 =	0.40	m
Pipe Lengths			Water Temp.	Pipe Roughnesses		
L1 =	1000	M	T(˚C) = 20	e1 =	0.00060	m
L2 =	4000	M	Viscosity	e2 =	0.00060	m
L3 =	2000	M	ν = 0.000001	e3 =	0.00060	m

Pipe#	e/D	Turb. F	L/D	h_f (m)	Velocity (m/sec)	N_R	Revised f *	Q (m^3/sec)
1	0.00200	0.0234	3333	21.0	2.30	6.89E+05	0.0238	
2	0.00120	0.0205	8000	1.0	0.35	1.73E+05	0.0221	
3	0.00150	0.0217	5000	19.0	1.85	7.41E+05	0.0221	
(using revised f value)					revised V	new N_R	new f	
1					2.28	6.84E+05	0.0238	0.161
2					0.33	1.66E+05	0.0222	0.065
3					1.84	7.34E+05	0.0221	0.231

* Revised "f" using the Swamee-Jain Equation

If P < WS2	∑Q =	-0.004	Flows Balance
If P > WS2	∑Q =	-0.135	

4.3.2 As a first guess, try P = 5150 ft (elevation of middle reservoir), or 5150.01 to avoid division by zero.

Three Reservoir Problem (Prob 4.3.2)

Reservoir Water Surface Elevations			Total Head at Junction (ft)	Pipe Diameters		
WS1 =	5200	Ft	P = 5150.01	D1 =	4.00	ft
WS2 =	5150	Ft	Trial until ∑Qs	D2 =	3.00	ft
WS3 =	5100	Ft	Balance	D3 =	5.00	ft
Pipe Lengths			Water Temp.	Pipe Roughnesses		
L1 =	6000	Ft	T(˚F) = 68	e1 =	0.00040	ft
L2 =	2000	Ft	Viscosity	e2 =	0.00040	ft
L3 =	8000	Ft	ν = 1.08E-05	e3 =	0.00040	ft

Pipe#	e/D	Turb. f	L/D	h_f (ft)	Velocity (ft/sec)	N_R	Revised f *	Q (ft^3/sec)
1	0.00010	0.0120	1500	50.0	13.38	4.96E+06	0.0124	
2	0.00013	0.0127	667	0.0	0.28	7.67E+04	0.0196	
3	0.00008	0.0115	1600	50.0	13.24	6.13E+06	0.0119	
(using revised f value)					revised V	new N_R	new f	
1					13.15	4.87E+06	0.0124	165
2					0.22	6.17E+04	0.0204	2
3					13.01	6.02E+06	0.0119	255

* Revised "f" using the Swamee-Jain Equation

If P < WS2 ∑Q = -88.5

If P > WS2 ∑Q = -91.7 To much outflow; elevation at junction is too high.

By trying a few different water surface elevations, flows were quickly balanced when P = 5141.8 ft. If the pressure head (P/γ) at the junction is 30 ft, then the elevation of the junction is P – 30 ft = **5111.8 ft** since the position head, pressure head, and velocity head must add to total head (P) at the junction. Recall that the velocity head is assumed to be negligible.

Three Reservoir Problem (Prob 4.3.2)

Reservoir Water Surface Elevations			Total Head at Junction (ft)	Pipe Diameters		
WS1 =	5200	Ft	P = 5141.8	D1 =	4.00	ft
WS2 =	5150	Ft	Trial until ∑Qs	D2 =	3.00	ft
WS3 =	5100	Ft	Balance	D3 =	5.00	ft

Pipe Lengths			Water Temp.	Pipe Roughnesses		
L1 =	6000	Ft	T(˚F) = 68	e1 =	0.00040	ft
L2 =	2000	Ft	Viscosity	e2 =	0.00040	ft
L3 =	8000	Ft	v = 0.0000108	e3 =	0.00040	ft

Pipe#	e/D	Turb. f	L/D	h_f (ft)	Velocity (ft/sec)	N_R	Revised f *	Q (ft^3/sec)
1	0.00010	0.0120	1500	58.2	14.44	5.35E+06	0.0124	
2	0.00013	0.0127	667	8.2	7.91	2.20E+06	0.0134	
3	0.00008	0.0115	1600	41.8	12.10	5.60E+06	0.0119	
(using revised f value)					revised V	new N_R	new f	
1					14.21	5.26E+06	0.0124	**179**
2					7.70	2.14E+06	0.0134	**54**
3					11.87	5.50E+06	0.0119	**233**

* Revised "f" using the Swamee-Jain Equation

If P < WS2 ∑Q = -0.2 Flows balance.

If P > WS2 ∑Q = -109.0

4.3.3 As a first guess, try P = 2080 m (elevation of middle reservoir), or 2080.01 to avoid division by zero.

Three Reservoir Problem (Problem 4.3.3)

Reservoir Water Surface Elevations			Total Head at Junction (m)		Pipe Diameters		
WS1 =	2100	M	P =	2080.01	D1 =	1.00	m
WS2 =	2080	M	Trial until ∑Qs Balance		D2 =	0.30	m
WS3 =	2060	M			D3 =	1.00	m

Pipe Lengths			Water Temp.		Pipe Roughnesses		
L1 =	5000	M	T(˚C) =	20	e1 =	0.000045	m
L2 =	4000	M	Viscosity		e2 =	0.000045	m
L3 =	5000	M	v =	0.000001	e3 =	0.000045	m

Pipe#	e/D	Turb. f	L/D	h_f (m)	Velocity (m/sec)	N_R	Revised f *	Q (m^3/sec)
1	0.000045	0.0103	5000	20.0	2.75	2.75E+06	0.0115	
2	0.000150	0.0130	13333	0.0	0.03	1.01E+04	0.0311	
3	0.000045	0.0103	5000	20.0	2.75	2.75E+06	0.0115	
(using revised f value)*					revised V	new N_R	new f	
1					2.62	2.62E+06	0.0115	**2.055**
2					0.02	6.52E+03	0.0352	**0.002**
3					2.62	2.62E+06	0.0115	**2.056**

* Revised "f" using the Swamee-Jain Equation

If P < WS2 ∑Q = 0.000

If P > WS2 ∑Q = -0.003 Flows balance.

No other trials are necessary since the flows balance when **P = 2080**, the elevation of the middle reservoir. In fact, the astute student may have guessed this based on the symmetry of the branching pipe system. However, the system design is probably not realistic since the pipe from the middle reservoir carries no water at these reservoir elevations. However, it is possible to have reservoir levels fluctuate, and note that the pipe from the middle reservoir is much smaller than the other two pipes. So even if the reservoir levels were quite different, it would not need to carry as much flow as the other two pipes. Now let's address the other questions. If the total head (elevation) of the junction is 2080 m, then **the pressure head (P/γ) at the junction is 10 m** (P – 2070 = 10 m) since the position (elevation) head, pressure head, and velocity head must add up to equal the total head (P) at the junction. Recall that the velocity head is assumed to be negligible. Finally, let's estimate the velocity head at the junction. Since all of the flow from the first pipe is passing through to the third pipe, the velocity head is roughly 0.35 m [$\mathbf{V^2/2g} = (2.62 \text{ m/s})^2/2g = \mathbf{0.350\ m}$]. The velocity is taken from the table above. You can readily see that the velocity head is not that significant even in the rare case of no mixing in the junction (i.e., pass through flow from pipe 1 to pipe 3).

4.3.4 The Hazen-Williams equation is easier to use on branching pipe systems because the roughness coefficient (C_{HW}) is not dependent on flow velocity. The spreadsheet below depicts the procedure. By trying a few different water surface elevations, flows are balanced when **P = 5141.9 ft. (Elev(J) = P – 30 = 5111.9 ft.)**

Three Reservoir Problem (Prob 4.3.4)

Reservoir Water Surface Elevations			Total Head at Junction (ft)
WS1 =	5200	ft	P = 5141.9
WS2 =	5150	ft	Trial until ∑Qs
WS3 =	5100	ft	Balance

Pipe Lengths			Pipe Diameters		
L1 =	6000	ft	D1 =	4.00	ft
L2 =	2000	ft	D2 =	3.00	ft
L3 =	8000	ft	D3 =	5.00	ft

Pipe#	C_{HW}	h_f (ft)	S*	R_h** (ft)	V*** (ft/sec)	Q (cfs)
1	140	58.1	0.00968	1.00	15.1	**190**
2	140	8.1	0.00405	0.75	7.9	**56**
3	140	41.9	0.00524	1.25	12.5	**245**

* $S = h_f/L$ (i.e., friction slope or EGL slope in this case)

If P < WS2; ∑Q = 0.5

** $R_h = D/4$ (Equation 3.26)

*** $V = 1.318CR_h^{0.63}S^{0.54}$ (Equation 3.25)

If P > WS2; ∑Q = -110.6

4.3.5 The Manning equation is easier to use on branching pipe systems because the roughness coefficient (n) is not dependent on flow velocity. The spreadsheet below depicts the procedure. By trying a few different water surface elevations, flows are balanced when **P = 5141.2 ft. (Elev(J) = P – 30 = 5111.2 ft.)**

Three Reservoir Problem (Prob 4.3.5)

Reservoir Water Surface Elevations			Total Head at Junction (ft)
WS1 =	5200	ft	P = 5141.2
WS2 =	5150	ft	Trial until ∑Qs
WS3 =	5100	ft	Balance

Pipe Lengths			Pipe Diameters		
L1 =	6000	ft	D1 =	4.00	ft
L2 =	2000	ft	D2 =	3.00	ft
L3 =	8000	ft	D3 =	5.00	ft

Pipe#	n	h_f (ft)	S*	R_h** (ft)	V*** (ft/sec)	Q (cfs)
1	0.011	58.8	0.00980	1.00	13.4	**168**
2	0.011	8.8	0.00440	0.75	7.4	**52**
3	0.011	41.2	0.00515	1.25	11.2	**221**

* $S = h_f/L$ (i.e., friction slope or EGL slope in this case)

If P < WS2; ∑Q = -0.5

** $R_h = D/4$ (Equation 3.26)

*** $V = (1.486/n)R_h^{(2/3)}S^{0.5}$ (Equation 3.29)

If P > WS2; ∑Q = 336.7

4.3.6 The energy level at the junction is 4085 m. ($V_J^2/2g + P_J/\gamma + h_J = 0 + 13.0\text{m} + 4072\text{m}$) Note that the velocity head at the junction is assumed negligible and $P_J/\gamma = (127{,}000\ \text{N/m}^2)/(9790\ \text{N/m}^3) = 13.0$ m. Now we realize that flow is from the J to the middle reservoir (2), and balancing energy would yield Q_2. Thus,

$\frac{V_J^2}{2g} + \frac{P_J}{\gamma} + h_J = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$; where $V_2 = P_2 = 0$; and $h_L = h_f = f(1000/0.20)(V^2/2g)$; where V is pipe velocity and we are ignoring minor losses. Assume complete turbulence: e/D = 0.003; thus; **f = 0.026** and $4085\text{m} = 4080\text{m} + 0.026(1000/0.2)((V_2)^2/2g)$; $V_2 = 0.869$ m/sec. Thus, $Q_2 = V_2 \cdot A_2 = 0.0273\ \text{m}^3$/sec. Now,

$\frac{V_J^2}{2g} + \frac{P_J}{\gamma} + h_J = \frac{V_3^2}{2g} + \frac{P_3}{\gamma} + h_3 + h_L$; where $V_3 = P_3 = 0$; and $h_L = h_f = f(3000/0.50)(V^2/2g)$; where V is pipe velocity; again ignoring minor losses. Assume complete turbulence: e/D = 0.0012; thus; **f = 0.0205** and $4085\text{m} = 4060\text{m} + 0.0205(3000/0.5)((V_3)^2/2g)$; $V_3 = 2.00$ m/sec. Thus, $Q_3 = V_3 \cdot A_3 = 0.393\ \text{m}^3$/sec. Now, Based on mass balance at the junction, $Q_1 = 0.027 + 0.393 = 0.420\ \text{m}^3$/s, leading to a final energy balance;

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_J^2}{2g} + \frac{P_J}{\gamma} + h_J + h_L$; where $V_1 = P_1 = 0$; and $h_L = h_f = f(2000/0.30)(V^2/2g)$; where V is pipe velocity; again ignoring minor losses. Assume complete turbulence: e/D = 0.002; thus; **f = 0.024** and $h_1 = 4085\text{m} + 0.024(2000/0.3)((V_1)^2/2g)$; where $V_1 = Q_1/A_1 = 5.94$ m/sec. Thus, $\mathbf{h_1 = 4373\ m}$

4.3.7 The flow to the junction from reservoir 1 is 75.0 cfs. Using the Hazen-Williams equation, we can determine the friction head loss and thus the energy level at the junction. Therefore, from equation (3.25)

$Q = 75.0\ \text{cfs} = VA = (1.318 C_{HW} R_h^{0.63} S^{0.54})(A) = (1.318)(150)(3\text{ft}/4)^{0.63}(h_f/8000)^{0.54}(\pi/4)(3\text{ft})^2$

$h_f = 49.7$ ft; Therefore, the energy level at the junction is 3200 – 49.7 = 3150.3 ft. Flow the reservoir 2 is

$Q = (1.318 C_{HW} R_h^{0.63} S^{0.54})(A) = (1.318)(150)(2.5\text{ft}/4)^{0.63}[(3150.3-3130)/2000]^{0.54}(\pi/4)(2.5\text{ft})^2 = 60.5$ cfs

Therefore, the flow to reservoir 3 is 75.0 – 60.5 = 14.5 cfs, and the friction head loss is found from

$Q = 14.5\ \text{cfs} = VA = (1.318 C_{HW} R_h^{0.63} S^{0.54})(A) = (1.318)(150)(2\text{ft}/4)^{0.63}(h_f/3000)^{0.54}(\pi/4)(2\text{ft})^2$

$h_f = 6.4$ ft; Therefore, the water surface elevation at reservoir 3 is 3150.3 - 6.4 **= 3143.9 ft. ≈ 3144 ft**

4.3.8 Letting the ground be the datum, assume the total energy level at the junction is 7 m (4 m above shower):

$Q_{AJ} = (1/n)A(R_h)^{2/3}S^{1/2} = (1/0.011)[(\pi/4)(0.03\text{m})^2](0.03/4)^{2/3}(1\text{m}/2\text{m})^{1/2} = 1.74$ L/sec; $Q_{BJ} = 0$; S = 0;

$Q_{JC} = (1/n)A(R_h)^{2/3}S^{1/2} = (1/0.011)[(\pi/4)(0.03\text{m})^2](0.03/4)^{2/3}(4\text{m}/5\text{m})^{1/2} = 2.20$ L/sec.

Since the outflow exceeds the inflow, assume a lower energy level at the junction, say 6.95 m.

$Q_{AJ} = (1/n)A(R_h)^{2/3}S^{1/2} = (1/0.011)[(\pi/4)(0.03\text{m})^2](0.03/4)^{2/3}(1.05\text{m}/2\text{m})^{1/2} = 1.78$ L/sec;

$Q_{BJ} = (1/n)A(R_h)^{2/3}S^{1/2} = (1/0.011)[(\pi/4)(0.03\text{m})^2](0.03/4)^{2/3}(0.05\text{m}/2\text{m})^{1/2} = 0.39$ L/sec.

$\mathbf{Q_{JC}} = (1/n)A(R_h)^{2/3}S^{1/2} = (1/0.011)[(\pi/4)(0.03\text{m})^2](0.03/4)^{2/3}(3.95\text{m}/5\text{m})^{1/2}$ **= 2.19 L/sec.**

The flows balance, so the assumed flow of **2.19 L/sec** to the shower is correct.

4.4.1 a) $\boldsymbol{P_F} = P_A - P_{AB} - P_{BC} - P_{CH} - P_{HF}$ = 489.5 – 80.3 – 119.4 – 5.9 – 116.5 **= 167.4 kPa**

This is the same pressure obtained in the example problem through a different path. It should not be surprising since energy was balanced as part of the process. Occasionally slight variations occur due to rounding error or computations end before the network is balanced completely.

b) **The lowest total energy in the system is at node F.** This is true because all flows move toward F (all network flows are incoming), and water always moves toward the point of least total energy.

c) **One possible solution is to increase some pipe sizes**, perhaps a few critical pipes where head losses are the greatest. It may also be possible to substitute newer/smoother pipes to reduce the friction loss.

d) The following spreadsheet represents computer software used to analyze Example 4.8.

Pipe Network Problem (Example 4.8)

Storage Tank Elevation (m)		Network Inflows (m^3/sec)	
A =	50.00	A =	0.300
Junction Elevations		**Network Outflows (m^3/sec)**	
All	0.00	C =	0.050
		F =	0.150
Roughness (e, in m)		G =	0.100
All	0.000260		

Pipe	*Q* (m^3/sec)	Length (m)	Diameter (m)	*e/D*	*f*	*K* (sec^2/m^5)
AB	0.200	300	0.30	0.00087	0.0190	193
AD	0.100	250	0.25	0.00104	0.0198	419
BC	0.080	350	0.20	0.00130	0.0210	1894
BG	0.120	125	0.20	0.00130	0.0210	676
GH	0.020	350	0.20	0.00130	0.0210	1894
CH	0.030	125	0.20	0.00130	0.0210	676
DE	0.100	300	0.20	0.00130	0.0210	1623
GE	0.000	125	0.15	0.00173	0.0226	3068
EF	0.100	350	0.20	0.00130	0.0210	1894
HF	0.050	125	0.15	0.00173	0.0226	3068

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1 *(clockwise)*	*AB*	0.200	193	7.74	38.7	0.205
	BG	0.120	676	9.74	81.2	0.125
	GE	0.000	3068	0.00	0.0	0.005
			$\sum h_{fc}$ =	17.48	119.9	$\equiv\sum(h_{fc}/Q_c)$
1 *(counter)*	*AD*	0.100	419	4.19	41.9	0.095
	DE	0.100	1623	16.23	162.3	0.095
			$\sum h_{fcc}$ =	20.42	204.2	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	-0.0045	

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)

2	*BC*	0.080	1894	12.12	151.5	0.077
(clockwise)	*CH*	0.030	676	0.61	20.3	0.027
			$\sum h_{fc}$ =	12.73	171.8	$\equiv\sum(h_{fc}/Q_c)$
2	*BG*	0.125	676	10.49	84.2	0.127
(counter)	*GH*	0.020	1894	0.76	37.9	0.023
			$\sum h_{fcc}$ =	11.25	122.1	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0025	

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
3	*GH*	0.023	1894	0.96	42.6	0.036
(clockwise)	*HF*	0.050	3068	7.67	153.4	0.063
			$\sum h_{fc}$ =	8.63	196.0	$\equiv\sum(h_{fc}/Q_c)$
3	*GE*	0.005	3068	0.06	14.0	-0.008
(counter)	*EF*	0.100	1894	18.94	189.4	0.087
			$\sum h_{fcc}$ =	19.00	203.3	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	-0.0130	

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1	*AB*	0.205	193	8.10	39.6	0.204
(clockwise)	*BG*	0.127	676	10.92	85.9	0.127
			$\sum h_{fc}$ =	19.01	125.5	$\equiv\sum(h_{fc}/Q_c)$
1	*AD*	0.095	419	3.82	40.0	0.096
(counter)	*DE*	0.095	1623	14.79	154.9	0.096
	EG	0.008	3068	0.22	25.9	0.008
			$\sum h_{fcc}$ =	18.83	220.8	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0003	

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2	*BC*	0.077	1894	11.37	146.7	0.080
(clockwise)	*CH*	0.027	676	0.51	18.6	0.030
			$\sum h_{fc}$ =	11.88	165.3	$\equiv\sum(h_{fc}/Q_c)$
2	*BG*	0.127	676	10.87	85.7	0.125
(counter)	*GH*	0.036	1894	2.39	67.2	0.033
			$\sum h_{fcc}$ =	13.26	153.0	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	-0.0022	

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
3	*GH*	0.033	1894	2.10	63.1	0.033
(clockwise)	*HF*	0.063	3068	12.17	193.2	0.063
	EG	0.008	3068	0.20	25.0	0.008
			$\sum h_{fc}$ =	14.48	281.4	$\equiv\sum(h_{fc}/Q_c)$
(counter)	*EF*	0.087	1894	14.34	164.8	0.087
			$\sum h_{fcc}$ =	14.34	164.8	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0002	

Pipe	Q (m^3/sec)	Q (L/sec)	Length (m)	Diameter (m)	h_f (m)	ΔP (kPa)
AB	0.2043	204.3	300	0.30	8.1	79.0
AD	0.0957	95.7	250	0.25	3.8	37.6
BC	0.0796	79.6	350	0.20	12.0	117.6
BG	0.1246	124.6	125	0.20	10.5	102.8
GH	0.0332	33.2	350	0.20	2.1	20.4
CH	0.0296	29.6	125	0.20	0.6	5.8
DE	0.0957	95.7	300	0.20	14.9	145.6
EG	0.0083	8.3	125	0.15	0.2	2.1
EF	0.0872	87.2	350	0.20	14.4	140.9
HF	0.0628	62.8	125	0.15	12.1	118.5

4.4.2 Flow rates could be assumed in each pipe and a traditional Hardy-Cross tabular method could be employed. However, since there is only one loop containing two pipes, a direct solution is possible.

a) For concrete (avg), e = 0.0012 ft, and $\nu = 1.08 \times 10^{-5}$ ft²/sec (68˚F). Assume complete turbulence:

$e/D_1 = 0.0012/1.5 = 0.0008$; thus; $\mathbf{f = 0.019}$ and $e/D_2 = 0.0012/2.0 = 0.0006$; thus; $\mathbf{f = 0.018;}$

$h_L = h_f = f(L/D)(V^2/2g)$; and based on conservation of energy, $h_{L1} = h_{L2}$. Therefore,

$[(0.019)(4000/1.5)](V_1)^2/2g = [(0.018)(3000/2.0)](V_2)^2/2g$;

$V_1 = Q_1/A_1 = Q_1/[(\pi/4)(1.5)^2] = 0.566{\cdot}Q_1$; and $V_2 = Q_2/A_2 = Q_2/[(\pi/4)(2.0)^2] = 0.318{\cdot}Q_2$; therefore

$[(0.019)(4000/1.5)](0.566{\cdot}Q_1)^2/2g = [(0.018)(3000/2.0)](0.318{\cdot}Q_2)^2/2g$; which results in

$Q_1 = 0.410Q_2$; from mass balance, $Q_1 + Q_2 = 50.0$ cfs; substituting yields $\mathbf{Q_2 = 35.5}$ **cfs;** $\mathbf{Q_1 = 14.5}$ **cfs**

Checking the friction factors; $N_R = DV/\nu$; $V_1 = 0.566{\cdot}Q_1 = 8.21$ ft/sec; and $V_2 = 0.318{\cdot}Q_2 = 11.3$ ft/sec;

$N_{R1} = D_1V_1/\nu = [(1.5)(8.21)/ (1.08 \times 10^{-5}) = 1.14 \times 10^{6;}$ f = 0.019 OK

$N_{R2} = D_2V_2/\nu = [(2.0)(11.3)/ (1.08 \times 10^{-5}) = 2.09 \times 10^{6;}$ f = 0.018 OK ; and the head loss is

$\mathbf{h_f} = [f(L/D)]{\cdot}(V^2/2g) = [(0.019)(4000/1.5)](8.21)^2/2g = \mathbf{53.0}$ **ft** (using pipe 1 parameters)

b) Find the equivalent pipe to replace Branches 1 and 2, let D = 2.5 ft and thus from e/D; f = 0.017

$[(D_E^5)/(f_E{\cdot}L_E)]^{1/2} = [(D_1^5)/(f_1{\cdot}L_1)]^{1/2} + [(D_2^5)/(f_2{\cdot}L_2)]^{1/2}$**;** from Equation (3.47)

$[(2.5^5)/(0.017{\cdot}L_E)]^{1/2} = [(1.5^5)/(0.019{\cdot}4000)]^{1/2} + [(2^5)/(0.018{\cdot}3000)]^{1/2}$**;** $L_E = 4870$ ft and from Table 3.4

$\mathbf{h_{fAB}} = [(0.0252{\cdot}f{\cdot}L)/D^5]Q^2 = [(0.0252{\cdot}0.017{\cdot}4870)/(2.5)^5](50.0)^2 = \mathbf{53.4}$ **ft**

Flow in pipe branches, $h_{f1} = 53.4$ ft $=[(0.0252{\cdot}0.019{\cdot}4000)/(1.5)^5]Q_1^2$; $\mathbf{Q_1 = 14.6}$ **cfs**

$h_{f2} = 53.4$ ft $= [(0.0252{\cdot}0.018{\cdot}3000)/(2.0)^5]Q_2^2$; $\mathbf{Q_2 = 35.4}$ **cfs**; same thing from $Q_1 + Q_2 = 50.0$ cfs

---

4.4.3 Flow rates could be assumed in each pipe and a traditional Hardy-Cross tabular method could be employed. However, since there is only one loop containing two pipes, a direct solution is possible.

a) For cast iron, e = 0.26 mm, and $\nu = 1.31 \times 10^{-6}$ m²/sec (water at 10°C). Assume complete turbulence:

$e/D_1 = 0.26mm/40mm = 0.0065$; thus; $\mathbf{f = 0.033}$ and $e/D_2 = 0.26mm/50mm = 0.0052$; thus; $\mathbf{f = 0.030;}$

$h_L = h_f = [f(L/D) + \sum K]{\cdot}(V^2/2g)$; and based on conservation of energy, $h_{L1} = h_{L2}$. Therefore,

$[(0.033)(25/0.04) + 2(0.2)](V_1)^2/2g = [(0.030)(30/0.05) + 2(0.2)](V_2)^2/2g$;

$V_1 = Q_1/A_1 = Q_1/[(\pi/4)(0.04)^2] = 796{\cdot}Q_1$; and $V_2 = Q_2/A_2 = Q_2/[(\pi/4)(0.05)^2] = 509{\cdot}Q_2$; therefore

(Problem 4.4.3 – continued)

$[(0.033)(25/0.04) + 2(0.2)](796 \cdot Q_1)^2/2g = [(0.030)(30/0.05) + 2(0.2)](509 \cdot Q_2)^2/2g$; which results in

$Q_1 = 0.598Q_2$; from mass balance, $Q_1 + Q_2 = 12$ L/s; substituting yields **$Q_2 = 7.51$ L/s; $Q_1 = 4.49$ L/s**

Checking the friction factors; $N_R = DV/\nu$; $V_1 = 796 \cdot Q_1 = 3.57$ m/sec; and $V_2 = 509 \cdot Q_2 = 3.82$ m/sec;

$N_{R1} = D_1V_1/\nu = [(0.04)(3.57)/ (1.31 \times 10^{-6}) = 1.09 \times 10^{5;}$ f = 0.033 OK

$N_{R2} = D_2V_2/\nu = [(0.05)(3.82)/ (1.31 \times 10^{-6}) = 1.46 \times 10^{5;}$ f = 0.0305 OK (close enough); and head loss is

$\mathbf{h_f} = [f(L/D) + \sum K] \cdot (V^2/2g) = [(0.033)(25/0.04) + 2(0.2)](3.57)^2/2g =$ **13.7 m** (using pipe 1 parameters)

b) Find the equivalent pipe to replace Branches 1 and 2, letting D = 0.06 m; thus from e/D; f = 0.029;

$[(D_E^5)/(f_E \cdot L_E)]^{1/2} = [(D_1^5)/(f_1 \cdot L_1)]^{1/2} + [(D_2^5)/(f_2 \cdot L_2)]^{1/2}$; from Equation (3.47)

$[(0.06^5)/(0.029 \cdot L_E)]^{1/2} = [(0.04^5)/(0.033 \cdot 25)]^{1/2} + [(0.05^5)/(0.030 \cdot 30)]^{1/2}$**;** $L_E = 30.2$ m. and from Table 3.4

$\mathbf{h_{fAB}} = [(0.0826 \cdot f \cdot L)/D^5]Q^2 = [(0.0826 \cdot 0.029 \cdot 30.2)/(0.06)^5](0.012)^2 =$ **13.4 m** (minor losses were ignored)

Flow in pipe branches, $h_{f1} = 13.4$ m $=[(0.0826 \cdot 0.033 \cdot 25)/(0.04)^5]Q_1^2$; **$Q_1 = 0.00449$ m³/s (4.49 L/sec)**

$h_{f2} = 13.4$ m $= [(0.0826 \cdot 0.030 \cdot 30)/(0.05)^5]Q_2^2$; **$Q_2 = 0.00751$ m³/s (7.51 L/sec)** ; or from $Q_1 + Q_2 = 12$ L/s

---

4.4.4 The following spreadsheet represents a tabular approach to the Hardy-Cross solution method.

Pipe Network Problem (Problem 4.4.4)

Storage Tank		Network Inflows	
Elevation (m)		(m^3/sec)	
A =	355.00	A =	1.000
Junction Elevations		Network Outflows	
(See problem writeup.)		(m^3/sec)	
Roughness (e, in m)		D =	0.550
All	0.000360	E =	0.450

Pipe	*Q* (m^3/sec)	Length (m)	Diameter (m)	*e/D*	*f**	*K* (sec^2/m^5)
AB	0.500	300	0.45	0.00080	0.0186	25
AC	0.500	300	0.45	0.00080	0.0186	25
BD	0.530	400	0.40	0.00090	0.0191	62
CE	0.470	400	0.40	0.00090	0.0191	62
CB	0.030	300	0.20	0.00180	0.0228	1764
ED	0.020	300	0.20	0.00180	0.0228	1764

** Equation 3.23 - hydraulically rough pipes (complete turbulance)*

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1	*AB*	0.500	25	6.25	12.5	0.510
(clockwise)						
			$\sum h_{fc}$ =	6.25	12.5	$\equiv\sum(h_{fc}/Q_c)$
1	*AC*	0.500	25	6.25	12.5	0.490
(counter)	*CB*	0.030	1764	1.59	52.9	0.020
			$\sum h_{fcc}$ =	7.84	65.4	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	-0.0102	

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2 (clockwise)	CB	0.020	1764	0.69	35.0	0.006
	BD	0.530	62	17.35	32.7	0.516
			$\sum h_{fc}$ =	18.05	67.7	$\equiv\sum(h_{fc}/Q_c)$
2 (counter)	CE	0.470	62	13.65	29.0	0.484
	ED	0.020	1764	0.71	35.3	0.034
			$\sum h_{fcc}$ =	14.35	64.3	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0140	

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1 (clockwise)	AB	0.510	25	6.51	12.8	0.504
			$\sum h_{fc}$ =	6.51	12.8	$\equiv\sum(h_{fc}/Q_c)$
1 (counter)	AC	0.490	25	6.00	12.2	0.496
	CB	0.006	1764	0.06	10.3	0.012
			$\sum h_{fcc}$ =	6.06	22.5	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0064	

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2 (clockwise)	CB	0.012	1764	0.26	21.5	0.011
	BD	0.516	62	16.45	31.9	0.515
			$\sum h_{fc}$ =	16.71	53.4	$\equiv\sum(h_{fc}/Q_c)$
2 (counter)	CE	0.484	62	14.47	29.9	0.485
	ED	0.034	1764	2.04	60.0	0.035
			$\sum h_{fcc}$ =	16.51	89.9	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0007	

Pipe	Q (m^3/sec)	Q (L/sec)	Length (m)	Diameter (m)	h_f (m)	ΔP (kPa)
AB	0.504	**504**	300	0.45	6.35	62.1
AC	0.496	**496**	300	0.45	6.16	60.3
BD	0.515	**515**	400	0.40	16.41	160.6
CE	0.485	**485**	400	0.40	14.51	142.1
CB	0.011	**11**	300	0.20	0.23	2.3
ED	0.035	**35**	300	0.20	2.12	20.8

Determine pressure at junction E and D (the demand points) by subtracting head losses on the flow path.

$H_E = H_A - h_{fAC} - h_{fCE}$ = 355m - 6.16m - 14.51m = 334.3m; and subtracting the position head yields

$(P/\gamma)_E = H_E - h_E$ = 334.3 - 314.1 = 20.2 m; $\mathbf{P_E}$ **=** (20.2 m)(9790 N/m^3) = **198 kPa (OK)**; Likewise,

H_D = 355m - 6.35m - 16.41m = 332.2m; $(P/\gamma)_D$ = 332.2 - 313.3 = 18.9 m; $\mathbf{P_D}$ **= 185 kPa (OK)**

4.4.5 The following spreadsheet is a tabular approach to the Hardy-Cross (Hazen-Williams) solution method.

Pipe Network Problem (Problem 4.4.5)

Storage Tank Elevations (m)		Network Inflows (m^3/sec)	
A =	355.00	A =	1.000

Junction Elevations (See problem writeup.)		Network Outflows (m^3/sec)	
Pipe Roughness (C_{HW})		D =	0.550
All	120	E =	0.450

Pipe	Q (m^3/sec)	Length (m)	Diameter (m)	C_{HW}	m	K* ($sec^{1.85}/m^{4.55}$)
AB	0.500	300	0.45	120	1.85	22
AC	0.500	300	0.45	120	1.85	22
BD	0.530	400	0.40	120	1.85	53
CE	0.470	400	0.40	120	1.85	53
CB	0.030	300	0.20	120	1.85	1159
ED	0.020	300	0.20	120	1.85	1159

* *Table 3.4;* $K = (10.7*L)/(D^{4.87}*C^{1.85})$

Loop	Pipe	Q (m^3/sec)	K ($sec^{1.85}/m^{4.55}$)	h_f** (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1 *(clockwise)*	*AB*	0.500	22	6.19	12.4	0.511
			$\sum h_{fc}$ =	6.19	12.4	$\equiv\sum(h_{fc}/Q_c)$
1 *(counter)*	*AC*	0.500	22	6.19	12.4	0.489
	CB	0.030	1159	1.76	58.8	0.019
			$\sum h_{fcc}$ =	7.96	71.2	$\equiv\sum(h_{fcc}/Q_{cc})$

***Table 3.4;* $h_f = KQ^{1.85}$

Equation 4.17b (correction for Hazen-Williams) **ΔQ=** -0.0114

Loop	Pipe	Q (m^3/sec)	K ($sec^{1.85}/m^{4.55}$)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2 *(clockwise)*	*CB*	0.019	1159	0.73	39.2	0.006
	BD	0.530	53	16.32	30.8	0.518
			$\sum h_{fc}$ =	17.05	70.0	$\equiv\sum(h_{fc}/Q_c)$
2 *(counter)*	*CE*	0.470	53	13.07	27.8	0.482
	ED	0.020	1159	0.83	41.7	0.032
			$\sum h_{fcc}$ =	13.90	69.5	$\equiv\sum(h_{fcc}/Q_{cc})$

ΔQ = 0.0122

Loop	Pipe	Q (m^3/sec)	K ($sec^{1.85}/m^{4.55}$)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1 *(clockwise)*	*AB*	0.511	22	6.46	12.6	0.506
			$\sum h_{fc}$ =	6.46	12.6	$\equiv\sum(h_{fc}/Q_c)$
1 *(counter)*	*AC*	0.489	22	5.94	12.1	0.494
	CB	0.006	1159	0.10	15.8	0.012
			$\sum h_{fcc}$ =	6.04	27.9	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0056	

Loop	Pipe	Q (m^3/sec)	K ($sec^{1.85}/m^{4.55}$)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2 *(clockwise)*	*CB*	0.012	1159	0.32	27.0	0.011
	BD	0.518	53	15.64	30.2	0.517
			$\sum h_{fc}$ =	15.96	57.2	$\equiv\sum(h_{fc}/Q_c)$
2 *(counter)*	*CE*	0.482	53	13.71	28.4	0.483
	ED	0.032	1159	2.01	62.5	0.033
			$\sum h_{fcc}$ =	15.72	90.9	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0009	

Pipe	Q (m^3/sec)	Q (L/sec)	Length (m)	Diameter (m)	h_f (m)	ΔP (kPa)
AB	0.506	**506**	300	0.45	6.33	61.9
AC	0.494	**494**	300	0.45	6.06	59.3
BD	0.517	**517**	400	0.40	15.59	152.6
CE	0.483	**483**	400	0.40	13.75	134.6
CB	0.011	**11**	300	0.20	0.28	2.8
ED	0.033	**33**	300	0.20	2.12	20.7

Pipe flows using the Hazen-Williams equation for friction loss do not differ significantly (< 2 L/s) from the Darcy-Weisbach approach (Prob 4.4.4). Head loss differences vary more (difficult to equate *e* and C_{HW}). Determine pressure at junction E and D (the demand points) by subtracting head losses on the flow path.
$H_E = H_A - h_{fAC} - h_{fCE}$ = 355m - 6.06m – 13,75m = 335.2m; and subtracting the position head yields $(P/\gamma)_E = H_E - h_E$ = 335.2 - 314.1 = 21.1 m; P_E = (21.1 m)(9790 N/m^3) = 207 kPa (OK); Likewise, H_D = 355m - 6.33m – 15.59m = 333.1m; $(P/\gamma)_D$ = 333.1 - 313.3 = 19.8 m; P_D = 194 kPa (OK)

4.4.6 It is clear from the solution table in Example 4.8 that **pipes DE and EF are the pipes that should be replaced by larger diameter pipes**. Both have head losses that are over 14 m; the next largest head loss is pipe BC with 12.2 m of head loss. Pipe AD, which has a diameter of 25 cm, discharges into pipes DE and EF, which have diameters of 20 cm. The head loss in AD is only 3.8 m, but some of that can be attributed to its shorter length than either DE or EF. **Let's replace DE**, since it has the highest head loss and it will not be as expensive to replace as EF since it is 50 m shorter.

(Problem 4.4.6 – continued)

The spreadsheet below depicts the results of an increase in pipe diameter for DE from 20 cm to 25 cm.

Pipe Network Problem (Problem 4.4.6)

Storage Tank		Network Inflows	
Elevations (m)		(m^3/sec)	
A =	50.00	A =	0.300
Junction Elevations		Network Outflows	
All	0.00	(m^3/sec)	
		C =	0.050
Roughness (e, in m)		F =	0.150
All	0.000260	G =	0.100

Pipe	*Q* (m^3/sec)	Length (m)	Diameter (m)	*e/D*	*f**	*K* (sec^2/m^5)
AB	0.200	300	0.30	0.00087	0.0190	193
AD	0.100	250	0.25	0.00104	0.0198	419
BC	0.080	350	0.20	0.00130	0.0210	1894
BG	0.120	125	0.20	0.00130	0.0210	676
GH	0.020	350	0.20	0.00130	0.0210	1894
CH	0.030	125	0.20	0.00130	0.0210	676
DE	0.100	300	0.25	0.00104	0.0198	503
GE	0.000	125	0.15	0.00173	0.0226	3068
EF	0.100	350	0.20	0.00130	0.0210	1894
HF	0.050	125	0.15	0.00173	0.0226	3068

** Equation 3.23 - hydraulically rough pipes (complete turbulance)*

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1 *(clockwise)*	*AB*	0.200	193	7.74	38.7	0.181
	BG	0.120	676	9.74	81.2	0.101
	GE	0.000	3068	0.00	0.0	-0.019
			$\sum h_{fc}$ =	17.48	119.9	$\equiv\sum(h_{fc}/Q_c)$
1 *(counter)*	*AD*	0.100	419	4.19	41.9	0.119
	DE	0.100	503	5.03	50.3	0.119
			$\sum h_{fcc}$ =	9.23	92.3	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0195	

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2 *(clockwise)*	*BC*	0.080	1894	12.12	151.5	0.071
	CH	0.030	676	0.61	20.3	0.021
			$\sum h_{fc}$ =	12.73	171.8	$\equiv\sum(h_{fc}/Q_c)$
2 *(counter)*	*BG*	0.101	676	6.84	68.0	0.110
	GH	0.020	1894	0.76	37.9	0.029
			$\sum h_{fcc}$ =	7.59	105.9	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0092	

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
3 (clockwise)	*GH*	0.029	1894	1.62	55.4	0.039
	HF	0.050	3068	7.67	153.4	0.059
	EG	0.019	3068	1.16	59.6	0.029
			$\sum h_{fc}$ =	10.45	268.4	$\equiv\sum(h_{fc}/Q_c)$
3 (counter)	*EF*	0.100	1894	18.94	189.4	0.091
			$\sum h_{fcc}$ =	18.94	189.4	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	-0.0093	

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1 (clockwise)	*AB*	0.181	193	6.31	34.9	0.183
	BG	0.110	676	8.15	74.2	0.112
			$\sum h_{fc}$ =	14.46	109.2	$\equiv\sum(h_{fc}/Q_c)$
1 (counter)	*AD*	0.119	419	5.98	50.1	0.117
	DE	0.119	503	7.18	60.1	0.117
	EG	0.029	3068	2.53	88.1	0.027
			$\sum h_{fcc}$ =	15.69	198.3	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	-0.0020	

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2 (clockwise)	*BC*	0.071	1894	9.48	134.0	0.073
	CH	0.021	676	0.29	14.0	0.023
			$\sum h_{fc}$ =	9.77	148.0	$\equiv\sum(h_{fc}/Q_c)$
2 (counter)	*BG*	0.112	676	8.45	75.6	0.109
	GH	0.039	1894	2.81	72.9	0.036
			$\sum h_{fcc}$ =	11.26	148.5	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	-0.0025	

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
3 (clockwise)	*GH*	0.036	1894	2.45	68.2	0.036
	HF	0.059	3068	10.78	181.8	0.059
	EG	0.027	3068	2.19	81.9	0.027
			$\sum h_{fc}$ =	15.42	331.9	$\equiv\sum(h_{fc}/Q_c)$
(counter)	*EF*	0.091	1894	15.59	171.8	0.091
			$\sum h_{fcc}$ =	15.59	171.8	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	-0.0002	

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1	*AB*	0.183	193	6.45	35.3	0.183

(clockwise)	*BG*	0.109	676	8.08	73.9	0.110
			$\sum h_{fc}$ =	14.53	109.2	$\equiv\sum(h_{fc}/Q_c)$
1	*AD*	0.117	419	5.78	49.2	0.117
(counter)	*DE*	0.117	503	6.94	59.1	0.117
	EG	0.027	3068	2.16	81.4	0.026
			$\sum h_{fcc}$ =	14.89	189.8	$\equiv\sum(h_{fcc}/Q_{cc})$

ΔQ = -0.0006

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2	*BC*	0.073	1894	10.16	138.7	0.073
(clockwise)	*CH*	0.023	676	0.37	15.7	0.023
			$\sum h_{fc}$ =	10.53	154.5	$\equiv\sum(h_{fc}/Q_c)$
2	*BG*	0.110	676	8.17	74.3	0.110
(counter)	*GH*	0.036	1894	2.48	68.5	0.036
			$\sum h_{fcc}$ =	10.64	142.8	$\equiv\sum(h_{fcc}/Q_{cc})$

ΔQ = -0.0002

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
3	*GH*	0.036	1894	2.45	68.1	0.036
(clockwise)	*HF*	0.059	3068	10.84	182.3	0.060
	EG	0.026	3068	2.06	79.6	0.026
			$\sum h_{fc}$ =	15.35	330.0	$\equiv\sum(h_{fc}/Q_c)$
(counter)	*EF*	0.091	1894	15.53	171.5	0.090
			$\sum h_{fcc}$ =	15.53	171.5	$\equiv\sum(h_{fcc}/Q_{cc})$

ΔQ = -0.0002

Pipe	Q (m^3/sec)	Q (L/sec)	Length (m)	Diameter (m)	h_f (m)	ΔP (kPa)
AB	0.1832	**183.2**	300	0.30	6.5	63.5
AD	0.1168	**116.8**	250	0.25	5.7	56.0
BC	0.0735	**73.5**	350	0.20	10.2	100.0
BG	0.1097	**109.7**	125	0.20	8.1	79.7
GH	0.0362	**36.2**	350	0.20	2.5	24.2
CH	0.0235	**23.5**	125	0.20	0.4	3.6
DE	0.1168	**116.8**	300	0.25	6.9	67.3
EG	0.0258	**25.8**	125	0.15	2.0	19.9
EF	0.0904	**90.4**	350	0.20	15.5	151.4
HF	0.0596	**59.6**	125	0.15	10.9	106.7

Now we can determine the pressure at junction F by subtracting head losses on the flow path.

$\mathbf{P_F} = P_A - P_{AB} - P_{BC} - P_{CH} - P_{HF}$ = 489.5 – 63.5 – 100.0 – 3.6 – 106.7 = **215.7 kPa**

This is a 48.3 kPa increase in the pressure at F by replacing one critical pipe in the system.

4.4.7 The following spreadsheet is a tabular approach to the Hardy-Cross (Hazen-Williams) solution method.

Pipe Network Problem (Problem 4.4.7)

Storage Tank	Network Inflows
Elevations (ft)	(ft^3/sec)
A = 475.20	A = 12.5
	Network Outflows
Junction Elevations	(ft^3/sec)
(See problem writeup.)	C = 3.50
Roughness (C_{HW})	D = 3.50
All 120	E = 5.50

Pipe	Q (ft^3/sec)	Length (ft)	Diameter (ft)	C_{HW}	*m*	*K** ($sec^{1.85}/ft^{4.55}$)
AB	11.00	600	1.50	120	1.85	0.056
AC	14.00	600	1.50	120	1.85	0.056
BD	7.00	800	1.25	120	1.85	0.182
CE	7.00	800	1.25	120	1.85	0.182
BF	4.00	400	1.00	120	1.85	0.269
CF	1.00	400	1.00	120	1.85	0.269
FG	5.00	800	1.25	120	1.85	0.182
GD	1.00	400	1.00	120	1.85	0.269
GE	4.00	400	1.00	120	1.85	0.269

* *Table 3.4;* $K = (4.73*L)/(D^{4.87}*C^{1.85})$

Loop	Pipe	*Q* (ft^3/sec)	*K* ($sec^{1.85}/ft^{4.55}$)	h_f** (ft)	h_f/Q (sec/ft^2)	New Q (ft^3/sec)
1	*AB*	11.00	0.06	4.74	0.43	10.85
(clockwise)	*BF*	4.00	0.27	3.50	0.88	3.85
			$\sum h_{fc}$ =	8.24	1.31	$\equiv\sum(h_{fc}/Q_c)$
1	*AC*	14.00	0.06	7.40	0.53	14.15
(counter)	*CF*	1.00	0.27	0.27	0.27	1.15
			$\sum h_{fcc}$ =	7.67	0.80	$\equiv\sum(h_{fcc}/Q_{cc})$

***Table 3.4;* $h_f = KQ^{1.85}$

Equation 4.17b (correction for Hazen-Williams) **ΔQ=** 0.146

Loop	Pipe	*Q* (ft^3/sec)	*K* ($sec^{1.85}/ft^{4.55}$)	h_f (ft)	h_f/Q (sec/ft^2)	New Q (ft^3/sec)
2	*BD*	7.00	0.18	6.65	0.95	7.09
(clockwise)						
			$\sum h_{fc}$ =	6.65	0.95	$\equiv\sum(h_{fc}/Q_c)$
2	*BF*	3.85	0.27	3.27	0.85	3.77
(counter)	*FG*	5.00	0.18	3.57	0.71	4.91
	GD	1.00	0.27	0.27	0.27	0.91
			$\sum h_{fcc}$ =	7.11	1.83	$\equiv\sum(h_{fcc}/Q_{cc})$

ΔQ = -0.089

Loop	Pipe	Q (ft^3/sec)	K ($sec^{1.85}/ft^{4.55}$)	h_f (ft)	h_f/Q (sec/ft^2)	New Q (ft^3/sec)
3 *(clockwise)*	*CF*	1.15	0.27	0.35	0.30	1.02
	FG	4.91	0.18	3.45	0.70	4.79
	GE	4.00	0.27	3.50	0.88	3.88
			$\sum h_{fc}$ =	7.30	1.88	$\equiv\sum(h_{fc}/Q_c)$
3 *(counter)*	*CE*	7.00	0.18	6.65	0.95	7.12
			$\sum h_{fcc}$ =	6.65	0.95	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.124	

Loop	Pipe	Q (ft^3/sec)	K ($sec^{1.85}/ft^{4.55}$)	h_f (ft)	h_f/Q (sec/ft^2)	New Q (ft^3/sec)
1 *(clockwise)*	*AB*	10.85	0.06	4.62	0.43	10.87
	BF	3.77	0.27	3.13	0.83	3.78
			$\sum h_{fc}$ =	7.75	1.26	$\equiv\sum(h_{fc}/Q_c)$
1 *(counter)*	*AC*	14.15	0.06	7.54	0.53	14.13
	CF	1.02	0.27	0.28	0.27	1.00
			$\sum h_{fcc}$ =	7.83	0.81	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ=	-0.019	

Loop	Pipe	Q (ft^3/sec)	K ($sec^{1.85}/ft^{4.55}$)	h_f (ft)	h_f/Q (sec/ft^2)	New Q (ft^3/sec)
2 *(clockwise)*	*BD*	7.09	0.18	6.81	0.96	7.06
			$\sum h_{fc}$ =	6.81	0.96	$\equiv\sum(h_{fc}/Q_c)$
2 *(counter)*	*BF*	3.78	0.27	3.16	0.84	3.81
	FG	4.79	0.18	3.29	0.69	4.81
	GD	0.91	0.27	0.23	0.25	0.94
			$\sum h_{fcc}$ =	6.68	1.77	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.025	

Loop	Pipe	Q (ft^3/sec)	K ($sec^{1.85}/ft^{4.55}$)	h_f (ft)	h_f/Q (sec/ft^2)	New Q (ft^3/sec)
3 *(clockwise)*	*CF*	1.00	0.27	0.27	0.27	1.00
	FG	4.81	0.18	3.33	0.69	4.81
	GE	3.88	0.27	3.30	0.85	3.87
			$\sum h_{fc}$ =	6.90	1.81	$\equiv\sum(h_{fc}/Q_c)$
3 *(counter)*	*CE*	7.12	0.18	6.87	0.96	7.13
			$\sum h_{fcc}$ =	6.87	0.96	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.006	

Pipe	Q (ft^3/sec)	Length (ft)	Diameter (ft)	K^* ($sec^{1.85}/ft^{4.55}$)	h_f (ft)	ΔP (psi)
AB	**10.87**	600	1.50	0.056	4.64	2.01
AC	**14.13**	600	1.50	0.056	7.53	3.26
BD	**7.06**	800	1.25	0.182	6.76	2.93
CE	**7.13**	800	1.25	0.182	6.88	2.98
BF	**3.81**	400	1.00	0.269	3.20	1.38
CF	**1.00**	400	1.00	0.269	0.27	0.12
FG	**4.81**	800	1.25	0.182	3.32	1.44
GD	**0.94**	400	1.00	0.269	0.24	0.10
GE	**3.87**	400	1.00	0.269	3.29	1.43

The pressure head at junction A is given as 45 psi. This translates into a pressure head of
$[(45 \text{ lb/in}^2)(144 \text{ in}^2/\text{ft}^2)]/(62.3 \text{ lb/ft}^2) = 104.0$ ft
This allows you to obtain the total head at junction A (h + P/γ) assuming the velocity head is negligible. Once the total head at junction A is determined, the total head at the other junctions is found by subtraction head losses from junction to junction using the pipe head losses from the above table. Finally, the pressure heads are determined by subtracting the elevations from the total heads. These can be transformed into pressures in psi using the appropriate conversions.

Junction	Elev. (ft)	P/γ (ft)	Total Head* (ft)	P/γ** (ft)	P (psi)
A	325.0	104.0	429.0	104.0	**45.0**
B	328.5		424.4	95.9	**41.5**
C	325.8		421.5	95.7	**41.4**
D	338.8		417.6	78.8	**34.1**
E	330.8		414.6	83.8	**36.3**
F	332.7		421.2	88.5	**38.3**
G	334.8		417.9	83.1	**35.9**

* Found by subtracting head losses in pipes from previous junction total head.
** Found by subtracting position head (i.e., elevation) from total head.

4.4.8 It is clear from the solution table in Example 4.9 that **pipes GD and BF are the pipes that should be replaced by larger diameter pipes.** Both have head losses that are over 20 m; the next largest head loss is pipe BC with 16.8 m of head loss. Pipe GD, which has a diameter of 25 cm, discharges into pipes DE and DC, which have diameters of 20 cm. Pipe BF, which has a diameter of 20 cm, discharges directly into node F which has the pressure problem. However, **let's replace GD** since it has the highest head loss and it is not likely to be as expensive to replace as BF since it is 100 m shorter. However, once the program is set up, both replacements can be attempted to see which one is more effective in increasing the pressure at F. Cost data would be needed to determine the most economical decision.

(Problem 4.4.8 – continued)

The spreadsheet below depicts the results of an increase in pipe diameter for GD from 25 cm to 30 cm.

Pipe Network Problem (Problem 4.4.8)

Storage Tank Elevations (m)		Network Inflows (m^3/sec)	
A =	85.00	A & G	unknown
G =	102.00		

Junction Elevations		Network Outflows (m^3/sec)	
(See problem writeup.)		C =	0.100
Roughness (e, in m)		F =	0.250
All	0.000260	E =	0.100

Pipe	*Q* (m^3/sec)	Length (m)	Diameter (m)	*e/D*	*f**	*K* (sec^2/m^5)
AB	0.200	300	0.30	0.00087	0.0190	193
BC	0.100	350	0.20	0.00130	0.0210	1894
BF	0.100	350	0.20	0.00130	0.0210	1894
CF	0.050	125	0.20	0.00130	0.0210	676
DC	0.050	300	0.20	0.00130	0.0210	1623
EF	0.100	300	0.20	0.00130	0.0210	1623
DE	0.200	125	0.20	0.00130	0.0210	676
GD	0.250	250	0.30	0.00087	0.0190	161

** Equation 3.23 - hydraulically rough pipes (complete turbulance)*

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1 (*clockwise*)	*BC*	0.100	1894	18.94	189.4	0.098
	CF	0.050	676	1.69	33.8	0.048
			$\sum h_{fc}$ =	20.63	223.2	$\equiv\sum(h_{fc}/Q_c)$
1 (*counter*)	*BF*	0.100	1894	18.94	189.4	0.102
			$\sum h_{fcc}$ =	18.94	189.4	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0020	

Loop	Pipe	*Q* (m^3/sec)	*K* (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2 (*clockwise*)	*DE*	0.200	676	27.05	135.3	0.154
	EF	0.100	1623	16.23	162.3	0.054
			$\sum h_{fc}$ =	43.28	297.6	$\equiv\sum(h_{fc}/Q_c)$
2 (*counter*)	*DC*	0.050	1623	4.06	81.2	0.096
	CF	0.048	676	1.55	32.4	0.094
			$\sum h_{fcc}$ =	5.61	113.6	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	0.0458	

Path (ABCDG)	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
(along flow path)	*AB*	0.200	193	7.74	38.7	0.179
	BC	0.098	1894	18.17	185.5	0.077
			$\sum h_{fc}$ =	25.91	224.2	$\equiv\sum(h_{fc}/Q_c)$
(opposite flow path)	*DC*	0.096	1623	14.90	155.5	0.117
	GD	0.250	161	10.08	40.3	0.271
			$\sum h_{fcc}$ =	24.98	195.8	$\equiv\sum(h_{fcc}/Q_{cc})$

Equation 4.18a* ***ΔQ = 0.0213

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1 *(clockwise)*	*BC*	0.077	1894	11.11	145.1	0.080
	CF	0.094	676	5.95	63.4	0.097
			$\sum h_{fc}$ =	17.06	208.5	$\equiv\sum(h_{fc}/Q_c)$
1 *(counter)*	*BF*	0.102	1894	19.72	193.2	0.099
			$\sum h_{fcc}$ =	19.72	193.2	$\equiv\sum(h_{fcc}/Q_{cc})$

ΔQ = -0.0033

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2 *(clockwise)*	*DE*	0.154	676	16.08	104.3	0.163
	EF	0.054	1623	4.77	88.0	0.063
			$\sum h_{fc}$ =	20.84	192.2	$\equiv\sum(h_{fc}/Q_c)$
2 *(counter)*	*DC*	0.117	1623	22.28	190.1	0.108
	CF	0.097	676	6.37	65.6	0.088
			$\sum h_{fcc}$ =	28.65	255.8	$\equiv\sum(h_{fcc}/Q_{cc})$

ΔQ = -0.0087

Path (ABCDG)	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
(along flow path)	*AB*	0.179	193	6.18	34.6	0.173
	BC	0.080	1894	12.09	151.3	0.075
			$\sum h_{fc}$ =	18.27	185.9	$\equiv\sum(h_{fc}/Q_c)$
(opposite flow path)	*DC*	0.108	1623	19.09	176.0	0.114
	GD	0.271	161	11.87	43.8	0.277
			$\sum h_{fcc}$ =	30.96	219.8	$\equiv\sum(h_{fcc}/Q_{cc})$

Equation 4.18a* ***ΔQ = 0.0053

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1 *(clockwise)*	*BC*	0.075	1894	10.54	141.3	0.078
	CF	0.088	676	5.28	59.8	0.092
			$\sum h_{fc}$ =	15.82	201.0	$\equiv\sum(h_{fc}/Q_c)$
1 *(counter)*	*BF*	0.099	1894	18.46	187.0	0.095
			$\sum h_{fcc}$ =	18.46	187.0	$\equiv\sum(h_{fcc}/Q_{cc})$

ΔQ = -0.0034

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2	*DE*	0.163	676	17.95	110.2	0.165
(clockwise)	*EF*	0.063	1623	6.42	102.1	0.065
			$\sum h_{fc}$ =	24.37	212.3	$\equiv\sum(h_{fc}/Q_c)$
2	*DC*	0.114	1623	21.00	184.6	0.111
(counter)	*CF*	0.092	676	5.69	62.1	0.089
			$\sum h_{fcc}$ =	26.70	246.7	$\equiv\sum(h_{fcc}/Q_{cc})$
				ΔQ =	-0.0025	

Path (ABCDG)	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
(along	*AB*	0.173	193	5.81	33.5	0.171
flow path)	*BC*	0.078	1894	11.52	147.7	0.076
			$\sum h_{fc}$ =	17.34	181.3	$\equiv\sum(h_{fc}/Q_c)$
(opposite	*DC*	0.111	1623	20.08	180.5	0.114
flow path)	*GD*	0.277	161	12.34	44.6	0.279
			$\sum h_{fcc}$ =	32.42	225.1	$\equiv\sum(h_{fcc}/Q_{cc})$
**Equation 4.18a*				***ΔQ =**	0.0024	

Pipe	Q (m^3/sec)	Q (L/sec)	Length (m)	Diameter (m)	h_f (m)	ΔP (kPa)
AB	**0.1710**	171.0	300	0.30	5.66	55.4
BC	**0.0756**	75.6	350	0.20	10.84	106.1
BF	**0.0953**	95.3	350	0.20	17.21	168.5
CF	**0.0892**	89.2	125	0.20	5.38	52.7
DC	**0.1136**	113.6	300	0.20	20.94	205.0
EF	**0.0654**	65.4	300	0.20	6.95	68.0
DE	**0.1654**	165.4	125	0.20	18.51	181.2
GD	**0.2790**	279.0	250	0.30	12.55	122.9

Junction	Elevation (m)	Energy Head (m)	Pressure Head (m)
A	48.00	85.00	**37.00**
B	46.00	79.34	**33.34**
C	43.00	68.51	**25.51**
D	48.00	89.45	**41.45**
E	44.00	70.94	**26.94**
F	48.00	62.13	**14.13**
G	60.00	102.00	**42.00**

The table above shows that replacing pipe GD with a 30 cm pipe produces the required results. That is, the pressure at junction F is now within the requirements (of 14 m of pressure head). Replacing pipe BF with a 25 cm pipe will not meet the pressure requirements, and thus the replacement of GD is the best choice.

4.4.9 From Table 3.4 (SI system), $h_f = [(10.7 \cdot L)/(D^{4.87} \cdot C^{1.85})]Q^{1.85} = K\,Q^{1.85}$.

In any network loop, the total head loss in the clockwise direction is the sum of the head losses in all pipes that carry flow in the clockwise direction around the loop. $\sum h_{fc} = \sum K_c Q_c^{1.85}$; and likewise for the counter-clockwise direction: $\sum h_{fcc} = \sum K_{cc} Q_{cc}^{1.85}$. Using the initially assumed flow rates, *Q*'s, it is not expected that these two values will be equal during the first trial. The difference, $\sum K_c Q_c^{1.85} - \sum K_{cc} Q_{cc}^{1.85}$ is the *closure error*. However, a flow correction ΔQ which, when subtracted from Q_c and added to Q_{cc}, will equalize the two head losses. Thus, $\sum K_c(Q_c - \Delta Q)^{1.85} = \sum K_{cc}(Q_{cc} + \Delta Q)^{1.85}$. A binomial series expansion yields:

$(Q_{cc} + \Delta Q)^{1.85} = (Q_{cc})^{1.85}(1+\Delta Q/Q_{cc})^{1.85} = (Q_{cc})^{1.85}[1 + 1.85(\Delta Q/Q_{cc}) + \{(1.85 \cdot 0.85)/2!\}(\Delta Q/Q_{cc})^2 + \ldots]$; or

$(Q_{cc} + \Delta Q)^{1.85} = (Q_{cc})^{1.85} + 1.85(Q_{cc})^{0.85}(\Delta Q)$; ignoring higher order terms. Substitute into the headloss e'qn

$\sum K_c[(Q_c)^{1.85} - 1.85(Q_c)^{0.85}(\Delta Q)] = \sum K_{cc}[(Q_{cc})^{1.85} + 1.85(Q_{cc})^{0.85}(\Delta Q)]$; and solving for ΔQ yields;

$\Delta Q = [\sum K_c[(Q_c)^{1.85} - \sum K_{cc}[(Q_{cc})^{1.85}] / [1.85\{\sum K_c(Q_c)^{0.85} + \sum K_{cc}(Q_{cc})^{0.85}\}]$; or

$\Delta Q = [\sum h_{fc} - \sum h_{fcc}] / [1.85\{\sum(h_{fc}/Q_c) + \sum(h_{fcc}/Q_{cc})\}]$; which is Equation (4.17b)

4.4.10 The computer software should return these results**.** The "k" values are approximate based on the equation used or reading the Moody diagram for complete turbulence.

Pipe	Length (m)	Diameter (m)	K (sec^2/m^5)	Flow (m^3/sec)	h_f (m)
AB	1200	0.50	54	**0.152**	**1.25**
FA	1800	0.40	261	**0.052**	**0.71**
BC	1200	0.10	247,900	**0.014**	**48.6**
BD	900	0.30	131	**0.138**	**2.49**
DE	1200	0.30	775	**0.040**	**1.24**
EC	900	0.10	185,900	**0.016**	**47.6**
FG	1200	0.60	21	**0.248**	**1.29**
GD	900	0.40	131	**0.152**	**3.03**
GH	1200	0.30	775	**0.096**	**7.14**
EH	900	0.20	4,880	**0.024**	**2.81**

4.4.11 The computer software should return these results**.**

Pipe	Length (m)	Diameter (m)	K ($sec^{1.85}/m^{4.55}$)	Flow (m^3/sec)	h_f (m)
AB	1200	0.50	46	**0.150**	**1.38**
FA	1800	0.40	202	**0.050**	**0.79**
BC	1200	0.10	115,500	**0.015**	**48.8**
BD	900	0.30	101	**0.135**	**2.49**
DE	1200	0.30	548	**0.038**	**1.29**
EC	900	0.10	86,650	**0.015**	**36.6**
FG	1200	0.60	19	**0.250**	**1.46**
GD	900	0.40	101	**0.153**	**3.13**
GH	1200	0.30	548	**0.097**	**7.32**
EH	900	0.20	2,960	**0.023**	**2.76**

4.4.12 All of the equations are written in the same format as in Example 4.10. The initial (trial) flow rates selected for the pipes are $Q_1 = 0.20$ m^3/sec, $Q_2 = 0.50$ m^3/sec, $Q_3 = 0.10$ m^3/sec, $Q_4 = 0.05$ m^3/sec, $Q_5 = 0.50$ m^3/sec, $Q_6 = 0.10$ m^3/sec, $Q_7 = 0.30$ m^3/sec, and $Q_8 = 0.25$ m^3/sec. The flow directions are designated in Figure 4.10c. The network outflows are $Q_c = 0.10$ m^3/sec, $Q_F = 0.30$ m^3/sec, and $Q_E = 0.10$ m^3/sec. Substituting these values into the equations formulated in Example 4.10 yields the following matrix:

$$\begin{bmatrix} -1.0 & 1.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & 0.0 & -1.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 1.0 & -1.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & -1.0 & 0.0 \\ 0.0 & 0.0 & -1.0 & 0.0 & -1.0 & -1.0 & 0.0 & 0.0 \\ 0.0 & 1900.0 & -380.0 & 0.0 & 678.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -163.0 & -678.0 & 326.0 & 406.8 & 0.0 \\ -77.6 & -1900.0 & 0.0 & 163.0 & 0.0 & 0.0 & 0.0 & 211.5 \end{bmatrix} \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4 \\ \Delta Q_5 \\ \Delta Q_6 \\ \Delta Q_7 \\ \Delta Q_8 \end{bmatrix} = \begin{bmatrix} -0.4000 \\ -0.0500 \\ -0.1000 \\ 0.1000 \\ 0.4000 \\ -625.5000 \\ 96.2550 \\ 469.2475 \end{bmatrix}$$

The first iteration yields discharge corrections of $\Delta Q_1 = 0.0715$ m^3/sec, $\Delta Q_2 = -0.2495$ m^3/sec, $\Delta Q_3 = -0.0790$ m^3/sec, $\Delta Q_4 = 0.0320$ m^3/sec, $\Delta Q_5 = -0.2675$ m^3/sec, $\Delta Q_6 = -0.0535$ m^3/sec, $\Delta Q_7 = -0.1535$ m^3/sec, and $\Delta Q_8 = -0.0215$ m^3/sec. Thus, for the second iteration we will use $Q_1 = 0.2715$ m^3/sec, $Q_2 = 0.2505$ m^3/sec, $Q_3 = 0.0210$ m^3/sec, $Q_4 = 0.0820$ m^3/sec, $Q_5 = 0.2325$ m^3/sec, $Q_6 = 0.0465$ m^3/sec, $Q_7 = 0.1465$ m^3/sec, and $Q_8 = 0.2285$ m^3/sec. The same procedure will be repeated until the all the corrections become negligible. The table below gives the final Q values for the pipe network.

Iteration Number	Q_1 (m^3/sec)	Q_2 (m^3/sec)	Q_3 (m^3/sec)	Q_4 (m^3/sec)	Q_5 (m^3/sec)	Q_6 (m^3/sec)	Q_7 (m^3/sec)	Q_8 (m^3/sec)
5	0.2300	0.1057	0.1243	0.1040	0.1097	0.0660	0.1660	0.2700

The resulting total heads are $H_A = 85$ m, $H_B = 74.74$ m, $H_C = 53.53$ m, $H_D = 71.16$ m, $H_E = 52.47$ m, $H_F = 45.37$ m, and $H_G = 102.00$ m. **The resulting pressure heads at nodes *A, B, C, D, E, F,* and *G*, respectively are 37.00 m, 28.74 m, 10.53 m, 23.16 m, 8.47 m, -2.63 m, and 42.00 m. Note that the pressure head drops below the threshold for two nodes.** Remedial action would need to be taken to increase the pressure at the critical nodes, most likely pipe replacement.

4.4.13 Based on Figure P4.4.13, the junction equations may be written as:

F1 = -Q(1) + Q(2) + Q(3) + QB
F2 = -Q(2) - Q(4) + Q(5) + QC -Q(7)
F3 = -Q(8) + Q(4) + Q(6) + QD
F4 = -Q(7) + QF - Q(6) - Q(3) - Q(5)

The loop equations may be written as:

F6 = K2*Q(2)*abs(Q(2)) + K5*Q(5)*ABS(Q(5)) - K3*Q(3)*ABS(Q(3))
F7 = -K4*Q(4)*ABS(Q(4)) + K6*Q(6)*ABS(Q(6)) - K5*Q(5)*ABS(Q(5))

(Problem 4.4.13 – continued)

The path equations may be written as:

F8=HA-K1*Q(1)*ABS(Q(1))-K2*Q(2)*ABS(Q(2))+K4*Q(4)*ABS(Q(4)) +K8*Q(8)*ABS(Q(8))-HG
F5=HA-K1*Q(1)*ABS(Q(1))-K2*Q(2)*ABS(Q(2))+K7*Q(7)*ABS(Q(7))-HE

Non-zero elements of Coefficient Matrix

A(1,1)=-1.	A(1,2)=1.0	A(1,3)=1.0	
A(2,2)=-1.	A(2,4)=-1.	A(2,5)=1.	A(2,7)=-1.
A(3,8)=-1.	A(3,4)=1.	A(3,6)=1.	
A(4,7)=-1.	A(4,6)=-1.	A(4,5)=-1.	A(4,3)=-1.

A(6,2)=2*K2*(Q(2))
A(6,5)=K5*2*(Q(5))
A(6,3)=-K3*2*(Q(3))
A(7,4)=-K4*2*(Q(4))
A(7,6)=K6*2*(Q(6))
A(7,5)=-K5*2*(Q(5))
A(8,1)=-K1*2*(Q(1))
A(8,2)=-K2*2*(Q(2))
A(8,4)=K4*2*(Q(4))
A(8,8)=K8*2*(Q(8))
A(5,1)=-K1*2*(Q(1))
A(5,2)=-K2*2*(Q(2))
A(5,7)=K7*2*(Q(7))

COEFFICIENT MATRIX

-1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000
0.000	-1.000	0.000	-1.000	1.000	0.000	-1.000	0.000
0.000	0.000	0.000	1.000	0.000	1.000	0.000	-1.000
0.000	0.000	-1.000	0.000	-1.000	-1.000	-1.000	0.000
-20.000	-6.000	0.000	0.000	0.000	0.000	20.000	0.000
0.000	6.000	-12.000	0.000	60.000	0.000	0.000	0.000
0.000	0.000	0.000	-6.000	-60.000	24.000	0.000	0.000
-20.000	-6.000	0.000	6.000	0.000	0.000	0.000	20.000

-F VALUES

1.0000 -4.0000 -1.0000 14.0000 -27.0000-291.0000 255.0000 10.0000

The table below gives the final Q values for the pipe network.

Iteration Number	Q_1 (cfs)	Q_2 (cfs)	Q_3 (cfs)	Q_4 (cfs)	Q_5 (cfs)	Q_6 (cfs)	Q_7 (cfs)	Q_8 (cfs)
5	8.3665	0.5149	1.8516	0.8766	1.7785	1.9828	6.3870	8.8594

The resulting total heads are H_A = 190.00 ft, H_B = 120.00 ft, H_C = 119.21 ft, H_D = 121.51 ft, H_E = 160.00 ft, H_F = 109.72 ft, and H_G = 200.00 ft.

4.5.1 a) Mass balance [Equation (4.23)], fluid elasticity [Equation (4.24)], Newton's 2nd law [Equation (4.25b)], and static pressure principles [Equation (4.26)].

b) Rapid valve closure ($t_c \le 2L/C$), a compressible fluid (although water is only slightly compressible), and an inviscid fluid (the equations developed are appropriate for the maximum pressure created by water hammer; the actual water hammer pressure rise will be less due to friction and other losses).

4.5.2 Rapid valve closures are those in which $t_c \le 2L/C$. The wave celerity (C) is dependent on the composite (water-pipe system) modulus of elasticity (E_c). Thus, solving for E_c with (k = 1.0 – 0.5·0.25 = 0.875)

$1/E_c = 1/E_b + (Dk)/(E_p e) = 1/(2.2 \times 10^9 \text{ N/m}^2) + [(0.5 \text{ m})(0.875)]/[(1.9 \times 10^{11} \text{ N/m}^2)(0.025 \text{ m})]$;

$E_c = 1.83 \times 10^9 \text{ N/ m}^2$; and $C = (E_c/\rho)^{1/2} = [(1.83 \times 10^9 \text{ N/ m}^2)/(998 \text{ kg/m}^3)(0.85)]^{1/2} = 1{,}470$ m/sec; now

2L/C = [2(500m)/(1470 m/s)] **= 0.680 sec** If the valve closes faster than this, it is a rapid closure.

4.5.3 Determine the composite modulus of elasticity (k = 1 – 0.5·0.25 = 0.875) and the wave speed:

$1/E_c = 1/E_b + (Dk)/(E_p e) = 1/(3.2 \times 10^5 \text{ psi}) + [(24 \text{ in.})(0.875)]/[(2.3 \times 10^7 \text{ psi})(1.5 \text{ in.})]$; $E_c = 2.68 \times 10^5$ psi

$C = (E_c/\rho)^{1/2} = [(2.68 \times 10^5 \text{ lb/in.}^2)(144 \text{ in.}^2/\text{ft}^2)/(1.94 \text{ slug/ft}^3)]^{1/2} = 4460$ ft/sec. Now, based on the wave travel time; 2L/C = [2(2,400 ft)]/(4460 ft/sec) = 1.08 sec > 1.05 sec; therefore it a is rapid valve closure. Now the maximum water hammer pressure at the valve can be calculated using Equation (4.25a, 4.25b, or 4.25c) based on an initial velocity of $V_0 = Q/A = 30 \text{ cfs}/[\pi\cdot(1 \text{ ft})^2] = 9.55$ ft/sec; thus

ΔP $= V_0(\rho\cdot E_c)^{1/2} = (9.55 \text{ ft/s})[(1.94 \text{ slug/ft}^3)(2.68 \times 10^5 \text{ lb/in.}^2)(144 \text{ in.}^2/\text{ft}^2)]^{1/2} = 8.26 \times 10^4 \text{ lb/ft}^2$ **(574 psi)**

If the flow rate was reduced by 20 cfs, $V_0 = Q/A = 10 \text{ cfs}/[\pi\cdot(1 \text{ ft})^2] = 3.18$ ft/sec; thus

ΔP $= V_0(\rho\cdot E_c)^{1/2} = 2.75 \times 10^4 \text{ lb/ft}^2$ (191 psi); and the pressure reduction is 574 – 191 = **383 psi**

4.5.4 Applying the energy equation from the surface of the reservoir (1) to the outlet of the pipe (2) yields;

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$; where $P_1 = P_2 = 0$; $h_L = h_f + [\sum K](V^2/2g)$; $V_1 = 0$ and $h_2 = 0$. Thus

$h_1 = [1 + f(L/D) + \sum K](V^2/2g)$; where $V_2 = V$ (pipe V); $K_e = 0.5$ (assume); $K_v = 10.0$; and assuming complete turbulence for the first trial: e/D = 0.045mm/300mm = 0.00015; thus; **f = 0.013,** and

$h_1 = 100 \text{ m} = [1+0.013(420/0.3)+0.5+10.0](V^2/2g)$; V = 8.12 m/sec; $N_R = DV/\nu = [(0.3)(8.12)]/(1.00\times10^{-6})$

$N_R = 2.44 \times 10^6$; From Moody; new **f = 0.014**; Thus, new V = 7.94 m/sec; and $Q = V\cdot A = 0.561 \text{ m}^3/\text{sec}$

Now determine the composite (water-pipe system) modulus of elasticity (k = 1.0) and wave speed:

$1/E_c = 1/E_b + (Dk)/(E_p e) = 1/(2.2 \times 10^9 \text{ N/m}^2)+[(0.3 \text{ m})]/[(1.9 \times 10^{11} \text{ N/m}^2)(0.01 \text{ m})]$; $E_c = 1.63 \times 10^9 \text{ N/m}^2$

$C = (E_c/\rho)^{1/2} = [(1.63 \times 10^9 \text{ N/m}^2)/(998 \text{ kg/m}^3)]^{1/2} = 1280$ m/sec; Now, based on the wave travel time; 2L/C = [2(420 m)]/(1280 m/sec) = 0.656 sec > 0.50 sec; therefore it a is rapid valve closure. Now, the maximum water hammer pressure at the valve can be calculated using Equation (4.25a, 4.25b, or 4.25c) as

ΔP $= E_c\cdot V_0/C = [(1.63 \times 10^9 \text{ N/m}^2)(7.94 \text{ m/sec})]/(1280 \text{ m/sec}) = 1.01 \times 10^7 \text{ N/m}^2$ **(10.1 MPa)**

The total (maximum) pressure the pipeline is exposed to can be determined from Equation (4.28):

$\mathbf{P_{max}} = \gamma H_0 + \Delta P = (9790 \text{ N/m}^3)(100 \text{ m}) + 1.01 \times 10^7 \text{ N/m}^2 = 1.11 \times 10^7 \text{ N/m}^2$ **(11.1 MPa)**

4.5.5 Applying the energy equation from the surface of reservoir (1) to the surface of reservoir (2) yields;

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$; where $P_1 = P_2 = 0$; $h_L = h_f + [\sum K](V^2/2g)$; $V_1 = V_2 = 0$ and $h_2 = 0$. Thus

$h_1 = [f(L/D) + \sum K](V^2/2g)$; where $V_2 = V$ (pipe V); $K_e = 0.5$ (assume); $K_v = 0.15$; and assuming complete turbulence for the first trial: e/D = 0.36mm/500mm = 0.00072; thus; **f = 0.0185,** and

$h_1 = 55\text{ m} = [0.0185(600/0.5)+0.5+0.15](V^2/2g)$; V = 6.87 m/sec; $N_R = DV/\nu = [(0.5)(6.87)]/(1.00\text{x}10^{-6})$

$N_R = 3.44 \times 10^6$; From Moody; new **f = 0.0185**; OK - Thus, V = 6.87 m/sec; and $Q = V \cdot A = 1.35\text{ m}^3/\text{sec}$

For a rigid pipe wall, $(Dk)/(E_pe) = 0$; therefore $E_c = E_b = 2.2 \times 10^9\text{ N/m}^2$

$C = (E_c/\rho)^{1/2} = [(2.2 \times 10^9\text{ N/m}^2)/(998\text{ kg/m}^3)]^{1/2} = 1480$ m/sec; Now, based on the wave travel time; 2L/C = [2(600 m)]/(1480 m/sec) = 0.811 sec > 0.65 sec; therefore it a is rapid valve closure. Now, the maximum water hammer pressure at the valve can be calculated using Equation (4.25a, 4.25b, or 4.25c) as

$\mathbf{\Delta P} = \rho \cdot C \cdot V_0 = (998\text{ kg/m}^3)(1480\text{ m/sec})(6.87\text{ m/sec}) = 1.01 \times 10^7\text{ N/m}^2$ **(10.1 MPa)**

4.5.6 Applying the energy equation from the surface of the reservoir (1) to the outlet of the pipe (2) yields;

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$; where $P_1 = P_2 = 0$; $h_L = h_f + [\sum K](V^2/2g)$; $V_1 = 0$ and $h_2 = 0$. Thus

$h_1 = [1 + f(L/D) + \sum K](V^2/2g)$; where $V_2 = V$ (pipe V); $K_e = 0.5$ (assume); $K_v = 0.15$; and assuming complete turbulence for the first trial: e/D = 0.00015ft/1.0ft = 0.00015; thus; **f = 0.013,** and

$h_1 = 98.4 = [1+0.013(1000/1.0)+0.5+0.15](V^2/2g)$; V = 20.8 ft/sec; $N_R = DV/\nu = [(1.0)(20.8)]/(1.08\text{x}10^{-5})$

$N_R = 1.93 \times 10^6$; From Moody; new **f = 0.014**; Thus, new V = 20.1 ft/sec; and $Q = V \cdot A = 15.8$ cfs

Now determine the composite (water-pipe system) modulus of elasticity (k = 1.0) and wave speed:

$1/E_c = 1/E_b + (Dk)/(E_pe) = 1/(3.2 \times 10^5\text{ psi}) + [(12\text{ in.})]/[(2.8 \times 10^7\text{ psi})(0.5\text{ in.})]$; $E_c = 2.51 \times 10^5$ psi

$C = (E_c/\rho)^{1/2} = [(2.51 \times 10^5\text{ lb/in.}^2)(144\text{ in.}^2/\text{ft}^2)/(1.94\text{ slug/ft}^3)]^{1/2} = 4320$ ft/sec; and now the maximum water hammer pressure at the valve can be calculated using Equation (4.25a, 4.25b, or 4.25c) as

$\mathbf{\Delta P} = V_0(\rho \cdot E_c)^{1/2} = (20.1\text{ ft/s})[(1.94\text{ slug/ft}^3)(2.51 \times 10^5\text{ lb/in.}^2)(144\text{ in.}^2/\text{ft}^2)]^{1/2} = 1.68 \times 10^5\text{ lb/ft}^2$ **(1170 psi)**

4.5.7 Use equation (4.28) to determine the allowable water hammer pressure based on the design pressure.

$P_{max} = \gamma H_0 + \Delta P$; $2.13 \times 10^6\text{ N/m}^2 = (9790\text{ N/m}^3)(40\text{ m}) + \Delta P$; thus, $\Delta P = 1.74 \times 10^6\text{ N/m}^2$

Based on Equation (4.25b); $\Delta P = \rho \cdot C \cdot V_0$; so solving for V_0 and C yields;

$V_0 = Q/A = (0.04\text{ m}^3/\text{s})/[(\pi/4)(0.20\text{ m})^2] = 1.27$ m/sec; and

$C = \Delta P/(\rho \cdot V_0) = (1.74 \times 10^6\text{ N/m}^2)/[(998\text{ kg/m}^3)(1.27\text{ m/sec})] = 1370$ m/sec.

Based on Equation (4.21); $C = (E_c/\rho)^{1/2}$; therefore

$C = 1370\text{ m/sec} = (E_c/\rho)^{1/2} = (E_c/998\text{ kg/m}^3)^{1/2}$; and thus $E_c = 1.87 \times 10^9\text{ N/m}^2$

Now determine the required pipe thickness based on Equation (4.22b); noting that (k = 1.0):

$1/E_c = 1/E_b + (Dk)/(E_pe)$

$1/(1.87 \times 10^9\text{ N/m}^2) = 1/(2.2 \times 10^9\text{ N/m}^2)+[(0.20\text{ m})]/[(1.6 \times 10^{11}\text{ N/m}^2)(e)]$;

e = 0.0156 m = 15.6 mm (roughly 16 mm)

4.5.8 Based on the hoop stress equation provided; $P\cdot D = 2\tau(e)$ where e = pipe thickness. Thus

$P = \Delta P = 2\tau(e)/D = 2(1.1 \times 10^8 \text{ N/m}^2)(e)/(2.0 \text{ m}) = (1.1 \times 10^8 \text{ N/m}^2)e$;

(Note that $P = \Delta P$ since the operational pressure is insignificant compared to the water hammer pressure.)

Based on Equation (4.25b); $\Delta P = \rho\cdot C\cdot V_0 = \rho\cdot V_0\cdot(E_c/\rho)^{1/2}$ since $C = (E_c/\rho)^{1/2}$. Squaring both sides yields $(\Delta P)^2 = \rho\cdot V_0^2\cdot E_c$. However, $\rho = 998 \text{ kg/m}^3$, and $V_0 = Q/A = (77.9 \text{ m}^3/\text{s})/[(\pi/4)(2.00 \text{ m})^2] = 24.8 \text{ m/sec}$; and substituting for E_c (since $1/E_c = 1/E_b + (Dk)/(E_pe)$) and noting that (k = 1.0) yields:

$(\Delta P)^2 = \rho\cdot V_0^2\cdot[1/\{1/E_b + (Dk)/(E_pe)\}] = (998)(24.8)^2[1/\{1/(2.2 \times 10^9 \text{ N/m}^2)+(2.0 \text{ m})/[(1.9 \times 10^{11} \text{ N/m}^2)(e)\}]$;

Substituting for ΔP from the hoop stress equation and solving the resulting implicit equation with computer algebra software (e.g., MathCad, Maple, or Mathematica), yields **$e \approx 0.32$ m = 32 cm**

Note that the thickness required is unreasonable. The next section of the book describes more cost effective alternatives to avoid water hammer damage.

4.5.9 Rapid valve closures are those in which $t_c \leq 2L/C$. The wave celerity (C) is dependent on the composite (water-pipe system) modulus of elasticity (E_c). For rigid pipe walls, $E_c = E_b = 2.2 \times 10^9 \text{ N/m}^2$; thus

$C = (E_c/\rho)^{1/2} = [(2.2 \times 10^9 \text{ N/m}^2)/(998 \text{ kg/m}^3)]^{1/2} = 1{,}480 \text{ m/sec}$; now

$2L/C = [2(700\text{m})/(1480 \text{ m/s})] = 0.946$ sec and therefore a 60 second closure time is not rapid.

Therefore, apply the Allievi equation to determine the water hammer pressure.

$N = [(\rho\cdot L\cdot V_0)/(P_0\cdot t)]^2 = [\{(998 \text{ kg/m}^3)(700 \text{ m})(24.8 \text{ m/sec})\}/\{(9790 \text{ N/m}^3)(150 \text{ m})(60 \text{ sec})\}]^2 = 0.0387$

$\Delta P = P_0[N/2 + \{N^2/4 + N\}^{1/2}] = 3.19 \times 10^5 \text{ N/m}^2$

Since the water hammer pressure is considerably smaller, we need to include the operational pressure in the determination of wall thickness. Thus,

$P_{max} = \gamma H_0 + \Delta P = (9790 \text{ N/m}^3)(150 \text{ m}) + 3.19 \times 10^5 \text{ N/m}^2 = 1.79 \times 10^6 \text{ N/m}^2$ (1.79 MPa)

Now, based on the hoop stress equation provided; $P\cdot D = 2\tau(e)$ where e = pipe thickness,

$P = P_{max} = 1.79 \times 10^6 \text{ N/m}^2 = 2(1.1 \times 10^8 \text{ N/m}^2)(e)/(2.0 \text{ m})$; **e = 0.0163 m = 1.63 cm**

4.5.10 Equation (4.25b) can be rearranged as $C = \Delta P/(\rho\cdot V_0)$; and substituting for ΔP from Equation (4.24) yields

$C = (E_c\cdot\Delta Vol)/(\rho\cdot V_0\cdot A\cdot L)$. Now we can substitute for ΔVol from Equation (4.23) which yields

$C = (E_c\cdot V_0\cdot A\cdot(L/C)]/(\rho\cdot V_0\cdot A\cdot L)$; and this reduces to

$C^2 = (E_c/\rho)$ or $C = (E_c/\rho)^{1/2}$ which is Equation (4.21).

4.6.1 a) Increased pressures are eliminated where the pipe meets the reservoir (the most vulnerable portion of the pipeline, as seen in Figure 4.12 – water hammer pressure is added to the highest static pressure here). But due to the increased water levels (and thus pressure) in the surge tank, the rest of the pipeline experiences higher pressures from the surge tank with the greatest additional pressure next to the surge tank.

b) Newton's second law, hydrostatic pressure, throttle loss, and pipeline losses.

c) Minor losses are neglected in Equation 4.31(including entrance loss if K_f only accounts for friction).

4.6.2 If the allowable surge tank rise (y_{max}) is 7.50 m, the damping factor (β) can be found from Equation (4.31)

$(y_{max} + h_L)/(\beta) = \ln [(\beta)/(\beta - y_{max})]$; or substituting, $(7.50m + 15.1m)/(\beta) = \ln [(\beta)/(\beta - 7.50m)]$

The damping factor is determined by using calculators or software that solve implicit equations. Thus,

$\beta = 7.97 = (LA)/(2gK_fA_s) = [(1500)(3.80)]/[2(9.81)(0.546)(A_s)$; therefore, $A_s = 66.8\ m^2$ and $\mathbf{D_s = 9.22\ m}$

4.6.3 Neglecting minor losses, determine the head loss and the pipeline friction factor.

$h_L = h_f = f(L/D)(V^2/2g)$; where $V = Q/A = (2.81\ m^3/sec)/[(\pi/4)(0.90m)^2] = 4.42$ m/sec;

$e/D = 0.045mm/900mm = 0.00005$; $N_R = DV/\nu = [(0.90)(4.42)]/(1.00x10^{-6}) = 3.98 \times 10^6$; **f = 0.0115**

$h_L = h_f = f(L/D)(V^2/2g) = [0.0115(425/0.90)[(4.42)^2/2g] = 5.41$ m; and now the pipeline friction factor is:

$K_f = h_L/V^2 = (5.41\ m)/[(4.42\ m/sec)^2] = 0.277\ sec^2/m$; and the damping factor is

$\beta = (LA)/(2gK_fA_s) = [(425m)(0.636m^2)]/[2(9.81m/sec^2)(0.277\ sec^2/m)(\pi/4)(2.0m)^2] = 15.8$ m;

To determine y_{max}, use Equation (4.33) which is: $(y_{max} + h_L)/(\beta) = \ln [(\beta)/(\beta - y_{max})]$; or substituting,

$(y_{max} + 5.41m)/(15.8\ m) = \ln [(15.8\ m)/(15.8\ m - y_{max})]$; $\mathbf{y_{max} \approx 9.80\ m}$

4.6.4 Neglecting minor losses, determine the head loss and the pipeline friction factor.

$h_L = h_f = f(L/D)(V^2/2g)$; where $V = Q/A = (350\ ft^3/sec)/[(\pi/4)(6.0\ ft)^2] = 12.4$ ft/sec;

$e/D = 0.0006ft/6.0ft = 0.0001$; $N_R = DV/\nu = [(6.0)(12.4)]/(1.08x10^{-5}) = 6.89 \times 10^6$; **f = 0.0125**

$h_L = h_f = f(L/D)(V^2/2g) = [0.0125(2500/6.0)[(12.4)^2/2g] = 12.4$ ft; and now the pipeline friction factor is:

$K_f = h_L/V^2 = (12.4\ ft)/[(12.4\ m/sec)^2] = 0.0806\ sec^2/ft$; and the damping factor is

$\beta = (LA)/(2gK_fA_s) = [(2500ft)(28.3ft^2)]/[2(32.2ft/sec^2)(0.0806\ sec^2/ft)(\pi/4)(20\ ft)^2] = 43.4$ ft;

To determine y_{max}, use Equation (4.33) which is: $(y_{max} + h_L)/(\beta) = \ln [(\beta)/(\beta - y_{max})]$; or substituting,

$(y_{max} + 12.4\ ft)/(43.4\ ft) = \ln [(43.4\ ft)/(43.4\ ft - y_{max})]$; $y_{max} \approx 25.0$ ft

and the surge tank height: $\mathbf{H_S} = H_B + y_{max} = 50.0\ ft + 25.0\ ft$ **= 75.0 ft**

4.6.5 Neglecting minor losses, determine the head loss and the pipeline friction factor.

$h_L = h_f = f(L/D)(V^2/2g)$; where $V = Q/A = (2.81\ m^3/sec)/[(\pi/4)(0.90m)^2] = 4.42$ m/sec;

$e/D = 0.045mm/900mm = 0.00005$; $N_R = DV/\nu = [(0.90)(4.42)]/(1.00x10^{-6}) = 3.98 \times 10^6$; **f = 0.0115**

$h_L = h_f = f(L/D)(V^2/2g) = [0.0115(425/0.90)[(4.42)^2/2g] = 5.41$ m; and now the pipeline friction factor is:

$K_f = h_L/V^2 = (5.41\ m)/[(4.42\ m/sec)^2] = 0.277\ sec^2/m$. Now determine the damping factor using y_{max}

by using Equation (4.33) which is: $(y_{max} + h_L)/(\beta) = \ln [(\beta)/(\beta - y_{max})]$; or substituting,

$(5\ m + 5.41m)/(\beta) = \ln [(\beta)/(\beta - 5\ m)]$; $\beta \approx 6.13$ m; and solving for A_s from $\beta = (LA)/(2gK_fA_s)$ yields

$6.13\ m = [(425m)(0.636m^2)]/[2(9.81m/sec^2)(0.277\ sec^2/m)(A_s)]$; $A_s = 8.11\ m^2$; or $\mathbf{D_s = 3.21\ m}$

4.6.6 Assuming complete turbulence, $e/D = 0.60mm/2000mm = 0.00030$; thus **f = 0.015**

$K_f = (f)(L)/(2gD) = [(0.015)(1500m)]/[2g(2m)] = 0.573\ sec^2/m$. Also,

$\beta = (LA)/(2gK_fA_s) = [(1500m)(3.14m^2)]/[2(9.81m/sec^2)(0.573\ sec^2/m)(\pi/4)(10.0m)^2] = 5.33$ m; now

$(y_{max} + h_L)/(\beta) = \ln [(\beta)/(\beta - y_{max})]$; or $(5\ m + h_L)/(5.33) = \ln [(5.33)/(5.33 - 5\ m)]$; $h_L = 9.83\ m = h_f$; and

$h_f = K_fV^2$; $V = 4.14$ m/sec, and $\mathbf{Q = AV = 13.0\ m^3/sec}$

Chapter 5 – Problem Solutions

5.1.1

Convert the pump discharge to cfs (ft^3/sec);

(2500 gpm)(1 cfs/449 gpm) = 5.57 cfs

$\mathbf{P_o} = \gamma Q H_P = (62.3\ lb/ft^3)(5.57\ ft^3/sec)(104\ ft)$

$\mathbf{P_o}$ = 36,100 ft-lb/sec(1 hp/550 ft·lb/sec) = 65.6 hp

$\mathbf{P_m} = \mathbf{P_o}/e$ = (65.6 hp)/(0.785) = 83.6 hp

$\mathbf{P_m}$ = 83.6 hp(1 kW/1.341 hp) **= 62.3 kW**

(Note: Conversion factors are in the book jacket)

5.1.2

$\mathbf{P_o} = \mathbf{P_m}(e)$ = 1000 watts(0.5) = 500 watts (N·m/sec)

$\mathbf{P_o}$ = 500 N·m/sec = $\gamma Q H_P = (9790\ N/m^3)(Q)(2\ m)$

Q = 0.025 m^3/s; Drawdown = Vol/area = (Q·t)/(area)

Drawdown = (0.025 m^3/sec)(86,400 sec)/(5000m^2)

Drawdown = 0.432 m (43.2 cm) during the first day

5.1.3

Balancing energy from reservoir A to reservoir B:

$H_A + H_P = H_B + h_L$; (Eq'n 4.2), $H_B – H_A$= 20m; and

$h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$; $K_d = 1.0$ (exit coef.)

V = Q/A = 4.10 m/s; e/D = 0.60mm/800mm = 0.00075

$N_R = DV/\nu = [(0.80)(4.10)]/(1.00\times10^{-6}) = 3.28 \times 10^6$;

From Moody; **f = 0.0185**; solving the energy eq'n;

H_P= 20m+[0.0185(100/0.80)+1.5]·[$(4.1)^2$/2g] = 23.3m

$\mathbf{P_o} = \gamma Q H_P = (9.79\ kN/m^3)(2.06\ m^3/sec)(23.3\ m)$

$\mathbf{P_o}$ = 470 kW; **e** = $\mathbf{P_o}/\mathbf{P_m}$ = 470/800 = **0.588 (58.8%)**

5.1.4

a) $H_P = H_R – H_S + h_L$

The pump supplies the energy to raise water to the higher reservoir and overcome the pipeline losses.

b) $H_P = P_3/\gamma – P_2/\gamma$

The pump adds energy in the form of pressure.

c) Angular momentum conservation and the definition of power (**P** = ω·T)

5.1.5

The torque on the exiting flow is $T = \rho Q r_o V_o \cos(\alpha_o)$

Where $Q = A\upsilon_{ro}$ and from Fig. 5.3, $\upsilon_{ro} = V_o \sin(\alpha_o)$

α_o = 90°- 55° = 35° from problem statement, thus

$\upsilon_{ro} = V_o\sin(\alpha_o)$ = (45 m/sec)(sin 35°) = 25.8 m/sec

$Q = A\upsilon_{ro} = 2\pi\ r_o(B)\upsilon_{ro}$ = 2π(0.5m)(0.2m)(25.8 m/s)

Q = 16.2 m^3/sec; and since $T = \rho Q r_o V_o \cos(\alpha_o)$

T = (998 kg/m^3)(16.2 m^3/sec)(0.5 m)(45 m/s)(cos 35°)

T = 298,000 N·m = 298 kN·m

5.1.6

$u_i = \omega r_i$ = (1800 rev/min)(1 min/60 sec)(2π rad/rev)
(0.333 ft) = 62.8 ft/sec; & u_o = 188 ft/sec

From Fig 5.3 w/α_i = 90°; $V_i = V_{ri} = (u_i)\{\tan(180°- \beta_i)\}$

thus V_{ri} = 62.8 ft/sec (tan 20°) = 22.9 ft/sec = υ_{ri}

Q = $A\upsilon_{ri}$ = 2π(0.333ft)(0.167ft)(22.9 ft/sec) **= 8.00 cfs**

$\upsilon_{ro} = Q/A_o$ = (8 cfs)/[(2π)(1 ft)(0.0625 ft)] = 20.5 ft/sec

From Fig 5.3, $\upsilon_{to} = \upsilon_{ro}$/tan 170° = -116 ft/sec

$V_{to} = u_o + \upsilon_{to}$ = 188 – 116 = 72.0 ft/sec; and

$\boldsymbol{\alpha_o} = \tan^{-1}(V_{ro}/V_{to}) = \tan^{-1}(20.5/72.0)$ **= 15.9°**

5.1.7

With no tangential velocity at the inlet, $V_{ti} = 0$ and from Fig. 5.3 we see that $\alpha_i = 90°$; $V_i = V_{ri} = \upsilon_{ri}$; thus

$V_i = V_{ri} = Q/A_{ri} = (70\ cfs)/[2\pi(1.0ft)(0.333ft)] = 33.5\ ft/s$

$\upsilon_i = V_i/\sin\beta_i = (33.5\ ft/s)/(\sin 120°) = 38.7\ ft/sec$; also from Fig. 5.3, note that $\upsilon_i\cos\beta_i = -u_i = -\omega r_i$; and

$(38.7\ ft/sec)(\cos 120°) = -(1.0\ ft)(\omega)$, $\omega = 19.4\ rad/sec$

$\boldsymbol{\omega} = (19.4\ rad/sec)(1\ rev/2\pi\ rad)(60s/1\ min)$ **= 185 rpm**

$u_o = \omega r_o = (19.4\ rad/sec)(2.5\ ft) = 48.5\ ft/sec$;

$\upsilon_{ro} = Q/A_o = (70cfs)/[(2\pi)(2.5ft)(0.333ft)] = 13.4\ ft/sec$

$\upsilon_{to} = \upsilon_{ro}/\tan\beta_o = (13.4)/\tan 135° = -13.4\ ft/sec$

$V_o = [\upsilon_{ro}^2 + (u_o + \upsilon_{to})^2]^{½} = [(13.4)^2 + (48.5-13.4)^2]^{½}$

$V_o = 37.6\ ft/sec$; $\alpha_o = \tan^{-1}[\upsilon_{ro}/(u_o + \upsilon_{to})]$

$\alpha_o = \tan^{-1}[13.4/(48.5 – 13.4)] = 20.9°$

$\mathbf{P_i} = \rho Q\omega[r_oV_o\cos(\alpha_o) - r_iV_i\cos(\alpha_i)] = (1.94\ slugs/ft^3)\cdot$ $(70\ ft^3/sec)(19.4\ rad/sec)[(2.5\ ft)(37.6\ ft/sec)\cos 20.9° - (1.0\ ft)(33.5\ ft/sec)\cos 90°] = 2.31 \times 10^5\ ft\text{-}lb/sec$

$\mathbf{P_i} = 2.31 \times 10^5\ ft\text{-}lb/sec(1\ hp/550\ ft\text{-}lb/sec)$ **= 420 hp**

5.1.8

Since $\alpha_i = 90°$; $V_i = V_{ri} = \upsilon_{ri}$; and $V_{ti} = 0$ (Fig. 5.3)

$\upsilon_{ri} = Q/A_i = (0.055\ m^3/s)/[(2\pi)(0.075m)(0.05m)] = 2.33m/s$

$\upsilon_{ti} = \upsilon_{ri}/\tan\beta_i = (2.33)/\tan 150° = -4.04\ m/sec$

$u_i = -\upsilon_{ti} = 4.04\ m/s$; and $u_i = \omega r_i$; therefore,

$\boldsymbol{\omega} = u_i/r_i = (4.04\ m/s)/(0.075m)$ **= 53.9 rad/s**

$\boldsymbol{\omega} = (53.9\ rad/sec)(1\ rev/2\pi\ rad)(60s/1\ min)$ **= 515 rpm**

$V_{ro} = Q/A_o = (0.055\ m^3/s)/[(2\pi)(0.15m)(0.03m)] = 1.95\ m/s$

$V_o = V_{ro}/\sin\alpha_o = (1.95)/\sin 22.4° = 5.12\ m/sec$

5.1.8 (continued)

$\mathbf{P_i} = \rho Q\omega[r_oV_o\cos(\alpha_o) - r_iV_i\cos(\alpha_i)] = (998\ kg/m^3)\cdot$ $(0.055\ m^3/s)(53.9\ rad/s)[(0.15m)(5.12\ m/s)\cos 22.4° - (0.075m)(2.33V_i)\cos 90°] = 2.10 \times 10^4\ N\text{-}m/sec$

$\mathbf{P_i} = 2.10 \times 10^3\ N\text{-}m/s(1\ kW/1000\ N\text{-}m/s)$ **= 0.210 kW**

5.5.1

Based on the pump curve, for Q = 15 cfs, $H_p = 259.5$ ft. From the system curve (Ex. 5.3), $h_f = 61.9$ft. for this Q. Applying Eq'n 5.19 (minor losses are now significant),

$H_p = H_s + h_L = H_s + h_f + h_v$;

259.5 ft = 120 ft + 61.9 ft + h_v; $\mathbf{h_v = 77.6\ ft}$

This is not an efficient system; too much energy is lost in the valve instead of being applied to the flow.

5.5.2

A spreadsheet is easily programmed to determine the relationship between Q and H_{SH} (system). Applying Equation 5.19 including minor losses, which are now significant ($h_v = 0.10(Q)^2$ where Q is in cfs), yields $H_{p(sys)} = H_s + h_L = H_s + h_f + h_v$ and the spreadsheet below.

Q	H_p	h_f	h_v	H_s	H_{SH}
(cfs)	(ft)	(ft)	(ft)	(ft)	(ft)
0	300.0	0.0	0.0	120.0	120.0
5	295.5	8.1	2.5	120.0	130.6
10	282.0	29.2	10.0	120.0	159.2
15	259.5	61.9	22.5	120.0	204.4
18	239.1*	86.8	32.4	120.0	239.2
20	225.5	105.4	40.0	120.0	265.4
25	187.5	159.3	62.5	120.0	341.8
30	138.0	223.2	90.0	120.0	433.2
35	79.5	296.9	122.5	120.0	539.4

*Interpolated from the pump characteristics.

From the table (or a graph), the intersection of the pump characteristic curve and the system curve is at:

Q = 18 cfs and $\mathbf{H_p \approx 239}$ ft

5.5.3

A spreadsheet is easily programmed to determine the relationship between Q and H_{SH} (system). Applying Equation 5.19 (but including minor losses) yields

$H_{SH} = H_s + h_L$; where $h_L = h_f + [\sum K](V)^2/2g$; and

$K_e = 0.5$, $K_v = 2.5$, and $K_d = 1.0$ (exit coefficient).

Also, with reference to Table 3.4, $h_f = KQ^2$ and

$K = (0.0826 \cdot f \cdot L)/(D^5) = (0.0826 \cdot 0.02 \cdot 3050)/[(0.5)^5]$;

K = 161. This leads to the spreadsheet below:

Single Pump and Pipeline Analysis (Prob 5.5.3)

Pipeline Data			Reservoir Data		
L =	3050	m	E_A =	45.5	m
D =	0.50	m	E_B =	52.9	m
f =	0.02		H_s =	7.4	m
$h_f = KQ^m$			Minor Losses		
m =	2.00		ΣK =	4.00	
K =	161.2		g =	9.81	m/sec^2

Q (m^3/s)	H_p (m)	h_f (m)	h_{minor} (m)	H_s (m)	H_{SH} (m)
0.00	91.4	0.0	0.0	7.4	7.4
0.15	89.8	3.6	0.1	7.4	11.1
0.30	85.1	14.5	0.5	7.4	22.4
0.45	77.2	32.7	1.1	7.4	41.1
0.595	66.3*	57.1	1.9	7.4	66.4
0.60	65.9	58.0	1.9	7.4	67.3
0.75	52.6	90.7	3.0	7.4	101.1
0.90	46.3	130.6	4.3	7.4	142.3
1.05	15.7	177.8	5.8	7.4	191.0

*Interpolated from the pump characteristics.

From the table (or a graph), the intersection of the pump characteristic curve and the system curve is at: **Q = 0.595 m^3/sec** and **H_p ≈ 66.3 m.** Note that minor losses are not significant, but they do change the match point slightly. Also, the velocity is:

V = Q/A = (0.59 m^3/s)/[π(0.25m)^2] **= 3.00 m/sec**

5.5.4

A spreadsheet is easily programmed to determine Q vs. H_{SH} (system) and this can be superimposed on Q vs. H_p (pump). Applying Equation 5.19 yields

$H_{SH} = H_s + h_L$; where $h_L = h_f = f(L/D)[(V)^2/2g]$; thus

Single Pump and Pipeline Analysis (Prob 5.5.4)

Pipeline Data			Reservoir Data		
L =	1000	m	E_A =	920.5	m
D =	0.40	m	E_B =	935.5	m
e =	0.045	mm	H_s =	15.0	m
e/D =	0.00011		Minor Losses		
T =	20°C		ΣK =	0.00	
ν =	1.00E-06		g =	9.81	m/sec^2

Q (m^3/s)	H_p (m)	V (m/sec)	N_R	f*	H_{SH} (m)
0.00	30.0	0.00	0.00E+00	0.0000	15.0
0.10	29.5	0.80	3.18E+05	0.0154	16.2
0.20	28.0	1.59	6.37E+05	0.0142	19.6
0.30	25.0	2.39	9.55E+05	0.0137	25.0
0.40	19.0	3.18	1.27E+06	0.0134	32.4
0.50	4.0	3.98	1.59E+06	0.0133	41.7

* Used Swamee-Jain Equation (3.14a).

From the table (or a graph), the intersection of the pump characteristic curve and the system curve is at:

Q = 300 liters/sec and H_p = 25.0 m

5.5.5

From Eq'n 5.19; $H_{SH} = H_s + h_f = H_s + f(L/D)[(V)^2/2g]$;

$H_{SH} = 14.9 + (0.019)(22.4/0.05)[(Q)^2/2g(A)^2]$; A=0.00196 m^2

$H_{SH} = 14.9 + 113{,}000(Q)^2$; with Q given in m^3/sec

$H_{SH} = 14.9 + 0.113(Q)^2$; with Q given in liters/sec

Also, $H_p = 23.9 - 7.59(Q)^2$; solving simultaneously yields **Q = 1.08 liters/sec and H_p = 15.0 m**

The same solution would result through the graphing procedure used in Example 5.3.

5.5.6

Find the equivalent pipe to replace Branches 1 and 2, (Eq'n 3.47) arbitrarily letting D = 3 ft and f = 0.02;

$[(D_E^5)/(f_E \cdot L_E)]^{1/2} = [(D_1^5)/(f_1 \cdot L_1)]^{1/2} + [(D_2^5)/(f_2 \cdot L_2)]^{1/2}$

$[(3^5)/(0.02 \cdot L_E)]^{1/2} = [(2^5)/(0.02 \cdot 100)]^{1/2} + [(1^5)/(0.02 \cdot 500)]^{1/2}$

L_E = 652 ft. Now find the equivalent pipe for the three pipes in series between the two reservoirs. However, since they all have the same diameter and f value:

$L_E = \Sigma L_i = 4000 + 652 + 1140 = 5792$ ft

A spreadsheet is easily programmed to determine the relationship between Q and H_{SH} (system). Applying Equation 5.19 (neglecting minor losses) yields

$H_{SH} = H_s + h_L = H_s + h_f$ and the spreadsheet below.

Single Pump and Pipeline Analysis (Prob 5.5.6)

Pipeline Data			Reservoir Data		
L =	5792	ft	E_A =	100	ft
D =	3.00	ft	E_D =	150	ft
f =	0.020		H_s =	50.0	ft
$h_f = KQ^m$			Minor Losses		
m =	2.00		ΣK =	0.00	
K =	0.0120		g =	32.2	ft/sec^2

Q (cfs)	H_p (ft)	h_f (ft)	h_{minor} (ft)	H_s (ft)	H_{SH} (ft)
0.0	60.0	0.0	0.0	50.0	50.0
10.0	55.0	1.2	0.0	50.0	51.2
13.5	52.2*	2.2	0.0	50.0	52.2
20.0	47.0	4.8	0.0	50.0	54.8
30.0	37.0	10.8	0.0	50.0	60.8
40.0	23.0	19.2	0.0	50.0	69.2
50.0	7.0	30.0	0.0	50.0	80.0

*Interpolated from the pump characteristics.

From the table (or a graph), the intersection of the pump characteristic curve and the system curve is at:

Q = 13.5 cfs and H_p = 52.2 ft. Friction loss (Table 3.4)

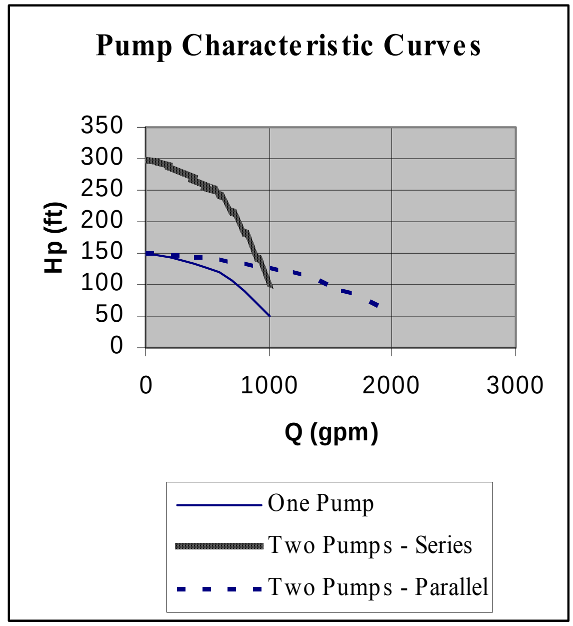
$h_{fBC} = (0.0252)(0.02)(652)(13.5)^2/(3)^5 = 0.246$ ft

$h_{fBC1} = 0.246 = (0.0252)(0.02)(100)Q_1^2/(2)^5$; **$Q_1$ = 12.5 cfs**

$h_{fBC2} = 0.246 = (0.0252)(0.02)(500)Q_2^2/(1)^5$; **$Q_2$ = 1.0 cfs**

5.6.1

The pump characteristic curves are plotted below. For two pumps in series, the heads are doubled for each value of flow. For two pumps in parallel, the flows are doubled for each value of head.



(d) If Q = 1700 gpm and Hp = 80 ft, you will need **two pumps in parallel.**

(e) If Q = 1700 gpm and Hp = 160 ft, you will need **four pumps, two parallel pipes with two pumps in series in each of the two parallel pipes.**

5.6.2

A spreadsheet is easily programmed to determine the relationship between Q and H_{SH} (system) and this can be superimposed on Q vs. H_p (pump) for the one or two pump series systems. Applying Equation 5.19 yields

$H_{SH} = H_s + h_L$; where $h_L = h_f = f(L/D)[(V)^2/2g]$ with

H_s = 25 m, L = 1000 m, D = 0.4 m, e = 0.045 mm;

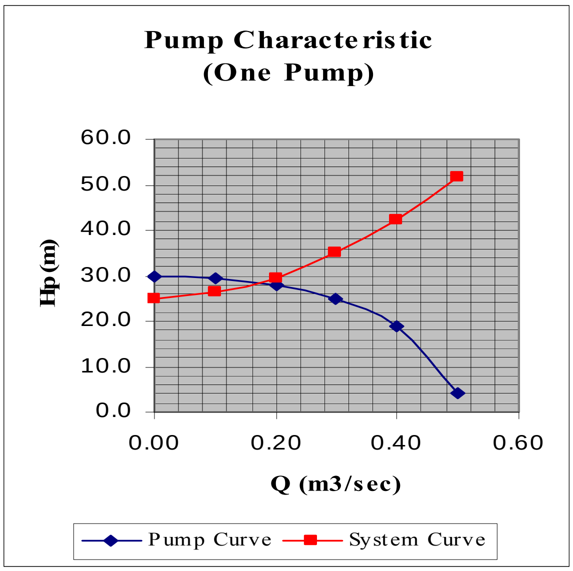
e/D = 0.00011; T = 20°C, and ν = 1.00 x 10^{-6} m^2/s

All this leads to the spreadsheet below:

5.6.2 (continued)

Q (m^3/s)	H_p (m)	V (m/sec)	N_R	f*	H_{SH} (m)
0.00	30.0	0.00	0.00E+00	0.0000	25.0
0.10	29.5	0.80	3.18E+05	0.0154	26.2
0.20	28.0	1.59	6.37E+05	0.0142	29.6
0.30	25.0	2.39	9.55E+05	0.0137	35.0
0.40	19.0	3.18	1.27E+06	0.0134	42.4
0.50	4.0	3.98	1.59E+06	0.0133	51.7

* Used Swamee-Jain Equation (3.24a).



From the graph above, **Q ≈ 0.165 m³/s; H_p ≈ 28.5 m**

For two pumps in series:

Q (m^3/s)	H_p (m)	V (m/sec)	N_R	f*	H_{SH} (m)
0.00	60.0	0.00	0.00E+00	0.0000	25.0
0.10	59.0	0.80	3.18E+05	0.0154	26.2
0.20	56.0	1.59	6.37E+05	0.0142	29.6
0.30	50.0	2.39	9.55E+05	0.0137	35.0
0.40	38.0	3.18	1.27E+06	0.0134	42.4
0.50	8.0	3.98	1.59E+06	0.0133	51.7

* Used Swamee-Jain Equation (3.24a).

From resulting graph, **Q ≈ 0.385 m³/s; H_p ≈ 41.0 m**

5.6.3

A spreadsheet is easily programmed to determine the relationship between Q and H_{SH} (system) and this can be superimposed on Q vs. H_p for the two parallel pump combination. Applying Equation 5.19 yields

$H_{SH} = H_s + h_L$; where $H_s = 878 - 772 = 106$ ft.

Also, with reference to Table 3.4, $h_f = KQ^{1.85}$ and

$K = (4.73 \cdot L)/(D^{4.87} \cdot C^{1.85}) = (4.73 \cdot 4860)/(2^{4.87} \cdot 100^{1.85})$;

K = 0.157. This leads to the spreadsheet below:

Pump Combinations (Prob 5.6.3)

Pipeline Data			Reservoir Data		
L =	4860	ft	E_A =	772	ft
D =	2.00	ft	E_D =	878	ft
C_{HW} =	100		H_s =	106.0	ft
$h_f = KQ^m$			Minor Losses		
m =	1.85		ΣK =	0.00	
K =	0.157		g =	32.2	ft/sec²

Q (cfs)	H_p (ft)	2Q (cfs)	h_f (ft)	H_s (ft)	H_{SH} (ft)
0	300	0	0	106	106
5	296	10	11	106	117
10	282	20	40	106	146
15	260	30	85	106	191
20	226	40	144	106	250
25	188	50	218	106	324
30	138	60	306	106	412
35	80	70	406	106	512

Recall that Qs are additive for parallel pump systems for each value of pump head. In this case the Qs are doubled for each value of Hp since they are identical pumps. Plotting the pump and system curves results in a plot with an intersection at:

Q ≈ 37.5 cfs; H_p ≈ 235 ft; V = 11.9 ft/sec

Note that minor losses would not change the answer significantly. For ∑K = 1.5 (entrance and exit loss), $(\sum K)(V)^2/2g$ = 3.3 ft << 235 ft (Hp)

5.6.4

A spreadsheet is programmed to determine the pump relationships (Q vs. H_p). For two in series, double the pump head for each value of Q, and for two in parallel, double the Q for each value of Hp. Applying Equation 5.19 (but including minor losses) yields

$H_{SH} = H_s + h_L$; where $h_L = h_f + [\sum K](V)^2/2g$; and

$K_e = 0.5$, $K_v = 2.5$, and $K_d = 1.0$ (exit coefficient).

Also, with reference to Table 3.4, $h_f = KQ^2$ and

$K = (0.0826 \cdot f \cdot L)/(D^5) = (0.0826 \cdot 0.02 \cdot 3050)/[(0.5)^5]$;

K = 161. This leads to the spreadsheets below:

Pump Combinations (Prob 5.6.4)

Two Pumps in Series

Q (m^3/s)	2(H_p) (m)	h_f (m)	h_{minor} (m)	H_s (m)	H_{SH} (m)
0.00	182.8	0.0	0.0	7.4	7.4
0.15	179.6	3.6	0.1	7.4	11.1
0.30	170.2	14.5	0.5	7.4	22.4
0.45	154.4	32.7	1.1	7.4	41.1
0.60	131.8	58.0	1.9	7.4	67.3
0.75	105.2	90.7	3.0	7.4	101.1
0.90	72.6	130.6	4.3	7.4	142.3
1.05	31.4	177.8	5.8	7.4	191.0

Two Pumps in Parallel

2(Q) (m^3/s)	H_p (m)	h_f (m)	h_{minor} (m)	H_s (m)	H_{SH} (m)
0.00	91.4	0.0	0.0	7.4	7.4
0.30	89.8	14.5	0.5	7.4	22.4
0.60	85.1	58.0	1.9	7.4	67.3
0.90	77.2	130.6	4.3	7.4	142.3
1.20	65.9	232.2	7.6	7.4	247.2

For two in series (graph): **Q ≈ 0.755 m^3/s; H_p ≈ 103 m**

Two in parallel (graph): **Q ≈ 0.680 m^3/s; H_p ≈ 84 m**

In this case, pumps in series provide more flow than the parallel combination because head losses due to friction are accumulating rapidly with increasing flow.

5.7.1

A spreadsheet is programmed to determine the system head for each pipe and the combined system. The pump curve is superimposed on the system curves to yield the system flow and individual pipe flows (see Ex. 5.5).

Pumps & Branching Pipes (Prob 5.7.1)

Pipeline # 1 Data			Pipeline # 2 Data		
L =	1000	m	L =	3000	m
D =	1.00	m	D =	1.00	m
f =	0.020		f =	0.020	
$h_{f1} = KQ_1^m$			$h_{f2} = KQ_2^m$		
m =	2.00		m =	2.00	
K =	1.65		K =	4.96	

Reservoir Data

E_B =	16.0	m	E_C =	22.0	m
E_A =	10.0	m	E_A =	10.0	m
H_{s1} =	6.0	m	H_{s2} =	12.0	m

Q (m^3/s)	H_p (m)	h_{f1} (m)	h_{f2} (m)	H_{SH1} (m)	H_{SH2} (m)
0.0	30.0	0.0	0.0	6.0	12.0
1.0	29.5	1.7	5.0	7.7	17.0
2.0	28.0	6.6	19.8	12.6	31.8
3.0	25.5	14.9	44.6	20.9	56.6
4.0	22.0	26.4	79.3	32.4	91.3
5.0	17.5	41.3	123.9	47.3	135.9
6.0	12.0	59.5	178.4	65.5	190.4
7.0	5.0	80.9	242.8	86.9	254.8

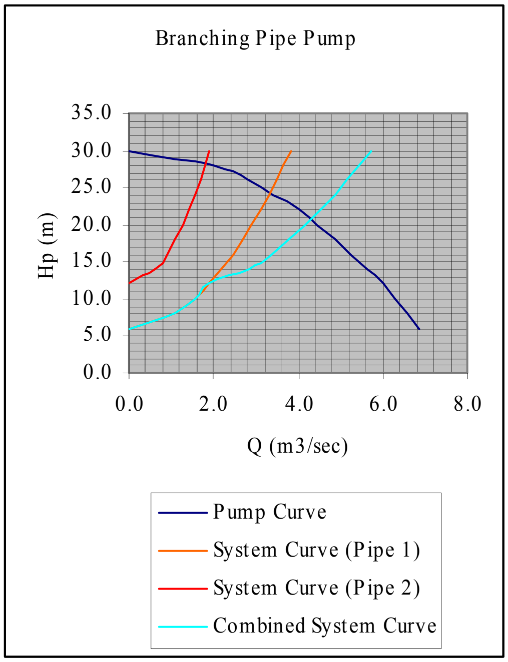
Plotting Data

Head (m)	Q_{pump}* (m^3/s)	Q_1** (m^3/s)	Q_2** (m^3/s)	Q_{sys} (m^3/s)
6.0	6.9	0.0		0.0
8.0	6.6	1.1		1.1
10.0	6.3	1.6		1.6
12.0	6.0	1.9	0.0	1.9
14.0	5.6	2.2	0.6	2.8
16.0	5.3	2.5	0.9	3.4
18.0	4.9	2.7	1.1	3.8
20.0	4.4	2.9	1.3	4.2
22.0	4.0	3.1	1.4	4.5
24.0	3.4	3.3	1.6	4.9
26.0	2.8	3.5	1.7	5.2
28.0	2.0	3.6	1.8	5.4
30.0	0.0	3.8	1.9	5.7

* Linear interpolation from pump data.

** Flows obtained from system equations.

5.7.1 (continued)



The points of intersection (match points) are:

$\mathbf{Q_{sys} = 4.3\ m^3/sec}$; $\mathbf{H_p = 20.8\ m}$, and for each pipe

$\mathbf{Q_1 = 3.0\ m^3/sec}$ **and** $\mathbf{Q_2 = 1.3\ m^3/sec}$ at that pump head.

5.7.2

A spreadsheet is programmed to determine the system head for the pipeline going from reservoir A to D. Only one pump is resident in this line. The pump curve is superimposed on the system curve to yield the pipeline flow noting that the flow changes after the junction, but the flow in pipeline BC is known. This process yields,

Pumps & Branching Pipes (Prob 5.7.2)

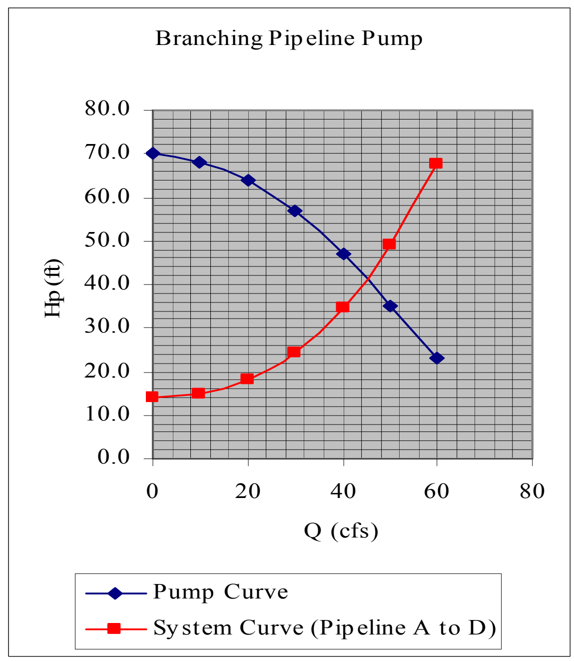
Pipeline AB Data			Pipeline BD Data		
L =	5000	ft	L =	5000	ft
D =	3.00	ft	D =	3.00	ft
f =	0.020		f =	0.020	
$h_f = KQ^m$			$h_f = K(Q\text{-}20)^m$		
m =	2.00		m =	2.00	
K =	0.0103		K =	0.0103	

Reservoir Data

E_D= 94.0 ft E_A= 80.0 ft

Q (cfs)	H_p (ft)	h_{fAB} (ft)	h_{fBD} (ft)	H_{s1} (ft)	H_{SH} (ft)
0	70.0	0.0		14.0	14.0
10	68.0	1.0		14.0	15.0
20	64.0	4.1	0.0	14.0	18.1
30	57.0	9.3	1.0	14.0	24.3
40	47.0	16.5	4.1	14.0	34.6
50	35.0	25.7	9.3	14.0	49.0
60	23.0	37.0	16.5	14.0	67.5

Note that the flow in pipe BD is (Q_{AB} – 20) since the flow in pipe AB is 20 cfs. Thus, friction losses do not occur in pipe BD until Q > 20 cfs. A plot yields



where $\mathbf{Q_{AB} = 45}$ **cfs** and $\mathbf{H_{pA} = 41.2}$ **ft**. Thus,

$\mathbf{Q_{BD}} = Q_{AB} - Q_{BD} = (45 – 20)$ **= 25 cfs**

Balancing energy between reservoir A and junction B,

$E_A + H_{pA} = E_B + h_{fAB}$ where E_B = total energy elev. at B,

$E_B = E_A + H_{pA} – K(Q_{AB})^2 = 80+41.2–(0.0103)(45)^2 = 100.3$ ft

Balancing energy from B to reservoir C (w/ $\mathbf{H_{pB} = 64}$ **ft** based on the pump data with Q = 20 cfs) yields

$\mathbf{E_C} = E_B + H_{pB} – K(Q_{BC})^2 = 100.3+64–(0.0103)(20)^2 = \mathbf{160.2\ ft}$

5.7.3

It is obvious from the system schematic that both pumps, acting in parallel, push flow through pipe 3. Thus, the pump characteristics of each can be added together as a parallel pump combination, graphed, and superimposed on the system curve (i.e., friction losses in pipe 3) to determine the total flow and the flow contributions from each pump. However, the pump characteristics must be reduced first; their total energy is not available to overcoming the friction losses in pipe 3. The two pumps have used their energy to overcome losses in sending flow through pipes 1 and 2 and raising the water to a higher elevation. With this information, a spreadsheet is programmed which yields:

Pumps & Branching Pipes (Prob 5.7.3)

Pipeline 1 Data			Pipeline 2 Data		
L =	8000	ft	L =	9000	ft
D =	2.00	ft	D =	2.00	ft
f =	0.020		f =	0.020	
$h_f = KQ^m$			$h_f = KQ^m$		
m =	2.00		m =	2.00	
K =	0.1250		K =	0.1406	
Pipeline 3 Data			Reservoir Data		
L =	15000	ft	E_A=	100.0	ft
D =	2.50	ft	E_B=	80.0	ft
f =	0.020		E_C=	120.0	ft
$h_f = KQ^m$					
m =	2.00		H_{sAC}=	20.0	
K =	0.0768		H_{sBC}=	40.0	

Q (cfs)	H_{p1} (ft)	Net H_{p1}* (ft)	H_{p2} (ft)	Net H_{p2}* (ft)	h_{f3} (ft)
0	200	180.0	150	110.0	0.0
10	195	162.5	148	93.9	7.7
15	188.8	140.7	145.5	73.9	17.3
20	180	110.0	142	45.8	30.7
25	168.8	70.7	137.5	9.6	48.0
30	155	22.5	132		69.1
40	120		118		122.9
50	75		100		192.0

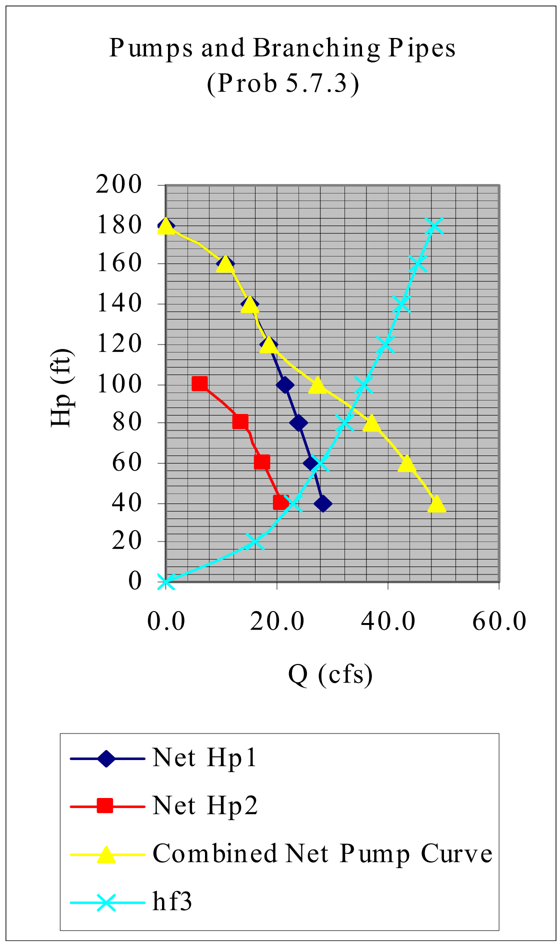
* Subtracting friction loss (pipes 1 & 2) and elev. rise (H_{sAC} & H_{sBC}).

Plotting Data

Head (ft)	Net Q_{p1}* (cfs)	Net Q_{p2}* (cfs)	Qsys (cfs)	Q_3** (cfs)
180	0.0		0.0	48.3
160	10.6		10.6	45.4
140	15.1		15.1	42.5
120	18.4		18.4	39.5
100	21.3	6.2	27.5	35.7
80	23.8	13.5	37.3	32.0
60	26.1	17.5	43.6	27.8
40	28.2	20.8	49.0	22.7
20				16.0
0				0.0

* Linear interpolation from net head pump data.

** Linearly interpolated from pipe 3 friction loss data.



The plot yields $Q_3 \approx$ **34 cfs** and $H_{pA} \approx$ **90 ft** with **$Q_1 \approx$ 22.5 cfs, $Q_2 \approx$ 11.5 cfs** reading across from the same head of 90 ft to the pump 1 and pump 2 curves.

5.9.1

$h_p \le [(P_{atm} - P_{vapor})/\gamma] - [H'_s + V^2/2g + h_L]$ where

$V = Q/A = (6.0\ ft^3/s)/[\pi\{(5/12)ft\}^2] = 11.0$ ft/sec

$P_{atm} = 14.7$ psi $= 2117$ lbs/ft^2; $P_{vapor} = 0.344$ psi $= 49.5$ lbs/ft^2

$(P_{atm} - P_{vapor})/\gamma = (2117 - 49.5)/(62.3) = 33.2$ ft, from book jacket. Also, $H'_s = 15$ ft, and h_L (suction side) is

$h_L = h_f + [\sum K](V)^2/2g = [f(L/D) + \Sigma K]\cdot[(V)^2/2g]$ and

$e/D = 0.00085/(5/12) = 0.00204$; $\nu = 1.08 \times 10^{-5}$ ft^2/s

$VD/\nu = (11.0)(10/12)/(1.08 \times 10^{-5}) = 8.49 \times 10^5$

And from the Moody diagram, f = 0.0235; thus

$h_L = [0.0235\{35/(10/12)\}+2.6]\cdot[(11.0)^2/2g] = 6.74$ ft, and

$h_p \le [(P_{atm} - P_{vapor})/\gamma] - [H'_s + V^2/2g + h_L]$

$\mathbf{h_p} \le [33.2ft] - [15.0ft + (11.0)^2/2g + 6.74ft] \mathbf{= 9.58\ ft}$

5.9.2

$h_p \le [(P_{atm} - P_{vapor})/\gamma] - [H'_s + V^2/2g + h_L]$ where

$V = Q/A = (0.120\ m^3/s)/[\pi(0.175m)^2] = 1.25$ m/sec

$(P_{atm} - P_{vapor})/\gamma = (101{,}400 - 4238)/(9771) = 9.94$ m

from Table 1.1, $H'_s = 6m$ (given), h_L (suction side) is

$h_L = h_f + [\sum K](V)^2/2g = [f(L/D) + \Sigma K]\cdot[(V)^2/2g]$ and

$e/D = 0.12/350 = 0.00034$; $T = 30^\circ C$, $\nu = 0.80 \times 10^{-6}$ m^2/s

$VD/\nu = (1.25m/s)(0.35m)/(0.80 \times 10^{-6}\ m^2/s) = 5.47 \times 10^5$

And from the Moody diagram, f = 0.0165; thus

$h_L = [0.0165(10/0.35)+3.7]\cdot[(1.25)^2/2g] = 0.332$ m, and

$h_p \le [(P_{atm} - P_{vapor})/\gamma] - [H'_s + V^2/2g + h_L]$

$\mathbf{h_p} \le [9.94m] - [6.00m + (1.25)^2/2g + 0.33m] \mathbf{= 3.53\ m}$

Since the pump is never more than 3 m above the supply reservoir, cavitation is not a problem.

5.9.3

$h_p \le [(P_{atm} - P_{vapor})/\gamma] - [H'_s + V^2/2g + h_L]$ where

$V = Q/A = (0.170\ m^3/s)/[\pi(0.125m)^2] = 3.46$ m/sec

$(P_{atm} - P_{vapor})/\gamma = (101{,}400 - 1226)/(9800) = 10.2$ m

from Table 1.1&1.2, $H'_s = 7.5m$, h_L (suction side) is

$h_L = h_f + [\sum K](V)^2/2g = [f(L/D) + \Sigma K]\cdot[(V)^2/2g]$ and

$h_L = [0.02(10/0.25)+3.07]\cdot[(3.46)^2/2g] = 2.36$ m, and

$h_p \le [(P_{atm} - P_{vapor})/\gamma] - [H'_s + V^2/2g + h_L]$

$\mathbf{h_p} \le [10.2m] - [7.5m + (3.46)^2/2g + 2.36m] \mathbf{= -0.27\ m}$

Since h_p is negative, the pump must be placed 0.27 m below the water surface elevation of the intake reservoir in order to avoid cavitation problems.

5.9.4

$h_p \le [(P_{atm} - P_{vapor})/\gamma] - V_i^2/2g - h_L - \sigma H_P$ where

$H_S + H_P = H_R + h_L$; (Eq'n 4.2), $H_R - H_S = 55$ m; and

$h_L = h_f + [\sum K](V)^2/2g$; $K_e = 0.5$; $K_d = 1.0$ (exit coef.)

$V = Q/A = 4.36$ m/s; $e/D = 0.60mm/800mm = 0.00075$

$VD/\nu = (4.36)(0.80)/(1.0 \times 10^{-6}) = 3.49 \times 10^6$

From Moody; **f = 0.0185**; solving the energy eq'n;

$H_P = 55m + [0.0185(250/0.80)+1.5]\cdot[(4.36)^2/2g] = 62.1m$

$(P_{atm} - P_{vapor})/\gamma = (101{,}400 - 2370)/(9790) = 10.1$ m

$h_p = -0.9$ m and $\sigma = 0.15$ (given), h_L (suction side) is

$h_L = [f(L/D)+\Sigma K](V^2/2g) = [0.0185(L/0.8)+0.5]\cdot[(4.36)^2/2g]$

Therefore, $h_p = [(P_{atm} - P_{vapor})/\gamma] - V_i^2/2g - h_L - \sigma H_P$.

$$-0.9m = 10.1m - (4.36)^2/2g - [0.0185(L/0.8)+0.5]\cdot[(4.36)^2/2g] - (0.15)(62.1m)$$

Therefore, **L = 10.3 m**

5.9.5

$h_p \le [(P_{atm} - P_{vapor})/\gamma] - V_i^2/2g - h_L - \sigma H_P$ where

$H_S + H_P = H_R + h_L$; (Eq'n 4.2), $H_R - H_S = 25$ m; and

$h_L = h_f + h_{suction} + [\sum K](V)^2/2g$; $K_d = 1.0$ (exit coef.)

$h_f = KQ^m = [(10.7 \cdot L)/(D^{4.87} \cdot C^{1.85})]Q^{1.85}$

$h_f = [(10.7 \cdot 300)/(0.15^{4.87} \cdot 120^{1.85})]0.04^{1.85} = 12.2$ m

$h_L = h_f + h_{suction} + V^2/2g$; $V = 2.26$ m/s (pipeline exit)

$H_P = 25\text{m} + [12.2 \text{ m} + 0.5 \text{ m} + (2.26)^2/2g] = 38.0$ m

$(P_{atm} - P_{vapor})/\gamma = (101{,}400 - 19{,}924)/(9643) = 8.45$ m

$V_i = Q/A = 1.57$ m/s (suction side); $\sigma = 0.075$ (given)

Therefore, $h_p \le [(P_{atm} - P_{vapor})/\gamma] - V_i^2/2g - h_L - \sigma H_P$.

$\mathbf{h_p} \le 8.45\text{m} - (1.57)^2/2g - 0.5\text{m} - (0.075)(38.0\text{m}) \mathbf{= 4.97\ m}$

5.9.6

Balancing energy from the water surface of the supply reservoir (s) to the to the inlet of the pump (i) yields:

$h_s + P_s/\gamma + V_s^2/2g = h_i + P_i/\gamma + V_i^2/2g + h_{L(suction)}$

Since $P_s/\gamma = V_s^2/2g = 0$ and $h_i - h_s = h_p$, thus

$(P_i/\gamma + V_i^2/2g) = -(h_p + h_{L(suction)})$; also

$h_p = [(P_{atm} - P_{vapor})/\gamma] - V_i^2/2g - h_L - \sigma H_P$ where

$H_P = 85$ m; and $V_i = Q/A = 5.94$ m/s (suction side)

$(P_{atm} - P_{vapor})/\gamma = (101{,}400 - 7{,}377)/(9732) = 9.66$ m

from Table 1.1 and 1.2. Also, $\sigma = 0.08$ (given)

therefore, $h_p = [(P_{atm} - P_{vapor})/\gamma] - V_i^2/2g - h_L - \sigma H_P$.

$h_p + h_L = [(P_{atm} - P_{vapor})/\gamma] - V_i^2/2g - \sigma H_P$.

$h_p + h_L = 9.66 \text{ m} - (5.94)^2/2g - (0.08)(85 \text{ m}) = 1.06$ m

and since, $(P_i/\gamma + V_i^2/2g) = -(h_p + h_{L(suction)})$; thus

$\mathbf{(P_i/\gamma + V_i^2/2g) = -(1.06\ m)}$

5.10.1

a) Converting gpm to cfs and rpm to rev/s yields

$S = [\omega(Q)^{1/2}]/(gH_p)^{3/4} \equiv [(\text{rev/s})(\text{ft}^3/\text{s})^{1/2}]/[(\text{ft/s}^2)(\text{ft})]^{3/4}$

$S \equiv (\text{ft/s})^{3/2}/(\text{ft/s})^{3/2}$; (ck), and in SI units

$S \equiv [(\text{rad/s})(\text{m}^3/\text{s})^{1/2}]/[(\text{m/s}^2)(\text{m})]^{3/4} \equiv (\text{m/s})^{3/2}/(\text{m/s})^{3/2}$

b) $N_s = [\omega(Q)^{1/2}]/(H_p)^{3/4} \equiv [(\text{rad/s})(\text{m}^3/\text{s})^{1/2}]/(\text{m})^{3/4}$

$N_s \equiv (\text{m/s})^{3/2}/(\text{m})^{3/4}$; No!!

$N_s = [\omega(\mathbf{P_i})^{1/2}]/(H_p)^{5/4} \equiv [(\text{rad/s})(\text{kW})^{1/2}]/(\text{m})^{5/4}$

$N_s \equiv (\text{kN} \cdot \text{m/s}^3)^{1/2}/(\text{m})^{5/4}$; No!!

c) $N_s = [\omega(\mathbf{P_i})^{1/2}]/(H_p)^{5/4} = [\omega(\gamma Q H_p)^{1/2}]/(H_p)^{5/4}$

$N_s = [\omega\gamma^{1/2}(Q)^{1/2}]/(H_p)^{3/4}$; close, but $N_s(Q) \neq N_s(\mathbf{P})$

d) Peripheral speed = $\omega(D/2)$ and $V = Q/A = 4Q/\pi D^2$

Therefore, the ratio is $4Q/[(\pi D^2)\omega(D/2)$ and dropping the constants yields: $Q/(\omega D^3)$

5.10.2

Converting rotational speed from rpm to rad/sec yields:

1720 rev/min$\cdot$[(2π rad)/(1rev)]$\cdot$[(1min)/(60sec)] = 180 rad/sec

And since geometrically similar pumps have the same specific speed, the pump head can be found using:

$N_s = [\omega(Q)^{1/2}]/(H_p)^{3/4}$

$50 = [(180)(12.7)^{1/2}]/(H_p)^{3/4}$; $\mathbf{H_p = 30.0\ m}$

To determine the power requirement, use the power specific speed equation with the head just obtained:

$N_s = [\omega(\mathbf{P_i})^{1/2}]/(H_p)^{5/4}$

$175 = [(180)(\mathbf{P_i})^{1/2}]/(30.0)^{5/4}$; $\mathbf{P_i = 4660\ kW}$

The same answer is obtained using $\mathbf{P_i} = \gamma Q H_p/e$

5.10.3

Based on the specific speed (unit discharge):

$N_s = [\omega(Q)^{1/2}]/(H_p)^{3/4}$

$68.6 = [(1800)(0.15)^{1/2}]/(H_p)^{3/4}$**;** $H_p = 22.0$ m

Now using; $N_s = [\omega(\mathbf{P_i})^{1/2}]/(H_p)^{5/4}$

$240 = [(1800)(\mathbf{P_i})^{1/2}]/(22.0)^{5/4}$; $\mathbf{P_i} = 40.4$ kW

And finally, using $\mathbf{P_i} = \gamma QH_p/e$ (watt = N·m/s)

40,400 N·m/s = (9790)(0.15)(22.0)/e; **e = 0.80**

5.10.4

Converting to the US unit system for specific speed:

1 cfs = 449 gpm and 12.5 cfs = 5610 gpm

And since geometrically similar pumps have the same specific speed, the pump head can be found using:

$N_{s(model)} = [\omega(Q)^{1/2}]/(H_p)^{3/4}$

$N_{s(model)} = [(1150)(449)^{1/2}]/(18)^{3/4} = 2790$

$N_{s(field)} = [\omega(Q)^{1/2}]/(H_p)^{3/4}$

$2790 = [\omega(5610)^{1/2}]/(95)^{3/4}$**;** $\boldsymbol{\omega = 1130}$ **rpm**

Also, $N_{s(model)} = [\omega(\mathbf{P_i})^{1/2}]/(H_p)^{5/4}$

$N_{s(model)} = [1150(3.1)^{1/2}]/(18)^{5/4} = 54.6$

Now, $N_{s(field)} = [\omega(\mathbf{P_i})^{1/2}]/(H_p)^{5/4}$

$54.6 = [1130(\mathbf{P_i})^{1/2}]/(95)^{5/4} = 54.6$; $\mathbf{P_i = 205}$ **hp**

The same answer is obtained using $\mathbf{P_i} = \gamma QH_p/e$

And based on Example 5.10:

$(Q/\omega D^3)_{model} = (Q/\omega D^3)_{field}$

$[449/\{(1150)(0.5)^3]_{model} = [5610/\{(1130)(D)^3]_{field}$

D = 1.17 ft

5.10.5

$N_{s(m)} = [\omega(Q)^{1/2}]/(H_p)^{3/4} = [(4500)(0.0753)^{1/2}]/(10)^{3/4} = 220$

For the prototype (H_p = 100 m based on scale factor)

$N_{s(p)} = [\omega(Q)^{1/2}]/(H_p)^{3/4}$

$220 = [2250(Q)^{1/2}]/(100)^{3/4}$**;** **Q = 9.56 m^3/sec**

$\mathbf{P_{i(m)}} = \gamma QH_p/e = [(9790)(0.0753)(10)]/0.89 = 8.28$ kW

$N_{s(m)} = [\omega(\mathbf{P_i})^{1/2}]/(H_p)^{5/4} = [(4500)(8.28)^{1/2}]/(10)^{5/4} = 728$

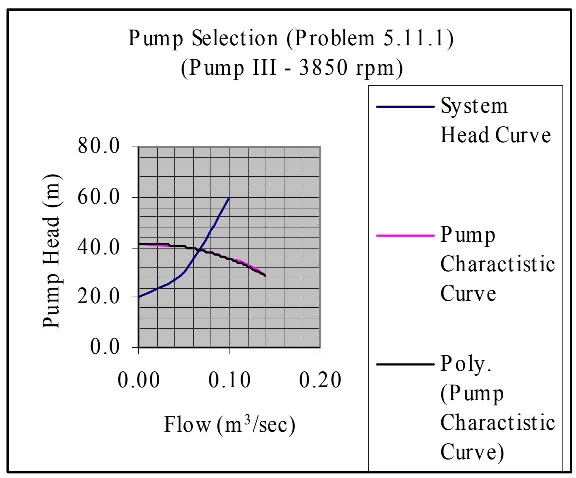
$N_{s(p)} = [\omega(\mathbf{P_i})^{1/2}]/(H_p)^{5/4}$

$728 = [2250(\mathbf{P_i})^{1/2}]/(100)^{5/4}$; $\mathbf{P_i} = 10{,}500$ kW

And finally, using $\mathbf{P_{i(p)}} = \gamma QH_p/e$ (watt = N·m/s)

10,500,000 N·m/s = (9790)(9.56)(100)/e; **e = 0.891**

5.11.1



a) Concave up; as Q increases, losses increase quickly.

b) Concave down; as Q increases, the pump is not able to overcome as much head ($\mathbf{P_o} = \gamma QH_p$)

c) The highest head is **41.5 m**. with **Q = 0**. This is the shut-off head. The pump is able to raise water to this height, but produce no flow (i.e., all of the energy is used up to maintain a 41.5 m column of water.)

d) The match point is **Q = 69 L/s; $\mathbf{H_p}$ = 39.5 m**; so this does not quite meet the design conditions.

5.11.2

A spreadsheet can be used to obtain Q vs. H_p (system). Applying Equation 5.19 (w/minor losses) yields $H_{SH} = H_s + h_L$; where $h_L = [f(L/D)+\sum K](V)^2/2g$; and $K_e = 0.5$, $K_v = 5(0.22)$, $K_v = 2(0.15)$, $K_d = 1.0$, and

Pump Selection (Prob 5.11.2)

Pipeline Data			Reservoir Data		
L =	300	M	E_A =	385.7	m
D =	0.20	M	E_B =	402.5	m
e =	0.360	Mm	H_s =	16.8	m
e/D =	0.00180		Minor Losses		
T =	20°C		ΣK =	2.90	
ν =	1.00E-06	m^2/sec	g =	9.81	m/sec^2

Q (m^3/s)	V (m/sec)	N_R	f*	h_L (m)	H_{SH} (m)
0.000	0.00	0.00E+00	0.0000	0.0	16.8
0.100	3.18	6.37E+05	0.0232	19.5	36.3
0.125	3.98	7.96E+05	0.0231	30.3	47.1
0.150	4.77	9.55E+05	0.0231	43.6	60.4

* Used Swamee-Jain Equation (3.24a).

From Figure 5.23, with Q = 125 L/sec and Hp = 47.1 m (from table above) **Pump IV** is the logical choice. Plotting Q vs H_{SH} (system) on Figure 5.24 (Pump IV):

ω = 4350 rpm, e ≈ 62%, Q ≈ 135 L/s, Hp ≈ 48 m

5.11.3

Applying Equation 5.19 (w/minor losses) yields $H_{SH} = H_s + h_L$; where $h_L = [f(L/D)+\sum K](V)^2/2g$; and $K_{suction} = 3.7$ and $K_d = 1.0$ (exit coefficient). Thus,

Pump Selection (Prob 5.11.3)

Pipeline Data			Reservoir Data		
L =	150	m	E_A =		m
D =	0.35	m	E_B =		m
e =	0.120	mm	H_s =	44.0	m
e/D =	0.00034		Minor Losses		
T =	20°C		ΣK =	4.70	
ν =	1.00E-06	m^2/sec	g =	9.81	m/sec^2

Q (m^3/s)	V (m/sec)	N_R	f*	h_L (m)	H_{SH} (m)
0.00	0.00	0.00E+00	0.0000	0.0	44.0
0.04	0.42	1.46E+05	0.0187	0.1	44.1
0.06	0.62	2.18E+05	0.0179	0.2	44.2
0.08	0.83	2.91E+05	0.0174	0.4	44.4
0.10	1.04	3.64E+05	0.0171	0.7	44.7
0.12	1.25	4.37E+05	0.0169	0.9	44.9
0.14	1.46	5.09E+05	0.0167	1.3	45.3

* Used Swamee-Jain Equation (3.24a).

Plotting Q vs Hp (system) on Figure 5.24 (Pump III):

ω = 4350 rpm, e ≈ 61%, Q ≈ 120 L/s, Hp ≈ 45 m

5.11.4

Applying Equation 5.19 (w/minor losses) yields $H_{SH} = H_s + h_L$; where $h_L = [f(L/D)+\sum K](V)^2/2g$; and $K_e = 0.5$, $K_v = 70.0$, $K_d = 1.0$, and thus,

Pump Selection (Prob 5.11.4)

Pipeline Data			Reservoir Data		
L =	100	m	E_A =		m
D =	0.15	m	E_B =		m
e =	0.150	mm	H_s =	20.0	m
e/D =	0.00100		Minor Losses		
T =	20°C		ΣK =	71.50	
ν =	1.00E-06	m^2/sec	g =	9.81	m/sec^2

Q (m^3/s)	V (m/sec)	N_R	f*	h_L (m)	H_{SH} (m)
0.00	0.00	0.00E+00	0.0000	0.0	20.0
0.01	0.57	8.49E+04	0.0227	1.4	21.4
0.02	1.13	1.70E+05	0.0214	5.6	25.6
0.03	1.70	2.55E+05	0.0209	12.6	32.6
0.04	2.26	3.40E+05	0.0206	22.3	42.3

* Used Swamee-Jain Equation (3.24a).

From Figure 5.23, with Q = 30 L/sec and Hp = 32.6 m. try **Pump I or II.** Plot Q vs H_{SH} (system) on Fig 5.24:

I ➔ ω = 3850 rpm, e ≈ 42%, Q ≈ 30 L/s, Hp ≈ 33 m

II ➔ ω = 3550 rpm, e ≈ 52%, Q ≈ 32 L/s, Hp ≈ 33 m

5.11.5

Determine Q vs. H_{SH} (system). Apply Equation 5.19

$H_{SH} = H_s + h_L$; where $h_L = h_f + [\sum K](V)^2/2g$; and

$K_e = 0.5$, $K_d = 1.0$, and $h_f = KQ^{1.85}$ (Table 3.4) where

$K = (4.73 \cdot L)/(D^{4.87} \cdot C^{1.85})$ and therefore:

Pump Selection (Prob 5.11.5)

Pipeline Data			Reservoir Data		
L =	8700	ft	E_A =	102	ft
D =	1.00	ft	E_D =	180	ft
C =	100		H_s =	78.0	ft
$h_f = KQ^m$			Minor Losses		
m =	1.85		ΣK =	1.50	
K =	8.21		g =	32.2	ft/sec^2

Q (gpm)	Q (cfs)	h_f (ft)	h_{minor} (ft)	H_s (ft)	H_{SH} (ft)
0	0.00	0.0	0.0	78.0	78.0
200	0.45	1.8	0.0	78.0	79.8
400	0.89	6.6	0.0	78.0	84.7
600	1.34	14.0	0.1	78.0	92.1
800	1.78	23.9	0.1	78.0	102.0
1000	2.23	36.1	0.2	78.0	114.3

Plotting Q vs H_{SH} on Figure 5.11.5: the match point is:

Q≈ 800 gpm(1.78 cfs)**, $H_p \approx$ 102 ft, $e_p \approx$ 82%, $P_i \approx$ 26 hp**

$\mathbf{P_o}$ = γQH_P = (62.3 lb/ft^3)(1.78 ft^3/sec)(102 ft)

$\mathbf{P_o}$ = 11,300 ft-lb/sec(1 hp/550 ft·lb/sec) **= 20.6 hp**

$\mathbf{e_p = P_o/P_i}$ = 20.6/26 = 79% **≈ 82%**

5.11.6

Applying Equation 5.19 (w/minor losses) yields

$H_{SH} = H_s + h_L$; where $h_L = [f(L/D)+\sum K](V)^2/2g$; and

$K_e = 0.5$, $K_v = 70.0$, $K_d = 1.0$, and $H_s = 40$m.

Try D = 20 cm, e/D = 0.045/200 = 0.000225

V = Q/A = 0.637 m/s, $N_R = 1.27 \times 10^5$ and f = 0.0185

H_{SH}= 40 + [71.5+0.0185(150/0.2)](0.637)^2/2g = 41.8m

Use pump I at 4050 rpm, Q = 20 L/s, $H_p \approx$ 45 m,

e ≈ 42%, $\mathbf{P_i} \approx$ 30 hp, Cost: C = 230

Now try D = 10 cm, e/D = 0.045/100 = 0.00045

5.11.6 (cont.)

V = Q/A = 2.55 m/s, $N_R = 1.27 \times 10^5$ and f = 0.0185

H_{SH} = 40 + [71.5+0.0185(150/0.1)](2.55)^2/2g = 72.9m

No single pump can achieve this head.

Try D = 13 cm, e/D = 0.045/130 = 0.00035

V = Q/A = 1.51 m/s, $N_R = 1.96 \times 10^5$ and f = 0.0185

H_{SH}= 40 + [71.5+0.0185(150/0.13)](1.51)^2/2g = 50.8m

Use pump I at 4350 rpm, Q = 20 L/s, $H_p \approx$ 52 m,

e ≈ 42%, $P_i \approx$ 35 hp, Cost: C = 133

Since cost increases as diameter increases, and the head increases dramatically (going off the pump charts) with diameters less than **13 cm**, use this as the optimum size.

5.11.7

Applying Equation 5.19 (w/out minor losses) yields

$H_{SH} = H_s + h_L$; where $h_L = [f(L/D)](V)^2/2g$; and thus

Pump Selection (Prob 5.11.7)

Pipeline Data			Reservoir Data		
L =	1500	m	E_A =		m
D =	0.40	m	E_B =		m
e =	0.045	mm	H_s =	15.0	m
e/D =	0.00011		Minor Losses		
T =	20°C		ΣK =	0.00	
ν =	1.00E-06	m^2/sec	g =	9.81	m/sec^2

Q (m^3/s)	V (m/sec)	N_R	f*	h_L (m)	H_{SH} (m)
0.00	0.00	0.00E+00	0.0000	0.0	15.0
0.20	1.59	6.37E+05	0.0142	6.9	21.9
0.30	2.39	9.55E+05	0.0137	15.0	30.0
0.40	3.18	1.27E+06	0.0134	26.0	41.0

* Used Swamee-Jain Equation (3.24a).

From Fig. 5.23, Q is too big. Use **two pumps in parallel** w/Q = 150 L/sec and Hp = 30.0m. Try Pump III or IV. Plot Q vs H_{SH} (system) on Fig 5.24:

III ➔ω = 4050rpm, e ≈ 52%, Q ≈ 150 L/s, Hp ≈ 32m

IV ➔ω = 3550 rpm, e≈ 61%, Q ≈ 150 L/s, Hp ≈ 30m

(Pump IV is best choice for efficiency)

Chapter 6 – Problem Solutions

6.1.1

a) unsteady, varied
b) steady, varied
c) steady, varied
d) unsteady, varied
e) steady, uniform
f) unsteady, varied

6.1.2

In natural channels, the bottom slope and cross sectional flow area are constantly changing so uniform flow is rare. It generally only occurs over short distances. Steady flow is not rare in natural steams. However, during and shortly after rainfall events the discharge is changing and produces unsteady flow.

6.2.1

For a trapezoidal channel, the area, wetted perimeter, and hydraulic radius (Table 6.1) are found to be:

$A = (b + my)y = [12 \text{ ft} + 1(8.0 \text{ ft})](8.0 \text{ ft}) = 160 \text{ ft}^2$

$P = b + 2y(1 + m^2)^{1/2} = 12 + 2(8.0)(1 + 1^2)^{1/2} = 34.6 \text{ ft}$

$R_h = A/P = (160 \text{ ft}^2)/(34.6 \text{ ft}) = 4.62 \text{ ft}$

Now applying Manning's equation (Eq'n 6.5b):

$Q = (1.49/n)(A)(R_h)^{2/3}(S)^{1/2}$

$2{,}200 = (1.49/n)(160)(4.62)^{2/3}(0.01)^{1/2}$; $\mathbf{n = 0.030}$

For a flow range of 2,000cfs < Q < 2,400 cfs;

the roughness coefficient has a range of :

0.0275 < n < 0.0330

6.2.2

Based on the side slope (m = 3) and Table 6.1;

$T = 2my$; $2 \text{ m} = 2(3)(y)$; $y = 0.333\text{m}$ (flow depth)

$A = my^2 = (3)(0.333 \text{ m})^2 = 0.333 \text{ m}^2$

$P = 2y(1 + m^2)^{1/2} = 2(0.333 \text{ m})(1 + 3^2)^{1/2} = 2.11 \text{ m}^2$

$R_h = A/P = (0.333 \text{ m}^2)/(2.11 \text{ m}) = 0.158 \text{ m}$

$Q = (1.00/n)(A)(R_h)^{2/3}(S)^{1/2}$ (Equation 6.5a)

$\mathbf{Q} = (1.00/0.013)(0.333)(0.158)^{2/3}(0.01)^{1/2} \mathbf{= 0.748 \text{ m}^3/\text{s}}$

6.2.3

For a trapezoidal channel, from Table 6.1:

$A = (b + my)y = [3 \text{ m} + 2(1.83 \text{ m})](1.83 \text{ m}) = 12.2 \text{ m}^2$

$P = b + 2y(1 + m^2)^{1/2} = 3 + 2(1.83)(1 + 2^2)^{1/2} = 11.2 \text{ m}$

$R_h = A/P = 1.09$ m. Applying Manning's equation

$Q = (1.0/n)(A)(R_h)^{2/3}(S)^{1/2}$; with n = 0.04 (Table 6.2)

$\mathbf{Q} = (1.0/0.04)(12.2)(1.09)^{2/3}(0.005)^{1/2} \mathbf{= 22.8 \text{ m}^3/\text{s}}$

Fig 6.4a, $(y_n/b)=0.61$; $nQ/[S^{1/2}b^{8/3}]=0.70$; $\mathbf{Q=23.1 \text{ m}^3/\text{s}}$

6.2.4

For a trapezoidal channel, from Table 6.1:

$A = (b + my)y = [4 \text{ m} + 4(y)](y) = 4y + 4y^2$

$P = b + 2y(1 + m^2)^{1/2} = 4 + 2y(1 + 4^2)^{1/2} = 4 + 8.25y$

Apply Manning's eq'n: $Q{\cdot}n/(S)^{1/2} = (A)^{5/3}(P)^{-2/3}$;

$(49.7)(0.024)/(0.002)^{1/2} = 26.7 = (4y+4y^2)^{5/3}(4+8.25y)^{-2/3}$

By successive substitution: $\mathbf{y_n = 2.00 \text{ m}}$; From Fig 6.4a

with $nQ/[(S^{1/2})(b^{8/3})] = 0.635$; $y_n/b = 0.50$; $\mathbf{y_n = 2.00 \text{ m}}$

6.2.5

Since $m = 1/(\tan 30°) = 1.73$; referring to Table 6.1:

$A = my^2 = 1.73y^2$; and $P = 2y(1 + m^2)^{1/2} = 4y$

Apply Manning's eq'n: $Q = (1.49/n)(A)^{5/3}(P)^{-2/3}(S)^{1/2}$

$4 = (1.49/0.02)(1.73y^2)^{5/3}(4y)^{-2/3}(0.006)^{1/2}$

$0.693 = (1.73y^2)^{5/3}/(4y)^{2/3} = 0.989y^{8/3}$: $\mathbf{y_n = 0.875}$ **ft**

6.2.6

For a circular channel, from Table 6.1:

$A = (1/8)(2\theta - \sin 2\theta)d_o^2 = (1/2)(2\theta - \sin 2\theta)$

$P = \theta d_o = 2\theta$; where θ is expressed in radians

Apply Manning's eq'n: $Q{\cdot}n/(S)^{1/2} = (A)^{5/3}(P)^{-2/3}$;

$(5.83)(0.024)/(0.02)^{1/2} = 0.989 = (A)^{5/3}(P)^{-2/3}$

$0.989 = [1/2(2\theta - \sin 2\theta)]^{5/3}(2\theta)^{-2/3}$; By successive substitution: $\boldsymbol{\theta = \pi/2}$; thus $\mathbf{y_n = 1.00}$ **m.** Or w/Fig 6.4b, $nQ/(k_m S^{1/2} d_o^{8/3}) = 0.155$, and $y_n/d_o = 0.5$, $y_n = 1.00$

6.2.7

$Q = 52$ m^3/min $= 0.867$ m^3/s; Table 6.1 can't be used.

$A = ½ (0.8)(0.8{\cdot}m) = 0.32{\cdot}m$; where m = slope

$P = 0.8 + (0.64 + 0.64{\cdot}m^2)^{1/2} = 0.8(1 + (1 + m^2)^{1/2})$

$R_h = A/P = (0.4{\cdot}m)/(1 + (1+m^2)^{1/2})$

Apply Manning's eq'n: $Q{\cdot}n/(S)^{1/2} = (A)(R_h)^{2/3}$;

$(0.867)(0.022)/(0.0016)^{1/2} = 0.477 = (A)(R_h)^{2/3}$

$0.477 = (0.32m)[(0.4m)/(1 + (1+m^2)^{1/2})]^{2/3}$

By successive approximation or computer software

m = 3.34 m/m (with n = 0.022)

6.2.8

For a circular channel (half full), from Table 6.1:

a) $A = (1/8)(2\theta - \sin 2\theta)d_o^2 = (\pi/8)d_o^2$;

$P = \theta d_o = (\pi/2)d_o$; where $\theta = \pi/2$ (in radians)

Apply Manning's eq'n: $Q = (1.49/n)(A)^{5/3}(P)^{-2/3}(S)^{1/2}$

$6 = (1.49/0.024)((\pi/8)d_o^2)^{5/3}((\pi/2)d_o)^{-2/3}(0.005)^{1/2}$

$d_o^{8/3} = 8.774$; $\mathbf{d_o = 2.26}$ **ft;** pipe size for half-full flow

b) $A = (\pi/4)d_o^2$; $P = (\pi)d_o$; Appling Manning's eq'n

$6 = (1.49/0.024)((\pi/4)d_o^2)^{5/3}(\pi d_o)^{-2/3}(0.005)^{1/2}$

$d_o^{8/3} = 4.39$; $\mathbf{d_o = 1.74}$ **ft;** pipe size for full flow

Using Figure 6.4b would produce the same results.

6.2.9

Applying Manning's: $Q = (1.0/n)(A)(R_h)^{2/3}(S)^{1/2}$; or

$Q = (1.0/n)A^{5/3}P^{-2/3}S^{1/2} = (S^{1/2}/n)(b{\cdot}y)^{5/3}(b+2y)^{-2/3}$

Assuming the slope and roughness remain constant:

$(b_1{\cdot}y_1)^{5/3}(b_1+2y_1)^{-2/3} = (b_2{\cdot}y_2)^{5/3}(b_2+2y_2)^{-2/3}$

$[(b_1{\cdot}y_1)/(b_2{\cdot}y_2)]^{5/3} = [[(b_1+2y_1)/(b_2+2y_2)]^{2/3}$

$[(36)/(8y_2)]^{5/3} = [(12+6)/(8+2y_2)]^{2/3}$

By successive substitution: $\mathbf{y_2 = 4.37}$ **m**

6.2.10

For Q = 100 cfs, S = 0.002 ft/ft, and n = 0.013

Rectangular channel: Try b = 10 ft, then y = 1.68 ft

Too shallow to be practical; **b = 5 ft, y = 3.14 ft** (ok)

Trapezoidal: Try b = 5 ft, and m = 2, then y = 1.91 ft

To meet 60% criteria; **b = 4 ft, m = 1, y = 2.41 ft (ok)**

6.3.1

Based on Example 6.4, $m = (3)^{1/2}/3 = 0.577$

which is a slope angle of 60˚ (Figure 6.6)

Also, $y = b\cdot[(3)^{1/2}/2] = 0.866(b)$ or $b = 1.15y$

For a trapezoidal channel,

$A = by + my^2$; Substituting for b and solving:

$100 = (1.15y)y + 0.577y^2 = 1.73y^2$; $\mathbf{y = 7.60\ m;}$

And since $b = 1.15y = 1.15(7.60\ m)$; $\mathbf{b = 8.74\ m}$

To check, determine the length of the channel sides.

Since it is a half hexagon, the sides should be equal

to the bottom width (see Figure 6.6).

$[(7.60m)^2 + \{(0.577)(7.60m)\}^2]^{1/2} = 8.77$ m (ck)

6.3.2

Based on Example 6.4, $A = by$ and $P = b + 2y$

Substituting for "b" yields: $P = A/y + 2y$;

To maximize Q for a given area, minimize P:

$dP/dy = -A/y^2 + 2 = 0$; $A/y^2 = 2$; or $(b\cdot y)/y^2 = 2$;

Therefore, b = 2y which is a half square.

6.3.3

For a circular channel (half full), from Table 6.1:

$A = (1/8)(2\theta - \sin 2\theta)d_o^{\,2} = (\pi/8)d_o^{\,2}$;

$P = \theta d_o = (\pi/2)d_o$; where $\theta = \pi/2$ (in radians)

Apply Manning's eq'n: $Q = (1.0/n)(A)^{5/3}(P)^{-2/3}(S)^{1/2}$

$1.0 = (1.0/0.011)((\pi/8)d_o^{\,2})^{5/3}((\pi/2)d_o)^{-2/3}(0.0065)^{1/2}$

$d_o^{\,8/3} = 0.876$; $\mathbf{d_o = 0.952\ m;}$ pipe size for half-full flow

6.3.4

Equation (4) is: $P = (2y)(2(1+m^2)^{1/2} - m)$ or

$P = 4y(1+m^2)^{1/2} - 2ym$; taking the first derivative

$dP/dm = 4ym(1+m^2)^{-1/2} - 2y = 0$

$4ym/(1+m^2)^{1/2} = 2y$; $2m = (1+m^2)^{1/2}$

$4m^2 = (1+m^2)$; $3m^2 = 1$; $m = 1/[(3)^{1/2}] = [(3)^{1/2}]/3$

6.3.5

Based on Example 6.4 and Table 6.1:

$A = my^2$ and $P = 2y(1 + m^2)^{1/2}$

Substituting for "m" yields: $P = 2y(1 + A^2/y^4)^{1/2}$

To maximize Q for a given area, minimize P:

$dP/dy = y(1 + A^2/y^4)^{-1/2}(-4A^2/y^5) + 2(1 + A^2/y^4)^{1/2} = 0$

$y(1 + A^2/y^4)^{-1/2}(2A^2/y^5) = (1 + A^2/y^4)^{1/2}$

Now substituting for A ($A = my^2$) yields

$y(1 + (my^2)^2/y^4)^{-1/2}(2(my^2)^2/y^5) = (1 + (my^2)^2/y^4)^{1/2}$

$y(1 + m^2)^{-1/2}(2m^2/y) = (1 + m^2)^{1/2}$

$y(2m^2/y) = (1 + m^2)$; $2m^2 = (1 + m^2)$; $\mathbf{m = 1;\ \theta = 45^\circ}$

6.3.6

From Example 6.4, $m = (3)^{1/2}/3 = 0.577$; and

$b = (2/3)(3)^{1/2}y = 1.15y$ Now using Table 6.1:

$A = (b + my)y = [1.15y + 0.577(y)](y) = 1.73y^2$

$P = b + 2y(1 + m^2)^{1/2} = 1.15y + 2y(1 + (0.577)^2)^{1/2} = 3.46y$

Apply Manning's eq'n: $Q\cdot n/(1.49)(S)^{1/2} = (A)^{5/3}(P)^{-2/3}$;

$(150)(0.013)/(1.49)(0.01)^{1/2} = 13.1 = (1.73y^2)^{5/3}(3.46y)^{-2/3}$

$12.0 = y^{8/3}$; $\mathbf{y_n = 2.54\ ft\ and\ b = 2.92\ ft}$;

6.4.1

a) From Eq'n 6.12 with $V = Q/A = 1.39$ m/sec

$\mathbf{N_f} = V/(gD)^{1/2} = 1.39/(9.81 \cdot 3.6)^{1/2} =$ **0.233 (subcritical)**

b) $\mathbf{y_c} = [(Q^2/(gb^2)]^{1/3} = [(15^2/(9.81 \cdot 3.0^2)]^{1/3}$ **= 1.37 m**

and **since 1.37 m < 3.6 m, flow is subcritical.**

6.4.2

For a rectangular channel, from Table 6.1:

$A = by = 4y$; $P = b + 2y = 4 + 2y$

Apply Manning's eq'n: $Q \cdot n/(S)^{1/2} = (A)^{5/3}(P)^{-2/3}$;

$(100)(0.013)/(0.01)^{1/2} = 13.0 = (4y)^{5/3}(4+2y)^{-2/3}$

By successive substitution: $\mathbf{y_n}$ **= 2.90 m**;

Fig 6.4 or computer software yields the same answer.

$\mathbf{y_c} = [(Q^2/(gb^2)]^{1/3} = [(100^2/(9.81 \cdot 4^2)]^{1/3}$ **= 3.99 m**

since $\mathbf{y_n < y_c}$, **flow is supercritical at normal depth.**

6.4.3

For a trapezoidal channel, from Table 6.1):

$A = (b + my)y = [16 + 3(4.5)](4.5) = 133 \text{ ft}^2$

$P = b + 2y(1 + m^2)^{1/2} = 16 + 2(4.5)(1 + 3^2)^{1/2} = 44.5$ ft

$R_h = A/P = (133 \text{ ft}^2)/(44.5 \text{ ft}) = 2.99$ ft

Using Manning's eq'n: $V = (1.49/n)(R_h)^{2/3}(S)^{1/2}$

$\mathbf{V} = (1.49/0.013)(2.99)^{2/3}(0.001)^{1/2}$ **= 7.53 ft/sec**

$N_f = V/(gD)^{1/2}$ where $D = A/T$; for Table 6.1

$T = b + 2my = 16 + 2(3)(4.5) = 43$ ft

$D = 133/43 = 3.09$ ft;

$\mathbf{N_f} = 7.53/(32.2 \cdot 3.09)^{1/2}$ **= 0.755 (subcritical)**

6.4.4

The flow classification will require the flow velocity.

$A = (\pi/8)d_o^2 = (\pi/8)(0.6)^2 = 0.141 \text{ m}^2$; and:

$P = (\pi/2)d_o = (\pi/2)(0.6) = 0.942$ m; $R_h = A/P = 0.150$ m

$V = (1/n)AR_h^{2/3}S^{1/2} = (1/0.013)(0.141)(0.150)^{2/3}(0.0025)^{1/2}$

$\mathbf{Q} = 0.153 \text{ m}^3/\text{sec}$; $V = Q/A = 0.153/0.141$ **= 1.09 m/s**

$N_f = V/(gD)^{1/2}$; $D = A/T = 0.141/0.6 = 0.235$ m;

$\mathbf{N_f} = (1.09)/(9.81 \cdot 0.235)^{1/2}$ **= 0.718 (<1, subcritical)**

6.4.5

Since $V = Q/A = 834/(10 \cdot 6) = 13.9$ ft/sec

$\mathbf{N_f} = V/(gD)^{1/2} = 13.9/(32.2 \cdot 6)^{1/2}$ **= 1.00 (critical flow)**

$\mathbf{E} = V^2/2g + y = (13.9)^2/(2 \cdot 32.2) + 6$ **= 9.00 ft**

$A = 60.0 \text{ ft}^2$; $P = b + 2y = 10 + 2(6) = 22.0$ ft;

$Q = (1.49/n)(A)^{5/3}(P)^{-2/3}(S)^{1/2}$;

$834 = (1.49/0.025)(60)^{5/3}(22)^{-2/3}(S)^{1/2}$; **S = 0.0143**

6.4.6

$N_f = V/(gD)^{1/2}$; where $V = Q/A$; $Q = 100 \text{ m}^2/\text{sec}$, and

$A = (b + my)y = [5 \text{ m} + 1(5 \text{ m})](5 \text{ m}) = 50 \text{ m}^2$

$V = Q/A = 100/50 = 2$ m/s; and $D = A/T$

$T = b + 2my = 5 \text{ m} + 2(1)(5 \text{ m}) = 15$ m

$D = A/T = 50/15 = 3.33$ m

$\mathbf{N_f} = V/(gD)^{1/2} = 2/(9.81 \cdot 3.33)^{1/2} =$ **0.350 (subcritical)**

$\mathbf{E} = V^2/2g + y = (2)^2/(2 \cdot 9.81) + 5$ **= 5.20 m**

The total energy head with respect to the datum is:

$\mathbf{H} = E_{WS} + V^2/2g = 50 \text{ m} + (2)^2/(2 \cdot 9.81)$ **= 50.2 m**

6.4.7

Using Internet freeware (plainwater.com):

y_n = 6.92 ft   and y_c = 3.90 ft

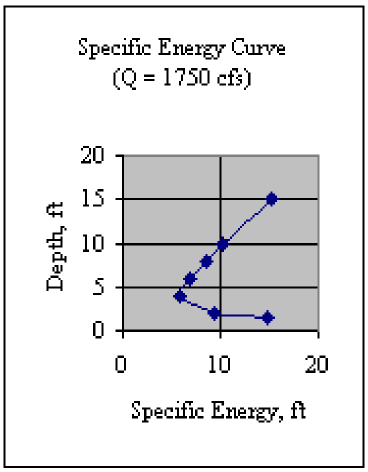
To determine a specific energy curve:

$E = Q^2/(2gA^2) + y = (1750)^2/[2g\{(40)(y)\}^2] + y$

Substituting different y's on either side of y_c= 3.90 ft

Q =	1750	cfs
b =	40.0	ft

y (ft)	E (ft)
1.5	14.7
2.0	9.4
3.9	5.9
6.0	6.8
8.0	8.5
10.0	10.3
15.0	15.1



6.4.8

a) $A = (b + my)y = [4\text{ m} + 1.5(3\text{ m})](3\text{ m}) = 25.5\text{ m}^2$

$E = Q^2/(2gA^2) + y = 50^2/[2\cdot 9.81(25.5)^2]+3\text{m} = 3.20\text{ m}$

thus, $3.2\text{m} = 50^2/(2g\{(4 + 1.5y)y\}^2) + y$; by trial; **y = 1.38 m**

b) At critical depth from Eq'n 6.13; $Q^2/g = DA^2 = A^3/T$

$A = (4 + 1.5y_c)y_c$, $T = b+2my = 4 + 2(1.5)y_c = 4 + 3y_c$

$Q^2/g = (50)^2/9.81 = 255 = A^3/T = [(4 + 1.5y_c)y_c]^3/(4 + 3y_c)$

By successive substitution, **y_c= 1.96m** (or use Fig 6.9)

c) From Manning's eq'n: $Q\cdot n/(S)^{1/2} = (A)^{5/3}(P)^{-2/3}$;

$(50)(0.022)/(0.0004)^{1/2} = 55 = [(4+1.5y_n)y_n]^{5/3}(4+3.61y_n)^{-2/3}$

By successive substitution: **y_n = 3.64 m**;

6.4.9

Using appropriate freeware (i.e., plainwater.com):

At the entrance to the transition, when S = 0.001:

y_i = 1.81 m; V_i = 1.67 m/sec; and at the transition exit,

when S = 0.0004: **y_e = 2.25 m**; V_e = 1.19 m/sec

Based on an energy balance: $H_i = H_e + h_L$ or

$V_i^2/2g + y_i + z_i = V_e^2/2g + y_e + z_e + h_L$;

$(1.67)^2/2g + 1.81\text{ m} + z_i = (1.19)^2/2g + 2.25\text{ m} + z_e + 0.02\text{ m}$

$z_i - z_e = \Delta z$ = 0.39 m (transition bottom elev. change)

6.4.10

Using appropriate freeware (i.e., plainwater.com):

At the entrance to the transition, when b = 12 ft:

y_i = 5.88 ft; V_i = 7.09 ft/sec; and at the transition exit,

when b = 6 ft: **y_e = 13.4 ft**; V_e = 6.23 ft/sec

Based on an energy balance: $H_i = H_e + h_L$ or

$V_i^2/2g + y_i + z_i = V_e^2/2g + y_e + z_e + h_L$; letting $z_e = 0$

$(7.09)^2/2g + 5.88\text{ ft} + z_i = (6.23)^2/2g + 13.4\text{ ft} + 0 + 1.5\text{ ft}$

z_i = 8.84 ft (the channel bottom height at the inlet)

The table below provides the transition properties.

$H = H_i - h_L$; (losses uniformly distributed in transition)

E = specific energy = H - z, and y is found from

$E = V^2/2g + y = Q^2/[2g\{(b)(y)\}^2] + y$

Section	Width, b (ft)	z (ft)	H (ft)	E (ft)	y (ft)
Inlet	12.0	8.84	15.50	6.66	5.88
25	10.5	6.63	15.13	8.50	7.94
50	9.0	4.42	14.75	10.33	9.80
75	7.5	2.21	14.38	12.17	11.70
Exit	6.0	0.00	14.00	14.00	13.40

6.5.1

Alternate depths have the same specific energy, one depth is subcritical and the other is supercritical. Sequent depths occur before and after a hydraulic jump and do not have the same specific energy.

6.5.2

The equation for specific energy is written as:

$E = V^2/2g + y = (Q/A)^2/2g + y$

It is evident from the equation that as the depth becomes very small (approaches zero), the velocity becomes very large (i.e., the area becomes very small) and thus the specific energy becomes very large. Likewise, it is evident from the equation that as the depth becomes very large, the velocity approaches zero (i.e., the area becomes very large) and thus the specific energy becomes equal to the depth. This will cause the specific energy curve to approach a 45° line (E = y).

6.5.3

Use eq'ns (6.20) for energy loss and (6.16) for flow:

$\mathbf{\Delta E} = (y_2 - y_1)^3/(4y_1y_2) = (3.1\text{-}0.8)^3/(4\cdot0.8\cdot3.1) = \mathbf{1.23\ m}$

$q^2/g = y_1 \cdot y_2[(y_1 + y_2)/2]$

$q^2/(9.81) = 0.8\cdot3.1[(0.8 + 3.1)/2]$; $q = 6.89\ m^3/sec\text{-}m$

$\mathbf{Q} = b\cdot q = (7\ m)(6.89\ m^3/sec\text{-}m) = \mathbf{48.2\ m^3/sec}$

The Froude numbers are determined as:

$N_{F1} = V_1/(gy_1)^{1/2}$; and $V_1 = q/y_1 = 6.89/0.8 = 8.61$ m/s

$\mathbf{N_{F1}} = 8.61/(9.81\cdot0.8)^{1/2}$ **= 3.07 (supercritical)**

$N_{F2} = V_2/(gy_2)^{1/2}$; and $V_2 = q/y_2 = 6.89/3.1 = 2.22$ m/s

$\mathbf{N_{F2}} = 2.22/(9.81\cdot3.1)^{1/2}$ **= 0.403 (subcritical)**

6.5.4

Use eq'ns (6.16) for depth and (6.20) for energy loss:

$q^2/g = y_1 \cdot y_2[(y_1+y_2)/2]$; $q = Q/b = 403/12 = 33.6$ cfs/ft

$(33.6)^2/(32.2) = y_1\cdot5[(y_1+5)/2]$;

By successive substitution: $\mathbf{y_1 = 2.00\ ft}$

$\Delta E = (y_2 - y_1)^3/(4y_1y_2) = (5.0 - 2.0)^3/(4\cdot5.0\cdot2.0)$

$\mathbf{\Delta E = 0.675\ ft}$

$N_{F1} = V_1/(gy_1)^{1/2}$; and $V_1 = q/y_1 = 33.6/2.0 = 16.8$ ft/s

$\mathbf{N_{F1}} = 16.8/(32.2\cdot2.0)^{1/2} =$ **2.09 (supercritical)**

$N_{F2} = V_2/(gy_2)^{1/2}$; and $V_2 = q/y_2 = 33.6/5.0 = 6.72$ ft/s

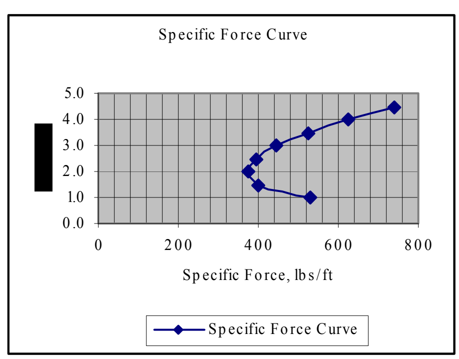
$\mathbf{N_{F1}} = 6.72/(32.2\cdot5.0)^{1/2}$ **= 0.530 (subcritical)**

6.5.5

Using Eq'ns (6.8) and (6.19): $E = V^2/2g + y$ and

$F_s = F + \rho qV = (\gamma/2)y_1^{\,2} + \rho(Q/b)V$

Depth (ft)	Area $(ft)^2$	V (ft/sec)	E (ft)	F_s (lbs/ft)
1.0	3.0	16.0	5.0	528
1.5	4.5	10.7	3.3	401
2.0	6.0	8.0	3.0	373
2.5	7.5	6.4	3.1	394
3.0	9.0	5.3	3.4	446
3.5	10.5	4.6	3.8	524
4.0	12.0	4.0	4.2	623
4.5	13.5	3.6	4.7	742



6.5.6

$y_c = [(Q^2/(gb^2)]^{1/3} = [(15^2/(9.81 \cdot 10^2)]^{1/3}$ **= 0.612 m**

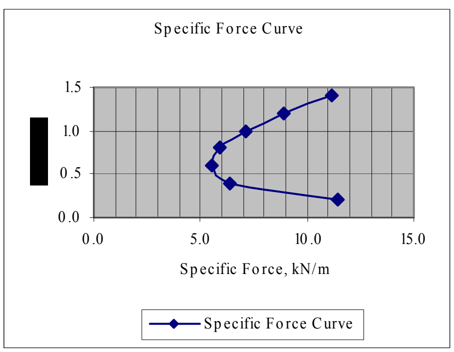
Minimum specific energy occurs at critical depth.

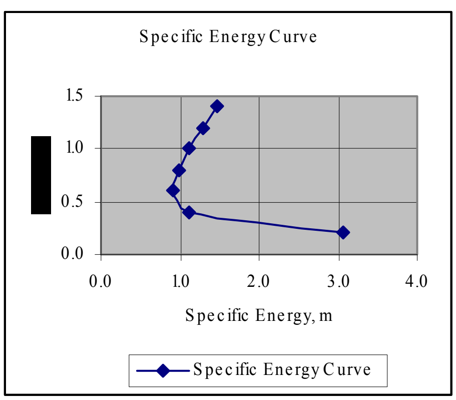
$E_{min} = V_c^2/2g + y_c = [15/(10 \cdot 0.612)]^2/2g + 0.612m$ **= 0.918m**

Using Eq'ns (6.8) and (6.19): $E = V^2/2g + y$ and

$F_s = F + \rho qV = (\gamma/2)y^2 + \rho(Q/b)V$; $\gamma = 9790\ N/m^3$
$\rho = 998\ kg/\ m^3$

Depth (m)	Area $(m)^2$	V (m/sec)	E (m)	F_s (kN/m)
0.2	2.0	7.50	3.1	11.4
0.4	4.0	3.75	1.1	6.4
0.6	6.0	2.50	0.9	5.5
0.8	8.0	1.88	1.0	5.9
1.0	10.0	1.50	1.1	7.1
1.2	12.0	1.25	1.3	8.9
1.4	14.0	1.07	1.5	11.2





From continuity, an increase in discharge will increase the depth. This will, in turn, increase the forces causing the specific force curve to move up and to the right.

6.8.1

a) reservoir flowing into a steep channel (S-2)

b) mild slope channel into a reservoir (M-2)

c) mild channel into a dam/reservoir (M-1)

d) horizontal into mild into reservoir (H-2 and M-2)

6.8.2

Channel and flow classification requires three depths: critical, normal, and actual. For normal depth, use the Manning eq'n (and Table 6.1), Fig 6.4, or computer software. Find critical depth from Eq'n 6.14 or software. $\mathbf{y_n}$ **= 3.99 ft**; $\mathbf{y_c}$ **= 3.20 ft.** Since $y_n > y_c$, the channel is mild. Since y/y_c and y/y_n are greater than 1.0; flow is Type 1; **classification is M-1** (see Fig 6.12)

6.8.3

Determine normal depth in both channels (upstream and downstream) by using the Manning equation (and Table 6.1), Figure 6.4 or computer software. Also, determine critical depth from Eq'n 6.10 or software.

$\mathbf{y_n}$ **(up) = 1.83 m**; $\mathbf{y_n}$ **(down) = 1.27 m,** $\mathbf{y_c}$ **= 1.34 m**

Since $y_n > y_c$; upstream channel is mild and flows to a steep channel. Flow will go through critical depth near the slope break (see Fig. 6.12); **classification is M-2.**

6.8.4

Channel and flow classification requires three depths: critical, normal, and actual. For normal depth, use the Manning eq'n (and Table 6.1), Fig 6.4, or computer software. Find critical depth from Eq'n 6.14 or software. $\mathbf{y_n}$ **= 0.871 m**; $\mathbf{y_c}$ **= 1.41 m.** Since $y_n < y_c$; channel is steep. Since y/y_c and y/y_n are less than 1.0; flow is Type 3; **classification is S-3** (see Fig. 6.12)

6.8.5

Compute normal and critical depth for the channel. Recall that for wide rectangular channels, the depth is very small in relation to its width. Thus, $R_h \approx y$ and the Manning equation may be written as formulated in Example 6.9. Solving for each yields: $\mathbf{y_n = 0.847\ m}$ and $\mathbf{y_c = 0.639\ m.}$

Note that computer software is not likely to give you the correct normal depth since the sides of the channel will be included in the wetted perimeter. Since $y_n > y_c$, the channel is mild. Since the depth of flow (0.73 m) is between critical and normal, it is a type-2 curve; the full **classification is M-2** (see Fig. 6.12). Therefore, the depth rises as you move upstream. The depth of flow 12 m upstream from the location where the depth is 0.73 m is **0.75 m** as computed in the following energy balance.

Section	y (m)	z (m)	A (m^2)	V (m/sec)	$V^2/2g$ (m)	R_h (m)	S_e*	$S_{e(avg)}$	$\Delta L \cdot S_{e(avg)}$ (m)	Total Energy (m)
1	0.73	0.000	0.730	2.192	0.245	0.730	1.64E-03	1.57E-03	0.019	0.994
2	**0.75**	0.012	0.750	2.133	0.232	0.750	1.50E-03	ΔL =	12	0.994

* The energy grade line slope is found using the Manning equation; in this case $S_e = n^2q^2/y^{10/3}$ (see Example 6.9).

6.8.6a

The complete solution to Example 6.9 is in the spreadsheet program results shown below. Refer to example 6.9 to determine the appropriate equations for the various cells.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Sec. #	**U/D**	**y** (m)	**z** (m)	**A** (m^2)	**V** (m/sec)	$V^2/2g$ (m)	**P** (m)	R_h (m)	S_e	$S_{e(avg)}$	h_L (m)	**Total E** (m)
1	D	2.00	0.000	12.00	1.042	0.0553	9.657	1.243	0.000508	0.000554	0.1042	2.159
2	U	**1.91**	0.188	11.29	1.107	0.0625	9.402	1.201	0.000601	ΔL =	188	2.160
	Note: The trial depth of 1.91 m is correct. Now balance energy between sections #2 and #3.											
2	D	1.91	0.188	11.29	1.107	0.0625	9.402	1.201	0.000601	0.000658	0.1547	2.315
3	U	**1.82**	0.423	10.59	1.180	0.0710	9.148	1.158	0.000716	ΔL =	235	2.314
	Note: The trial depth of 1.82 m is correct. Now balance energy between sections #3 and #4.											
3	D	1.82	0.423	10.59	1.180	0.0710	9.148	1.158	0.000716	0.000779	0.2532	2.567
4	U	**1.74**	0.748	9.99	1.252	0.0798	8.921	1.120	0.000842	ΔL =	325	2.568
	Note: The trial depth of 1.74 m is correct. Now balance energy between sections #4 and #5.											
4	D	1.74	0.748	9.99	1.252	0.0798	8.921	1.120	0.000842	0.000920	0.8528	3.421
5	U	**1.66***	1.675	9.40	1.330	0.0902	8.695	1.081	0.000998	ΔL =	927	3.425

* Normal depth appears to have been obtained, however it took a long distance upstream of the obstruction before this occurred even though the dam only raised the water level 0.34 m above normal depth.

6.8.6b

The complete solution to Example 6.9 is in the spreadsheet program results shown below based on the direct step method. It compares very closely to the solution using the standard step method.

Sec #	**U/D**	**y** (m)	**A** (m^2)	**P** (m)	$\mathbf{R_h}$ (m)	**V** (m/sec)	$\mathbf{V^2/2g}$ (m)	**E** (m)	$\mathbf{S_e}$	$\mathbf{\Delta L}$ (m)	Distance to Dam (m)
1	D	2.00	12.00	9.657	1.243	1.042	0.0553	2.0553	0.000508		0
2	U	**1.91**	11.29	9.402	1.201	1.107	0.0625	1.9725	0.000601	186	**186**
	Determine how far upstream it is before the depth reduces to 1.91 m.										
2	D	1.91	11.29	9.402	1.201	1.107	0.0625	1.9725	0.000601		186
3	U	**1.82**	10.59	9.148	1.158	1.180	0.0710	1.8910	0.000716	239	**424**
	Determine how far upstream it is before the depth reduces to 1.82 m.										
3	D	1.82	10.59	9.148	1.158	1.180	0.0710	1.8910	0.000716		424
4	U	**1.74**	9.99	8.921	1.120	1.252	0.0798	1.8198	0.000842	322	**746**
	Determine how far upstream it is before the depth reduces to 1.74 m.										
4	D	1.74	9.99	8.921	1.120	1.252	0.0798	1.8198	0.000842		746
5	U	**1.66**	9.40	8.695	1.081	1.330	0.0902	1.7502	0.000998	870	**1,616**
	Normal depth has been obtained.										

6.8.7a

The complete solution to Example 6.10 is in the spreadsheet program results shown below. Refer to example 6.10 to determine the appropriate equations for the various cells. Normal depth and critical depth computations may also be programmed into the spreadsheet.

Section	y (ft)	z (ft)	A (ft^2)	V (ft/sec)	$V^2/2g$ (ft)	R_h (ft)	S_e	$S_{e(avg)}$	$\Delta L \cdot S_{e(avg)}$ (ft)	Total Energy (ft)
1	2.76	10.000	24.90	7.431	0.857	1.571	6.59E-03	ΔL =	2	13.617
2	**2.66**	9.976	23.46	7.885	0.966	1.524	7.73E-03	7.16E-03	0.014	13.616
2	2.66	9.976	23.46	7.885	0.966	1.524	7.73E-03	ΔL =	5	13.602
3	**2.58**	9.916	22.34	8.280	1.065	1.486	8.82E-03	8.27E-03	0.041	13.602
3	2.58	9.916	22.34	8.280	1.065	1.486	8.82E-03	ΔL =	10	13.561
4	**2.51**	9.796	21.39	8.651	1.162	1.452	9.92E-03	9.37E-03	0.094	13.562
4	2.51	9.796	21.39	8.651	1.162	1.452	9.92E-03	ΔL =	40	13.468
5	**2.42***	9.316	20.18	9.166	1.305	1.409	1.16E-02	1.08E-02	0.430	13.471

* The final depth of flow computed 57 feet downstream from the reservoir is within 1% of the normal depth (2.40 ft). Note that the largest changes in depth occur quickly once the water enters the steep channel and approaches normal depth asymptotically as depicted in the S-2 curves in Figures 6.12 and 6.13a.

6.8.7b

The complete solution to Example 6.10 is in the spreadsheet program results shown below based on the direct step method. It compares very closely to the solution using the standard step method.

Sec #	U/D	y (ft)	A (ft^2)	P (ft)	R_h (ft)	V (ft/sec)	$V^2/2g$ (ft)	E (ft)	S_e	ΔL (ft)	Distance to Dam (ft)
1	U	2.76	24.90	15.843	1.571	7.431	0.8575	3.6175	0.00659		0
2	D	**2.66**	23.46	15.396	1.524	7.885	0.9655	3.6255	0.00773	1.66	**1.66**
	Determine how far downstream it is before the depth reduces to 2.66 ft.										
2	U	2.66	23.46	15.396	1.524	7.885	0.9655	3.6255	0.00773		1.66
3	D	**2.58**	22.34	15.038	1.486	8.280	1.0646	3.6446	0.00882	5.12	**6.78**
	Determine how far downstream it is before the depth reduces to 2.58 ft.										
3	U	2.58	22.34	15.038	1.486	8.280	1.0646	3.6446	0.00882		6.78
4	D	**2.51**	21.39	14.725	1.452	8.651	1.1621	3.6721	0.00992	10.4	**17.2**
	Determine how far downstream it is before the depth reduces to 2.51 ft.										
4	U	2.51	21.39	14.725	1.452	8.651	1.1621	3.6721	0.00992		17.2
5	D	**2.42**	20.18	14.323	1.409	9.166	1.3047	3.7247	0.01159	42.3	**59.5**
	Determine how far downstream it is before the depth reduces to 2.42 ft.										

6.8.8

Compute normal and critical depth for the channel. For normal depth, use the Manning equation (and Table 6.1), Fig 6.4, or computer software. Find critical depth from Equation 6.10 (or 6.13) or computer software.

Solving for both depths yields: $\mathbf{y_n = 1.12}$ **m**, $\mathbf{y_c = 1.53}$ **m, and y = 1.69 m (given).**

Since $y_n < y_c$; the channel is steep. Since y/y_c and y/y_n are greater than 1.0; the flow is Type 1;

Therefore, the complete **classification is S-1** (see Fig. 6.12) and depth decreases going upstream.

The depth of flow 6.0 m upstream from the location where the depth is 1.69 m is **1.62 m** as computed in the following energy balance.

Section	y (m)	z (m)	A (m^2)	V (m/sec)	$V^2/2g$ (m)	R_h (m)	S_e	$S_{e(avg)}$	$\Delta L \cdot S_{e(avg)}$ (m)	Total Energy (m)
1	1.69	0.000	11.3	3.10	0.488	1.16	9.56E-04	1.04E-03	0.006	2.185
2	**1.61**	0.024	10.6	3.29	0.551	1.11	1.13E-03	ΔL =	6	2.185

The total energy is found using Equation 6.26b with losses added to the downstream section (#1). The area and hydraulic radius are solved using formulas from Table 6.1. The slope of the energy grade line is found from the Manning equation: $S_e = (n^2V^2)/(R_h)^{4/3}$

6.8.9

Compute normal and critical depth for the channel. For normal depth, use the Manning equation (and Table 6.1), Fig 6.4, or computer software. Find critical depth from Equation 6.14 or computer software. Solving for each yields: **y_n = 0.780 m** and **y_c = 0.639 m.**

Since $y_n > y_c$, the channel is mild. Since the depth of flow (5.64 m) is greater than critical and normal, it is a type-1 curve; the full **classification is M-1** (see Fig. 6.12). Thus, the depth falls as you move upstream. The depth of flow upstream at distances of 300 m, 900 m, 1800 m, and 3000 m from the control section (where the depth is 5.64 m) are shown in the spreadsheet program results below. (Note: $R_h = A/P$ and $S_e = n^2V^2/R_h^{(4/3)}$; Manning Eq'n)

Water Surface Profile (Problem 6.8.9)

Q =	16.00	m^3/sec	y_c =	0.639	m
S_o =	0.0016		y_n =	0.780	m
n =	0.015		g =	9.81	m/sec^2
b =	10	m			

Section	y (m)	z (m)	A (m^2)	V (m/sec)	$V^2/2g$ (m)	R_h (m)	S_e	$S_{e(avg)}$	$\Delta L \cdot S_{e(avg)}$ (m)	Total Energy (m)
1	5.64	0.000	56.40	0.284	0.004	2.650	4.94E-06	5.59E-06	0.002	5.646
2	5.16	0.480	51.60	0.310	0.005	2.539	6.24E-06	ΔL =	300	5.645
2	5.16	0.480	51.60	0.310	0.005	2.539	6.24E-06	8.55E-06	0.005	5.650
3	4.20	1.440	42.00	0.381	0.007	2.283	1.09E-05	ΔL =	600	5.647
3	4.20	1.440	42.00	0.381	0.007	2.283	1.09E-05	2.28E-05	0.021	5.668
4	2.77	2.880	27.70	0.578	0.017	1.782	3.47E-05	ΔL =	900	5.667
4	2.77	2.880	27.70	0.578	0.017	1.782	3.47E-05	2.91E-04	0.349	6.016
5	1.10	4.800	11.00	1.455	0.108	0.902	5.47E-04	ΔL =	1,200	6.008

6.8.10

Compute normal and critical depth for the channel. For normal depth, use the Manning equation (and Table 6.1), Fig 6.4, or computer software. Find critical depth from Equation 6.14 or computer software. Solving for each yields: **y_n = 0.217 m** and **y_c = 0.639 m.**

Since $y_n < y_c$, the channel is steep. By referring to Figure 6.13a and 6.13c, we can see that the water surface profile will **go through critical depth** as it starts down the back side of the dam (which is a steep channel) and this becomes the control section. Calculations begin here, where the depth is known, and proceed downstream where the depth decreases as it approaches normal depth making it a type-2 curve: the full **classification is S-2** (see Fig. 6.12). The depth of flow downstream at distances of 0.3 m, 2.0 m, 7.0 m, and 30.0 m from the **control section (where the depth is critical, 0.639 m)** are shown in the spreadsheet program results below. (Note: $R_h = A/P$ and $S_e = n^2V^2/R_h^{(4/3)}$; derived from the Manning Eq'n)

Water Surface Profile (Problem 6.8.10)

Q =	16.0	m^3/sec	y_c =	0.639	m
S_o =	0.100		y_n =	0.217	m
n =	0.015		g =	9.81	m/sec^2
b =	10	m			

Section	y	z	A	V	$V^2/2g$	R_h	S_e	$S_{e(avg)}$	$\Delta L \cdot S_{e(avg)}$	Total Energy
	(m)	(m)	(m^2)	(m/sec)	(m)	(m)			(m)	(m)
1	0.639	5.000	6.390	2.504	0.320	0.567	3.01E-03	ΔL =	0.3	5.959
2	**0.539**	4.970	5.390	2.968	0.449	0.487	5.18E-03	4.09E-03	0.001	5.959
2	0.539	4.970	5.390	2.968	0.449	0.487	5.18E-03	ΔL =	1.7	5.958
3	**0.426**	4.800	4.260	3.756	0.719	0.393	1.10E-02	8.11E-03	0.014	5.959
3	0.426	4.800	4.260	3.756	0.719	0.393	1.10E-02	ΔL =	5.0	5.945
4	**0.326**	4.300	3.260	4.908	1.228	0.306	2.63E-02	1.87E-02	0.093	5.947
4	0.326	4.300	3.260	4.908	1.228	0.306	2.63E-02	ΔL =	23.0	5.854
5	**0.233**	2.000	2.330	6.867	2.403	0.223	7.86E-02	5.25E-02	1.206	5.843

6.8.11

Compute normal and critical depth for the channel. For normal depth, use the Manning equation (and Table 6.1), Fig 6.4, or computer software. Find critical depth from Equation 6.14 or computer software. Solving for each yields: **y_n = 2.687 ft** and **y_c = 1.459 ft.**

Since $y_n > y_c$, the channel is mild. Since the depth of flow (5.00 ft) is greater than critical and normal, it is a type-1 curve; the full **classification is M-1** (see Fig. 6.12). Thus, the depth falls as you move upstream. The depth of flow upstream at distances of 200 ft, 500 ft, and 1,000 ft from the control section (where the depth is 5.00 ft) are shown in the spreadsheet program results below. (Note: $R_h = A/P$; $S_e = n^2V^2/2.22R_h^{(4/3)}$ from the Manning Eq'n)

Water Surface Profile (Problem 6.8.11)

Q =	50.0	ft^3/sec				y_c =	1.459	ft
S_o =	0.001		b =	5	ft	y_n =	2.687	ft
n =	0.015		m =	0	rectangle	g =	32.2	ft/sec^2

Section	y	z	A	V	$V^2/2g$	R_h	S_e	$S_{e(avg)}$	$\Delta L \cdot S_{e(avg)}$	Total Energy
	(ft)	(ft)	(ft^2)	(ft/sec)	(ft)	(ft)			(ft)	(ft)
1	5.00	0.000	25.00	2.000	0.062	1.667	2.05E-04	2.14E-04	0.043	5.105
2	**4.84**	0.200	24.20	2.066	0.066	1.649	2.22E-04	ΔL =	200	5.106
2	4.84	0.200	24.20	2.066	0.066	1.649	2.22E-04	2.37E-04	0.071	5.177
3	**4.60**	0.500	23.00	2.174	0.073	1.620	2.52E-04	ΔL =	300	5.173
3	4.60	0.500	23.00	2.174	0.073	1.620	2.52E-04	2.81E-04	0.140	5.314
4	**4.23**	1.000	21.15	2.364	0.087	1.571	3.10E-04	ΔL =	500	5.317

6.8.12

Computing normal and critical depths yields: $\mathbf{y_n = 2.190}$ **m** and $\mathbf{y_c = 1.789}$ **m. (Rf: plainwater.com)**

Since $y_n > y_c$, the channel is mild. Since the depth of flow (5.8 m) is greater than both, it is a type-1 curve; the full **classification is M-1** (see Fig. 6.12). Thus, the depth falls as you move upstream. The water surface profile is given below. (Note: $R_h = A/P$ and $S_e = n^2V^2/R_h^{(4/3)}$; from the Manning Eq'n)

Water Surface Profile (Problem 6.8.12)

Q =	44.0	m^3/sec			y_c =	2.19	m
S_o =	0.004		b =	3.6	m	y_n =	1.789	m
n =	0.015		m =	2		g =	9.81	m/sec^2

Section	y (m)	z (m)	A (m^2)	V (m/sec)	$V^2/2g$ (m)	R_h (m)	S_e	$S_{e(avg)}$	$\Delta L \cdot S_{e(avg)}$ (m)	Total Energy (m)
1	5.80	0.000	88.16	0.499	0.013	2.985	1.30E-05	2.24E-05	0.006	5.818
2	4.79	1.000	63.13	0.697	0.025	2.523	3.18E-05	ΔL =	250	5.815
2	4.79	1.000	63.13	0.697	0.025	2.523	3.18E-05	6.32E-05	0.016	5.831
3	3.77	2.000	42.00	1.048	0.056	2.053	9.47E-05	ΔL =	250	5.826
3	3.77	2.000	42.00	1.048	0.056	2.053	9.47E-05	2.43E-04	0.061	5.887
4	2.73	3.000	24.73	1.779	0.161	1.565	3.92E-04	ΔL =	250	5.891
4	2.73	3.000	24.73	1.779	0.161	1.565	3.92E-04	1.22E-03	0.220	6.112
5	1.84	3.720	13.40	3.285	0.550	1.132	2.06E-03	ΔL =	180*	6.110

*Note: The last interval only needed to be 180 m to get within 2% of the normal depth.

6.8.13

Computing normal and critical depths yields: $\mathbf{y_n = 1.233}$ **m** and $\mathbf{y_c = 1.534}$ **m. (Rf: plainwater.com)**

Since $y_n < y_c$, the channel is steep. Since the depth of flow (3.4 m) is greater than both, it is a type-1 curve; the full **classification is S-1** (see Fig. 6.12). Thus, the depth falls as you move upstream. The first 150 m of the water surface profile is given below. It is left up to the student and the instructor to determine where and at what depth the hydraulic jump occurs. (Note: $R_h = A/P$ and $S_e = n^2V^2/R_h^{(4/3)}$; from the Manning Eq'n)

Section	y (m)	z (m)	A (m^2)	V (m/sec)	$V^2/2g$ (m)	R_h (m)	S_e	$S_{e(avg)}$	$\Delta L \cdot S_{e(avg)}$ (m)	Total Energy (m)
1	3.40	0.000	28.56	1.225	0.077	1.954	1.04E-04	1.18E-04	0.006	3.482
2	3.19	0.200	26.13	1.340	0.091	1.863	1.32E-04	ΔL =	50	3.481
2	3.19	0.200	26.13	1.340	0.091	1.863	1.32E-04	1.52E-04	0.008	3.489
3	2.98	0.400	23.78	1.472	0.110	1.771	1.71E-04	ΔL =	50	3.490
3	2.98	0.400	23.78	1.472	0.110	1.771	1.71E-04	1.99E-04	0.010	3.500
4	2.76	0.600	21.42	1.634	0.136	1.672	2.27E-04	ΔL =	50	3.496

6.9.1

Equation (6.29) yields:

$R_h = [0.022 \cdot 4.0)/(1.49 \cdot 0.0011^{1/2})]^{3/2} = 2.38$ ft

Also, $A = Q/V_{max} = 303/4.0 = 75.8$ ft^2.

Hence P = A/R = 75.8/2.38 = 31.8 ft.

Now from Table 6.1 (with m = 3):

$A = (b + 3y)y = 75.8$ ft^2 ; and

$P = b + 2y(1 + 3^2)^{1/2} = 31.8$ ft

Solving these two equations simultaneously we obtain

b = 3.53 ft and y = 4.47 ft. Also,

T = b + 2my = 3.53 + 2(3)4.47 = 30.4 ft;

D = A/T = 75.8/30.4 = 2.49 ft; and finally

$N_F = V/(gD)^{1/2} = 4.0/(32.2 \cdot 2.49)^{1/2} = 0.447$; ok

Using Equation (6.28) with C = 1.6 (by interpolation),

$F = (C \cdot y)^{1/2} = (1.6 \cdot 4.47)^{1/2} = 2.67$ ft.

Design Depth = y + F = 4.47 + 2.67 = 7.14 ft

Use a design depth of 7.25 ft or 7.5 ft and a bottom width of 3.5 ft for practicality in field construction.

6.9.2

This is an analysis problem, not a design problem.
For the given channel, determine the normal depth using the Manning equation (and Table 6.1), Fig 6.4, or computer software.

y_n = 1.98 m (Rf: plainwater.com)

The corresponding velocity (Q/A) is 1.40 m/sec.

This is smaller than V_{MAX} = 1.8 m/sec (Table 6.7).

Therefore, the channel can carry the design discharge of 11 m^3/sec without being eroded.

6.9.3

From Table 6.2, n = 0.013. Equation (6.30) yields

$b/y = 2[(1 + m^2)^{1/2} - m] = 2[(1 + 1.5^2)^{1/2} - 1.5] = 0.606$

Then from Equation (6.31)

$$y = \frac{\left[(b/y) + 2\sqrt{1+m^2}\right]^{1/4}}{[(b/y)+m]^{5/8}}\left(\frac{Q \cdot n}{k_M\sqrt{S_0}}\right)^{3/8}$$

$$y = \frac{\left[0.606 + 2\sqrt{1+1.5^2}\right]^{1/4}}{[(0.606)+1.5]^{5/8}}\left(\frac{342\ (0.013)}{1.49\sqrt{0.001}}\right)^{3/8}$$

y = 4.95 ft. Then **b** = 0.606 (4.95) = **3.00 ft**. Also,

T = b + 2my = 3.00 + 2(1.5)4.95 = 17.9 ft;

A = (b + my)y = [3.00 + (1.5)4.95]4.95 = 51.6 ft^2

D = A/T = 2.88 ft; V = Q/A = 6.63 ft/s; and finally

$N_F = V/(gD)^{1/2} = 6.63/(32.2 \cdot 2.88)^{1/2} = 0.688$; ok

Using Figure 6.15, the height of lining above the free surface is 1.0 ft. Also, the freeboard (height of banks) above the free surface is 2.7 ft. Therefore,

Design Depth = y + F = 4.95 + 2.7 = 7.65 ft

Use a design depth of 7.75 ft or 8.0 ft and a bottom width of 3.0 ft for practicality in field construction.

6.9.4

Due to the limitation on the flow depth, the section will no longer be the most efficient hydraulically. Applying the Manning equations yields:

$Q = (1.49/n)AR_h^{2/3}S^{1/2} = (1.49/n)(A)^{5/3}(P)^{-2/3}(S)^{1/2}$

$342 = (1.49/0.013)[\{b+(1.5)(3.5)\}3.5]^{5/3}[b+2(3.5) \cdot (1+1.5^2)^{1/2}]^{-2/3}(0.001)^{1/2}$

By successive substitution or computer software;
b = 9.97 ft or 10 ft. The corresponding Froude number is 0.70 and is acceptable. The freeboard is still 2.7 feet based on Figure 6.15. .

Chapter 7 – Problem Solutions

7.1.1

Volume of solids (Vol_s) is 3,450 ml (3,450 cm^3),

since 1 ml of water ≈ 1 cc of water. Therefore,

Vol_s= 3,450cm^3(1m/100cm)^3(35.3 ft^3/1m^3) = 0.122 ft^3

Sample volume: Vol = (1 ft)[π(0.25 ft)^2] = 0.196 ft^3

From Equation 7.1:

α = Vol_v/Vol = (0.196-0.122)/0.196 **= 0.378**

7.1.2

Equation (7.1) states that: α = Vol_v/Vol;

From the definition of density:

ρ_b = mass/Vol; thus Vol = mass/ρ_b; also

ρ_s = mass/Vol_s = mass/(Vol - Vol_v); thus

(Vol - Vol_v) = mass/ρ_s; Vol_v = Vol – (mass/ρ_s)

Substituting into the original equation yields:

α = Vol_v/Vol = [Vol – (mass/ρ_s)]/(mass/ρ_b)

α = [(mass/ρ_b) - (mass/ρ_s)]/(mass/ρ_b) **= 1 – (ρ_b/ρ_s)**

7.1.3

The sample volume is: Vol = 65.0 N/γ_k;

where γ_k is the specific weight of kerosene.

Using the same reasoning, the void volume is:

Vol_v = (179 N - 157 N)/γ_k = 22.0 N/γ_k

Finally, using Equation Equation 7.1;

α = Vol_v/Vol = (22.0 N/γ_k)/(65.0 N/γ_k) **= 0.338**

7.1.4

a) The texture, porosity, grain orientation, and packing may be markedly different from in-situ conditions.

b) Some disturbance always occurs when removing a sample from a well or borehole. In addition, wall effects in the sample tube and the direction of flow in the field vs. the direction of flow through the sample in the lab are likely to skew the test results.

c) V_s = V/α = Q/(A·α); using α = 0.35 (Table 7.1):
   V_s = Q/(A·α) = 10.7/[(12.6)(0.35)] = 2.43 cm/min
   t = L/V_s = (30cm)/(2.43 cm/min) **= 12.3 min**

d) **V (actual) > V_s** since the flow path is tortuous.

7.1.5

Using Equation 7.4, try K = 10^{-6} m/sec = 10^{-4} cm/sec

(from Table 7.2 for very fine sands, low end of range)

Q = KA(dh/dL); where A = π(5 cm)^2 = 78.5 cm^2; thus

Q = (10^{-4}cm/s)(78.5cm^2)(40/30) = 0.0105cm^3/s (ml/s)

Q = Vol/t; or t = Vol/Q = (50 ml)/(0.0105 ml/s)

t = 4760 sec = **79.3 min ≈ 80 min per test**

To determine the tracer time, we need to determine the seepage velocity. The seepage velocity is the average speed that the water moves between two points.

From Table 7.1, use a porosity of 0.30.

V_s = V/α = Q/(A·α) = (0.0105 cm^3/s)/[(78.5 cm^2)(0.30)

V_s = 4.46 x 10^{-4} cm/sec

t = ΔL/V_s = (30 cm)/(4.46 x 10^{-4} cm/sec)(1 hr/3600 sec)

t = 18.7 hrs.

7.1.6

From Eq'n 7.3, $K = (Cd^2\gamma)/\mu$ or $Cd^2 = K\mu/\gamma$. Since the sand characteristics are constant: $K_{20}\mu_{20}/\gamma_{20} = K_5\mu_5/\gamma_5$

thus, $K_5 = (\mu_{20}/\mu_5)(\gamma_5/\gamma_{20})K_{20}$ (using Tables 1.2 & 1.3)

$K_5 = (1.002\text{x}10^{-3}/1.518\text{x}10^{-3})(9808/9790)(1.8 \text{ cm/min})$

$K_5 =(1.002\text{x}10^{-3}/1.518\text{x}10^{-3})(9808/9790)(1.81 \text{ cm/min})$

K_5 = 1.20 cm/min **= 2.00 x 10^{-4} m/sec**

$Q = KA(dh/dL) = (1.20 \text{ cm/min})(12.6 \text{ cm}^2)(14.1/30)$

Q = 7.11 cm^3/min **= 35.6 cm^3/5min**

7.1.7

The apparent (Darcy) velocity can be computed as:

$V = K(dh/dL) = (0.0164 \text{ ft/sec})(0.02) = 3.28 \text{ x } 10^{-4} \text{ ft/sec}$

($K = 5.0 \text{ x } 10^{-3}$ m/sec = 0.0164 ft/sec; from Table 7.2).

$V_s = V/\alpha = (3.28 \text{ x } 10^{-4} \text{ ft/sec})/(0.275) = 1.19 \text{ x } 10^{-3} \text{ ft/sec}$

(α from Table 7.1). Now determine the travel time

$t = \Delta L/V_s = [(82 \text{ ft})/(1.19 \text{ x } 10^{-3} \text{ ft/sec})](1 \text{ hr}/3600 \text{ sec})$

t = 19.1 hours (gw moves slowly, even in course soils)

7.1.8

The apparent (Darcy) velocity can be computed as:

$V = K(dh/dL) = (1.00 \text{ x } 10^{-5} \text{ m/s})(1/50) = 2.00 \text{ x } 10^{-7} \text{ m/sec}$

The slope of the water table is the hydraulic gradient; not the slope of the confining bedrock layer. The flow rate through each weep hole requires the continuity equation (Q = AV) where the area is in the aquifer where V was found, not the weep hole. Therefore,

$Q = AV = [(1\text{m})(3\text{m})](2.00 \text{ x } 10^{-7} \text{ m/sec})$

Q = 6.00 x 10^{-7} m^3/sec **= 0.600 cm^3/sec**

7.1.9

Find the apparent (Darcy) velocity from: $V = K(dh/dL)$. The hydraulic gradient (dh/dL) is the slope of the water table in the vicinity of the cross section located at the 5200 ft contour, which will be used to obtain the flow area. Since there is a 20 ft drop (dh) in the water table over a 1100 ft length (dL) from contour 5210′ to 5190′,

$V = K(dh/dL) = (0.00058 \text{ ft/s})(20\text{ft}/1100 \text{ ft}) = 1.05 \text{ x } 10^{-5} \text{ ft/s}$

Thus, $Q = AV = [1/2(1500 \text{ ft})(40 \text{ ft})](1.05 \text{ x } 10^{-5} \text{ ft/s})$

Q = 0.315 cfs (ft^3/sec)

7.1.10

Groundwater moves from the unconfined aquifer to the confined since there is a hydraulic gradient (dh/dL) in that direction [i.e., the water level (energy head) in the unconfined well is 2 m higher than the water level in the confined well]. The flow rate from Darcy's law is:

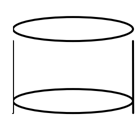
$Q = KA(dh/dL) = (0.305 \text{ m/day})(1 \text{ m}^2)(2 \text{ m}/1.5 \text{ m})$

Q = 0.407 m^3/day per sq. meter (of semi-impervious layer)

7.2.1

Cylinder

r = radius



b = height

$A = 2\pi rb$

Variable separate and integrate Equation 7.5:

$(Q/r)dr = (2\pi Kb)dh$ ➔ w/ Q, K, and b constant yields

$Q \ln r = 2\pi Kb(h)$ ➔ $h = h_w$ at $r = r_w$ & $h = h_o$ at $r = r_o$

$Q \ln(r_o/r_w) = 2\pi Kb(h_o-h_w)$ or $Q = 2\pi Kb(h_o-h_w)/\ln(r_o/r_w)$

Variable separate and integrate of Equation 7.13:

$(Q/r)dr = (2\pi K)h\, dh$ ➔ w/ Q, K, and b constant yields

$Q \ln r = 2\pi K(h^2/2)$➔ $h = h_w$ at $r = r_w$ & $h = h_o$ at $r = r_o$

$Q \ln(r_o/r_w) = \pi K(h_o^2-h_w^2)$ or $Q = \pi K(h_o^2-h_w^2)/\ln(r_o/r_w)$

7.2.2

Apply Equation (7.14); $Q = \pi K(h_o^2 - h_w^2)/\ln(r_o/r_w)$

for each radius of influence. Obtain K from Table 7.2.

$Q_{350} = \pi(1.00\text{x}10^{-3})(50^2-40^2)/\ln(350/0.10) = 0.346\ m^3/s$

$Q_{450} = \pi(1.00\text{x}10^{-3})(50^2-40^2)/\ln(450/0.10) = 0.336\ m^3/s$

About a 3% change in Q for a 12.5% range of r_o.

7.2.3

Substituting values into Eq'n 7.6 yields:

$Q = 2\pi Kb(h_o-h_w)/\ln(r_o/r_w)$ ➔ (Note: 449 gpm = 1.0 cfs)

$(1570/449) = 2\pi(4.01\text{x}10^{-4})(100)(350-250)/\ln(r_o/1.33)$

r_o = 1790 ft

7.2.4

Apply Equation (7.14) and obtain K from Table 7.2.

$Q = \pi K(h_o^2-h_w^2)/\ln(r_o/r_w)$

$0.200 = \pi(1.00\text{ x}10^{-3})(40^2 - h_w^2)/\ln(400/0.15)$

h_w = 33.1 m. Therefore, **s_w** = 40.0 – 33.1 **= 6.9 m**

7.2.5

Note that Q = 30 m^3/hr = 8.33 x 10^{-3} m^3/sec, and

T = K·b = (1.3 x 10^{-4} m/s)(10 m) = 1.3 x 10^{-3} m^3/s-m

Substituting values into Eq'n 7.11 yields:

$s = s_{ob} + (Q/2\pi T)[\ln(r_{ob}/r)]$ ➔ Here, the observation well is the pumped well where drawdown is known.

$s = 15 + [8.33\text{ x }10^{-3}/(2\pi \cdot 1.3\text{ x }10^{-3})][\ln(0.15/30)]$

s = 9.60 m (drawdown 30 m from the pumped well)

7.2.6

Using Eq'n 7.11; $s = s_{ob} + (Q/2\pi T)[\ln(r_{ob}/r)]$ yields:

$\mathbf{s_1} = 2.72 + [2150/(2\pi \cdot 880)][\ln(80/100)]$ **= 2.63 m**

Using Eq'n 7.11 again for the second well acting alone:

$\mathbf{s_2} = 2.72 + [2150/(2\pi \cdot 880)][\ln(80/140)]$ **= 2.50 m**

$s_{Total} = \mathbf{s_1} + \mathbf{s_2}$ = 2.63 + 2.50 **= 5.13 m**

7.2.7

Apply Equation (7.12) using both observation wells

$s = s_{ob} + (Q_1/2\pi T)[\ln(r_{1o}/r_1)] + (Q_2/2\pi T)[\ln(r_{2o}/r_2)]$

$$0.242 = 1.02 + (2950/2\pi T)[\ln(50/180)] + (852/2\pi T)[\ln(90/440)]$$

0.242 = 1.02 – (601/T) – (215/T); **T = 1050 m^2/day**

7.2.8

For existing (well #1) drawdown, apply Eq'n (7.15) to determine the drawdown at the well:

$h_w^2 = h_o^2 - [Q/(\pi \cdot K)]\cdot[\ln(r_o/r_w)]$

$h_w^2 = 130^2 - [3.5/(\pi \cdot 0.00055)]\cdot[\ln(500/0.5)] = 2910\ ft^2$

h_w = 53.9 ft; thus, $\mathbf{s_{w1}}$ = 130 – 53.9 = 76.1 ft

Based on the hint given in the Problem, Equation (7.17) will be applied using the radius of influence as the observation well with zero drawdown. Therefore,

$h^2 = h_{ob}^2 - \Sigma[Q_i/(\pi \cdot K)]\cdot[\ln(r_{io}/r_i)]$; $h_{ob}^2 = 130^2$; r_{io}= 500;

$$h^2 = 130^2 - [3.5/(\pi \cdot 0.00055)]\cdot[\ln(500/0.5)] - [3.5/(\pi \cdot 0.00055)]\cdot[\ln(500/250)]$$

h = 38.7 ft; Therefore, the drawdown for both wells is

$\mathbf{s_{w(1+2)}}$ = 130 – 38.7 = 91.3 ft; and the added drawdown

$\mathbf{\Delta s_w}$ = 91.3 – 76.1 **= 15.2 ft** of additional drawdown

7.2.9

The aquifer T (or K) is not given. Therefore use Eq'n (7.11), with the observation well data to obtain T.

$s = s_{ob} + (Q/2\pi T)[\ln(r_{ob}/r)]$; Use (s, r) for closer well.

$s - s_{ob} = 2\text{ m} = [2000/(2\pi T)]\cdot[\ln(160/20)]$;

$T = 331\text{ m}^3/\text{day-m}$; Now use Eq'n (7.11) again with the furthest observation well and the radius of influence.

$s = s_{ob} + (Q/2\pi T)[\ln(r_{ob}/r)]$; Use (s, r) for rad. of infl.

$0.0 = 1.0 + [2000/(2\pi \cdot 331)]\cdot[\ln(160/r_o)]$; $\mathbf{r_o = 453\text{ m}}$

Alternative solution: Use Equation (7.8) with both of the observation wells; use the inner one for (r_w, h_w):

$h - h_w = (h_o - h_w)[\ln(r/r_w)/\ln(r_o/r_w)]$;

$249\text{-}247 = (250\text{-}247)[\ln(160/20)/\ln(r_o/20)]$; $\mathbf{r_o = 453\text{ m}}$

7.2.10

Since the slope is known, apply Eq'n 7.5 directly:

$Q = 2\pi rbK(dh/dr) = 2\pi(90)(50)(7.55 \times 10^{-4})(0.0222)$

Q = 0.474 ft^3/sec (449 gpm/ 1cfs) **= 213 gpm**

7.2.11

Apply Eq'n (7.16) twice to find two different flow rates using the two design conditions as observation well data and the radius of influence as the other data point in the equation. The higher Q will govern the design.

$h^2 = h_{ob}^2 - (Q/\pi K)[\ln(r_{ob}/r)]$; for design condition #1

$8.2^2 = (8.2 - 1.5)^2 - (Q_1/0.0001\pi)[\ln(30/150)]$

$Q_1 = 4.36 \times 10^{-3}\text{ m}^3/\text{sec}$; and for design condition #2

$8.2^2 = (8.2 - 3.0)^2 - (Q_2/0.0001\pi)[\ln(3.0/150)]$

$Q_2 = 3.23 \times 10^{-3}\text{ m}^3/\text{sec}$; for design condition #2

Thus, the design $\mathbf{Q = 4.36 \times 10^{-3}\text{ m}^3/\text{sec}}$**.**

7.2.12

Use Eq'n (7.11), w/observation well data to obtain T.

$s = s_{ob} + (Q/2\pi T)[\ln(r_{ob}/r)]$; Use (s, r) for closer well.

$s - s_{ob} = 42.8\text{ ft} = [Q/(2\pi T)]\cdot[\ln(1000/500)]$;

To obtain Q from the data, find the seepage velocity:

$V_s = \Delta L/t = 500/49.5 = 10.1\text{ ft/hr} = 2.81 \times 10^{-3}\text{ ft/sec}$

likely a sand/gravel mixture based on Table 7.2

Thus, the apparent (Darcy) velocity can be obtained as:

$V = \alpha\cdot V_s = (0.26)(2.81 \times 10^{-3}\text{ ft/sec}) = 7.31 \times 10^{-4}\text{ ft/sec}$

Now we can use Darcy's velocity to obtain the Q:

$Q = AV$; area is a cylindrical surface of radius 750 ft

$Q = (2\pi rb)V = 2\pi(750\text{ ft})(20\text{ ft})(7.31 \times 10^{-4}\text{ ft/sec})$

Q = 68.9 cfs; now substituting into Eq'n (7.11) yields

$s - s_{ob} = 42.8\text{ ft} = [68.9/(2\pi T)]\cdot[\ln(1000/500)]$;

$\mathbf{T = 0.178\text{ ft}^2/\text{sec}}$ Note: An average Darcy velocity was used in the calculations, but the velocity change is not linear between observation wells. It is left to the student to determine a more accurate estimate of T.

7.3.1

Applying Equations (7.20) and (7.21) produces

$u = (r^2S)/(4Tt) = (100^2\cdot 0.00025)/(4\cdot 25\cdot t) = 0.025/t$; and

$s = [Q_w/(4\pi T)]W(u) = [300/(4\pi\cdot 25)]W(u) = 0.955\cdot W(u)$.

Thus, the drawdowns for the various times are:

t (hr)	u	W(u)	s (m)
10	0.0025	5.437	**5.19**
50	0.0005	7.024	**6.71**
100*	0.00025	7.738	**7.39**

*Note that the drawdown increase has slowed considerably after 100 hours of pumping.

7.3.2

Applying Eq'n (7.21) yields: $s = [Q_w/(4\pi T)]W(u)$

$3.66 = [50{,}000/(4\pi \cdot 12{,}000)]W(u)$; $W(u) = 11.0$

From Table 7.3, $u = 9.0 \times 10^{-6}$, and Eq'n (7.20) yields:

$u = (r^2S)/(4Tt)$: $9.0 \times 10^{-6} = (300^2 \cdot 0.0003)/(4 \cdot 12000 \cdot t)$

Solving for t, we obtain **t = 62.5 days.**

7.3.3

Use superposition for unsteady flow in an aquifer with multiple wells. For the first well, Eq'n (7.27) yields

$u_1 = (r_1^2S)/(4Tt) = (100^2 \cdot 0.00025)/(4 \cdot 25 \cdot 96) = 2.60 \times 10^{-4}$

From Table 7.3, $W(u_1) = 7.697$. Now applying

Eq'n (7.26): $s = [1/(4\pi T)]\{Q_1 \cdot W(u_1) + Q_2 \cdot W(u_2)\}$

$10.5 = [1/(4\pi \cdot 25)]\{300 \cdot 7.697 + 200 \cdot W(u_2)\}$

$W(u_2) = 4.948$. From Table 7.3, $u_2 = 4.0 \times 10^{-3}$

Now Eq'n (7.27) yields: $u_2 = (r_2^2S)/(4Tt)$

$4.0 \times 10^{-3} = (r_2^2 \cdot 0.00025)/(4 \cdot 25 \cdot 96)$; $\mathbf{r_2 = 392\ m}$

7.3.4

Use superposition for unsteady flow in an aquifer with multiple wells and start times: Eq'ns (7.28) and (7.26):

$u_1 = (r_1^2S)/(4Tt) = (300^2 \cdot 0.0005)/(4 \cdot 10000 \cdot 3) = 3.75 \times 10^{-4}$

From Table 7.3: $W(u_1) = 7.319$ and

$s_1 = [Q_1 \cdot W(u_1)]/(4\pi T) = [40{,}000 \cdot 7.319]/(4\pi \cdot 10000) = 2.33$ ft

$u_2 = (r_2^2S)/(4Tt) = (300^2 \cdot 0.0005)/(4 \cdot 10000 \cdot 1.5) = 7.50 \times 10^{-4}$

From Table 7.3: $W(u_2) = 6.621$ and

$s_2 = [Q_2 \cdot W(u_2)]/(4\pi T) = [40{,}000 \cdot 6.621]/(4\pi \cdot 10000) = 2.11$ ft

Total drawdown = s = 2.33 + 2.11= 4.44 ft

7.3.5

Use superposition for unsteady well flow: Eq'ns (7.24) and (7.25) with N = 2, $Q_o = 0$, $Q_1 = 800\ m^3/hr$, $Q_2 = 500\ m^3/hr$, $t_o = 0$, and $t_1 = 48$ hrs.

For k = 1: $u_1 = (r_1^2S)/[4T(t - t_o)] = (50^2 \cdot 0.00025)/(4 \cdot 40 \cdot 72)$

$u_1 = 5.43 \times 10^{-5}$; From Table 7.3, $W(u_1) = 9.248$.

For k = 2: $u_2 = (r_2^2S)/[4T(t - t_1)] = (50^2 \cdot 0.00025)/(4 \cdot 40 \cdot 24)$

$u_2 = 1.63 \times 10^{-4}$; From Table 7.3, $W(u_2) = 8.196$. Thus,

$s = [1/(4\pi T)]\{[Q_1 - Q_0]W(u_1) + [Q_2 - Q_1]W(u_2)\}$

$\mathbf{s} = [1/(4\pi \cdot 40)]\{[800](9.248)+[-300](8.196))\}$ **= 9.83 m**

7.3.6

Use Eq'ns (7.30) through (7.33) for unsteady radial flow in unconfined aquifers: $T = Kh_o = 5(500) = 2500\ ft^2/day$

$u_a = (r^2S_a)/[4Tt] = (50^2 \cdot 0.0005)/(4 \cdot 2500 \cdot 2) = 6.25 \times 10^{-5}$

$u_y = (r^2S_y)/[4Tt] = (50^2 \cdot 0.10)/(4 \cdot 2500 \cdot 2) = 1.25 \times 10^{-2}$;

$\eta = r^2/(h_o^2) = (50^2)/(500^2) = 0.01$; Therefore, $1/u_a = 16{,}000$, $1/u_y = 80$, and from Fig 7.8, $W(u_a, u_y, \eta) \approx 4$; Eqn (7.33) gives

$s = [Q_w/(4\pi T)]W(u_a, u_y, \eta) = [10{,}000/(4\pi \cdot 2500)](4)$

s = 1.27 ft

7.4.1

Using Equation (7.34) with observation well #1 represented by the pumped well yields

$T = (Q_w/2\pi)[\ln(r_1/r_2)/(s_2 - s_1)]$

$\mathbf{T} = (0.1/2\pi)[\ln(0.2/50)/(10-30)]$ $\mathbf{= 4.39 \times 10^{-3}\ m^2/sec}$

Now using Equation (7.6) noting that Kb = T

$Q = 2\pi T(h_o - h_w)/\ln(r_o/r_w)$

$0.1 = 2\pi(4.39 \times 10^{-3})(30)/\ln(r_o/0.2)$; $\mathbf{r_o = 785\ m}$

7.4.2

Using Equation (7.37) with observation wells at the pumped well and the radius of influence yields

$K = [Q_w/\pi(h_2^2 - h_1^2)]\ln(r_2/r_1)$; Note: 449 gpm = 1 cfs

$\mathbf{K} = [(7.5/449)/\pi(60^2 - 25^2)]\ln(300/1) = \mathbf{1.02 \times 10^{-5}\ ft/s}$

Use the same equation again with either known point:

$K = [Q_w/\pi(h_o^2 - h_1^2)]\ln(r_o/r_1)$;

1.02×10^{-5} ft/s = $[(7.5/449)/\pi(60^2 - h_1^2)]\ln(300/150)$

$h_1 = 56.9$ ft; Drawdown: $\mathbf{s_1} = 60.0 - 56.9 = \mathbf{3.1\ ft}$

7.4.3

A plot of *s* versus *r* is displayed below. From the best fit line, $\Delta^* s = 0.85 - 0.71 = 0.14$ m (r = 1 to 10). Then Equation (7.36) yields the transmissivity.

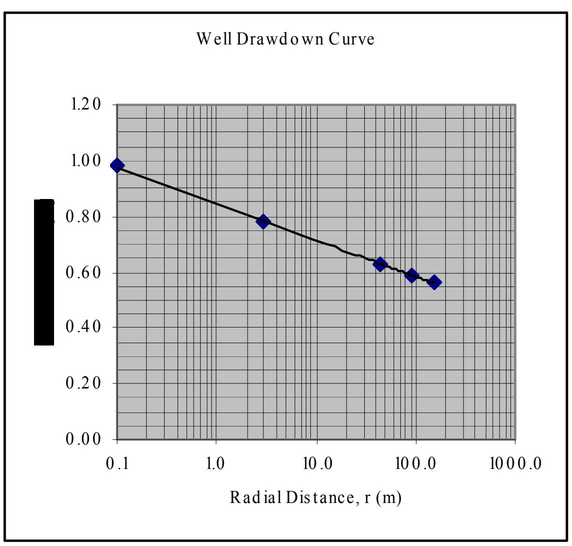
$\mathbf{T} = [2.30 \cdot Q_w/(2\pi \cdot \Delta^* s)] = [2.30 \cdot 14.9/(2\pi \cdot 0.14)] = \mathbf{39.0\ m^2/hr}$

Now use Eq'n (7.11) with any observation well:

$s = s_{ob} + (Q/2\pi T)[\ln(r_{ob}/r)]$; $s = 0.5$ m

$0.5 = 0.63 + [14.9/2\pi \cdot 39.0)][\ln(45/r)]$; $\mathbf{r = 381\ m}$

(can also be approximated from the plot below)



7.4.4

A plot of *s* versus h^2 is displayed below. From the best fit line, $\Delta^* h^2 = 1710 - 1630 = 80$ ft^2 (r = 10 ft to 1 ft). Then Eq'n (7.39) yields the coefficient of permeability

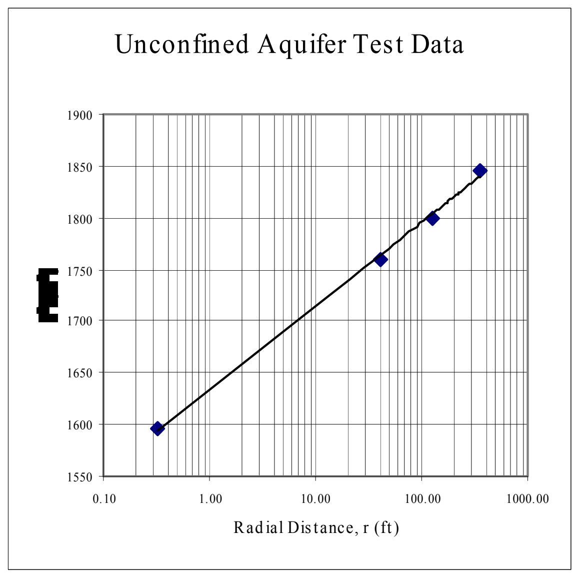
$\mathbf{K} = [2.30 \cdot Q_w/(\pi \cdot \Delta^* h^2)] = [2.30 \cdot 1300/(\pi \cdot 80)] = \mathbf{11.9\ ft/hr}$

Now apply Eq'n (7.16) with any observation well using $s = 2.5$ ft; $h = h_o - s = 46 - 2.5 = 43.5$ ft. Let $r_{ob} = 40$ ft,

$h_{ob} = 46 - 4.05 = 41.95$ ft: $h^2 = h_{ob}^2 - (Q/\pi K)[\ln(r_{ob}/r)]$;

$43.5^2 = 41.95^2 - (1300/\pi \cdot 11.9)[\ln(40/r)]$; $\mathbf{r = 1800\ ft}$

(can also be approximated from the plot below)



7.4.5

The *storage coefficient* (also referred to as *storage constant*, or *storativity*), *S*, is an aquifer parameter linking the changes in the volume of water in storage to the changes in the piezometric head (confined aquifers) or changes in the water table elevation (unconfined aquifers). Changes in the piezometric head or water table elevation only occur under non-equilibrium conditions. Therefore, a non-equilibrium aquifer test is needed to determine its magnitude.

7.4.6

To derive Eq'n (7.43), start with Eq'n (7.40):

$$s = \frac{2.30\ Q_w}{4\ \pi\ T}\left[\log\frac{2.25\ T\ t}{r^2\ S}\right]$$

On the s vs. t plot, the x-intercept is found (i.e., time when drawdown is zero). Thus, setting $t = t_o$ and $s = 0$ yields

$$0 = \frac{2.30\ Q_w}{4\ \pi\ T}\left[\log\frac{2.25\ T\ t_0}{r^2\ S}\right]; \text{ but } 2.30 \cdot Q_w/(4\pi T) \neq 0$$

therefore; $0 = \log[(2.25 \cdot T \cdot t_o)/(r^2 \cdot S)]$;

$1 = (2.25 \cdot T \cdot t_o)/(r^2 \cdot S)$; and $\mathbf{S = (2.25 \cdot T \cdot t_o)/(r^2)}$

To derive Eq'n (7.47), start with Eq'n (7.40):

$$s = \frac{2.30\ Q_w}{4\ \pi\ T}\left[\log\frac{2.25 \cdot T/(r^2/t)}{S}\right]; \text{ or}$$

On s vs. r^2/t plot, the x-intercept is found (i.e., r^2/t when drawdown is zero). Setting $r^2/t = (r^2/t)_o$ and $s = 0$ yields

$$0 = \frac{2.30\ Q_w}{4\ \pi\ T}\left[\log\frac{2.25 \cdot T/(r^2/t)_o}{S}\right]; \text{ but}$$

$2.30 \cdot Q_w/(4\pi T) \neq 0$ thus; $0 = \log[\{2.25 \cdot T/(r^2/t)_o\}/S]$;

$1 = [2.25 \cdot T/(r^2/t)_o]/S]$; and $\mathbf{S = (2.25 \cdot T)/(r^2/t)_o}$

7.4.7

A plot of *s* versus *t* is displayed below. From the best fit line, $\Delta^o s = 1.62 - 1.10 = 0.52$ m (t = 10 to 1). Then Equation (7.42) yields the transmissivity.

$\mathbf{T} = [2.30 \cdot Q_w/(4\pi \cdot \Delta^o s)] = [2.30 \cdot 6.00/(4\pi \cdot 0.52)] = \mathbf{2.11\ m^2/hr}$

Using $t_o = 0.0082$ hr from the plot, Eq'n (7.43) yields:

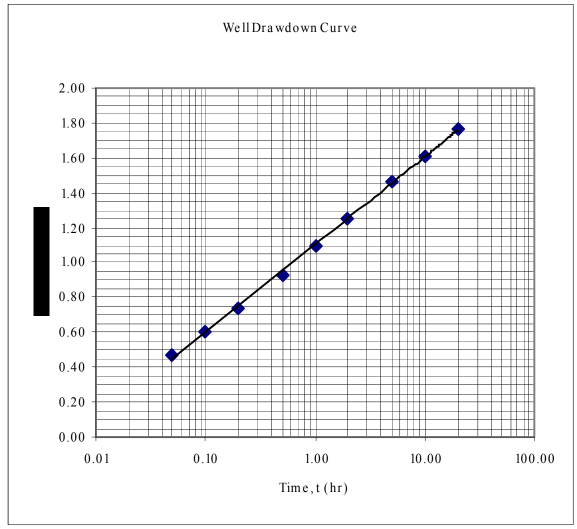
$\mathbf{S} = (2.25 \cdot T \cdot t_o)/(r^2) = (2.25 \cdot 2.11 \cdot 0.0082)/(22^2) = \mathbf{8.04 \times 10^{-5}}$

To determine the drawdown after 50 hours, use the nonequilibrium relationship; Equation (7.40):

$s = [2.30 \cdot Q_w/(4\pi T)]\log[2.25Tt/(r^2S)]$

$s = [2.30 \cdot 6.0/(4\pi \cdot 2.11)]\log[2.25 \cdot 2.11 \cdot 50/(22^2 \cdot 8.04 \times 10^{-5})]$

s = 1.97 m (can be approximated from plot below)



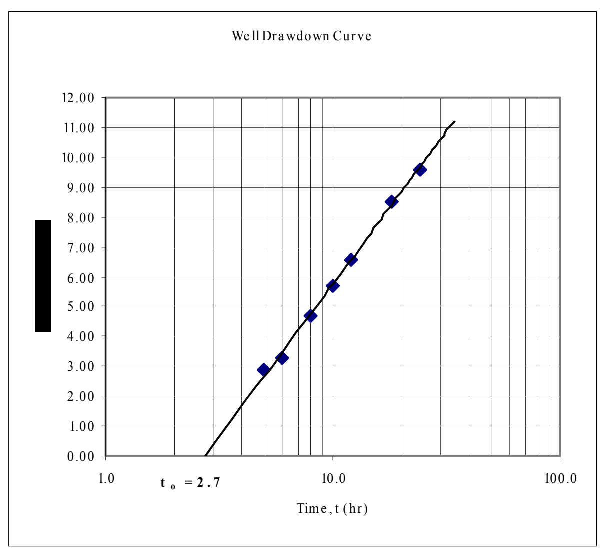
7.4.8

Confined aquifer procedures are applicable due to small drawdowns compared to the aquifer thickness. Plot and eliminate first four plotting values (t = 1 to 4) since they depart from a straight line. A plot of *s* versus *t* (below) yields: $\Delta^o s = 10.8 - 0.4 = 10.4$ m (t = 30 to 3). Use Eq'n (7.42) for T (then K) and Eq'n (7.43) for S.

$T = [2.30 \cdot Q_w/(4\pi \cdot \Delta^o s)] = [2.30 \cdot 1.25/(4\pi \cdot 10.4)] = 0.0220\ ft^2/s$

$\mathbf{K} = T/b = \mathbf{2.44 \times 10^{-4}\ ft/sec}$; w/$t_o$ = 2.7 hr = 9720 sec

$\mathbf{S} = (2.25 \cdot T \cdot t_o)/(r^2) = (2.25 \cdot 0.022 \cdot 9720)/(120^2) = \mathbf{0.0334}$



7.4.9

The r^2/t values are first calculated for both observation wells as listed in the last two columns of the table below. Then all the available data are plotted as shown in the figure below and a best fit line is drawn. From this line we obtain $\Delta^+ s$ = 3.05 – 1.85 = 1.20 m (r2/t = 100 to 1000) and $(r^2/t)_o$ = 34,000 m^2/hr.

t (hr)	s_1 (m)	s_2 (m)	r_1^2/t (m^2/hr)	r_2^2/t (m^2/hr)
0.05	0.77	0.53	8000	12500
0.1	1.13	0.9	4000	6250
0.2	1.5	1.26	2000	3125
0.5	1.98	1.75	800	1250
1	2.35	2.11	400	625
2	2.72	2.48	200	313
5	3.2	2.97	80	125
10	3.57	3.33	40	63
20	3.93	3.7	20	31

Solving Equations (7.46) and (7.47) yields

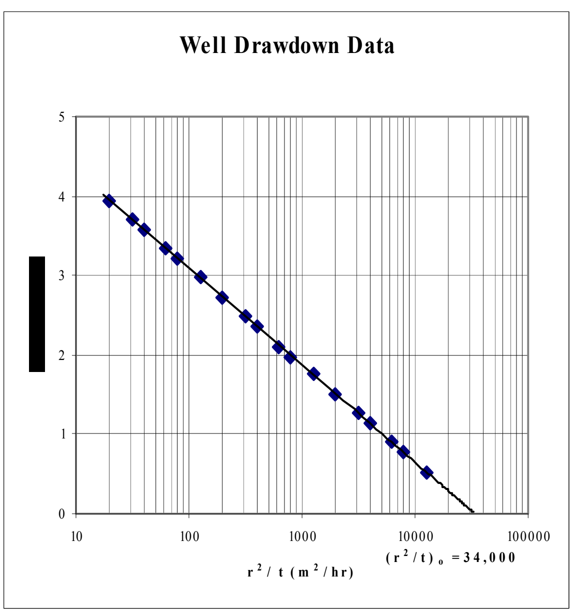
T = $[2.30 \cdot Q_w/(4\pi \cdot \Delta^+ s)] = [2.30 \cdot 10/(4\pi \cdot 1.20)]$ **= 1.53 m^2/hr**

S = $(2.25 \cdot T)/(r^2/t)_o = (2.25 \cdot 1.53)/(34{,}000)$ **= 1.01 x 10^{-4}**

$s = [2.30 \cdot Q_w/(4\pi T)]\log[2.25Tt/(r^2S)]$; Eq'n (7.40)

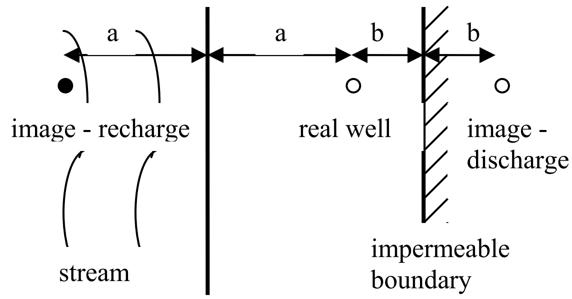
$s = [2.30 \cdot 10/(4\pi \cdot 1.53)]\log[2.25 \cdot 1.53 \cdot 240/(20^2 \cdot 1.01 \times 10^{-4})]$

s = 5.16 m (can be approximated from plot below)



7.5.1

Here is the equivalent hydraulic system:



7.5.2

The key to this problem is finding the well's radius of influence. Substituting values into Eq'n 7.6 yields:

$Q = 2\pi Kb(h_o - h_w)/\ln(r_o/r_w)$ ➔ $h_o - h_w = s_w - s_o$

$20{,}000 = 2\pi(20)(25)(30)/\ln(r_o/0.5)$; **$r_o$ = 55.7 ft**

Thus, the boundary won't impact well performance.

7.5.3

In this case, the equivalent hydraulic system is an image well placed across the boundary at the same distance (30 m) from the boundary. Using the principle of superposition, the drawdown 30 m away is:

$s_{30} = s_{real} + s_{image}$ = 9.60 m + 9.60 m = 19.2 m

The drawdown caused by the pumped (real) well at the well itself can be solved using Eq'n 7.11 and yields (noting that Q = 30 m^3/hr = 8.33 x 10^{-3} m^3/sec):

$s = s_{ob} + (Q/2\pi T)[\ln(r_{ob}/r)]$; use original obs. well

$s = 9.60 + [8.33 \times 10^{-3}/(2\pi \cdot 1.3 \times 10^{-3})][\ln(30/0.15)] = 15.0$ m

The image well drawdown at the pumped (real) well:

$s = 9.60 + [8.33 \times 10^{-3}/(2\pi \cdot 1.3 \times 10^{-3})][\ln(30/60)] = 8.9$ m

$s_w = s_{real} + s_{image}$ = 15.0 m + 8.9 m = 23.9 m

7.5.4

The equivalent hydraulic system is an image (recharge) well placed across the boundary at the same distance (60 m) from the boundary. Drawdowns are solved at all locations using Eq'n (7.11) based on an infinite aquifer assumption (noting Q = 30 m^3/hr = 8.33 x 10^{-3} m^3/sec):

$s = s_{ob} + (Q/2\pi T)[\ln(r_{ob}/r)]$;

s_w = 15.0 m (given; used as observation well for others)

s_{30} = 15.0+[8.33 x 10^{-3}/(2π ·1.3 x 10^{-3})][ln(0.15/30)]= 9.6 m

s_{60} = 15.0+[8.33 x 10^{-3}/(2π ·1.3 x 10^{-3})][ln(0.15/60)]= 8.9 m

s_{90} = 15.0+[8.33 x 10^{-3}/(2π ·1.3 x 10^{-3})][ln(0.15/90)]= 8.5 m

s_{120}= 15.0+[8.33 x 10^{-3}/(2π ·1.3 x 10^{-3})][ln(0.15/120)]= 8.2 m

Now use the principle of superposition for drawdowns noting that the recharge will produces buildup:

s (at well) = s_w - s_{120} = 15.0 - 8.2 = **6.8 m**

s (boundary) = s_{60} – s_{60} = 8.9 - 8.9 = **0.0 m**

s (at mid point) = s_{30} – s_{90} = 9.6 - 8.5 = **1.1 m**

7.5.5

The equivalent hydraulic system is an image (recharge) well 600 m on the other side of the boundary. The drawdown caused by the pumped (real) well at the irrigation well using Eq'n 7.11 yields:

$s_{real} = s_{ob\text{-}r} + (Q/2\pi T)[\ln(r_{ob}/r)]$; observ. well at boundary

s_{real}= $s_{ob\text{-}r}$ + [Q/(2π ·0.0455)][ln(600/300)]; Drawdown caused by the image well at the irrigation well:

s_{image} = $s_{ob\text{-}i}$ + [-Q/(2π ·0.0455)][ln(600/900)];

s_{Total} = s_{real} + s_{image} (Note: $s_{ob\text{-}r}$ + $s_{ob\text{-}I}$ = 0.0; s_{Total}= 5.0 ft)

s_{Total}= 5.0 = [Q/(2π ·0.0455)][ln(600/300) - ln(600/900)];

Q = 1.30 cfs

7.5.6

The equivalent hydraulic system is an image (recharge) well placed 500 m on the opposite side of the boundary (i.e. 1000 ft from the real well). Determine drawdowns using Eq'ns (7.20) and (7.21). For the real well:

u = (r^2S)/(4Tt) = (100^2·0.00025)/(4·25·50) = 5.0 x 10^{-4}

from Table 7.3: W(u) = 7.024; therefore,

s = [Q_w/(4πT)]W(u) = [300/(4π·25)](7.024) = 6.71 m

For the image well (900 ft from drawdown location):

u = (r^2S)/(4Tt) = (900^2·0.00025)/(4·25·50) = 4.05 x 10^{-2}

from Table 7.3: W(u) = 2.670; therefore,

s = [Q_w/(4πT)]W(u)= [-300/(4π·25)](2.670) = -2.55 m

The total drawdown is: **s = 6.71 – 2.55 = 4.16 m**

7.5.7

The equivalent hydraulic system: an image well placed 330 ft across the boundary. Determine the drawdowns w/Eq'ns (7.20) and (7.21). The real well:

u = (r^2S)/(4Tt) = (330^2·0.00023)/(4·250·50) = 5.01 x 10^{-4}

from Table 7.3: W(u) = 7.022; therefore,

s = [Q_w/(4πT)]W(u) = [10,600/(4π·250)](7.024) = 23.7 ft

For the image (recharge) well, the drawdown is the same since it is the same distance away:

The total drawdown is: **s = 2(23.7 ft) = 47.4 ft**

To determine how long it would take the drawdown to reach 70 ft at the boundary, split the drawdown in half and attribute this impact to the real well.

s = [Q_w/(4πT)]W(u); 29.5 = [10,600/(4π·250)]W(u);

W(u) = 8.743; from Table 7.3; u = 8.99 x 10^{-5}

u = (r^2S)/(4Tt); 8.99 x 10^{-5} = (330^2·0.00023)/(4·250·t)

t = 279 hr = 11.6 days

7.8.1

a) When we add 5 new streamlines, we no longer have square cells in the flow net. You must bisect all of the cells with new equipotential lines. Recomputing, **q** = K(m/n)H = (2.14)(10/26)(50) **= 41.2 m^3/day** per meter which is the same answer as the example.

b) If you have done a good job on your new flow net, the answers should be fairly close.

7.8.2

The head drop (loss) between equipotential lines is:

$\Delta h = H/n = (160\text{ ft})/16 = 10.0$ ft. from Eq'n (7.54)

Energy head at location #1: $\mathbf{H_1}$ = 160 – 2(10) **= 140 ft**
For velocity at #1, use Darcy's equation with the energy difference between the previous and next equipotential lines. Also, measure the distance between the two lines using the arc distance through point 1 which yields:
$V_1 = K(\Delta h/\Delta s)_1 = (7.02)[(150 - 130)/200] = 0.702$ ft/day

From this apparent velocity, find the seepage velocity.

$\mathbf{V_{S1}} = V_1/\alpha = (0.702)/0.40$ **= 1.76 ft/day**

Flow direction: always parallel to nearby streamlines.

Energy head at location #2: $\mathbf{H_2}$ = 160 – 3(10) **= 130 ft**
$V_2 = K(\Delta h/\Delta s)_2 = (7.02)[(140 - 120)/70] = 2.01$ ft/day

$\mathbf{V_{S2}} = V_2/\alpha = (2.01)/0.40$ **= 5.03 ft/day**

Energy head at location #4: $\mathbf{H_4}$ = 160–14.5(10) **= 15 ft**
$V_4 = K(\Delta h/\Delta s)_4 = (7.02)[(20 - 10)/120] = 0.585$ ft/day

$\mathbf{V_{S4}} = V_4/\alpha = (0.585)/0.40$ **= 1.46 ft/day**

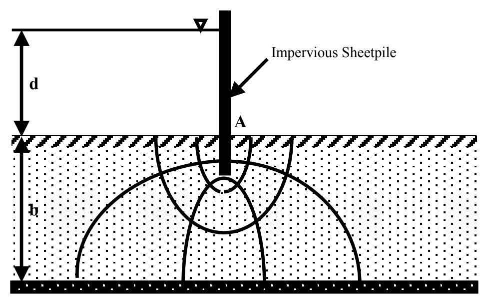
Note: The seepage velocity is greater in areas where the flow net cells are small (Δh occurs over a shorter distance). Also, pressure heads at locations 1, 2, and 4 can be found from ($P/\gamma = H - h$) where h is the position head measured on the scale drawing from a datum.

7.8.3

See the flow net below. Applying Equation 7.56:
q = K(m/n)H = (0.195)(3/5)(3.5) **= 0.410 m^3/day** per meter of sheetpile (**0.0171 m^3/hour-m)**.
$\Delta h = d/n = (3.5\text{ m})/5 = 0.70$ m. The velocity @exit "A"

$V_A = K(\Delta h/\Delta s)_A = (0.195)[(0.70)/(1.0)] = 0.137$ m/day

where Δs is the distance through the last cell next to the sheet pile. The seepage velocity is now found as
$\mathbf{V_{SA}} = V_A/\alpha = (0.137)/0.35 = 0.391$ m/day **(4.53 x 10^{-6} m/s)**



7.8.4

A flow net is sketched on the figure in the book giving 3 flow lines & 6 equipotential drops. Apply Eq'n 7.56:
q = K(m/n)H = (4.45 x 10^{-7})(3/6)(20) = 4.45 x 10^{-6} m^3/s-m
Q = (4.45 x 10^{-6} m^3/s-m)(100m)(86,400s/day) **= 38.4 m^3/day**
The head drop (loss) between equipotential lines is:
$\Delta h = H/n = (20\text{ m})/6 = 3.33$ m. At the middle of dam:
H = 20 – 3(3.33) **= 10.0 m.** The Darcy velocity is:
$V = K(\Delta h/\Delta s) = (4.45 \times 10^{-7})[(13.3-6.67)/15] = 1.98 \times 10^{-7}$ m/s
with Δs =15 m coming from the scale drawing. To obtain seepage velocity; $\mathbf{V_S} = V/\alpha$ **= 4.40 x 10^{-7} m/s**

7.8.5

A flow net is sketched on the figure in the book giving 3 flow lines & 7.5 equipotential lines. Apply Eq'n 7.56:
q = K(m/n)H = (4.45 x 10^{-7})(3/7.5)(20) **= 3.56 x 10^{-6} m^3/s-m**
Seepage Reduction: (4.45-3.56)/4.45 = 0.20 (**20%**)

7.9.1

From Table 7.2, silt has a mid-range permeability of 5.0 x 10^{-8} m/sec. The flow net sketched in Figure 7.29 provides the (m/n) ratio. Applying Eq'n 7.56:

q = K(m/n)H = (5.0 x 10^{-8})(4/19)(7.0) = 7.37 x 10^{-8} m^3/s-m

Q = (7.37 x 10^{-8} m^3/s-m)(80m)(86,400s/day) **= 0.509 m^3/day**

The head drop (loss) between equipotential lines is:
$\Delta h = H/n$ = (7.0 m)/19 = 0.368 m. At point "D":

H = 7.0 – 14.5(0.368) **= 1.66 m**. Using Darcy's eq'n:
V= K($\Delta h/\Delta s$)= (5.0 x 10^{-8})[(0.368)/1.0] = 1.84 x 10^{-8} m/s

where Δh is the equipotential drop from the 14th to the 15th equipotential line and the distance (Δs = 1.0 m) is measured from the scale drawing. Thus, $\mathbf{V_S} = V/\alpha$ **= 4.09 x 10^{-8} m/s** where the porosity is 0.45 from Table 7.1.

7.9.2

A flow net sketched in Figure 7.30 with three flow channels yields approximately 11 equipotential drops. Regardless of the number of flow channels, the (m/n) ratio is approximately (3/11). Applying Eq'n 7.56:

q = K(m/n)H; 0.005 = K(3/11)(4.24); K = 4.32 x 10^{-3} m/day

K = (4.32 x 10^{-3} m/day)(1 day/86,400 sec) **= 5.0 x 10^{-8} m/s**

From Table 7.2, this permeability indicates a silt soil.

7.9.3

A flow net sketched in Figure P7.9.3 with three flow channels yields approximately 13 equipotential drops. Regardless of the number of flow channels, the (m/n) ratio is approximately (3/13). Applying Eq'n 7.56:

q = K(m/n)H = (2.0 x 10^{-6})(3/13)(25) = 1.15 x 10^{-5} m^3/sec-m
where H = 25 m from the drawing; less than dam height.
q =(1.15 x 10^{-5} m^2/sec-m)(86,400 sec/day)**= 0.994 m^3/day-m**
Both answers are per meter width of dam.

7.9.4

A flow net sketched in Figure P7.9.3 with three flow channels yields approximately 15 equipotential drops. Regardless of the number of flow channels, the (m/n) ratio is approximately (3/15). Applying Eq'n 7.56:

q = K(m/n)H = (2.0 x 10^{-6})(3/15)(25) = 1.00 x 10^{-5} m/sec

q = (1.00 x 10^{-5} m/sec)(86,400 sec/day) **= 0.864 m^3/day**

Both answers are per meter width of dam.

7.9.5

Applying Eq'n 7.56 using the flow net provided:

q = K(m/n)H = (3.28 x 10^{-6})(4/31)(30) = 1.27 x 10^{-5} ft^3/s-ft

Q = (1.27 x 10^{-5} ft^3/s-ft)(90ft)(86,400s/day) **= 98.8 ft^3/day**

Point "D", as defined in Figure 7.29, needs to be located in Figure P7.9.5 before the seepage velocity can be computed at this location. However, first we will locate Point "C" as defined in Figure 7.29. Using the upstream depth as a scale, the distance x (from toe of dam to the point where the phreatic line pierces the downstream slope in Figure P7.9.5) is measured to be approximately 60 ft. Therefore, Point "D" is $[(180 - \theta)/400]\cdot x$ = $[(180 - 16)/400]\cdot 60$ft = 24.6 ft down the embankment from "C." This equation comes from Figure 7.29. The head drop (loss) between equipotential lines is: $\Delta h = H/n$ = (30 ft)/31 = 0.968 ft. At point "D": **H** = 30.0 – 23.5(0.968) **= 7.25 ft.** since there are 23.5 equipotential drops from the reservoir to Point D. The Darcy (apparent) velocity is:
V= K($\Delta h/\Delta s$)= (3.28 x 10^{-6})[(0.968)/4.0] = 7.94 x 10^{-7} ft/s

where Δh is the equipotential drop from the 23rd to the 24th equipotential line and the distance (Δs = 4.0 ft) is measured from the scale drawing. Thus, the seepage velocity is found to be: $\mathbf{V_S} = V/\alpha$ **= 1.99 x 10^{-6} ft/s**
The porosity is 0.40 given in the problem statement.

Chapter 8 – Problem Solutions

8.3.1

Find all vertical forces on the dam per unit width:

$W = \gamma(\text{Vol}) = (2.5)(9.79)[(33)(5) + ½(33)(22)]$

$W = 12{,}900$ kN/m. Full uplift pressure at the heel:

$P_{heel} = \gamma H = (9.79)(30) = 294 \text{ kN/m}^2$

With no tailwater, $P_{toe} = 0$; thus the uplift force is:

$F_u = ½(294)(5 + 22) = 3{,}970$ kN/m

The hydrostatic force on the dam face, Eq'n (2.12) is:

$F_{HS} = \gamma \bar{h} A = (9.79)(15)(30) = 4410$ kN/m

The force ratio against sliding [Eq'n (8.1)] is < 1.3:

FR_{slide} $= [(0.6)(12{,}900\text{-}3970)]/(4410)$ **= 1.21 (not safe)**

8.3.2

Weight in sections per unit width (rectangle & triangle)

$W_r = \gamma(\text{Vol}) = (2.5)(9.79)[(33)(5)] = 4{,}040$ kN/m

$W_t = (2.5)(9.79)[½(33)(22)] = 8{,}880$ kN/m

The full uplift pressure at the heel is:

$P_{heel} = \gamma H = (9.79)(33) = 323 \text{ kN/m}^2$

With no tailwater, $P_{toe} = 0$; thus the uplift force is:

$F_u = ½(323)(5 + 22) = 4{,}360$ kN/m

The hydrostatic force on the dam face, Eq'n (2.12) is:

$F_{HS} = \gamma \bar{h} A = (9.79)(16.5)(33) = 5330$ kN/m

Thus, the force ratio against overturning, Eq'n (8.2), is

FR_{over} $= [(4040\cdot 24.5)+(8880\cdot 14.7)]/[(4360\cdot 18)+(5330\cdot 11)]$

FR_{over} = 1.67 (< 2.0, not safe)

8.3.3

Find all forces on the dam per unit width:

(The weight is determined in simple geometric sections, which includes one rectangle and two triangles. Note that the base of the dam is 26.5 ft long and is broken into three parts: 7.5 ft, 4 ft, and 15 ft.)

$W_r = \gamma(\text{Vol}) = (2.65)(62.3)[(30)(4)] = 19{,}800$ lbs/ft

$W_{t1} = (2.65)(62.3)[½(30)(7.5)] = 18{,}600$ lbs/ft

$W_{t2} = (2.65)(62.3)[½(30)(15)] = 37{,}100$ lbs/ft

$W_{total} = 19{,}800 + 18{,}600 + 37{,}100 = 75{,}500$ lbs/ft

One third uplift pressure at the heel:

$P_{heel} = (1/3)\gamma H = (1/3)(62.3)(27) = 561 \text{ lbs/ft}^2$

With no tailwater, $P_{toe} = 0$; thus the uplift force is:

$F_u = ½(561)(7.5 + 4 + 15) = 7{,}430$ lbs/ft

The hydrostatic force components on the dam face.

$(F_{HS})_V = (62.3)[½(27)(6.75)] = 5{,}680$ lbs/ft

$(F_{HS})_H = \gamma \bar{h} A = (62.3)(27/2)(27) = 22{,}700$ lbs/ft

Therefore, the resultant of all vertical forces is:

$\Sigma F_V = 75{,}500 + 5{,}680 – 7{,}430 = 73{,}800$ lbs/ft

The force ratio against sliding, Eq'n (8.1), is

FR_{slide} $= [(0.65)(73{,}800)]/(22{,}700)$ **= 2.11 (safe)**

The force ratio against overturning, Eq'n (8.2), is

FR_{over} $= [(19{,}800\cdot 17)+(18{,}600\cdot 21.5)+(37{,}100\cdot 10)+$

$(5{,}680\cdot(26.5 – 2.25))]/[(22{,}700\cdot 9)+(7{,}430\cdot 17.7)]$

FR_{over} = 3.71 (safe)

Both criteria are significantly exceeded.

8.3.4

Find all forces on the dam per unit width:

(The weight is determined in simple geometric sections, which includes one rectangle and two triangles. Note that the base of the dam is 98 m long and is broken into three parts: 30 m, 8 m, and 60 m)

$W_r = \gamma(\text{Vol}) = (2.63)(9.79)[(30)(8)] = 6{,}180$ kN/m

$W_{t1} = (2.63)(9.79)[½(30)(30)] = 11{,}600$ kN/m

$W_{t2} = (2.63)(9.79)[½(30)(60)] = 23{,}200$ kN/m

$W_{total} = 6{,}180 + 11{,}600 + 23{,}200 = 41{,}000$ kN/m

Determine the 60% uplift pressure at the heel:

$P_{heel} = (0.60)\gamma H = (0.6)(9.79)(27.5) = 161$ kN/m^2

Note: $P_{toe\text{-}drain} = 0$, thus the uplift force is:

$F_u = ½(161)(30 + 8 + 30) = 5{,}470$ kN/m

The hydrostatic force components on the dam face.

$(F_{HS})_V = (9.79)[½(27.5)(27.5)] = 3{,}700$ kN/m

$(F_{HS})_H = \gamma \bar{h} A = (9.79)(27.5/2)(27.5) = 3{,}700$ kN/m

Therefore, the resultant of all vertical forces is:

$\Sigma F_V = 41{,}000 + 3{,}700 – 5{,}470 = 39{,}200$ kN/m

The force ratio against sliding, Eq'n (8.1), is

$\mathbf{FR_{slide}}= [(0.53)(39{,}200)]/(3{,}700)$ **= 5.61**

The force ratio against overturning, Eq'n (8.2), is

$\mathbf{FR_{over}} = [(6{,}180\cdot 64)+(11{,}600\cdot 78)+(23{,}200\cdot 40)+$

$(3{,}700\cdot(98 – 9.17))]/[(5{,}470\cdot 75.3)+(3{,}700\cdot 9.17)]$

$\mathbf{FR_{over} = 5.73}$

Note: the moment arm for the uplift force is:

98 – (68/3) = 75.3 m

Both force ratios are high – dam is safe.

8.3.5

Find all vertical forces on the dam per unit width:

Weight in sections per unit width (rectangle & triangle)

$W_r = \gamma(\text{Vol}) = (2.5)(9.79)[(33)(5)] = 4{,}040$ kN/m

$W_t = (2.5)(9.79)[½(33)(22)] = 8{,}880$ kN/m

Full uplift pressure at the heel:

$P_{heel} = \gamma H = (9.79)(30) = 294$ kN/m^2

With no tailwater, $P_{toe} = 0$; thus the uplift force is:

$F_u = ½(294)(5 + 22) = 3{,}970$ kN/m

Therefore, the resultant of all vertical forces is:

$R_V = 4{,}040 + 8{,}880 - 3970 = 8{,}950$ kN/m

Find distance of R_υ from the center line ("e", Fig 8.4)

using the principle of moments (clockwise positive):

Resultant moment = Component moment; $R_V\cdot e = \Sigma M_{CL}$

$8950\cdot e = [3970\cdot(13.5\text{-}9.0)\text{-}8880\cdot(14.7\text{-}13.5)\text{-}4{,}040\cdot(13.5\text{-}2.5)]$

e = - 4.16 m; Note: Half of dam bottom is 13.5 m.

Since the counter-clockwise moments produced by the component forces exceeds the clockwise moments, the location of R_V is on the upstream side of the centerline (i.e., to produce a counter-clockwise moment).

Therefore, P_H will be greater than P_T.

From Eq'n (8.3) revised: $P_T = (R_\upsilon/B)(1 - 6e/B)$

$\mathbf{P_T} = (8950/27)(1 - 6(4.16)/27)$ **= 25.0 kN/m²**

From Eq'n (8.4) revised: $P_H = (R_V/B)(1 + 6e/B)$

$\mathbf{P_H} = (8950/27)(1 + 6(4.16)/27)$ **= 638 kN/m²**

The pressure distribution in Figure 8.4 is reversed with P_H exceeding P_T. However, P_T and P_H remains positive, which is desirable.

8.3.6

Find all vertical forces on the dam per unit width:

Weight in sections per unit width (rectangle & 2 triangles)

$W_r = \gamma(Vol) = (2.65)(62.3)[(30)(4)] = 19{,}800$ lbs/ft

$W_{t1} = (2.65)(62.3)[½(30)(7.5)] = 18{,}600$ lbs/ft

$W_{t2} = (2.65)(62.3)[½(30)(15)] = 37{,}100$ lbs/ft

$W_{total} = 19{,}800 + 18{,}600 + 37{,}100 = 75{,}500$ lbs

One third uplift pressure at the heel:

$P_{heel} = (1/3)\gamma H = (1/3)(62.3)(27) = 561$ lbs/ft^2

With no tailwater, $P_{toe} = 0$; thus the uplift force is:

$F_u = ½(561)(7.5 + 4 + 15) = 7{,}430$ lbs/ft

The water weight on the upstream side of dam.

$(F_{HS})_V = (62.3)[½(27)(6.75)] = 5{,}680$ lbs/ft

Therefore, the resultant of all vertical forces is:

$R_V = 75{,}500 + 5{,}680 – 7{,}430 = 73{,}800$ lbs/ft

Find distance of R_V from the center line ("e", Fig 8.4)

using the principle of moments (clockwise positive):

Resultant moment = Component moment; $R_V \cdot e = \Sigma M_{CL}$

$$73{,}800 \cdot e = [-19{,}800(13.25-9.5)-18{,}600(13.25-(2/3)7.5) +37{,}100(13.25-(2/3)15)+7{,}430(13.25-(1/3)26.5) -5{,}680(13.25-(1/3)6.75)]; \quad e = -1.85 \text{ ft.}$$

Since the counter-clockwise moments produced by the component forces exceeds the clockwise moments, the location of R_V is on the upstream side of the centerline (i.e., to produce a counter-clockwise moment).

Therefore, P_H will be greater than P_T.

From Eq'n (8.3) revised: $P_T = (R_\upsilon/B)(1 - 6e/B)$

$\mathbf{P_T}$ = (73,800/26.5)(1 - 6(1.85)/26.5) **= 1620 lbs/ft^2**

From Eq'n (8.4) revised: $P_H = (R_V/B)(1 + 6e/B)$

$\mathbf{P_H}$ = (73,800/26.5)(1 + 6(1.85)/26.5) **= 3950 lbs/ft^2**

8.3.7

To avoid tension in the base of a concrete dam P_H must be kept positive. If R_V is in the middle third of the base, $e \le B/6$. Letting e = B/6 in Equation (8.4):

$P_H = (R_V/B)(1 - 6e/B) = (R_V/B)(1 – 6(B/6)/B) = 0$

Thus, any value of e < B/6 keeps P_H positive.

8.3.8

Eq'n (8.6), $R = r\cdot\gamma\cdot h$, $\gamma = 62.3$ lb/ft^3, r = arch radius.

Based on Figure 8.5(a), r = (width/2)/(sin (120°/2))

a) @ 25 ft, width = 15 ft, r = 8.66 ft, **R = 37,200 lbs/ft***

b) @ 50 ft, width = 30 ft, r = 17.3 ft, **R = 47,400 lbs/ft**

c) @ 75 ft, width = 45 ft, r = 26.0 ft, **R = 30,800 lbs/ft**

*Subtract freeboard to get water depth (h);
i.e., @ 25 ft dam height, h = 100 – 25 – 6 = 69 ft

8.3.9

Stress: $\sigma = F/A$, where F = R (force on the abutment) and A is the abutment thickness per unit height.
From Eq'n (8.6), $R = r\cdot\gamma\cdot h$ where r = arch radius.
Based on Figure 8.5 and the central arch (rib) angle;

r = (width/2)/(sin (θ/2)) = 75m/(sin 75°) = 77.6 m

a) Crest: h = 0, thus **σ = 0**

b) Midheight (39 m): h = 39 – 3(freeboard) = 36 m

R = (77.6)(9.79)(36) = 2.73 x 10^4 kN/m

σ = F/A = (2.73 x 10^4)/[(11.8+4)/2] **= 3.46 x 10^3 kN/m^2**

c) Dam base (78 m): h = 78 – 3(freeboard) = 75 m

R = (77.6)(9.79)(75) = 5.70 x 10^4 kN/m

σ = F/A = (5.70 x 10^4)/[11.8] **= 4.83 x 10^3 kN/m^2**

8.5.1

Applying Equations (8.7) and (8.8a) yields:

$H = (3/2)y_c + x$;

$1.52 = (3/2)y_c + 1.05$ m; $\mathbf{y_c = 0.313}$ **m**

$q = [g(y_c)^3]^{1/2}$;

$q = [9.81(0.313)^3]^{1/2} = 0.548$ m^3/sec per m width

Q = bq = (0.548)(4) **= 2.19 $\mathbf{m^3}$/sec**

8.5.2

Applying Equations (8.8b) yields:

$q = 3.09\ H_s^{3/2}$; where q = Q/b

q = 30.9/4.90 = 6.31 m^3/sec per m width. Thus,

$6.31 = 3.09\ H_s^{3/2}$; $H_s = 1.61$ m. Now the elevation of the weir crest is: $\mathbf{WS_{elev}}$ = 96.1 - 1.61 **= 94.5 m**

8.5.3

Applying Equations (8.7) and (8.8a) yields:

$H = (3/2)y_c + x$;

$1.89 = (3/2)y_c + 1.10$ m; $y_c = 0.527$ m

$q = [g(y_c)^3]^{1/2}$;

$q = [9.81(0.527)^3]^{1/2} = 1.20$ m^3/sec per m width

Q = bq = (1.20)(3.05) **= 3.66 $\mathbf{m^3}$/sec**

Alternatively, we could use Equation (8.8c):

$q = 1.70\ H_s^{3/2} = 1.70(1.89\text{-}1.1)^{3/2} = 1.19$ m^3/sec-m

Q = bq = (1.19)(3.05) **= 3.63 $\mathbf{m^3}$/sec** (roughly the same)

Now, to determine the velocity of flow over the weir:

V = Q/A = 3.66/[(0.527)(3.05)] **= 2.28 m/sec**

8.5.4

a) Start with Eq'n (6.11) using critical depth subscripts,

$V_c/[g{\cdot}D_c]^{1/2} = 1$; or $D_c = V_c^2/g$.

However, for rectangular channels, $D_c = y_c$ and

$V_c = Q/A = Q/(b{\cdot}y_c) = q/y_c$. Therefore,

$y_c = q^2/gy_c^2$ or $y_c = [q^2/g]^{1/3}$ which is Eq'n (6.14)

b) Starting with Eq'n (8.8a), we have

$q = [g(y_c)^3]^{1/2} = [g\{2/3(H_s)\}^3]^{1/2}$;

Substituting g = 9.81 in SI units yields

$q = [9.81\{2/3(H_s)\}^3]^{1/2} = 1.70\ H_s^{3/2}$

c) If losses are not ignored, Equation 8.7 becomes

$H = (3/2)y_c + x + h_L$; therefore

$H - x = H_s = (3/2)y_c + h_L$; $y_c = (2/3)(H_s - h_L)$

Substituting into Equation (8.8a) yields

$q = [g(y_c)^3]^{1/2} = [g\{2/3(H_s - h_L)\}^3]^{1/2}$;

Clearly this decreases the discharge requiring a reduced discharge coefficient.

8.5.5

Applying Equations (8.7) and (8.8a) yields:

$H = (3/2)y_c + x$; $2.00 = (3/2)y_c + 0.78$ m; $y_c = 0.813$ m

$q = [g(y_c)^3]^{1/2} = [9.81(0.813)^3]^{1/2} = 2.30$ m^3/sec-m

Q = bq = (2.30)(4) **= 9.20 $\mathbf{m^3}$/sec**

Including the upstream velocity head in Eq'n (8.8c):

$q = 1.70\ H_s^{3/2}$; where $H_s = H - x + (q/H)^2/2g$

$q = 1.70[(2.0 - 0.78 + (q/2)^2/2{\cdot}9.81]^{3/2}$; solving implicit equation, q = 2.52 m^3/sec-m; **Q** = bq **= 10.1 $\mathbf{m^3}$/sec**

8.5.6

For a frictionless weir, Equation (8.8c) is written as:

$q = 1.70\ H_s^{3/2}$, where H_s is the vertical distance from the weir crest to the upstream water level (ignoring the approach velocity). Using the true energy level given:

$q = C_d\ H_s^{3/2}$; $2.00 = C_d\ (2.70 - 1.40)^{3/2}$; $\mathbf{C_d = 1.35}$

Applying an energy balance at the weir and upstream:

$E_{up} = (3/2)y_c + x + h_L$; and from Equation (6.14):

$y_c = [(2.00)^2/9.81]^{1/3} = 0.742$ m, therefore

$E_{up} = 2.70\text{ m} = (3/2)(0.742) + 1.40 + h_L$; $\mathbf{h_L = 0.187\ m}$

8.5.7

Apply the Manning equation to get the maximum Q:

$Q = (1.49/n)(A)(R_h)^{2/3}(S)^{1/2} = (1.49/n)(A)^{5/3}(P)^{-2/3}(S)^{1/2}$

$Q = (1.49/0.013)(6\cdot 15)^{5/3}(15+2\cdot 6)^{-2/3}(0.002)^{1/2} = 1030$ cfs

From Equation (6.14):

$y_c = [Q^2/gb^2]^{1/3} = [(1030)^2/\{32.2(15)^2]^{1/3} = 5.27$ ft

Finally, applying Equations (8.7) yields:

$H = (3/2)y_c + x$; $10 = (3/2)(5.27) + x$; $\mathbf{x = 2.10\ ft}$

8.6.1

From Equation (8.9) w/negligible approach velocity:

a) $\mathbf{Q} = CLH_s^{3/2} = (3.42)(31.2)(3.25)^{3/2} = \mathbf{625\ cfs}$

b) $\mathbf{Q} = CLH_s^{3/2} = (3.47)(31.2)(4.10)^{3/2} = \mathbf{899\ cfs}$

$V_a = Q/A = 899/[(31.2)(34.1)] = 0.845$ ft/sec

From Equation (8.10): $H_a = H_s + V_a^2/2g$

$H_a = 4.10 + (0.845)^2/2\cdot 32.2 = 4.11$ ft/sec

c) $\mathbf{Q} = CLH_a^{3/2} = (3.47)(31.2)(4.11)^{3/2} = \mathbf{902\ cfs}$

8.6.2

From Equation (8.9) including the approach velocity:

$H_a = [Q/(CL)]^{2/3} = [400/(2.22\cdot 80)]^{2/3} = 1.72$ m

From Equation (8.10): $H_a = H_s + V_a^2/2g$, where

$V_a = Q/A = 400/[(80)(6+H_s)] = 5/(6+H_s)$; thus

$1.72 = H_s + [5/(6+H_s)]^2/2\cdot 9.81$; $\mathbf{H_s = 1.70\ m}$

% error = [(1.72 – 1.70)/1.70] **= 1.18%**

8.6.3

Ignoring the approach velocity underestimates the spillway discharge. For a 2% underestimation error, we obtain the ratio: $\dfrac{Q_{ignore}}{Q_{include}} = \dfrac{CLH_s^{3/2}}{CLH_a^{3/2}} = 0.98$;

Reducing yields: $\dfrac{H_s^{3/2}}{(H_s + V_a^2/2g)^{3/2}} = 0.98$

$\dfrac{H_s}{(H_s + V_a^2/2g)} = 0.987$; $H_s = 0.987(H_s + V_a^2/2g)$

$H_s = 76.9[0.987\cdot(V_a^2/2\cdot 32.2)]$; $\boldsymbol{V_a = 0.921\cdot H_s^{1/2}}$

8.6.4

From Equation (8.9) w/negligible approach velocity:

a) $\mathbf{Q} = CLH_s^{3/2} = (1.96)(21)(3.1)^{3/2} = \mathbf{225\ m^3/sec}$

From Equation (8.10): $H_a = H_s + V_a^2/2g$, where

$V_a = Q/A = Q/[(21)(15+3.1)] = Q/380$; thus

b) $\mathbf{Q} = CLH_s^{3/2} = (1.96)(21)[3.1+ (Q/380)^2/2g]^{3/2}$

solving the implicit equation yields: $\mathbf{Q = 227\ m^3/sec}$

Note: the implicit equation may be solved by successive substitution, numerical techniques, computer algebra software, MathCAD, or even some calculators.

8.6.5

Eq'n (8.9): $\mathbf{L = Q}/CH_s^{3/2} = 214/(2.22)(1.86)^{3/2}$ **= 38.0 m**

Fig 8.12: $a/H_s = 0$; $a = 0.0$; $b/H_s = 0.199$; $b = 0.370$ m

$r_1 = 0.837$ m; $r_2 = 0.0$ m; $K = 0.534$; $P = 1.776$

Also, $(y/H_s) = -K(x/H_s)^P = -0.534(x/H_s)^{1.776}$

For tangency point: $\dfrac{d(y/H_s)}{d(x/H_s)} = -PK(x/H_s)^{P-1}$;

$-1 = -1.776(0.534)(x/H_s)^{0.776}$; Hence

$x/H_s = 1.07$, $\quad \mathbf{x_{P.T.} = 1.99}$ **m**

$y/H_s = -0.602$, $\quad \mathbf{y_{P.T.} = -1.12}$ **m**: The profile is:

x/H_s	x	y/H_s	y
0.25	0.465	-0.0455	-0.0846
0.50	0.930	-0.156	-0.290
0.75	1.40	-0.320	-0.595
1.00	1.86	-0.534	-0.993

8.6.6

Eq'n (8.9): $\mathbf{Q} = CLH_s^{3/2} = (4.02)(104)(7.2)^{3/2}$ **= 8100 cfs**

Fig 8.12: $a/H_s = 0.175$; $a = 1.26$ ft; $b/H_s = 0.282$; $b = 2.03$ ft

$r_1 = 3.60$ ft; $r_2 = 1.44$ ft; $K = 0.5$; $P = 1.85$

Also, $(y/H_s) = -K(x/H_s)^P = -0.5(x/H_s)^{1.85}$

For tangency point: $\dfrac{d(y/H_s)}{d(x/H_s)} = -PK(x/H_s)^{P-1}$;

$-1.5 = -1.85(0.5)(x/H_s)^{0.85}$; Hence

$x/H_s = 1.77$, $\quad \mathbf{x_{P.T.} = 12.7}$ **ft**

$y/H_s = -1.44$, $\quad \mathbf{y_{P.T.} = -10.4}$ **ft** The profile is:

x/H_s	x	y/H_s	y
0.50	3.6	-0.139	-1.00
1.00	7.2	-0.500	-3.60
1.25	9.0	-0.756	-5.44
1.50	10.8	-1.06	-7.63
1.75	12.6	-1.41	-10.2

8.7.1

The solution table with $S_o = 0.05$ is as follows:

Δx (ft)	Δy (ft)	y (ft)	A (ft^2)	Q (ft^3/s)	V (ft/s)	Q_1+Q_2 (ft^3/s)
-	-	5.01	50.1	637	12.7	-
5	-2.44	**7.45**	74.5	478	6.42	1115

V_1+V_2 (ft/s)	ΔQ (ft^3/s)	ΔV (ft/s)	R_h (ft)	S_f (ft)	Δy (ft)
-	-	-	-	-	-
19.1	159	6.31	2.99	0.0007	-2.44

The new depth is **7.45 ft.**, somewhat less than 7.73 ft.

8.7.2

a) Prior to Equation (8.14), we see that an assumption was made concerning the size of the angle θ. It is stated that sin θ = S_o for a reasonably small angle. That same assumption can be applied to cos θ. That is, for reasonably small angles, cos θ = 1.00. Thus, the cos θ can be replaced by 1.00.

b) For uniform flow, Equation (6.1a) is written as:
$F_1 + W \sin\theta - F_2 - F_f = 0$
For uniform flow, $F_1 = F_2$; thus, $F_f = W \sin\theta$
The weight component can be expressed as:
$W = \gamma \cdot A \cdot \Delta x$, where Δx is the channel length.
Also, for small angles, sin θ = S_o. Thus,
$F_f = W \sin\theta = \gamma \cdot A \cdot \Delta x \cdot S_o$
Lastly, we note that for non-uniform flow, as we have in side channel spillways, $S_o \neq S_f$. Thus, to determine the friction loss in non-uniform flow:
$F_f = \gamma \cdot A \cdot \Delta x \cdot S_f$; where S_f is the EGL slope.

c) $(S_o - \sin\theta) / (\sin\theta) \leq 0.01$; For a 10% slope:
$S_o = 0.10$; $\theta = 5.71°$; $\sin\theta = 0.0995$; and
$(S_o - \sin\theta)/(\sin\theta) = (0.10-0.0995)/0.0995 = 0.005 \leq 0.01$
Any slope larger than 14.1% (8.03°); exceeds 1% error.

8.7.3

From Equation (8.9) w/negligible approach velocity: $Q = CLH_s^{3/2}$: $36.0 = C(30)(0.736)^{3/2}$; $C = 1.90$. Thus, the flow 10 m upstream: $Q = (1.90)(20)(0.736)^{3/2} = 24.0$ m^3/sec; Also, $y_c = [(Q^2/(gb^2)]^{1/3} = [(36^2/(9.81 \cdot 3.0^2)]^{1/3} = 2.45$ m.

The solution table (using Excel) is given below. The depth 10 m upstream from the end of the channel is **3.87 m**.

Δx (m)	Δy (m)	y (m)	A (m^2)	Q (m^3/s)	V (m/s)	Q_1+Q_2 (m^3/s)	V_1+V_2 (m/s)	ΔQ (m^3/s)	ΔV (m/s)	R_h (m)	S_f (m)	Δy (m)
-	-	2.45	7.34	36.0	4.90	-	-	-	-	-	-	-
10	-1.42	3.87	11.6	24.0	2.07	60.0	6.97	12.0	2.84	1.08	0.0015	-1.42

8.7.4

$Q = CLH_s^{3/2}$: $637 = (3.70)(25)H_s^{3/2}$; $H_s = 3.62$ ft. Thus, the flow 5 ft upstream ($Q_5 = 510$ cfs) and 10 ft upstream ($Q_{10} = 382$ cfs) from proportions or Equation (8.11). Also, $y_c = [(Q^2/(gb^2)]^{1/3} = [(637^2/(32.2 \cdot 8.0^2)]^{1/3} = 5.82$ m.

The solution table (using Excel) is given below. The computed depths are **8.66 ft** and **9.45 ft.**

Δx (ft)	Δy (ft)	y (ft)	A (ft^2)	Q (ft^3/s)	V (ft/s)	Q_1+Q_2 (ft^3/s)	V_1+V_2 (ft/s)	ΔQ (ft^3/s)	ΔV (ft/s)	R_h (ft)	S_f (ft)	Δy (ft)
-	-	5.82	46.5	637	13.7	-	-	-	-	-	-	-
5	-2.84	8.66	69.3	510	7.36	1147	21.0	127	6.33	2.74	0.0011	-2.84
5	-0.79	9.45	75.6	382	5.06	892	12.4	127	2.30	2.81	0.0005	-0.79

8.7.5 The solution table (using Excel) is given below. The computed depths are **11.5 m**, **12.2 m**, and **12.4 m.**

Δx (m)	Δy (m)	y (m)	A (m^2)	Q (m^3/s)	V (m/s)	Q_1+Q_2 (m^3/s)	V_1+V_2 (m/s)	ΔQ (m^3/s)	ΔV (m/s)	R_h (m)	S_f (m)	Δy (m)
-	-	9.80	45.1	243	5.38	-	-	-	-	-	-	-
30	-1.72	11.5	53.0	162	3.05	404	8.43	80.9	2.33	1.92	0.0009	-1.72
30	-0.69	12.2	56.2	81	1.44	243	4.49	80.9	1.61	1.94	0.0002	-0.69
30	-0.18	12.4	57.0	0	0.00	81	1.44	80.9	1.44	1.94	0.0000	-0.18

8.7.6 Find critical depth for the side-channel from Eq'n 6.13 or computer software with one vertical side and one side slope of 2(V) to 1(H). Thus, $\mathbf{y_c}$**= 5.22 m.** The solution table (using Excel) is given below.

Δx (m)	Δy (m)	y (m)	A (m^2)	Q (m^3/s)	V (m/s)	Q_1+Q_2 (m^3/s)	V_1+V_2 (m/s)	ΔQ (m^3/s)	ΔV (m/s)	R_h (m)	S_f (m)	Δy (m)
-	-	5.22	59.0	400	6.78	-	-	-	-	-	-	-
20	-2.54	7.76	92.7	300	3.24	700	10.0	100	3.54	3.63	0.0003	-2.54
20	-0.61	8.37	101.2	200	1.98	500	5.21	100	1.26	3.79	0.0001	-0.61
20	-0.30	8.67	105.5	100	0.95	300	2.92	100	1.03	3.86	0.0000	-0.30
20	-0.09	8.76	106.8	0	0.00	100	0.95	100	0.95	3.88	0.0000	-0.09

8.8.1

The maximum negative pressure head allowable

(from the book jacket) is: $(P_{vapor} - P_{atm})/\gamma$ =

$(2.37 \times 10^3 - 1.014 \times 10^5)N/m^2/9790\ N/m^3 = -10.1$ m

Thus, balancing energy from reservoir to crown:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_c^2}{2g} + \frac{P_c}{\gamma} + h_c + h_L$$ referring to Ex. 8.4:

$0 + 0 = [(6.37)^2/2g](1+0.1+0.8+0.25) + (-10.1) + (h_c - h_1)$

$\mathbf{h_c - h_1 = 5.65}$ **m**

8.8.2

a) The maximum negative pressure head allowable (values from book jacket): $(P_{vapor} - P_{atm})/\gamma$ =

$(2.37 \times 10^3 - 1.014 \times 10^5)N/m^2/9790\ N/m^3 = -10.1$ m

b) Since the loss occurs throughout the bend, it is not quite appropriate to include the entire bend loss. But in design, it is safer (more conservative) to do so.

c) If the reservoir level fell below the crown by 8 m, $P_c/\gamma = -12.4$ m based on redoing the calculations in Example 8.4. This would fall below the vapor pressure of water (roughly -10.3 m depending on water temperature). The rule of thumb does not work because the losses and velocity head are large.

8.8.3

Balancing energy from reservoir to downstream pool

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$$

$0 + 0 + 368 = 0 + 0 + 335 + 10.5 + V^2/2g$;

$V^2/2g = 22.5$ ft., V = 38.1 ft/sec, **Q = AV = 686 cfs**

Balancing energy from reservoir to siphon crown:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_c^2}{2g} + \frac{P_c}{\gamma} + h_c + h_L; \quad h_L = h_f + h_e$$

$0 + 0 + 1.5 = 22.5 + P_c/\gamma + 0 + 3.5$; $\mathbf{P_c/\gamma = -24.5}$ **ft**

8.8.4

Balancing energy from reservoir to downstream pool:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L; \quad V = Q/A = 4.53 \text{ m/s}$$

$0 + 0 + h_1 = 0 + 0 + h_2 + (0.2+0.02(60/0.3)+1.0)[4.53^2/2g]$

$h_1 - h_2 = 5.44$ m; thus $\mathbf{h_c - h_2}$ = 1.2 + 5.44 = **6.64 m** (i.e., the elev. difference between crown and downstream pool)

Balancing energy from reservoir to siphon crown:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_c^2}{2g} + \frac{P_c}{\gamma} + h_c + h_L; \quad h_L = h_f + h_e$$

$0 = [4.53^2/2g] + P_c/\gamma + 1.2 + (0.2+0.02(10/0.3))[4.53^2/2g]$

$P_c/\gamma = -3.15$ m. $\mathbf{P_c}$ = (-3.15m)(9.79 kN/m^3) = **-30.8 kN/m^3**

8.8.5

Balancing energy from reservoir to reservoir yields:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L$$

$0 + 0 + 52.5 = 0 + 0 + 0 + 3.5[V^2/2g]$; $V^2/2g = 15$ ft.

Balancing energy from reservoir to siphon crown:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_c^2}{2g} + \frac{P_c}{\gamma} + h_c + h_L; \quad h_L = h_f + h_e$$

$0 + 0 + 52.5 = 15 + P_c/\gamma + 60 + [0.5 + 0.5](15)$

$\mathbf{P_c/\gamma = -37.5}$ **ft** < -33.8 ft (vacuum), cavitation danger

---

8.8.6

Balancing energy from reservoir to reservoir yields:

$h_1 - h_2 = 163.3 – 154.4 = 8.9 \text{ m} = [\Sigma K + f(L/D)](V^2/2g)$

$8.9 = [0.25+1.0+0.7+0.022(36.6/D)][\{4(5.16)/\pi D^2\}^2/2\cdot 9.81]$

D = 0.915 m; $V = Q/A = 5.16/(\pi\cdot 0.915^2/4) = 7.85$ m/s

Balancing energy from reservoir to siphon crown:

$h_1 = h_c + V^2/2g + P_c/\gamma + [\Sigma K + f(L/D)](V^2/2g)$

$163.3 = h_c+7.85^2/2g+(-10.1)+$
$[0.25+0.7+0.022(7.62/0.915)[7.85^2/2g]$

$\mathbf{h_c = 166.7}$ **m, MSL**

8.8.7

The siphon length: $L = 3.2 + 30 + 15 = 48.2$ m

Balancing energy from reservoir to reservoir yields:

$h_s+30 = [0.5 + 0.3 + 1.0]\,(V^2/2g) + h_f$; from Manning Eq'n.

$h_f = L(nV/R^{2/3})^2 = [(L\cdot n^2)/R^{4/3}]V^2$; therefore,

$h_s + 30 = [1.8/2g + (L\cdot n^2)/R^{4/3}]V^2$

$h_s + 30 = [0.092 + 0.030/R^{4/3}]V^2$ (1)

Balancing energy from reservoir to siphon crown:

$h_s = [1.5/2g + (3.2\cdot n^2)/R^{4/3}]V^2 + P_c/\gamma$

$h_s = [0.076 + (0.002)/R^{4/3}]V^2 - 8\text{m}$ (2)

also, $V = Q/A = 20/A$ (3)

Solving the three equations simultaneously yields:

$A = 1.65\ \text{m}^2$, $V = 12.1$ m/sec, $h_s = 4.5$ m, $R = 0.31$

8.9.1

a) Check full pipe flow since that controls the design:

$h_L = H + S_oL - D = 3.2 + (0.003)(40) - 1.25 = 2.07$ m

Also, from Equation (8.18) with $K_e = 0.2$, we have

$h_L = [K_e + \{n^2L/R_h^{4/3}\}(2g) + 1]\{8Q^2/(\pi^2 gD^4)\}$

$h_L = [1.2+\{0.024^2(40)/(1.25/4)^{4/3}\}2g]\{8(5.25)^2/(\pi^2 g\cdot 1.25^4)\}$

$h_L = 3.10$ m > 2.07 m (pipe diameter is too small)

b) Changing the slope and entrance condition.

$h_L = H + S_oL - D = 3.2 + (0.01)(40) - 1.25 = 2.35$ m

Also, from Equation (8.18) with $K_e = 0.2$, we have

$h_L = [K_e + \{n^2L/R_h^{4/3}\}(2g) + 1]\{8Q^2/(\pi^2 gD^4)\}$

$h_L = [1.2+\{0.024^2(40)/(1.25/4)^{4/3}\}2g]\{8(5.25)^2/(\pi^2 g\cdot 1.25^4)\}$

$h_L = 3.10$ m > 2.35 m (pipe diameter still too small)

Losses are too large to overcome w/small changes.

8.9.2

Balancing energy from "1" to "2" yields:

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L;$$

$0 + 0 + h_1 = V_2^2/2g + 0 + 0$; let $h_1 = h$ and $V_2 = V$;

$h = V^2/2g$; thus, $V = (2gh)^{1/2}$. Since $Q = VA$

$Q = A(2gh)^{1/2}$; and accounting for losses with a coefficient of discharge, C_d: **$Q = C_dA(2gh)^{1/2}$**

The variable h represents the height of the water surface above the center of the orifice opening.

8.9.3

Since the outlet is submerged, the hydraulic operation category is (a) and the energy balance is:

$H_{up} + S_oL = H_{down} + h_L$; where H = stream depths

$4.05 + (0.03)(15) = 3.98 + h_L$; $h_L = 0.52$ m

The head loss from hydraulic category (a) is:

$h_L = [K_e + \{n^2L/R_h^{4/3}\}(2g) + 1]\{Q^2/(2g\cdot A^2)\}$;

where $R_h = A/P = (4\ \text{m}^2)/(8\ \text{m}) = 0.5$ m. Thus,

$0.52 = [0.5+\{0.013^2(15)/(0.5)^{4/3}\}(2g)+1]\{Q^2/(2g(4^2)\}$

$Q = 10.0\ \text{m}^3/\text{sec}$

8.9.4

Assuming hydraulic operation category (c):

$Q = C_dA(2gh)^{1/2}$; and h = depth above culvert center

$Q = (0.6)(4)(2\cdot 9.81\cdot 3.05)^{1/2} = 18.6\ \text{m}^3/\text{sec}$

Determine normal depth to see if pipe will flow full.

$nQ/(k_M S_o^{1/2} b^{8/3}) = (0.013\cdot 18.6)/(1.0\cdot 0.03^{1/2}\cdot 2^{8/3}) = 0.220$

From Fig. 6.4: $y_n/b = 0.53$; $y_n = (2)(0.54) = 1.08$ m

Since $y_n < 2$ m, flow is category (c) and **$Q = 18.6\ \text{m}^3/\text{s}$**

8.9.5

Since the outlet is submerged, the hydraulic operation category is (a) and the energy balance is:
$HW = TW + h_L$; $h_L = 544.4 - 526.4 = 18.0$ ft

The head loss from hydraulic category (a) is:

$h_L = [K_e + \{n^2L/R_h^{4/3}\}(2g) + 1]\{8Q^2/(\pi^2 \cdot g \cdot D^4)\}$;

$18 = [0.5+\{0.013^2(330)/(D/4)^{4/3}\}(2g)+1]\{8 \cdot 654^2/(\pi^2 \cdot g \cdot D^4)\}$

Solving the implicit equation: **D = 6.67 ft**

For a twin barrel culvert, each barrel conveys 327 cfs

$18 = [0.5+\{0.013^2(330)/(D/4)^{4/3}\}(2g)+1]\{8 \cdot 327^2/(\pi^2 \cdot g \cdot D^4)\}$

Solving the implicit equation: **D = 5.00 ft**

8.9.6

Based on the design conditions (submerged inlet, but unsubmerged outlet) the culvert must be hydraulic operation category (b) or (c). For category (b) or outlet control, the headloss is determined to be:
$h_L = [K_e + \{n^2L/(D/4)^{4/3}\}(2g) + 1]\{8Q^2/(\pi^2 gD^4)\}$

$h_L = [0.2 + \{0.013^2(20)/(1.5/4)^{4/3}\}(2 \cdot 9.81) + 1] \cdot \{8(9.5)^2/(\pi^2 \cdot 9.81 \cdot 1.5^4)\}$
$h_L = 2.13$ m

An energy balance (H = upstream depth) yields:
$H + S_oL = D + h_L$; $H + (0.02)(20) = 1.5 + 2.13$

$H = 3.23$ m (Required depth for outlet control)

If the culvert is operating under partially full flow (inlet control or hydraulic category (c)), then
$Q = C_dA(2gh)^{1/2}$ with $h = H - D/2 = H - 0.75$.

$9.5 = (0.95)[\pi(1.5)^2/4][2 \cdot 9.81 \cdot (H - 0.75)]^{1/2}$;

$H = 2.38$ m (Required depth for inlet control)

Thus, the culvert will operate under outlet control,

and the upstream depth will be **H = 3.23 m.**

8.9.7

Based on the design conditions (submerged inlet, but unsubmerged outlet) the culvert must be hydraulic operation category (b) or (c). For category (b), an energy balance (H = upstream depth) yields:
$H + S_oL = D + h_L$; $2 + 0.1 \cdot 60 = D + h_L$; $h_L = 8.0 - D$

and h_L for the full pipe flow (outlet control) is:

$h_L = [K_e + \{n^2L/(D/4)^{4/3}\}(2g) + 1]\{8Q^2/(\pi^2 gD^4)\}$

$h_L = [0.5+\{0.024^2(60)/(D/4)^{4/3}\}(2g)+1]\{8(2.5)^2/(\pi^2 gD^4)\}$

Equating the two h_L expressions above yields

$8.0 - D = [1.5 + 4.31/D^{4/3}]\{0.516/D^4\}$; $D = 0.84$ m

If the culvert is operating under partially full flow (inlet control or hydraulic category (c)), then

$Q = C_dA(2gh)^{1/2}$ with $h = 2.0 - D/2$. Therefore,

$2.5 = (0.6)(\pi D^2/4)[2 \cdot 9.81 \cdot (2.0 - D/2)]^{1/2}$; $D = 0.99$ m

Thus, the culvert will operate under inlet control, and

the required **D = 0.99 m (or 1.0 m).**

8.9.8

a) Partially full flow (hydraulic category (c)):

$Q = C_dA(2gh)^{1/2}$ with $h = H - D/2 = 5.5 - 2.0 = 3.5$ ft

Q $= (0.6)[(4)(4)[(2 \cdot 32.2)(3.5)]^{1/2}$ **= 144 cfs**

b) Open channel (hyd. category (d)); y_c at entrance:

$y_c = [Q^2/(gb^2)]^{1/3}$; $4.0 = [Q^2/(32.2 \cdot 4^2)]^{1/3}$; **Q = 182 cfs**

Determine normal depth (from Fig 6.4): $y_n = 1.48$ ft
OK – goes from y_c to y_n in culvert barrel.
c) Open channel (hyd. category (d)); y_c at entrance:

$y_c = [Q^2/(gb^2)]^{1/3}$; $2.5 = [Q^2/(32.2 \cdot 4^2)]^{1/3}$; **Q = 89.7 cfs**

$y_n = 0.90$ ft; OK – from y_c to y_n in culvert barrel.

8.10.1

a) Find sequent depth from Figure 8.20(b):

$N_{f1} = V_1/(gd_1)^{1/2}$: Eq'n (6.12); need d_1:

$Q = AV$; $350 = [35 \cdot d_1](30)$; $d_1 = 0.33$ ft

Thus, $N_{f1} = 30/(32.2 \cdot 0.33)^{1/2} = 9.2$

From Figure 8.20(b): $TW/d_1 = 12.5$; $TW = 4.1$ ft

and since $TW/d_2 = 1.0$; $\mathbf{d_2 = 4.1}$ **ft**

Note: Figure 8.20(b) is graph form of Eq'n (6.17)

b) Find length of the jump from Figure 8.20(d):

$L/d_2 = 2.7$; **L = 11.1 ft**

c) From Eq'n 6.20: $\mathbf{\Delta E} = (d_2 - d_1)^3/4d_1d_2$ **= 9.9 ft**

d) Specific energy is found from Eq'n 6.8:

$E_1 = d_1 + V_1^2/2g = 0.33 + 30^2/2g = 14.3$ ft

$V_2 = Q/A_2 = 350/[35 \cdot 4.1] = 2.44$ ft/sec

$E_2 = d_2 + V_2^2/2g = 4.1 + 2.44^2/2g = 4.4$ ft

Efficiency = $E_2/E_1 = 4.4/14.3$ **= 0.31 or 31%**

8.10.2

a) Use Equation (6.12) to obtain the Froude number

$N_{f1} = V_1/(gd_1)^{1/2} = 15/(9.81 \cdot 0.2)^{1/2} = 10.7$

Based on N_{f1} and V_1 (< 20 m/s), choose **Type III**:

b) Find sequent depth from Figure 8.20(b):

$TW/d_1 = 15$; $TW = 3.0$ m; $TW/d_2 = 1.0$; $\mathbf{d_2 = 3.0}$ **m**

Note: Figure 8.20(b) is graph form of Eq'n (6.17)

c) Find length of the jump from Figure 8.20(d):

$L/d_2 = 2.8$; **L = 8.4 m**

d) From Eq'n 6.20: $\mathbf{\Delta E} = (d_2 - d_1)^3/4d_1d_2$ **= 9.1 m**

8.10.3

a) From problem 8.10.2, determine channel width

$Q = A_1V_1$; $22.5 = (b \cdot 0.2)(15)$; $b = 7.5$ m

For this problem: $V_1 = 45/(7.5 \cdot 0.25) = 24$ m/sec

Use Equation (6.12) to obtain the Froude number

$N_{f1} = V_1/(gd_1)^{1/2} = 24/(9.81 \cdot 0.25)^{1/2} = 15.3$

Based on N_{f1} and V_1 (> 20 m/s), choose **Type II:**

b) Find sequent depth from Figure 8.21(b):

$TW/d_1 = 22$; $TW = 5.5$ m; $TW/d_2 = 1.05$; $\mathbf{d_2 = 5.2}$ **m**

c) Find length of the jump from Figure 8.21(c):

$L/d_2 = 4.3$; **L = 22.4 m**

d) From Eq'n 6.20: $\mathbf{\Delta E} = (d_2 - d_1)^3/4d_1d_2$ **= 23.3 m**

e) Specific energy is found from Eq'n 6.8:

$E_1 = d_1 + V_1^2/2g = 0.25 + 24^2/2g = 29.6$ m

$V_2 = Q/A_2 = 45/[7.5 \cdot 5.2] = 1.15$ m/sec

$E_2 = d_2 + V_2^2/2g = 5.2 + 1.15^2/2g = 5.27$ m

Efficiency = $E_2/E_1 = 5.27/29.6$ **= 0.18 or 18%**

Chapter 9 – Problem Solutions

9.1.1

An equal pressure surface exists at 1 and 2, thus

$P_{water} = (6 \text{ in.})(\gamma)$; on the right side which equals

$P_{oil} = (8.2 \text{ in.})(\gamma_{oil})$; on the left side. Therefore

$(6 \text{ in.})(\gamma) = (8.2 \text{ in.})(\gamma_{oil})$

S.G (oil) = $(\gamma_{oil})/(\gamma) = 6/8.2$ **= 0.732**

9.1.2

An equal pressure surface exists at 1 and 2, thus

$P_A + (y)(\gamma) = (h)(\gamma_{Hg})$; where $\gamma_{Hg} = (SG)(\gamma)$ and

$P_A + (1.24)(9.79) = (1.02)(13.6)(9.79)$

$\mathbf{P_A}$= 0.12 lb/in^2 **= 124 kN/m^2**

If a piezometer was used: $P_A = (\gamma)(h)$, or

h $= P_A/\gamma = (124 \text{ kN/m}^2)/(9.79 \text{ kN/m}^3)$ **= 12.7 m**

This is an impractical tube length to measure pressure and shows why piezometers are rarely used in practice.

9.1.3

Since $P = \gamma\cdot h$; pressure can be expressed as the height of any fluid. In this case (referring to Figure 9.2),

$P = (\gamma_{Oil})\Delta h = (S.G.)_{Oil}(\gamma)(\Delta h)$; where $\Delta h = (\Delta l)(\sin\theta)$

$\Delta h = (0.15 \text{ m})(\sin 15°) = 0.0388$ m. Therefore,

$P = (S.G.)_{Oil}(\gamma)(\Delta h)$;

$323 \text{ N/m}^2 = (S.G.)_{Oil}(9{,}790 \text{ N/m}^3)(0.0388 \text{ m})$

$\mathbf{(S.G.)_{Oil} = 0.850}$

9.2.1

Use the piezometer to determine the pressure:

$P/\gamma = 3.20$ m; **P** $= (3.20 \text{ m})(9.79 \text{ kN/m}^3)$ **= 31.3 kPa**

Use the Pitot tube to determine the velocity:

$P/\gamma + V^2/2g = 3.30$ m; $V^2/2g = 3.30 - 3.20 = 0.10$ m

V $= [(2\cdot 9.81)(0.10 \text{ m})]^{1/2}$ = **1.38 m/sec**

Since the stagnation pressure is a point pressure, the velocity is a point (centerline, not average) velocity.

9.2.2

Referring to Figure 9.4(b), let *x* be the distance from position 1 to the interface between the water and the manometry fluid and let γ_m be the specific weight of the manometry fluid. Applying manometry principles:

$P_1 - \gamma x + \gamma_m\Delta h - \gamma\Delta h + \gamma x = P_o$; or

$P_o - P_1 = \Delta P = \Delta h\,(\gamma_m - \gamma)$

Substituting this into Equation (9.1a) yields:

$V^2 = 2g\Delta h[(\gamma_m - \gamma)/\gamma]$; since $SG = \gamma_m/\gamma$

$\mathbf{V = [2g\Delta h(SG - 1)]^{1/2}}$

9.2.3

Applying Equation (9.1b) yields:

V $= [2g(\Delta P/\gamma)]^{1/2}$ where $\Delta P = SG\cdot\gamma\cdot\Delta h$; thus

$V = [2g(SG\cdot\Delta h)]^{1/2} = [(2\cdot 32.2)(13.6)(5.65/12)]^{1/2}$

V = 20.3 ft/sec; Point velocity at tip of Pitot tube.

Q $= AV = [\pi(5)^2/4](20.3)$ = **399 cfs**. This is a flow estimate since the velocity is not an average velocity.

9.2.4

Applying Equation (9.1b) yields:

a) $V = [2g(\Delta P/\gamma)]^{1/2}$ where $\Delta P = \gamma \cdot \Delta h$; thus

$V = [2g(\Delta h)]^{1/2} = [(2 \cdot 9.81)(0.334)]^{1/2}$; $\mathbf{V = 2.56\ m/s}$

b) $V = [2g(\Delta P/\gamma)]^{1/2}$ where $\Delta P = \Delta h\,(\gamma_m - \gamma)$; thus

$V = [2g(\Delta h)(\gamma_m - \gamma)/\gamma]^{1/2} = [2g(\Delta h)(SG - 1]^{1/2}$

$V = [(2 \cdot 9.81)(0.334)(13.6 - 1)]^{1/2}$; $\mathbf{V = 9.09\ m/s}$

9.2.5

Applying Equation (9.1b) yields:

a) $V = [2g(\Delta P/\gamma_{oil})]^{1/2}$ where $\Delta P = \gamma_{oil} \cdot \Delta h$; thus

$V = [2g(\Delta h)]^{1/2} = [(2 \cdot 32.2)(18.8/12)]^{1/2}$; $\mathbf{V = 10.0\ ft/s}$

b) $V = [2g(\Delta P/\gamma_{oil})]^{1/2}$ where $\Delta P = \Delta h(\gamma_m - \gamma_{oil})$; thus

$V = [2g(\Delta h)(\gamma_m - \gamma_{oil})/\gamma_{oil}]^{1/2}$

$V = [2g(\Delta h)\{(SG_m/SG_{oil})\text{-}1\}]^{1/2}$

$V = [(2 \cdot 32.2)(18.8/12)\{(13.6/0.85) - 1\}]^{1/2}$

$\mathbf{V = 38.9\ ft/s}$

9.2.6

Applying Equation (9.1b) yields:

$V = [2g(\Delta P/\gamma)]^{1/2}$ where $\Delta P = \Delta h\,(\gamma_m - \gamma)$; thus

$V = [2g(\Delta h)(\gamma_m - \gamma)/\gamma]^{1/2} = [2g(\Delta h)(SG - 1]^{1/2}$

$V = [(2 \cdot 9.81)(0.25)(13.6 - 1)]^{1/2}$;

$\mathbf{V = 7.86\ m/s}$ Note that this is a point velocity that exists at the tip of the Pitot tube probe.

Q $= AV = [\pi(10)^2/4](7.86)$ $\mathbf{= 617\ m^3/s}$.

Note that this is a flow estimate since the velocity is not an average velocity.

9.3.1

$A_1 = (\pi/4)(0.5)^2 = 0.196\ m^2$; $A_2 = (\pi/4)(0.2)^2 = 0.0314\ m^2$

Equation (9.5b) yields: $C_d = 1/[(A_1/A_2)^2 - 1]^{1/2}$

$C_d = 1/[(0.196/0.0314)^2 - 1]^{1/2} = 0.162$

From Equation (9.5c): $Q = C_d A_1[2g(\Delta P/\gamma)]^{1/2}$;

Q $= (0.162)(0.196)[2g(290\text{-}160)/9.79)]^{1/2}$ $\mathbf{= 0.513\ m^3/s}$

9.3.2

From manometry principles: $\Delta P = \Delta h(\gamma_{Hg} - \gamma)$ or

$\Delta P/\gamma = \Delta h(SG - 1) = 2.01(13.6 - 1) = 25.3$ ft

Therefore, $\Delta(P/\gamma + z) = 25.3 + 0.80 = 26.1$ ft

Equation (9.5b) and Equation (9.5a) yield:

$C_d = 1/[(A_1/A_2)^2 - 1]^{1/2} = 1/[(8\text{ in}/4\text{ in})^4 - 1]^{1/2} = 0.258$

$Q = C_d A_1[2g\{\Delta(P/\gamma + z)]^{1/2}$;

Q $= (0.258)[\pi/4(8/12)^2][2 \cdot 32.2(26.1)]^{1/2}$ $\mathbf{= 3.69\ ft^3/sec}$

9.3.3

$A_1 = (\pi/4)(0.2)^2 = 0.0314\ m^2$; $A_2 = (\pi/4)(0.1)^2 = 0.00785\ m^2$

Equation (9.5b) yields: $C_d = 1/[(A_1/A_2)^2 - 1]^{1/2}$

$C_d = 1/[(0.0314/0.00785)^2 - 1]^{1/2} = 0.258$

From Equation (9.6b): $Q = C_v C_d A_1[2g(\Delta P/\gamma)]^{1/2}$;

To obtain C_v: $V_2 = Q/A_2 = 0.082/0.00785 = 10.4$ m/sec

$N_{R2} = V_2 d_2/\nu = (10.4)(0.10)/(1.00 \times 10^{-6}) = 1.04 \times 10^6$

From Fig. 9.8 with $d_2/d_1 = 0.5$; $C_v = 0.991$; Now

$Q = C_v C_d A_1[2g(\Delta P/\gamma)]^{1/2}$;

$0.082 = (0.991)(0.258)(0.0314)[2g(\Delta P/9{,}790)]^{1/2}$;

$\mathbf{\Delta P = 52.1 \times 10^3\ Pa = 52.1\ kPa}$

9.3.4

$A_1 = (\pi/4)(0.4)^2 = 0.126\ m^2$; $A_2 = (\pi/4)(0.16)^2 = 0.0201\ m^2$

Equation (9.5b) yields: $C_d = 1/[(A_1/A_2)^2 - 1]^{1/2}$

$C_d = 1/[(0.126/0.0201)^2 - 1]^{1/2} = 0.162$

From manometry principles: $\Delta P = \Delta h(\gamma_{Hg} - \gamma)$ or

$\Delta P/\gamma = \Delta h(SG - 1) = 0.20(13.6 - 1) = 2.52$ m

Therefore, $\Delta(P/\gamma + z) = 2.52 + (-0.25) = 2.27$ m

Applying Eq'n (9.6a) and assuming $C_v = 0.99$ yields

$Q = C_vC_dA_1[2g\{\Delta(P/\gamma + z)\}]^{1/2}$;

$Q = (0.99)(0.162)(0.126)[2g(2.27)]^{1/2} = 0.135\ m^3/sec$

Verifying the assumed C_v: $N_{R2} = V_2d_2/\nu$

$N_{R2} = [(0.135/0.0201)(0.16)]/(1.00 \times 10^{-6}) = 1.07 \times 10^6$

From Fig. 9.8 with $d_2/d_1 = 0.4$; $C_v = 0.994$; Now

Q = (0.994/0.99)(0.135) **= 0.136 m^3/sec**

9.3.5

$A_1 = (\pi/4)(1.75)^2 = 2.41\ ft^2$; $A_2 = (\pi/4)(1.0)^2 = 0.785\ ft^2$

Equation (9.5b) yields: $C_d = 1/[(A_1/A_2)^2 - 1]^{1/2}$

$C_d = 1/[(2.41/0.785)^2 - 1]^{1/2} = 0.345$

Now use Equation (9.6a) to determine $\Delta P/\gamma$:

$Q = C_vC_dA_1[2g\{\Delta(P/\gamma + z)\}]^{1/2}$

$12.4 = (0.675)(0.345)(2.41)[2\cdot 32.2\{\Delta P/\gamma + (-0.75)\}]^{1/2}$

$\Delta P/\gamma = 8.33$ ft

Based on manometry: $\Delta P = \Delta h(\gamma_{Hg} - \gamma)$

Therefore $\Delta P/\gamma = \Delta h(SG - 1)$;

$8.33 = \Delta h(13.6 - 1)$;

Δh = 0.661 ft = 7.93 in. (minimum U-tube length)

9.3.6

From manometry principles: $\Delta P = \Delta h(\gamma_{Hg} - \gamma)$ or

$\Delta P/\gamma = \Delta h(SG - 1) = 0.09(13.6 - 1) = 1.13$ m

Applying Eq'n (9.6b): $Q = C_vC_dA_1[2g(\Delta P/\gamma)]^{1/2}$;

$0.00578 = (0.605)C_d[(\pi/4)(0.10)^2][2\cdot 9.81(1.13)]^{1/2}$

$C_d = 0.258$ Applying Equation (9.5b) yields

$C_d = 1/[(A_1/A_2)^2 - 1]^{1/2} = 1/[(D_1/D_2)^4 - 1]^{1/2}$

$0.258 = 1/[(0.10/D_2)^4 - 1]^{1/2}$; **$D_2$ = 0.0500 m (5.00 cm)**

9.3.7

Equation (9.8) yields: $C_d = R/2D = 80/(2\cdot 75) = 0.533$

Now apply Eq'n 9.7: $Q = C_dA[2g(\Delta(P/\gamma)]^{1/2}$

$51/60 = (0.533)(\pi/4)(0.75)^2[2\cdot 9.81(\Delta P/\gamma)]^{1/2}$

$\Delta P/\gamma = 0.664$ m; from manometry: $\Delta P = \Delta h(\gamma_{Hg} - \gamma)$

Therefore $\Delta P/\gamma = \Delta h(SG - 1)$;

$0.664 = \Delta h(13.6 - 1)$; **Δh = 0.0526 m = 5.26 cm**

9.3.8

Equation (9.8) yields: $C_d = R/2D = 80/(2\cdot 75) = 0.533$

From manometry principles: $\Delta P = \Delta h(\gamma_{Hg} - \gamma)$ or

$\Delta P/\gamma = \Delta h(SG - 1) = 0.0526(13.6 - 1) = 0.663$ m

Also, $\Delta z = D\cdot \sin 45° = (0.75)(0.707) = 0.530$ m

Therefore, $\Delta(P/\gamma + z) = 0.663 + 0.530 = 1.19$ m

Now apply Eq'n 9.7 modified for height change:

$Q = C_dA[2g\{\Delta(P/\gamma + z)\}]^{1/2}$

$0.850 = C_d(\pi/4)(0.75)^2[2\cdot 9.81(1.19)]^{1/2}$

C_d = 0.398

9.4.1

Applying Eq'n (9.10b) yields: $C = 1.78 + 0.22(H/p)$

$C = 1.78 + 0.22[(4.4 - 3.1)/3.1] = 1.87\ m^{0.5}/s$

Applying Eq'n (9.9) yields: $Q = CLH^{3/2}$

Q $= (1.87)(4.8)(4.4 - 3.1)^{3/2}$ **= 13.3 m³/sec**

For the contracted weir and same upstream depth,

Applying Eq'n (9.11) yields: $Q = C[L-(n\cdot H/10)]H^{3/2}$

Q $= (1.87)[2.4 - (2\cdot 1.3/10)](1.30)^{3/2}$ = **5.93 m³/sec**

9.4.2

Applying Eq'n (9.14) yields: $Q = 2.49H^{2.48}$

$25.6 = 2.49H^{2.48}$; $H = 2.56$ ft

Applying Eq'n (9.12a) yields: $Q = 3.33(L - 0.2H)H^{2.48}$

$33.3 = 3.33[L - 0.2(2.56)](2.56)^{2.48}$; **L = 1.48 ft**

9.4.3

Applying Eq'n (9.15): $Q = 3.367LH^{3/2}$

requires BG units. Thus, H = 0.259 m = 0.850 ft

and Q = 0.793 m^3/sec (35.3 cfs/1 cms) = 28.0 cfs

Now applying Eq'n (9.15) yields: $Q = 3.367LH^{3/2}$

$28.0 = 3.367(L)(0.850)^{3/2}$; **L** = 10.6 ft **= 3.24 m**

9.4.4

Applying Eq'n (9.19):

$Q = 0.433(2g)^{1/2}[y_1/(y_1 + h)]^{1/2}LH^{3/2}$

$Q = 0.433(2\cdot 9.81)^{1/2}[1.4/(1.4 + 1.0)]^{1/2}(3)(0.4)^{3/2}$

Q = 1.11 m³/sec

9.4.5

$H_b/H_a = 0.5/1.0 = 0.5$, thus flow is not submerged.

Applying Eq'n (9.24), which requires BG units

$Q = (3.6875W + 2.5)H_a^{1.6}$

$Q = [3.6875(15\text{ ft}) + 2.5)(3.28\text{ ft})^{1.6} = 387$ cfs **= 11.0 m³/s**

9.4.6

Applying Eq'n (9.10b) yields: $C = 1.78 + 0.22(H/p)$

$C = 1.78 + 0.22[(2.2-1.5)/1.5] = 1.88\ m^{0.5}/s$

Applying Eq'n (9.9) yields: $Q = CLH^{3/2}$

Q $= (1.88)(4.5)(2.2 - 1.5)^{3/2}$ **= 4.95 m³/sec**

For the same Q and upstream depth of 1.8 m:

$Q = CLH^{3/2}$ where $H = 1.8 - p$

$4.95 = [1.78 + 0.22\{(1.8 - p)/p\}](4.5)(1.8 - p)^{3/2}$

Solving the implicit equation (with calculator, iteration, MathCad or other software) yields: **p = 1.11 m**

9.4.7

For the existing weir Eq'n (9.10a) yields:

$C = 3.22+0.4(H/p) = 3.22+0.40[1.0/3.5] = 3.33\ ft^{0.5}/s$

Eq'n (9.9): $Q_1 = CLH^{3/2} = (3.33)L(1)^{3/2} = 3.33\cdot L\ ft^3/sec$

Applying Eq'ns (9.9) and (9.10a) for previous weir:

$Q_2 = CLH^{3/2} = [3.22 + 0.40(H/1.75)]LH^{3/2}$;

Since the flow rate has remained constant, equating Qs

$3.33 = [3.22 + 0.40(H/1.75)]H^{3/2}$ ➔Ls have canceled.

Solving the implicit equation (with calculator, iteration, MathCad or other software) yields: H = 0.98 m

Thus, **Δh** = (3.5 + 1) – (1.75 + 0.98) **= 1.77 ft**

9.4.8

Determine the unsubmerged flow with Eq'n (9.24)

$$Q_u = 4WH_a^{1.522 W^{0.026}} = 4(8)(2.5)^{1.522(8)^{0.026}} = 139 \ cfs$$

Now determine the flow correction: $Q = Q_u - Q_c$

$129 = 139 - Q_c$; $Q_c = 10$ cfs Based on Fig (9.15):

$Q_c = 10$ cfs $= 5.4(CF)$: CF = 1.85 cfs;

and from Fig (9.15): $H_b/H_a = 0.80$; $\mathbf{H_b = 2.0}$ **ft**

9.4.9

Applying Eq'n (9.11) yields: $Q = C[L-(nH/10)]H^{3/2}$

$Q = 1.86[1.0-(2 \cdot 0.6/10)](0.6)^{3/2} = 0.761 \ m^3/sec$

For the replacement weir, Eq'ns (9.9) and (9.10b):

$Q = [1.78 + 0.22(H/p)]LH^{3/2}$

$0.761 = [1.78 + 0.22\{(2.3-p)/p\}](4)(2.3 - p)^{3/2}$

Solving the implicit equation (with calculator, iteration, MathCad or other software) yields: **p = 2.08 m**

9.4.10

Rearranging Equation (9.16) and noting $\gamma = \rho g$:

$(\rho/\gamma)q^2(1/y_2 - 1/y_1) = (y_1^2/2) - (y_2^2/2) - y_1 h + h^2/2$

$(2q^2/g)[(y_1 - y_2)/y_2 y_1] = y_1^2 - y_2^2 - 2y_1 h + h^2$

Substituting Eq'n (9.17): $y_2 = (y_1 – h)/2 = H/2$

$(2q^2/g)[\{y_1 - (y_1/2 - h/2)\}/\{(H/2)y_1\}]$
$\quad = y_1^2 - (H/2)^2 - 2y_1 h + h^2$

$(2q^2/g)[(y_1 + h)/\{(H)y_1\}] =$
$\quad = (y_1 - h)(y_1 - h) - (H/2)^2 = (H)^2 - (H^2/4)$

$(2q^2/g)[(y_1 + h)/\{(H)y_1\}] = 3/4(H)^2$

$$\mathbf{q = 0.433(2g)^{1/2}[y_1/(y_1 + h)]^{1/2}(H)^{3/2}}$$

9.4.11

Substituting h = 0 and g = 9.81 into Eq'n (9.19) yields:

$Q = 0.433(2 \cdot 9.81)^{1/2}[y_1/y_1]^{1/2}LH^{3/2} = \mathbf{1.92LH^{3/2}}$

and h ➔ ∞ and g = 9.81 into Eq'n (9.19) yields:

$\mathbf{Q} = 0.433(2 \cdot 9.81)^{1/2}[1/2]^{1/2}LH^{3/2} = \mathbf{1.36LH^{3/2}}$

For BG: $\mathbf{Q = 3.47LH^{3/2}}$ **and** $\mathbf{Q = 2.46LH^{3/2}}$

9.4.12

Starting with Eq'n (6.14): $y_c = [Q^2/gb^2]^{1/3}$

where b = L (weir width). Thus, $y_c = [Q^2/gL^2]^{1/3}$

Balancing energy between the free water surface (1) upstream of the weir and the top of the weir (2).

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L \text{ : assume } h_L = 0,$$

$V_1 = P_1 = 0$; $h_1 = H$ using the weir crest as a datum

also; $V_2 = V_c$; $P_2 = 0$; $h_2 = y_c$.

$0 + 0 + H = V_c^2/2g + 0 + y_c + 0$;

but at critical depth $V_c/(gy_c)^{1/2} = 1$; or $V_c^2/2g = y_c/2$;

Substituting this into the energy balance yields:

$H = y_c + y_c/2 = (3/2)y_c$; or $y_c = (2/3)H$.

Now returning to the Eq'n (6.14):

$y_c = [Q^2/gL^2]^{1/3}$; or $Q^2 = gL^2 y_c^3$

and finally, $Q = g^{1/2}Ly_c^{3/2}$;

substituting $y_c = (2/3)H$ yields

$Q = g^{1/2}L[(2/3)H]^{3/2} = g^{1/2}(2/3)^{3/2}LH^{3/2}$

$\mathbf{Q = 3.09LH^{3/2}}$

For the BG unit system. In general, $\mathbf{Q = CLH^{3/2}}$

where C is calibrated and accounts for any losses.

9.4.13

The general form of the V-notch weir equation is:

$Q = CH^x$ Taking the natural log of the equation:

$\ln Q = \ln C + x \ln H$

Now substitute the experimental data into the equation:

Test 1: $\ln(0.0220) = \ln C + x \ln(0.300)$

$-3.82 = \ln C + x(-1.20)$

Test 2: $\ln(0.132) = \ln C + x \ln(0.600)$

$-2.02 = \ln C + x(-0.511)$

Solving the two equations simultaneously :

$X = 2.61$, $C = 0.503$

The discharge equation for the V-notch weir is:

$Q = 0.503H^{2.61}$ in SI form

In BG units:

$H_1 = 0.984$ ft, $Q_1 = 0.777$ cfs

$H_2 = 1.97$ ft, $Q_2 = 4.66$ cfs

Now substitute the experimental data into the equation:

Test 1: $\ln(0.777) = \ln C + x \ln(0.984)$

$-0.252 = \ln C + x(-0.0161)$

Test 2: $\ln(4.66) = \ln C + x \ln(1.97)$

$1.54 = \ln C + x(0.678)$

Solving the two equations simultaneously :

$X = 2.58$, $C = 0.810$

The discharge equation for the V-notch weir is:

$Q = 0.810H^{2.58}$ in BG form

Chapter 10 – Problem Solutions

10.2.1

Applying Manning's eq'n: $Q = (1/n)AR^{2/3}S^{1/2}$

$A = 8\ m^2$, $P = 2(2\ m) + 4\ m = 8\ m$, and

$R_h = A/P = 1.0\ m$. Hence,

$Q_p = (1/0.025)(8)(1)^{2/3}(0.001)^{1/2} = 10.1\ m^3/sec$

For geometric similarity, there is no time scaling.

Thus, $Q_p/Q_m = (L_p^3/T_p)/(L_m^3/T_m) = (L_p^3)/(L_m^3)$;

and $Q_p/Q_m = L_r^3$; substituting we have

$(10.1)/(0.081) = L_r^3$; and $L_r = 5$

Therefore, the model depth and width are:

$y_p/y_m = b_p/b_m = L_r$; $\mathbf{y_m = 0.400\ m}$ and $\mathbf{b_m = 0.800\ m}$

10.2.2

The pond depth is 5 m. The surface area is:

Width: $W_p = 10m + 2(3)(5m) = 40\ m$

Length: $L_p = 40m + 2(3)(5m) = 70\ m$

Area: $A_p = (40)(70) = 2{,}800\ m^2$

The volume of the detention pond (prototype) is:

$Vol_p = LWd + (L+W)zd^2 + (4/3)z^2d^3$

$Vol_p = (40)(10)(5) + (40+10)(3)(5)^2 + (4/3)(3)^2(5)^3$

$Vol_p = 7{,}250\ m^3$; Based on geometric similarity and using Equations (10.2), (10.3), and (10.4):

$d_p/d_m = L_r$; $\mathbf{d_m} = d_p/L_r = 5/15\ \mathbf{= 0.333\ m}$

$A_p/A_m = L_r^2$; $\mathbf{A_m} = A_p/L_r^2 = 2{,}800/(15)^2\ \mathbf{= 12.4\ m^2}$

$Vol_p/Vol_m = L_r^3$;

$\mathbf{Vol_m} = Vol_p/L_r^3 = 7{,}250/(15)^3\ \mathbf{= 2.15\ m^3}$

10.2.3

Kinematic similarity seems appropriate since time is involved, but not force. For a length scale,

$L_p/L_m = [(3.20\ mi)(5280\ ft/mi)]/(70\ ft) = 241$

Use $\mathbf{L_r = 250} = L_p/L_m$; thus $L_m = 67.6$ ft. Now,

$Q_p/Q_m = (L_p^3/T_p)/(L_m^3/T_m) = (L_r^3/T_r)$

$2650/Q_m = (250^3/10)$; $\mathbf{Q_m = 1.70 \times 10^{-3}\ cfs}$

10.2.4

Kinematic similarity seems appropriate since time is involved, but not force. To determine a time scale,

$Q = C_dA(2gh)^{1/2}$ where Q = gate discharge. Thus,

$Q_p/Q_m = (C_{d(p)}/C_{d(m)})(A_p/A_m)[(2g_p/2g_m)(h_p/h_m)]^{1/2}$

$Q_r = L_r^3/T_r = (A_p/A_m)[(h_p/h_m)]^{1/2} = L_r^2(L_r)^{1/2}$

since $C_{d(p)}/C_{p(m)} = 1$ and $2g_p/2g_m = 1$. Therefore,

$L_r^3/T_r = L_r^2(L_r)^{1/2}$; $T_r = L_r^{1/2} = (150)^{1/2} = 12.2$; Hence,

$T_r = T_p/T_m$; $12.2 = T_p/(18.3)$; $\mathbf{T_p = 223\ min\ (3.72\ hrs)}$

10.2.5

Kinematic similarity seems appropriate since time is involved, but not force. Using Eq'n (10.10)

$N_p/N_m = 1/T_r$; $400/1200 = 1/T_r$; $T_r = 3$;

From Eq'n (10.8): $Q_r = L_r^3/T_r = (5)^3/3 = 41.7$.

Thus, $Q_p/Q_m = Q_r$; $(1)/Q_m = 41.7$; $\mathbf{Q_m = 0.0240\ m^3/s}$

From Example 10.4: $H_r = L_r^2/T_r^2 = (5)^2/(3)^2 = 2.78$

and $H_r = H_p/H_m$; $2.78 = 30/H_m$; $\mathbf{H_m = 10.8\ m}$

10.2.6

Kinematic similarity seems appropriate since time is involved, but not force. First find the time ratio and then use this to determine the flow rate in the model.

$T_r = L_r^{1/2} = (50)^{1/2} = 7.07$; then using Equation (10.8):

$Q_r = L_r^3/T_r = (50)^3/7.07 = 17{,}700$; from which

$Q_p/Q_m = Q_r$; $1{,}150/Q_m = 17{,}700$; $\mathbf{Q_m = 0.0650\ m^3/s}$

To determine the flow depth on the toe of the model spillway, determine spillway length and use continuity:

Spillway length: $L_r = 50 = L_p/L_m = 100/L_m$; $L_m = 2$ m

$Q = AV$; $0.0650 = (2\cdot d_m)(3)$; $d_m = 0.0108$ m

For prototype spillway: $d_r = 50 = d_p/d_m = d_p/0.0108$

Thus, $d_p = 0.540$ m and $V_r = L_r/T_r = 50/7.07 = 7.07$

Finally, $V_p/V_m = V_r$; $V_p/3.00 = 7.07$; $V_p = 21.2$ m/s

The Froude numbers may now be determined:

Model: $\mathbf{N_F} = V/(gd)^{1/2} = 3/(g\cdot 0.0108)^{1/2}$ **= 9.22**

Prototype: $\mathbf{N_F} = V/(gd)^{1/2} = 21.2/(g\cdot 0.540)^{1/2}$ **= 9.21**

10.2.7

Dynamic similarity is used since force is involved.

From Equation (10.13): $F_r = F_p/F_m = \rho_r L_r^4 T_r^{-2}$.

To determine the time ratio, use Equation (10.6):

$V_r = L_r/T_r$; $7.75 = 20/T_r$; $T_r = 2.58$.

Therefore, from Equation (10.13)

$\mathbf{F_r} = \rho_r L_r^4 T_r^{-2} = (1.00)(20)^4(2.58)^{-2}$ $\mathbf{= 2.40 \times 10^4}$

Also, based on Equation (10.8)

$Q_r = L_r^3/T_r$; $Q_r = (20)^3/(2.58)_r = 3{,}100$. Finally,

$Q_p/Q_m = Q_r$; $\mathbf{Q_p} = (10.6)(3{,}100)$ $\mathbf{= 3.29 \times 10^4}$ **cfs**

10.2.8

Dynamic similarity is used since force is involved.

From the problem statement, $L_r = 50$, and from

(a) Eq'n (10.6): $V_r = L_r/T_r$; $1/50 = 50/T_r$; $\mathbf{T_r = 2{,}500}$

(b) Eq'n (10.13): $\mathbf{F_r} = \rho_r L_r^4 T_r^{-2} = (1)(50)^4(2500)^{-2}$ **= 1.00**

(c) Eq'n (10.16): $\mathbf{P_r} = F_r L_r/T_r = (1)(50)/(2500)$ **= 0.0200**

(d) $E = (1/2)MV^2$; Thus, using Eq'n (10.14)

$\mathbf{E_r} = M_r V_r^2 = (F_r T_r^2 L_r^{-1})(L_r T_r^{-1})^2 = F_r L_r = (1)(50)$ **= 50.0**

10.2.9

Dynamic similarity is used since force is involved.

From the problem statement, $L_r = 30$, and from

Eq'n (10.6): $V_r = L_r/T_r$; $10 = 30/T_r$; $T_r = 3.00$

Assume the model uses sea water, from Eq'n (10.13):

$F_r = \rho_r L_r^4 T_r^{-2} = (1)(30)^4(3.00)^{-2} = 9.00 \times 10^4$. Thus,

$F_p/F_m = F_r$; $F_p = (0.510)(9.00 \times 10^4) = 4.59 \times 10^4$ lbs

The prototype length is: $L_r = L_p/L_m$; $L_p = 3\cdot 30 = 90$ ft

Thus, $\mathbf{F_p/L_p} = 4.59 \times 10^4/90$ **= 510 lbs/ft**

10.2.10

Dynamic similarity is used since force is involved.

The force on the model is: $F_m = 1.5$ N·m/(1 m) = 1.5 N

Also, $F_r = F_p/F_m$, and from Newton's 2nd law, $F = ma$

$F_r = F_p/F_m, = (\rho_p Vol_p a_p)/(\rho_m Vol_m a_m)$ where $a = g$. Thus,

$F_r = Vol_p/Vol_m = L_r^3$. Therefore, $F_p = F_r\cdot F_m = (L_r^3)(F_m)$

$F_p = (125)^3(1.5\ N) = 2{,}930$ kN. Prototype moment is

$F_p(L_m\cdot L_r) = (2930\ kN)(1m)(125)$ $\mathbf{= 3.66 \times 10^5}$ **kN·m**

10.3.1

Since inertial and viscous forces dominate, the Reynolds number law governs the flow.

Based on the problem statement: $\rho_r = 1$, $\mu_r = 1$

From Eq'n (10.19), $(\rho_r L_r V_r)/\mu_r = L_r V_r = 1$,

therfore, $V_r = 1/L_r = 1/10 = 0.10$

By definition: $V_r = V_p/V_m$

$\mathbf{V_m} = V_p/V_r = 5/0.10$ **= 50.0 m/sec**

10.3.2

The Reynolds number law (for $\rho_r = 1$, $\mu_r = 1$) is

$(\rho_r L_r V_r)/\mu_r = L_r V_r = 1$. Therefore,

a) $\mathbf{V_r} = 1/L_r = \mathbf{L_r^{-1}}$

b) Eq'n (10.6): $V_r = L_r/T_r$; $L_r^{-1} = L_r/T_r$; $\mathbf{T_r = L_r^2}$

c) Eq'n (10.7): $a_r = L_r/T_r^2 = L_r/(L_r^2)^2$; $\mathbf{a_r = L_r^{-3}}$

d) Eq'n (10.8): $\mathbf{Q_r} = L_r^3/T_r = L_r^3/L_r^2 = \mathbf{L_r}$

e) Eq'n (10.13): $\mathbf{F_r} = \rho_r L_r^4 T_r^{-2} = L_r^4(L_r^2)^{-2} = \mathbf{1}$

f) Eq'n (10.16): $\mathbf{P_r} = F_r L_r T_r^{-1} = (1)(L_r)(L_r^2)^{-1} = \mathbf{L_r^{-1}}$

10.3.3

Given: $L_r = D_p/D_m = (4\text{ ft})/(0.5\text{ ft}) = 8.0$, $\rho_r = 0.8$, and

$\mu_r = \mu_p/\mu_m = (9.93 \times 10^{-5})/(2.09 \times 10^{-5}) = 4.75$

From Equation (10.19): $(\rho_r L_r V_r)/\mu_r = 1$

$(0.8)(8.0)(V_r)/(4.75) = 1$; $V_r = 0.742 = V_p/V_m$;

$V_p = Q_p/A_p = 125/[(\pi/4)(4)^2] = 9.95$ ft/sec

$V_r = 0.742 = V_p/V_m = 9.95/V_m$; $V_m = 13.4$ ft/sec

Thus, $\mathbf{Q_m} = A_m \cdot V_m = [(\pi/4)(0.5)^2](13.4)$ **= 2.63 cfs**

10.3.4

Since inertial and viscous forces dominate, the Reynolds number law governs the flow.

Based on the problem statement: $\rho_r = 1$, $\mu_r = 1$

From Eq'n (10.19), $(\rho_r L_r V_r)/\mu_r = L_r V_r = 1$,

therfore, $V_r = 1/L_r = 1/20 = 0.05$

Since $V_r = V_p/V_m$; $\mathbf{V_p} = V_m \cdot V_r = 20 \cdot 0.05$ **= 1.00 m/sec**

From Table 10.2: $F_r = F_p/F_m = 1$, and by definition, Torque is force times distance: $T = F \cdot L$;

thus, $T_r = T_p/T_m = F_p \cdot L_p/F_m \cdot L_m = L_r = 20$ which yields

$\mathbf{T_p} = T_m \cdot T_r = (10\text{ N}\cdot\text{m})(20)$ **= 200 N·m**

10.3.5

Given: $L_r = 10$, $\rho_r = \rho_p/\rho_m = 969/998 = 0.971$, and

$\mu_r = \mu_p/\mu_m = (3.35 \times 10^{-4})/(1.00 \times 10^{-3}) = 0.335$

From Equation (10.19): $(\rho_r L_r V_r)/\mu_r = 1$

$(0.971)(10)(V_r)/(0.335) = 1$; $V_r = 0.0345 = V_p/V_m$;

Since $Q = AV$; dimensionally, $Q_r = (L_r)^2(V_r)$ or

$Q_r = (10)^2(0.0345) = 3.45$. Thus, $Q_r = Q_p/Q_m$ or

$\mathbf{Q_m} = Q_p/Q_r = 5.00/3.45$ **= 1.45 m³/sec**

10.3.6

(a) From Eq'n (10.19), $(\rho_r L_r V_r)/\mu_r = L_r V_r = 1$,

$V_r = 1/25 = 0.04$; $\mathbf{V_m} = V_p/V_r = 5/0.04$ **= 125 m/sec**

(b) Air model, $\rho_r = \rho_p/\rho_m = 1030/1.204 = 855$, and

$\mu_r = \mu_p/\mu_m = (1.57 \times 10^{-3})/(1.82 \times 10^{-5}) = 86.3$

$V_r = \mu_r/(\rho_r L_r) = 86.3/(855 \cdot 25) = 0.00404$

$\mathbf{V_m} = V_p/V_r = 5/0.00404$ **= 1,240 m/sec**

10.4.1

Based on the Froude number law, assuming $\rho_r = 1$, use of Table 10.3 produces: $T_m = T_p/T_r = T_p/L_r^{1/2}$

$\mathbf{T_m}$ = 1 day/$(1000)^{1/2}$ **= 0.0316 day (45.5 min)**

10.4.2

The Froude number law (for $g_r = 1$, $\rho_r = 1$) is

$(V_r)/(g_r^{1/2}L_r^{1/2}) = (V_r)/(L_r^{1/2}) = 1$. Therefore,

a) $\mathbf{V_r = L_r^{1/2}}$

b) Eq'n (10.6): $V_r = L_r/T_r$; $L_r^{1/2} = L_r/T_r$; $\mathbf{T_r = L_r^{1/2}}$

c) Eq'n (10.7): $a_r = L_r/T_r^2 = L_r/(L_r^{1/2})^2$; $\mathbf{a_r = 1}$

d) Eq'n (10.8): $\mathbf{Q_r} = L_r^3/T_r = L_r^3/L_r^{1/2} = \mathbf{L_r^{5/2}}$

e) Eq'n (10.13): $\mathbf{F_r} = \rho_r L_r^4 T_r^{-2} = L_r^4(L_r^{1/2})^{-2} = \mathbf{L_r^3}$

f) Eq'n (10.16): $\mathbf{P_r} = F_r L_r T_r^{-1} = (L_r^3)(L_r)(L_r^{1/2})^{-1} = \mathbf{L_r^{7/2}}$

10.4.3

The Froude number law (for $g_r = 1$, $\rho_r = 1$) yields:

$\mathbf{Q_m} = Q_p/Q_r = Q_p/L_r^{5/2} = 14{,}100/(10)^{5/2}$ **= 44.6 cfs,** and

$\mathbf{V_p} = Q_p/A_p = 14{,}100/[(100)(2.60)]$ **= 54.2 ft/sec**

$\mathbf{V_m} = V_p/V_r = V_p/L_r^{1/2} = 54.2/(10)^{1/2}$ **= 17.1 ft/sec**

10.4.4

Determine the unit flow rate for the prototype:

$q_p = Q_p/b = 3600/300 = 12.0$ m^2/sec. Also,

$q_r = L_r^2/T_r = L_r^2/L_r^{1/2} = L_r^{3/2}$, therefore

$\mathbf{q_m} = q_p/q_r = q_p/L_r^{3/2} = 12.0/(20)^{3/2}$ **= 0.134 m²/sec**

Since the model is 1 m wide, this is the flow rate.

10.4.5

a) The Froude number law (for $g_r = 1$, $\rho_r = 1$) yields:

$\mathbf{Q_m} = Q_p/Q_r = Q_p/L_r^{5/2} = (2650)/(25)^{5/2}$ **= 0.848 cfs**

b) $\mathbf{V_m} = V_p/V_r = V_p/L_r^{1/2} = 32.8/(25)^{1/2}$ **= 6.56 ft/sec**

c) $V_p = Q_p/A_p$; $32.8 = 2650/[(82)(y_p)]$; $y_p = 0.985$ ft

$\mathbf{N_f} = (V_p)/(g_p \cdot y_p)^{1/2} = (32.8)/(32.2\cdot 0.985)^{1/2}$ **= 5.82**

The model Froude number is the same. Let's verify.

Since the model channel width is (82 ft)/L_r = 3.28 ft,

$V_m = Q_m/A_m$; $6.56 = 0.848/[(3.28)(y_m)]$; $y_m = 0.0394$ ft

Alternatively, the model depth is $0.985/L_r = 0.0394$ ft,

$\mathbf{N_f} = (V_m)/(g_m \cdot y_m)^{1/2} = (6.56)/(32.2\cdot 0.0394)^{1/2}$ **= 5.82**

d) Based on Fig. 8.21, for $N_f = 5.82$, $TW/d_1 = 8$. Thus

TW = (8)(0.985) **= 7.88 ft =** $\mathbf{y_2}$**.**

10.4.6

a) $\mathbf{V_p} = V_m \cdot V_r = V_m \cdot L_r^{1/2} = 3.54(50)^{1/2}$ **= 25.0 m/sec**

b) $V_p = Q_p/A_p$; $25.0 = 1200/[(50)(y_p)]$; $y_p = 0.960$ m

$\mathbf{N_f} = (V_p)/(g_p \cdot y_p)^{1/2} = (25.0)/(9.81\cdot 0.960)^{1/2}$ **= 8.15**

c) Based on Fig 8.21, for $N_f = 8.15$, $TW/d_1 = 11$. Thus,

TW = (11)(0.96) **= 10.6 m =** $\mathbf{y_2}$**.**

d) From Eq'ns (6.20) and (5.4), $\Delta E = (y_2 - y_1)^3/(4y_1y_2)$

$\Delta E = (10.6 - 0.96)^3/(4\cdot 0.96\cdot 10.6) = 22.0$ m (headloss).

P = $\gamma Q H_P = (9.79)(1200)(22.0)$ **= 2.58 x 10⁵ kW**

e) Based on Equation (6.8), the approach energy is:

$E = V^2/2g + y = (25.0)^2/2(9.81) + 0.960 = 32.8$ m.

The energy removal efficiency is:

e = $\Delta E/E = 22.0/32.8$ **= 0.671 (67.1%)**

10.5.1

From Equation (10.8): $Q_r = L_r^3/T_r$; and from the Weber number law (with $\rho_r = 1$, $\sigma_r = 1$);

$T_r = L_r^{3/2}$; Equation (10.29). Therefore

$\mathbf{Q_r} = L_r^3/T_r = L_r^3/L_r^{3/2} = L_r^{3/2} = (1/5)^{3/2} = \mathbf{0.0894}$

From Equation (10.13) [or from Eq'n (10.24)]:

$\mathbf{F_r} = \rho_r L_r^4 T_r^{-2} = (1)L_r^4(L_r^{3/2})^{-2} = L_r = \mathbf{0.200}$

10.5.2

From the Weber number law (with $\rho_r = 1$);

$(\rho_r V_r^2 L_r)/\sigma_r = V_r^2 L_r/\sigma_r = [(L_r/T_r)^2 L_r]/\sigma_r = 1$. Thus

$(L_r)^3/(\sigma_r T_r^2) = 1$ or $(10)^3/(\sigma_r \cdot 2^2) = 1$; $\sigma_r = 250$

Finally, $\boldsymbol{\sigma_p} = \sigma_r \cdot \sigma_m = 250 \cdot 150 = \mathbf{3.75 \times 10^4}$ **dyn/cm**

From Equation (10.13) [or from Eq'n (10.24)]:

$\mathbf{F_r} = \rho_r L_r^4 T_r^{-2} = (1)(10)^4(2)^{-2} = \mathbf{2{,}500}$

10.5.3

From Equation (10.8): $Q_r = L_r^3/T_r$; and from the Weber number law (with $\rho_r = 1$, $\sigma_r = 1$); $V_r = 1/L_r^{1/2}$; and $T_r = L_r^{3/2}$; Eq'ns (10.28) and (10.29). Therefore

a) $\mathbf{Q_r} = L_r^3/T_r = L_r^3/L_r^{3/2} = L_r^{3/2} = (100)^{3/2} = \mathbf{1{,}000}$

b) From Eq'n (3.11), $E = ½ MV^2$. Dimensionally,

$E_r = (F_r T_r^2 L_r^{-1})(L_r/T_r)^2 = F_r L_r = (\rho_r L_r^4 T_r^{-2})(L_r)$

$\mathbf{E_r} = \rho_r L_r^5(L_r^{3/2})^{-2} = \rho_r L_r^2 = (1)(100)^2 = \mathbf{10{,}000}$

c) From $p = F/A$ we have dimensionally, $p_r = F_r/L_r^2$

$\mathbf{p_r} = (\rho_r L_r^4 T_r^{-2})/(L_r)^2 = \rho_r L_r^2 (L_r^{3/2})^{-2} = \rho_r L_r^{-1} = \mathbf{0.01}$

d) From Eq'n (10.16): $P_r = F_r L_r/T_r = (\rho_r L_r^4 T_r^{-2})(L_r/T_r)$

$\mathbf{P_r} = \rho_r L_r^5 T_r^{-3} = \rho_r L_r^5 (L_r^{3/2})^{-3} = \rho_r L_r^{1/2} = (1)(100)^{1/2} = \mathbf{10}$

10.7.1

To satisfy both Reynolds and Froude number laws, based on Equation (10.30):

$\nu_r = L_r^{3/2} = (100)^{3/2} = 1000$

By definition; $\mu_r = \nu_r \cdot \rho_r$

Also $\rho_m = 0.9\rho_p$; thus $\rho_r = 1.11$

Therefore, $\mu_r = \nu_r \cdot \rho_r = (1000)(1.11) = 1{,}110$

Finally, $\boldsymbol{\mu_m} = \mu_p/\mu_r = 1.00 \times 10^{-3}/1{,}110$

$\boldsymbol{\mu_m} = \mathbf{9.01 \times 10^{-7}}$ **N·sec/m²**

10.7.2

The Froude number law (for $g_r = 1$, $\rho_r = 1$) is

$(V_r)/(g_r^{1/2} L_r^{1/2}) = (V_r)/(L_r^{1/2}) = 1$;

Thus $V_r = L_r^{1/2}$ and

$T_r = L_r/V_r = L_r/ L_r^{1/2} = L_r^{1/2}$.

Finally,

$F_r = \rho_r L_r^4 T_r^{-2} = (1)L_r^4(L_r^{1/2})^{-2}$

Thus, $F_r = L_r^3 = (250)^3$ and

$\mathbf{F_p} = F_m \cdot F_r = (10.7\ N)(250)^3$

$\mathbf{F_p} = \mathbf{1.67 \times 10^8}$ **N**

10.7.3

Based on the Froude number law (Table 10.3)

$V_r = L_r^{1/2} = (L_r^{1/2}) = (100)^{1/2} = 10$

Also, $\mathbf{V_m} = V_p/V_r = 1.5/(10)$

$\mathbf{V_m} = \mathbf{0.15}$ **m/sec**

10.7.4

Based on the Froude number law (Table 10.3)

$V_r = L_r^{1/2} = (150)^{1/2} = 12.2$; and

$T_r = L_r^{1/2} = (150)^{1/2} = 12.2$

The Reynolds number of the model is

$N_R = (V_m L_m)/\nu_m = (1)(0.1)/(1.00 \times 10^{-6})$

$N_R = 1.00 \times 10^5$

Thus, $C_D = 0.25$ and

$D_m = C_{Dm}[(1/2)\rho_m A_m V_m^2]$

$D_m = 0.25[(1/2)1000\cdot(0.1\cdot0.02)(1.0)^2]$

$N_R = 0.25$ N

$F_{Wm} = F_{Tm} - D_m = 0.3 - 0.25 = 0.05$ N

$\mathbf{V_p} = V_m \cdot V_r = (1)(12.2) = \mathbf{12.2\ m/sec}$

$A_p = A_m \cdot L_r^2 = (0.002\ m^2)(150)^2 = 45\ m^2$

The Reynolds number of the prototype is

$N_R = (V_p L_p)/\nu_p = (12.2)(15)/(1.00 \times 10^{-6})$

$N_R = 1.83 \times 10^8$

Thus, $C_D = 0.25$ and

$D_p = C_{Dp}[(1/2)\rho_p A_p V_p^2]$

$D_p = 0.25[(1/2)1000\cdot(45)(12.2)^2]$

$N_R = 837$ kN

$F_{Wp} = F_{Wm} \cdot F_r$

where $F_r = Lr^3$ (Table 10.3)

$F_{Wp} = (0.05)(150)^3 = 169$ kN

$F_{Tp} = D_p + F_{Wp}$

$F_{Tp} = 837 + 169$

$\mathbf{F_{Tp} = 1,006\ kN}$

10.8.1

For an undistorted model: $X_r = Y_r = L_r = 100$.

Thus, from Eq'n (10.33); $n_r = L_r^{1/6} = (100)^{1/6} = 2.15$

$\mathbf{n_m} = n_p/n_r = 0.045/2.15 = \mathbf{0.021.}$ Based on Ex 10.8:

$V_r = R_{hr}^{1/2} = Y_r^{1/2} = L_r^{1/2} = (100)^{1/2} = 10.0$

Also, $V_p = Q_p/A_p = 94.6/(1.2\cdot20) = 3.94$ ft/sec. Thus,

$\mathbf{V_m} = V_p/V_r = 3.94/10 = \mathbf{0.394\ ft/s}$

10.8.2

Based on roughness, $n_r = n_p/n_m = 0.035/0.018 = 1.94$

Based on Ex. 10.8: $n_r = Y_r^{2/3}/X_r^{1/2} = (80)^{2/3}/X_r^{1/2} = 1.94$

Thus, $\mathbf{X_r = 91.6.}$ Also from Ex. 10.8 (same Y_r)

$\mathbf{V_m} = V_p/V_r = 4.25/(80)^{1/2} = \mathbf{0.475\ m/s}$

10.8.3

For the same scale, $Y_r = X_r = L_r = 400$. Thus,

$n_r = Y_r^{2/3}/X_r^{1/2} = L_r^{1/6} = (400)^{1/6} = 2.71$

$\mathbf{n_m} = n_p/n_r = 0.035/2.71 = \mathbf{0.013}$

Very low – would be very hard to get a channel bed of granular material w/this low an n-value.

$V_r = R_{hr}^{1/2} = Y_r^{1/2} = L_r^{1/2} = (400)^{1/2} = 20.0$

$\mathbf{V_m} = V_p/V_r = 4.25/20 = \mathbf{0.213\ m/s}$ (low, but ok)

$Q_r = X_r Y_r^{3/2} = L_r^{5/2} = (400)^{5/2} = 3.20 \times 10^6$

$\mathbf{Q_m} = Q_p/Q_r = 850/(3.20 \times 10^6) = \mathbf{2.65 \times 10^{-4}\ m^3/s}$**, or**

(**0.266 L/sec**, low but ok. Check the Reynolds number.)

$\mathbf{N_r} = V_m Y_m/\nu = (0.213)(4/400)/1.1 \times 10^{-6} = \mathbf{1,940}$

Reynolds number is too low to keep flow turbulent.

10.8.4

For a large width to depth ratio, $R_{hr} = Y_r$.

Also, since gravitational forces dominate, we have

$N_f = V_r/[g_r^{1/2} \cdot R_{hr}^{1/2}] = 1.00$; $V_r = R_{hr}^{1/2} = Y_r^{1/2}$.

Using Manning's eq'n: $V_r = V_p/V_m = (1/n_r)R_{hr}^{2/3}S_r^{1/2}$;

but since $S_r = Y_r/X_r$, $V_r = Y_r^{1/2}$, and $R_{hr} = Y_r$;

$Y_r^{1/2} = (1/n_r)Y_r^{2/3}(Y_r/X_r)^{1/2}$. Rearranging and

substituting ($w/n_r = 0.031/0.033 = 0.939$) yields:

$\mathbf{Y_r} = (n_r \cdot X_r^{1/2})^{3/2} = [(0.939)(300)^{1/2}]^{3/2} = \mathbf{65.6}$

Since $Q_r = A_r \cdot V_r = (X_r \cdot Y_r)(Y_r^{1/2}) = (X_r)(Y_r^{3/2})$

$Q_r = (300)(65.6)^{3/2} = 1.59 \times 10^5$. Therefore,

$\mathbf{Q_p} = Q_m \cdot Q_r = (0.052)(1.59 \times 10^5) = \mathbf{8.27 \times 10^3\ m^3/sec}$

10.8.5

For a large width to depth ratio, $R_{hr} = Y_r = 65$.

Also, since gravitational forces dominate, we have

$N_f = V_r/[g_r^{1/2} \cdot R_{hr}^{1/2}] = 1.00$; $V_r = R_{hr}^{1/2} = Y_r^{1/2} = (65)^{1/2}$.

Using Manning's eq'n: $V_r = V_p/V_m = (1/n_r)R_{hr}^{2/3}S_r^{1/2}$;

but since $S_r = Y_r/X_r$, $V_r = Y_r^{1/2}$, and $R_{hr} = Y_r$;

$Y_r^{1/2} = (1/n_r)Y_r^{2/3}(Y_r/X_r)^{1/2}$. Rearranging and

substituting ($w/n_r = 0.03/0.02 = 1.5$) yields:

$\mathbf{X_r} = Y_r^{4/3}/n_r^2 = (65)^{4/3}/(1.5)^2 = \mathbf{116.}$ Since,

$V_r = R_{hr}^{1/2} = Y_r^{1/2} = (65)^{1/2}$ and $V_r = X_r/T_r$

$\mathbf{T_r} = X_r/V_r = 116/(65)^{1/2} = \mathbf{14.4}$. Also,

$Q_r = A_r \cdot V_r = (X_r \cdot Y_r)(Y_r^{1/2}) = (X_r)(Y_r^{3/2})$

$Q_r = (116)(65)^{3/2} = 60{,}800$. And since,

$Q_r = Q_p/Q_m$; $\mathbf{Q_m} = 10{,}600/60{,}800 = \mathbf{0.174\ cfs}$

10.9.1

Based on the problem statement: $q = f(H, g, h)$ or

$f'(q, H, g, h) = 0$ with $n = 4$, $m = 2$. Thus, there are

$n - m = 2$ dimensionless groups, and $Ø(\Pi_1,\Pi_2) = 0$.

Taking H and g as the repeating variables, we have

$\Pi_1 = H^a g^b q^c$ and $\Pi_2 = H^d g^e h^f$. From the Π_1 group,

$L^0T^0 = L^a(L/T^2)^b[L^3/(T \cdot L)]^c$. For L: $0 = a + b + 2c$; and

T: $0 = -2b - c$. Hence, $b = -(1/2)c$ and $a = -(3/2)c$.

yielding, $\Pi_1 = H^{-(3/2)c}g^{-(1/2)c}q^c = [q/(H^{3/2}g^{1/2})]^c$

From the Π_2 group: $L^0T^0 = L^d(L/T^2)^e(L)^f$. For L:

$0 = d + e + f$; and for T: $0 = -2e$. So, $e = 0$ and $d = -f$.

Hence, $\Pi_2 = H^{-f}g^0h^f = [h/H]^f$

$Ø(\Pi_1,\Pi_2) = Ø[q/(H^{3/2}g^{1/2}), h/H] = 0$; which restated is

$\mathbf{q = H^{3/2}g^{1/2}\ Ø'(h/H)}$ (i.e., identical to Ex. 10.9)

10.9.2

Based on the problem statement: $P = f(\omega, T)$ or

$f'(P, \omega, T) = 0$ with $n = 3$, $m = 3$. Thus, there are

$n - m = 0$ dimensionless groups, so all of the variables

are repeating and the relationship can't be reduced.

Hence, $\Pi = P^a\omega^bT^c$; which yields,

$F^0L^0T^0 = (F \cdot L/T)^a(1/T)^b(F \cdot L)^c$. For F: $0 = a + c$; for

L: $0 = a + c$; and finally for T: $T = -a - b$. Hence,

$c = -a$; and $b = -a$ yielding

$\Pi = P^a\omega^{-a}T^{-a} = [P/(\omega \cdot T)]^a$; which can be expressed as

$\mathbf{P = \omega \cdot T}$ Note that this is Equation (5.3) and contains no dimensional groupings since the number of variables matched the number of dimensions.

10.9.3

Based on the problem statement:

$\Delta P_l = f(D, V, \rho, \mu)$ or $f'(\Delta P_l, D, V, \rho, \mu) = 0$

with n = 5, m = 3. There are n – m = 2 dimensionless groups, and therefore $Ø(\Pi_1,\Pi_2) = 0$.

Taking D, V, and ρ as the repeating variables, we have

$\Pi_1 = D^aV^b\rho^c\mu^d$; $\Pi_2 = D^eV^f\rho^g\Delta P_l^h$.

Also note that based on Newton's 2nd Law (F = ma), the dimension for force in the MLT system of units is:

$F = (M)(L/T^2) = M{\cdot}L/T^2$ and thus pressure drop (force per unit area) per unit length can be expressed as:

$\Delta P_l = (M{\cdot}L/T^2)/L^3 = M/L^2T^2$

And the units for viscosity are:

$\mu = F{\cdot}T/L^2 = (M{\cdot}L/T^2){\cdot}T/L^2 = M/L{\cdot}T$

Therefore, from the Π_1 group,

$M^0L^0T^0 = L^a(L/T)^b[M/L^3]^c[M/L{\cdot}T]^d$

For M: $0 = c + d$; for L: $0 = a + b - 3c - d$; and for T: $0 = -b - d$; Hence, $c = -d$; $b = -d$; and $a = -d$.

yielding, $\Pi_1 = D^{-d}V^{-d}\rho^{-d}\mu^d = [DV\rho/\mu]^{-d}$

Π_2 group: $M^0L^0T^0 = L^e(L/T)^f[M/L^3]^g[M/L^2T^2]^h$

For M: $0 = g + h$; for L: $0 = e + f - 3g - 2h$; and for T: $0 = -f - 2h$; Hence, $g = -h$; $f = -2h$; and $e = h$.

yielding, $\Pi_2 = D^hV^{-2h}\rho^{-h}\Delta P_l^h = [\Delta P_l{\cdot}D/(V^2\rho)]^h$

Therefore, $Ø(\Pi_1,\Pi_2) = Ø[DV\rho/\mu, \Delta P_l{\cdot}D/(V^2\rho)] = 0$;

which may be restated as:

$\boldsymbol{\Delta P_l = (V^2\rho/D)\, Ø'(DV\rho/\mu)}$

10.9.4

Based on the problem statement:

$q = f(h, g, H, \rho, \mu)$ or $f'(q, h, g, H, \rho, \mu) = 0$

which results in n = 6, m = 3. There are n – m = 3 dimensionless groups, and therefore

$Ø(\Pi_1,\Pi_2,\Pi_3) = 0$.

Taking h, g, and μ as the repeating variables, we have

$\Pi_1 = h^ag^b\mu^cq^d$ $\Pi_2 = h^eg^f\mu^gH^h$ $\Pi_3 = h^ig^j\mu^k\rho^s$

Therefore, from the Π_1 group,

$F^0L^0T^0 = L^a(L/T^2)^b[FT/L^2]^c[L^3/T]^d$

For F: $0 = c$; for L: $0 = a + b - 2c + 3d$; and for T: $0 = -2b + c - d$; Hence, $c = 0$; $d = -2b$; and $a = 5b$.

yielding, $\Pi_1 = h^{5b}g^b\mu^0q^{-2b}$ or $\boldsymbol{\Pi_1 = [gh^5/q^2]}$

Π_2 group: $F^0L^0T^0 = L^e(L/T^2)^f[FT/L^2]^g[L]^h$

For F: $0 = g$; for L: $0 = e + f - 2g + h$; and for T: $0 = -2f + g$; Hence, $g = 0$; $f = 0$; and $e = -h$.

yielding, $\Pi_2 = h^{-h}g^0\mu^0H^h$ or $\boldsymbol{\Pi_2 = [H/h]}$

For the Π_3 group, we need density in FLT units.

Based on Newton's 2nd Law (F = ma), we have,

$F = (M)(L/T^2)$ or $M = F{\cdot}T^2/L$ making the density units $\rho = (F{\cdot}T^2/L)/L^3 = F{\cdot}T^2/L^4$. Therefore,

Π_3 group: $F^0L^0T^0 = L^i(L/T^2)^j[FT/L^2]^k[F{\cdot}T^2/L^4]^s$

For F: $0 = k + s$; for L: $0 = i + j - 2k - 4s$; and for T: $0 = -2j + k + 2s$; Hence, $k = -s$; $j = (½)s$; and $i = (3/2)s$

yielding, $\Pi_3 = h^{(3/2)s}g^{(1/2)s}\mu^{-s}\rho^s$ or $\Pi_3 = [\rho g^{1/2}h^{3/2}/\mu]$

which may also be expressed as: $\boldsymbol{\Pi_3 = [\rho(gh^3)^{1/2}/\mu]}$

10.9.5

Based on the problem statement:

$V = f(D, g, \rho, \mu, \sigma)$ or $f'(V, D, g, \rho, \mu, \sigma) = 0$

which results in n = 6, m = 3. There are n – m = 3 dimensionless groups, and thus $Ø(\Pi_1,\Pi_2,\Pi_3) = 0$.

Taking D, ρ, and μ as the repeating variables, we have

$\Pi_1 = D^a\rho^b\mu^cV^d$ $\Pi_2 = D^e\rho^f\mu^hg^i$ $\Pi_3 = D^j\rho^k\mu^s\sigma^t$

Also note that based on Newton's 2nd Law (F = ma), the dimension for force in the MLT system of units is:

$F = (M)(L/T^2) = M{\cdot}L/T^2$; thus units for viscosity are

$\mu = F{\cdot}T/L^2 = (M{\cdot}L/T^2){\cdot}T/L^2 = M/(LT)$

For surface tension, $\sigma = F/L = (M{\cdot}L/T^2)/L = M/T^2$

Therefore, from the Π_1 group,

$M^0L^0T^0 = L^a(M/L^3)^b[M/(LT)]^c[L/T]^d$

For M: $0 = b + c$; for L: $0 = a - 3b - c + d$; and for T: $0 = -c - d$; Hence, $c = -b$; $d = b$; and $a = b$.

yielding, $\Pi_1 = D^b\rho^b\mu^{-b}V^b$ or $\mathbf{\Pi_1 = [VD\rho/\mu]}$

Π_2 group: $M^0L^0T^0 = L^e(M/L^3)^f[M/(LT)]^h[L/T^2]^i$

For M: $0 = f + h$; for L: $0 = e - 3f - h + i$; and for T: $0 = -h - 2i$; Hence, $f = -h$; $i = -(1/2)h$; and $e = -(3/2)h$

yielding, $\Pi_2 = D^{-(3/2)h}\rho^{-h}\mu^hg^{-(1/2)h}$ or $\mathbf{\Pi_2 = [\mu/\{\rho(D^3g)^{1/2}\}]}$

Π_3 group: $M^0L^0T^0 = L^j(M/L^3)^k[M/(LT)]^s[M/T^2]^t$

For M: $0 = k + s + t$; for L: $0 = j - 3k - s$; and for T: $0 = -s - 2t$; Hence, $s = -2t$; $k = t$; and $j = t$

yielding, $\Pi_2 = D^t\rho^t\mu^{-2t}\sigma^t$ or $\mathbf{\Pi_3 = [D\rho\sigma/\mu^2]}$

Thus, $\mathbf{V = (\mu/D\rho)\ Ø'[\mu/\{\rho(D^3g)^{1/2}\}]\ Ø''[D\rho\sigma/\mu^2]}$

or, $\mathbf{V = (\mu/D\rho)\ Ø'''[\sigma/\{\rho g D^2\}]}$

10.9.6

Based on the problem statement:

$V = f(d, g, \rho, \varepsilon, \mu, \theta)$ or $f'(V, d, g, \rho, \varepsilon, \mu, \theta) = 0$

with n = 6, m = 3 (θ is dimensionless) and n – m = 3 dimensionless groups, and thus $Ø(\Pi_1,\Pi_2,\Pi_3) = 0$.

Taking d, g, and ρ as the repeating variables, we have

$\Pi_1 = d^ag^b\rho^cV^d$ $\Pi_2 = d^eg^f\rho^h\varepsilon^i$ $\Pi_3 = d^jg^k\rho^s\mu^t$

Also note that based on Newton's 2nd Law (F = ma), the dimension for force in the MLT system of units is:

$F = (M)(L/T^2) = M{\cdot}L/T^2$; thus units for viscosity are

$\mu = F{\cdot}T/L^2 = (M{\cdot}L/T^2){\cdot}T/L^2 = M/(LT)$

Therefore, from the Π_1 group,

$M^0L^0T^0 = L^a(L/T^2)^b[M/L^3]^c[L/T]^d$

For M: $0 = c$; for L: $0 = a + b - 3c + d$; and for T: $0 = -2b - d$; Hence, $c = 0$; $b = -(1/2)d$; and $a = -(1/2)d$.

yielding, $\Pi_1 = d^{-(1/2)d}g^{-(1/2)d}\rho^0V^d$ or $\mathbf{\Pi_1 = [V/(dg)^{1/2}]}$

Π_2 group: $M^0L^0T^0 = L^e(L/T^2)^f[M/L^3]^h[L]^i$

For M: $0 = h$; for L: $0 = e + f - 3h + i$; and for T: $0 = -2f$; Hence, $h = 0$; $f = 0$; and $i = -e$

yielding, $\Pi_2 = d^e\varepsilon^{-e}$ or $\mathbf{\Pi_2 = [d/\varepsilon]}$

Π_3 group: $M^0L^0T^0 = L^j(L/T^2)^k[M/L^3]^s[M/(LT)]^t$

For M: $0 = s + t$; for L: $0 = j + k - 3s - t$; and for T: $0 = -2k - t$; Hence, $t = -s$; $k = (1/2)s$; and $j = (3/2)s$

yielding, $\Pi_2 = d^{(3/2)s}g^{(1/2)s}\rho^s\mu^{-s}$ or $\mathbf{\Pi_3 = [(d^3g)^{1/2}\rho/\mu]}$

Other repeating variables may be selected, but they should yield the same trio of dimensionless groups.

Chapter 11 – Problem Solutions

11.1.1

a) Clouds – water (vapor) holding element
b) Precipitation – liquid transport
c) Interception/Depression storage/Snow pack – water (or ice) holding
d) Evaporation – vapor transport
e) Infiltration – liquid transport
f) Evapotranspiration – vapor transport
g) Aquifer – water holding/liquid transport
h) Surface runoff – liquid transport
i) River – liquid transport/water holding
j) Lake/Ocean – water holding element

11.1.2

a) Clouds – water (vapor) droplets condense around dust and oceanic salt particles
b) Precipitation – droplets pick up air pollutants
c) Interception/Depression storage – water picks up natural and anthropogenic pollutants
d) Evaporation – salts/minerals left behind
e) Infiltration – water dissolves and transports salts, minerals, and nutrients in the soil
f) Evapotranspiration – nutrients are transported into plants from the ground water
g) Aquifer – water dissolves and/or transports minerals/nutrients/pollutants in the aquifer
h) Surface runoff – water transports minerals, organics, and anthropogenic pollutants
i) Lakes/Rivers/Oceans – natural/anthropogenic pollutants are received, stored, and transported

---

11.1.3

Unique solutions depending on watershed chosen.

11.1.4

Apply Equation (11.1) using units of inches:

$P + Q_i - Q_o - I - E - T = \Delta S$, where

P = 4 in. (rainfall during month)

Q_i = [(10 gal/min)(60 min)(1 ft^3/7.48 gal)
(12 in./1 ft)/[(30ft)(10 ft)] = 3.2 in.

Q_o = 0 (no water drained during month); T = 0,

I = ? (leakage?); E = (8 in.)(1.25) = 10 in.; thus

4 in. + 3.2 in. - 0 - I - 10 in. - 0 = -5.0; I = 2.2 in.

Since I = 2.2 in., **there is a leak.** Gallons of water lost:

(2.2 in.)(1 ft/12 in.)[(30 ft)(10 ft)] (7.48 gal/1 ft^3) =

411 gal; **Leak =** 411 gal/30 days **= 13.7 gal/day**

11.1.5

Apply Equation (11.1) using units of cubic meters:

$P + Q_i - Q_{o1} - Q_{o2} - I - E - T = \Delta S$, where

P = (0.11 m)(40 hec)(10,000 m^3/1 hec) = 44,000 m^3

Q_i = [(1.9 m^3/s)(1 day)(86,400 sec/day) = 164,160 m^3

Q_{o1} = [(0.8 m^3/s)(1 day)(86,400 sec/day) = 69,120 m^3

Q_{o2} = [(1.9 m^3/s)(1 day)(86,400 sec/day) = 164,160 m^3

E = (0.03 m)(40 hec)(10,000 m^3/1 hec) = 12,000 m^3

ΔS = (0.155 m)(40 hec)(10,000 m^3/1 hec) = 62,000 m^3

where Q_{o2} = city water outflow. Eq'n 11.1 yields

44 + 164 - 69 - 164 - I - 12 - 0 = -62**; I = 25,000 m^3**

11.1.6

The control volume is the watershed with precipitation minus interception (P - Int) entering the control volume and infiltration (I) leaving. Apply Eq'n (11.1):

$P - Int - I = \Delta S$, w/P = 1.1 in., Int = 0.10 in.,

I = 0.65 in. Therefore, $P - Int - I = \Delta S$;

$1.1 - 0.10 - 0.65 = \Delta S = 0.35$ in. (runoff depth)

Rainfall (P) and Runoff (ΔS) in acre-feet:

P =(1.1 in)(1ft/12 in)(150sq.mi)(640ac/1sq.mi.)**= 8,800 ac-ft**

ΔS =(0.35in)(1ft/12in)(150sq.mi)(640ac/1sq.mi)**=2,800 ac-ft**

11.1.7

The control volume (CV) for the watershed results in surface reservoir storage (ΔS) only and infiltration as an outflow. Groundwater outflow is outside the CV. Thus,

$P + Q_i - Q_o - I - E - T = \Delta S$, where I = 0.560 km^3

$Q_i = 0.0$ (inflow to reservoirs is inside the CV)

$P = (0.74\text{ m})(1\text{ km}/1000\text{ m})(6200\text{ km}) = 4.59\text{ km}^3$

$E + T = (0.35\text{ m})(1\text{ km}/1000\text{ m})(6200\text{ km}) = 2.17\text{ km}^3$

$Q_o = (75.5\text{ m}^3/\text{s})(1\text{ km}^3/1\text{x}10^9\text{ m}^3)(1\text{ yr})(365\text{ days}/1\text{ yr})(86{,}400\text{ sec/day}) = 2.38\text{ km}^3$.

Therefore, the change of storage in the watershed is:

$4.59 + 0.0 – 2.38 – 0.56 – (2.17) = \mathbf{-0.52\ km^3 = \Delta S}$

The control volume (CV) for the drainage basin results in infiltration (I) being within the CV and groundwater outflow as a new boundary transfer. Thus, the change of storage (ΔS, surface water and groundwater) is:

$P + Q_i – Q_{o1} – Q_{o2} - I - E - T = \Delta S$,

$4.59 + 0.0 – 2.38 – 0.200 – 0.00 – (2.17) = \mathbf{-0.16\ km^3 = \Delta S}$

11.2.1

Lifting takes place by mechanical means or through a thermodynamic process. *Orographic precipitation* occurs through mechanical lifting of clouds moving over mountain ranges. Thermodynamic lifting produces c*onvective precipitation,* rising air masses in the tropics and over large cities due to solar heat gain. *Cyclonic precipitation* occurs when warm, moisture laden air masses rise over colder heavier air masses producing frontal storms (again through thermodynamic lifting).

11.2.2

a) Virginia is closer to a moisture source.

b) Both states are close to a moisture source, but Maine is further north where there is less evaporation.

c) It is close to a moisture source and the Sierra Nevada Mountains produce orographic precipitation.

d) Nevada is further from a moisture source and on the leeward side of the coastal mountains which extracted a lot of the moisture out of the clouds moving inland.

e) The Appalachian Mountains in western North Carolina produce orographic precipitation.

f) The Rocky Mountain states are far from a moisture source and on the leeward side of the coastal ranges in California which extract a lot of moisture.

11.2.3

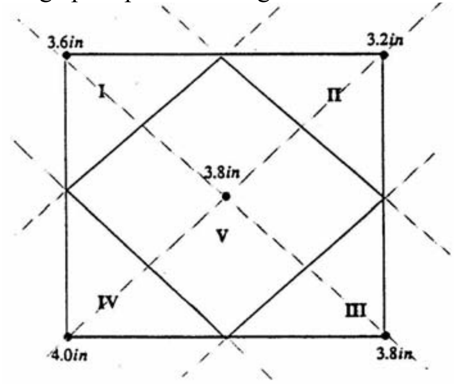
The average weighted precipitation is found from:

$P_{avg} = [(52\text{ km}^2)(12.4\text{ cm}) + (77\text{ km}^2)(11.4\text{ cm}) + (35\text{ km}^2)(12.6\text{ cm}) + (68\text{ km}^2)(9.9\text{ cm})]/(232\text{ km}^2)$

$\mathbf{P_{avg} = 11.4\ cm}$

11.2.4

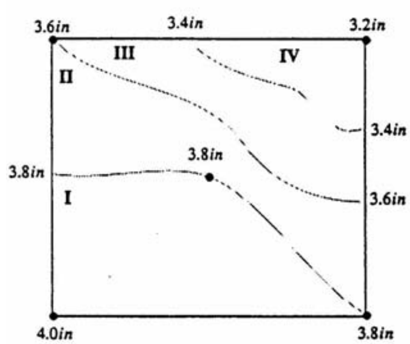
The average precipitation using the Thiessen method:



Area	Gage Precip.	Area Factor	Weighted Precip.
I	3.6	0.125	0.45
II	3.2	0.125	0.40
III	3.8	0.125	0.48
IV	4.0	0.125	0.50
V	3.8	0.500	1.90
Total		1.00	**3.73 in.**

11.2.5

The average precipitation using the isohyetal method:

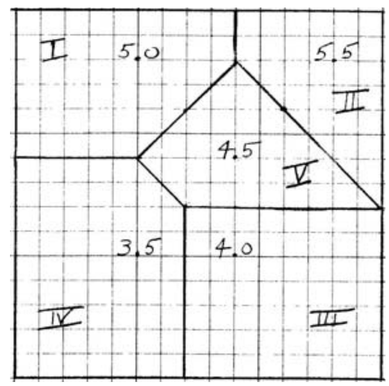


Area	Average Precip.	Area Factor	Weighted Precip.
I	3.85*	0.125	0.45
II	3.70	0.125	0.40
III	3.50	0.125	0.48
IV	3.35	0.125	0.50
Total		1.00	**3.73 in.**

*Avg. rainfall in this interval is closer to 3.8 in. than 4.0 in. since the 4.0 isohyet is outside the area of interest.

11.2.6

The average precipitation using the Thiessen method:

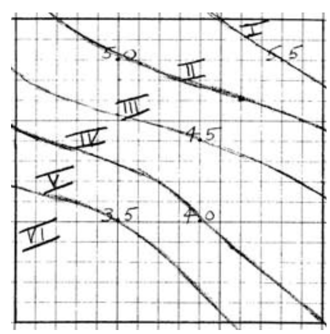


Area	Gage Precip.	Area Factor*	Weighted Precip.
I	5.0	0.204	1.02
II	5.5	0.133	0.73
III	4.0	0.249	1.00
IV	3.5	0.271	0.95
V	4.5	0.142	0.64
Total		1.000	**4.34 cm**

*The total area is 225 squares or 22,500 hectares.

11.2.7

The average precipitation using the isohyetal method:

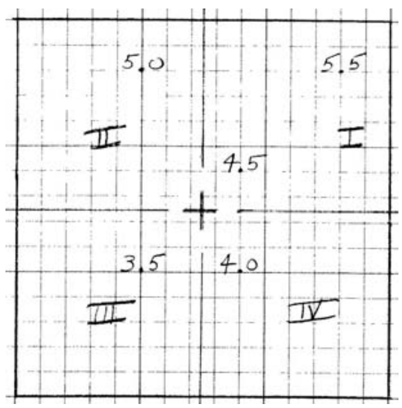


Area	Average Precip.	Area Factor*	Weighted Precip.
I	5.60*	0.036	0.20
II	5.25	0.133	0.70
III	4.75	0.200	0.95
IV	4.25	0.244	1.04
V	3.75	0.187	0.70
VI	3.40	0.200	0.68
Total		1.000	**4.27 cm**

*Avg. rainfall in this interval is closer to 5.5 cm than 6.0 cm since the 6.0 cm isohyet is outside the area of interest.

11.2.8

Using the inverse-distance weighting method:



Quadrant	Distance (d) to centroid*	Gage Precip.	Weighting Factor**	Weighted Precip.
I	2,120 m	4.5	0.497	2.24
II	6,040 m	5.0	0.061	0.31
III	3,540 m	3.5	0.178	0.62
IV	2,910 m	4.0	0.264	1.06
Total			1.000	**4.23 cm**

*Distances are found using Pythagorean theorem.
**Weighting factor in quadrant 1:

$$w_1 = (1/d_1^2) / [1/d_1^2+1/d_2^2+1/d_3^2+1/d_4^2]$$

11.2.9

One method is to use the average of the surrounding gages to estimate the storm precipitation depth:

$P_x = (6.02 + 6.73 + 5.51)/3$; $\mathbf{P_x = 6.09}$ **in.**

Another possibility is to use the annual precipitations to establish a storm weighting factor:

Wt. = [(6.02/61.3) + (6.73/72.0) + (5.51/53.9)]/3

Wt. = 0.098

$P_x = (0.098)(53.9)$; $\mathbf{P_x = 5.28}$ **in.**

The weighting method is a better estimate than the average method. It takes into consideration that Station X receives less precipitation than the nearby stations.

11.3.1

t_1 (hr)	t_2 (hr)	P_1/P_T	P_2/P_T	P_1 (in.)	P_2 (in.)	ΔP (in.)	i (in./hr)
0	0.5	0.000	0.005	0.000	0.030	0.030	0.060
0.5	1	0.005	0.010	0.030	0.060	0.030	0.060
1	1.5	0.010	0.015	0.060	0.090	0.030	0.060
1.5	2	0.015	0.020	0.090	0.120	0.030	0.060
2	2.5	0.020	0.026	0.120	0.156	0.036	0.072
2.5	3	0.026	0.032	0.156	0.192	0.036	0.072
3	3.5	0.032	0.037	0.192	0.222	0.030	0.060
3.5	4	0.037	0.043	0.222	0.258	0.036	0.072
4	4.5	0.043	0.050	0.258	0.300	0.042	0.084
4.5	5	0.050	0.057	0.300	0.342	0.042	0.084
5	5.5	0.057	0.065	0.342	0.390	0.048	0.096
5.5	6	0.065	0.072	0.390	0.432	0.042	0.084
6	6.5	0.072	0.081	0.432	0.486	0.054	0.108
6.5	7	0.081	0.089	0.486	0.534	0.048	0.096
7	7.5	0.089	0.102	0.534	0.612	0.078	0.156
7.5	8	0.102	0.115	0.612	0.690	0.078	0.156
8	8.5	0.115	0.130	0.690	0.780	0.090	0.180
8.5	9	0.130	0.148	0.780	0.888	0.108	0.216
9	9.5	0.148	0.167	0.888	1.002	0.114	0.228
9.5	10	0.167	0.189	1.002	1.134	0.132	0.264
10	10.5	0.189	0.216	1.134	1.296	0.162	0.324
10.5	11	0.216	0.250	1.296	1.500	0.204	0.408
11	11.5	0.250	0.298	1.500	1.788	0.288	0.576
11.5	12	0.298	0.600	1.788	3.600	1.812	3.624
12	12.5	0.600	0.702	3.600	4.212	0.612	1.224
12.5	13	0.702	0.751	4.212	4.506	0.294	0.588
13	13.5	0.751	0.785	4.506	4.710	0.204	0.408
13.5	14	0.785	0.811	4.710	4.866	0.156	0.312
14	14.5	0.811	0.830	4.866	4.980	0.114	0.228
14.5	15	0.830	0.848	4.980	5.088	0.108	0.216
15	15.5	0.848	0.867	5.088	5.202	0.114	0.228
15.5	16	0.867	0.886	5.202	5.316	0.114	0.228
16	16.5	0.886	0.895	5.316	5.370	0.054	0.108
16.5	17	0.895	0.904	5.370	5.424	0.054	0.108
17	17.5	0.904	0.913	5.424	5.478	0.054	0.108
17.5	18	0.913	0.922	5.478	5.532	0.054	0.108
18	18.5	0.922	0.930	5.532	5.580	0.048	0.096
18.5	19	0.930	0.939	5.580	5.634	0.054	0.108
19	19.5	0.939	0.948	5.634	5.688	0.054	0.108
19.5	20	0.948	0.957	5.688	5.742	0.054	0.108

Etc.

$\mathbf{i_{(peak)} = 3.624}$ **in/hr** (vs. 4.56 in/hr); at the same time.

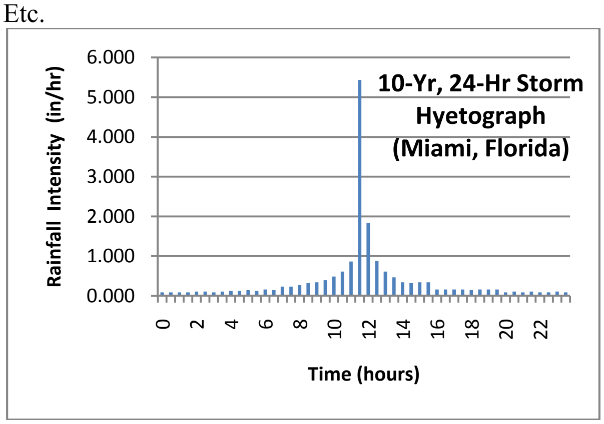
11.3.2

10-Year, 24-Hour Storm Hyetograph (Miami, FL)

P_T = 9 in. Type III Δt = 0.5 hr.

t_1 (hr)	t_2 (hr)	P_1/P_T	P_2/P_T	P_1 (in.)	P_2 (in.)	ΔP (in.)	i (in./hr)
0	0.5	0.000	0.005	0.000	0.045	0.045	0.090
0.5	1	0.005	0.010	0.045	0.090	0.045	0.090
1	1.5	0.010	0.015	0.090	0.135	0.045	0.090
1.5	2	0.015	0.020	0.135	0.180	0.045	0.090
2	2.5	0.020	0.026	0.180	0.234	0.054	0.108
2.5	3	0.026	0.032	0.234	0.288	0.054	0.108
3	3.5	0.032	0.037	0.288	0.333	0.045	0.090
3.5	4	0.037	0.043	0.333	0.387	0.054	0.108
4	4.5	0.043	0.050	0.387	0.450	0.063	0.126
4.5	5	0.050	0.057	0.450	0.513	0.063	0.126
5	5.5	0.057	0.065	0.513	0.585	0.072	0.144
5.5	6	0.065	0.072	0.585	0.648	0.063	0.126
6	6.5	0.072	0.081	0.648	0.729	0.081	0.162
6.5	7	0.081	0.089	0.729	0.801	0.072	0.144
7	7.5	0.089	0.102	0.801	0.918	0.117	0.234
7.5	8	0.102	0.115	0.918	1.035	0.117	0.234
8	8.5	0.115	0.130	1.035	1.170	0.135	0.270
8.5	9	0.130	0.148	1.170	1.332	0.162	0.324
9	9.5	0.148	0.167	1.332	1.503	0.171	0.342
9.5	10	0.167	0.189	1.503	1.701	0.198	0.396
10	10.5	0.189	0.216	1.701	1.944	0.243	0.486
10.5	11	0.216	0.250	1.944	2.250	0.306	0.612
11	11.5	0.250	0.298	2.250	2.682	0.432	0.864
11.5	12	0.298	0.600	2.682	5.400	2.718	5.436
12	12.5	0.600	0.702	5.400	6.318	0.918	1.836
12.5	13	0.702	0.751	6.318	6.759	0.441	0.882
13	13.5	0.751	0.785	6.759	7.065	0.306	0.612
13.5	14	0.785	0.811	7.065	7.299	0.234	0.468
14	14.5	0.811	0.830	7.299	7.470	0.171	0.342

Etc.



11.3.3

Since small storms are "nested" within the 24-hr storm, we can easily estimate the 6-hr storm hyetograph. The 6 hour time period that contains the greatest percentage of rainfall depth is from hour 9 to hour 15 (0.856 – 0.147 = 0.709 or 70.9% of the rainfall). Using the 24-hr storm ratio intervals, but setting the time at hour 9 to zero and hour 15 to hour 6 yields the following:

10-year, 6-Hour Storm Hyetograph (Va. Beach. VA)

P = 6 in. Δt = 0.5 hr.

t_1 (hr)	t_2 (hr)	P_1/P_T	P_2/P_T	P_1 (in.)	P_2 (in.)	ΔP (in.)	i (in./hr)
0	0.5	0.147	0.163	0.882	0.978	0.096	0.192
0.5	1	0.163	0.181	0.978	1.086	0.108	0.216
1	1.5	0.181	0.203	1.086	1.218	0.132	0.264
1.5	2	0.203	0.236	1.218	1.416	0.198	0.396
2	2.5	0.236	0.283	1.416	1.698	0.282	0.564
2.5	3	0.283	0.663	1.698	3.978	2.280	4.560
3	3.5	0.663	0.735	3.978	4.410	0.432	0.864
3.5	4	0.735	0.776	4.410	4.656	0.246	0.492
4	4.5	0.776	0.804	4.656	4.824	0.168	0.336
4.5	5	0.804	0.825	4.824	4.950	0.126	0.252
5	5.5	0.825	0.842	4.950	5.052	0.102	0.204
5.5	6	0.842	0.856	5.052	5.136	0.084	0.168

Note: Using the intensities in the table, it is easily determined that this storm produces 4.254 inches of rainfall, or 70.9 percent of the 24-hr depth of 6.0 inches.

11.3.4

The total depth of rainfall is:

P(total) = ∑P = 1.50 in.

The maximum intensity (between time 18 and 21):

i = (0.18 in.)/[(3/60) hr]; **i = 3.6 in./hr**

The rainfall volume is:

Vol = [(1.5 in.)/(12 in./ft)] (150 sq mi)(640 ac/sq mi)

Vol = 12,000 ac. ft.

11.4.1

Advantages: Bridges constrict the flow, minimizing the width of the x-section. Also, it is very helpful to have a bridge to stand on (rather than a boat) when taking stream velocity measurements. This is especially true during flood events, and high flow measurements are needed to obtain a full and useful rating curve.

Disadvantages: Flow rates are often high through constricted sections, so velocity measurements must be accurate. Also, the channel x-section under bridges are not always stable (scour and deposition).

11.4.2

USGS provides information on the gage location, the drainage area, period of record, gage type and datum, special remarks, and extremes outside the period of record. Daily mean flows are reported (for the water year from October 1 to September 30) along with summary statistics. Other data includes lake elevations, ground water elevations, precipitation information, and water quality information at selected gages.

11.4.3

A spreadsheet solution is appropriate. Most areas are trapezoids and $Q = AV_{avg}$ in each section.

V	1	2	3	4	5	6	7	8	9	10	11
0.2y	0.2*	2.0	3.3	4.3	4.5	4.7	4.8	4.4	4.2	3.8	3.0
0.8y		1.4	2.3	3.3	3.7	3.9	3.8	3.6	3.4	2.0	1.2
d	1.0	1.6	1.8	2.0	2.0	2.0	2.0	2.0	2.0	1.6	0.6
Vavg	0.2	1.7	2.8	3.8	4.1	4.3	4.3	4.0	3.8	2.9	2.1
A	0.5	1.3	1.7	1.9	2.0	2.0	2.0	2.0	2.0	1.8	1.1
Q	0.1	2.2	4.8	7.2	8.2	8.6	8.6	8.0	7.6	5.2	2.3

Q(total) = ∑Q = 62.8 m^3/sec

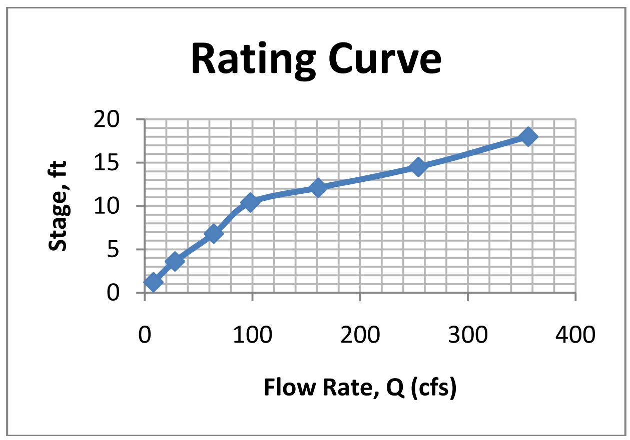
11.4.4

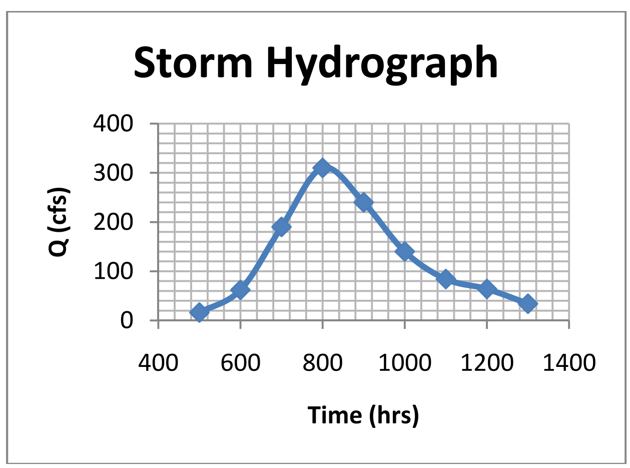
A spreadsheet solution is appropriate. Most areas are trapezoids and $Q = AV_{avg}$ in each section.

V	1	2	3	4	5	6	7	8	9	10	11
0.2y	0.1*	0.3	0.4	0.5	0.6	0.7	0.7	0.6	0.5	0.4	0.3
0.8y		0.1	0.2	0.3	0.4	0.5	0.5	0.4	0.3	0.2	0.1
d	1.8	3.6	4.2	4.8	4.8	4.8	4.8	4.8	4.8	3.6	1.0
Vav	0.1	0.2	0.3	0.4	0.5	0.6	0.6	0.5	0.4	0.3	0.2
A	1.8	5.4	7.8	9.0	9.6	9.6	9.6	9.6	9.6	8.4	4.6
Q	0.2	1.1	2.3	3.6	4.8	5.8	5.8	4.8	3.8	2.5	0.9

Q(total) = ∑Q = 35.6 cubic feet per second

11.4.5





Q(peak) ≈ 310 cfs

11.5.1

a) Yes. Since the rainfall would occur over a shorter period of time, the flow rates would increase more rapidly on the rising limb and give a larger peak Q.

b) As water infiltrates into the ground, the water table would rise and contribute more flow to the stream.

c) Each flow value represents the average flow over a 6-hour time interval. To obtain a volume using the average flows, we multiply them by time intervals (or add them and multiply the sum by the time interval according to the distributive law). This is merely a finite difference method used to determine the area under the curve (numerical integration).

d) No. The sum the runoff values would likely double, but the sum would be multiplied by a 3-hour time increment instead of a 6-hour time increment so the depth of rainfall would remain unchanged.

11.5.2

The unit hydrograph (UH) is tabulated below, a **4-hour UH** since effective precipitation occurs over 4 hours. The 2.5-inch rain produces 1.5 in. of runoff depth.

Hour	Rainfall (in)	Stream Flow (cfs)	Base Flow (cfs)	Direct Runoff (cfs)	Unit Hyd (cfs)	Hours After Start
0		20	20	0	**0**	**0**
	1.25					
2		90	22	68	**45**	**2**
	1.25					
4		370	24	346	**231**	**4**
6		760	26	734	**489**	**6**
8		610	28	582	**388**	**8**
10		380	30	350	**233**	**10**
12		200	32	168	**112**	**12**
14		130	34	96	**64**	**14**
16		90	36	54	**36**	**16**
18		60	38	22	**15**	**18**
20		40	40	0	**0**	**20**
Sum	2.5			2420		
			Runoff	1.50	inches	

11.5.3

The unit hydrograph (UH) is tabulated below, a **1-hour UH** since effective precipitation occurs over 1 hour. The 1.0-cm rain produces 0.73 cm. of runoff depth.

Hour	Rainfall (cm)	Stream Flow (m^3/s)	Base Flow (m^3/s)	Direct Runoff (m^3/s)	Unit Hyd (m^3/s)	Hours After Start
8		1.6	1.6	0.0		
9		1.4	1.4	0.0	**0.0**	**0**
	1					
10		4.6	1.6	3.0	**4.1**	**1**
11		9.7	1.8	7.9	**10.8**	**2**
12		13.0	2.0	11.0	**15.1**	**3**
13		10.5	2.2	8.3	**11.4**	**4**
14		7.8	2.4	5.4	**7.4**	**5**
15		5.8	2.6	3.2	**4.4**	**6**
16		4.5	2.8	1.7	**2.3**	**7**
17		3.0	3.0	0.0	**0.0**	**8**
18		2.9	2.9	0.0		
19		2.8	2.8	0.0		
Sum	1			40.5		
			Runoff	0.73	cm	

11.5.4

The unit hydrograph (UH) is tabulated below, a **3-hour UH** since effective precipitation occurs over 3 hours.

Hour	Rainfall (in.)	Stream Flow (cfs)	Base Flow (cfs)	Direct Runoff (cfs)	**Unit Hyd (cfs)**	**Hours After Start**
400		110	110	0	**0**	
	0.2					
600		100	100	0	**0**	**0**
	1.4					
800		1200	106	1094	**1257**	**2**
	0.7					
1000		2000	112	1888	**2170**	**4**
1200		1600	118	1482	**1703**	**6**
1400		1270	124	1146	**1317**	**8**
1600		1000	130	870	**1000**	**10**
1800		700	136	564	**648**	**12**
2000		500	142	358	**411**	**14**
2200		300	148	152	**175**	**16**
2400		180	154	26	**30**	**18**
200		160	160	0	**0**	**20**
Sum	2.3			7580		
			Runoff	0.87	in.	

11.5.5

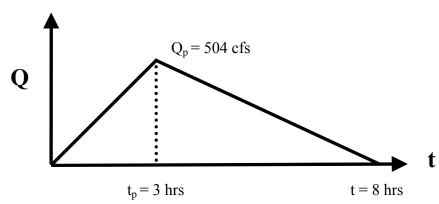
The storm hydrograph is depicted below. The runoff volume can be found by determine the triangular area under the storm hydrograph.

R/O volume = [(1/2)(8 hrs)(3600 sec/hr)(504 ft^3/sec)]

R/O volume = 7.26 x 10^6 ft^3

R/O depth = (7.26 x 10^6 ft^3)/[(1000 acres)(43,560 ft^3/ac)

R/O depth = 0.167 ft = 2.00 in.



Since the storm produces 2.00 inches of runoff, **the unit hydrograph is a triangle half as high (Q_p = 252 cfs)**. Also, it is a **2-hour unit hydrograph** since the excess rainfall occurs during a 2 hour period.

11.5.6

Subracting losses from the rainfall yields 1.5 cm of runoff in the 1st hour and 2.0 cm in the 2nd hour. Thus, multiplying the unit hydrograph by these amounts and lagging the second hour of runoff by an hour yields:

Hour	Unit Hyd. (m^3/s)	1.5×UH (m^3/s)	2×UH (m^3/s)	Base Flow (m^3/s)	**Stream Flow (m^3/s)**
0	0	0		5	**5**
0.5	10	15		5	**20**
1	30	45	0	5	**50**
1.5	70	105	20	5	**130**
2	120	180	60	5	**245**
2.5	100	150	140	5	**295**
3	70	105	240	5	**350**
3.5	40	60	200	5	**265**
4	20	30	140	5	**175**
4.5	0	0	80	5	**85**
5	0	0	40	5	**45**
5.5	0	0	0	5	**5**

11.5.7

The resulting stream flow is tabulated below.

Hour	Unit Hyd. (m^3/s)	3×UH* (m^3/s)	3×UH* (m^3/s)	4×UH (m^3/s)	Base Flow (m^3/s)	**Stream Flow (m^3/s)**
0	0	0			150	**150**
6	38	114			157	**271**
12	91	273	0		165	**438**
18	125	375	114		172	**661**
24	110	330	273	0	180	**783**
30	78	234	375	152	187	**948**
36	44	132	330	364	195	**1021**
42	26	78	234	500	202	**1014**
48	14	42	132	440	210	**824**
54	7	21	78	312	217	**628**
60	0	0	42	176	225	**443**
66			21	104	200	**325**
72			0	56	180	**236**
78				28	170	**198**
84				0	160	**160**

*Assumes 3 inches of runoff occurs in each 6-hour period of the first 12-hour period.

11.5.8

In order to use a 1-hour unit hydrograph (U.H.) as a design tool, runoff depths must be available in 1 hour increments. Hence, the runoff depths are 1.0 inch in the first hour and 0.5 inches in the second hour. The predicted stream flow is tabulated below.

Hour	Unit Hyd. (cfs)	1×UH (cfs)	0.5×UH (cfs)	Base Flow (cfs)	**Stream Flow (cfs)**
0	0	0		10	**10**
1	60	60	0	10	**70**
2	100	100	30	10	**140**
3	80	80	50	10	**140**
4	50	50	40	10	**100**
5	20	20	25	10	**55**
6	0	0	10	10	**20**
7			0	10	**10**
				Σ =	545

The runoff volume is:

Vol = (545 ft^3/s)(1 hr)(3600 sec/hr)(1 ac/43,560 ft^2)

Vol = 45.0 ac-ft

11.5.9

In order to use a 12-hour unit hydrograph (UH_{12}) as a design tool, runoff depths must be available in 12 hour increments. Hence, the runoff depths are 0.5 inch in the first 12 hours and 0.5 inches in the second 12 hours. The predicted 24-hr unit hydrograph (UH_{24}) is:

Hour	UH_{12} (m^3/s)		$0.5{\times}UH_{12}$ (m^3/s)	$0.5{\times}UH_{12}$ (m^3/s)	**UH_{24} (m^3/s)**
0	0		0.0		**0.0**
6	38		19.0		**19.0**
12	91		45.5	0.0	**45.5**
18	125		62.5	19.0	**81.5**
24	110		55.0	45.5	**100.5**
30	78		39.0	62.5	**101.5**
36	44		22.0	55.0	**77.0**
42	26		13.0	39.0	**52.0**
48	14		7.0	22.0	**29.0**
54	7		3.5	13.0	**16.5**
60	0		0.0	7.0	**7.0**
66				3.5	**3.5**
72				0.0	**0.0**

To find the UH_6, place the UH_{12} in the far right column in the table below and work numerically backwards. Some instability in the numerical process occurs in the hydrograph tail, so it should be graphed and adjusted so that the resulting UH_6 produces one inch of runoff. A 3 hour interval would produce less numerical instability. The asterisks below depict where the instability occurs.

Hour	**UH_6 (m^3/s)**		$0.5{\times}UH_6$ (m^3/s)	$0.5{\times}UH_6$ (m^3/s)	UH12 (m^3/s)
0	**0**		0.0		0
6	**76**		38.0	0	38
12	**106**		53.0	38.0	91
18	**144**		72.0	53.0	125
24	**76**		38.0	72.0	110
30	**80**		40.0	38.0	78
36	**8**		4.0	40.0	44
42	*		22.0	4.0	26
48	*		-8.0	22.0	14
54	*		15.0	-8.0	7
60	*		-15.0	15.0	0

11.5.10

In order to use original storm flows to predict the future design storm, we need to formulate a unit hydrograph. Since it is a 2 hour storm of uniform intensity, a 2-hour unit hydrograph (UH_2) is formulated by dividing the original flows by two. With this a design tool, stream flows are predicted in the normal way by multiplying 2-hour runoff depths by the unit hydrograph (linearity) and adding the resulting flows (superposition).

Hour	Q (m^3/s)	UH_2 (m^3/s)	$1.5{\times}UH_2$ (m^3/s)	$3.0{\times}UH_2$ (m^3/s)	New Q (m^3/s)	
4:00	0	0	0		0	
5:00	160	80	120		120	
6:00	440	220	330	0	330	
7:00	920	460	690	240	930	
8:00	860	430	645	660	1305	
9:00	720	360	540	1380	**1920**	**Peak Q**
10:00	580	290	435	1290	1725	

11.6.1

Based on Table 11.3, we have B soils. Based on Table 11.4, CN = 61 for the open space and CN = 88 for the industrial park. The area-weighted composite curve number representing the whole watershed is:

CN = [(169 ac)(61) + (31 ac)(88)]/200 ac = 65

Then based on Figure 11.8 and the location of Chicago, the 10-year, 24-hour rainfall depth is 4.0 inches. Next, using Equations 11.4 and 11.3, we have

S = {1000 – 10(65)}/65 = 5.4 in.

$R = [P - 0.2(S)]^2/[P + 0.8(S)] =$

R = $[4.0 - 0.2(5.4)]^2/[4.0 + 0.8(5.4)]$ **= 1.0 in.**

Alternatively, Figure 11.18 yields the same answer.

The volume of runoff is:

Vol. = (1.0 in.)(1 ft/12 in.)(200 ac) **= 16.7 ac. ft.**

11.6.2

Based on Table 11.3, we have B soils (Drexel), C soils (Bremer) and A soils (Donica). Based on Table 11.4, CN = 61 for the golf course - B soils, CN = 74 for the golf course - C soils, CN = 94 for the commercial area - C soils, and CN = 54 for the residential area – A soils. The area-weighted composite curve number representing the whole watershed is:

CN = [(8 hec)(61) + (12 hec)(74) + (30 hec)(94) + (50 hec)(54)]/100 hec = 69.0

For a 15 cm (5.91 in.) storm, use Eq'ns 11.4 and 11.3:

S = {1000 – 10(69.0)}/69.0 = 4.49 in.

R = $[5.91 – 0.2(4.49)]^2/[5.91 + 0.8(4.49)]$ **= 2.64 in.**

Thus the runoff is 2.64 in. (**6.71 cm**).

Alternatively, Figure 11.18 yields the same answer.

The volume of runoff is:

Vol. = (0.0671 m)(100 hec)(10,000 m^2/1 hec)

Vol. = 67,100 m^3

11.6.3

The solution procedure follows Example 11.6 except the P_1/P_T and P_2/P_T columns which come from Table 11.1.

t_1 (hr)	t_2 (hr)	P_1/P_T	P_2/P_T	P_1 (in.)	P_2 (in.)	R_1 (in.)	R_2 (in.)	ΔR (in.)
0	2	0.000	0.023	0.00	0.14	0.00	0.00	0.00
2	4	0.023	0.048	0.14	0.29	0.00	0.00	0.00
4	6	0.048	0.080	0.29	0.48	0.00	0.00	0.00
6	8	0.080	0.120	0.48	0.72	0.00	0.00	0.00
8	10	0.120	0.181	0.72	1.09	0.00	0.04	0.04
10	12	0.181	0.663	1.09	3.98	0.04	1.58	1.54
12	14	0.663	0.825	3.98	4.95	1.58	2.32	0.74
14	16	0.825	0.881	4.95	5.29	2.32	2.59	0.27
16	18	0.881	0.922	5.29	5.53	2.59	2.80	0.20
18	20	0.922	0.953	5.53	5.72	2.80	2.95	0.15
20	22	0.953	0.977	5.72	5.86	2.95	3.07	0.12
22	24	0.977	1.000	5.86	6.00	3.07	3.18	0.12

Note: P_T = 6.0 in, CN = 74, and S = 3.51 in.

11.6.4

For the sheet flow segment by using Equation (11.5)

$T_{t1} = [0.007\cdot(0.15\cdot200)^{0.8}]/\{(3.4)^{0.5}\cdot(0.02)^{0.4}\} = 0.276$ hrs

For the shallow concentrated flow, from V = L/V:

T_{t2} = [600 ft/(2 ft/sec)](1 hr/3600 sec) = 0.083 hrs

For the channel flow segment, R_h = A/P = D/4 = 0.50 ft.

Then from Manning's Equation (11.8)

$V = (1.49/0.013)\cdot(0.50)^{2/3}\cdot(0.01)^{1/2} = 7.22$ ft/sec

and

T_{t3} = 2,000/[(7.22 ft/sec)(3600 sec/hr)] = 0.077 hr

Thus, $\mathbf{T_c}$ = 0.276+0.083+0.077 = 0.436 hrs (**26.2 min.**)

11.6.5

The time-to-peak requires the time of concentration. Applying Equation (11.9):

$T_c = [L^{0.8}\cdot(S+1)^{0.7}]/(1140\cdot Y^{0.5})$

requires the maximum potential retention (*S*).

Thus, from Equation (11.4)

S = (1000/CN) - 10 = (1000/92) - 10 = 0.87 in.

where the curve number is found in Table 11.4 (commercial with B soils). Substituting into Equation (11.9) yields

$T_c = [(3000)^{0.8}\cdot(0.87+1)^{0.7}]/(1140\cdot(2.5)^{0.5}) = 0.52$ hours

Apply Equation (11.13) to estimate the time-to-peak.

$\mathbf{T_p}$ = $0.67\cdot T_c$ = 0.67(0.52) **= 0.35 hours**

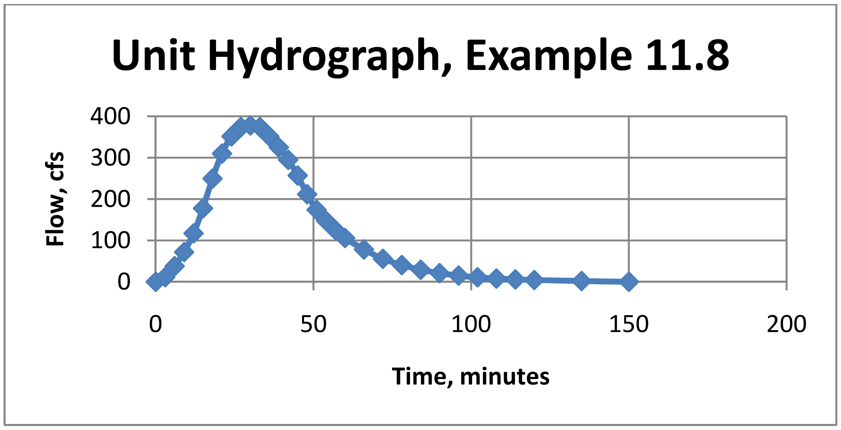
The peak discharge for the watershed is calculated from Equation (11.14) as

$q_p = (K_p\cdot A)/T_p$

q_p = 484·[100 acres (1 sq.mi./640 acres)]/0.35 hrs

$\mathbf{q_p}$ **= 216 cfs**

11.6.6



The unit hydrograph is graphed above. To determine the volume of runoff, we must determine the area under the hydrograph curve. This can be done through numerical integration (i.e., multiply each flow by time interval associated with it). Note that we can't sum all the flows and multiply by three minutes (distributive law) since the time interval changes to six minutes two hours into the hydrograph. Performing the numerical integrations yields:

Vol = [(4,445 ft^3/sec)(3 min)+(268 ft^3/sec)(6 min)+ (2 ft^3/sec)(15 min)](60 sec/1 min)(1 acre/43,560 ft^2) **= 20.6 acre-ft**

Runoff depth = Volume/Drainage Area = [(20.6 ac-ft)/(250 acres)](12 in./ft) **= 0.99 in. ≈ 1.0 in.** (i.e., unit hydrograph).

11.6.7

The area weighted composite curve number: CN = [(125 ac)(94) + (125 acres)(90)]/250 acres = 92.0

Thus, from Equation (11.4): S = (1000/CN) - 10 = (1000/92) - 10 = 0.870 in.

Now applying Equation (11.9): $T_c = [(4500)^{0.8}\cdot(0.870+1)^{0.7}]/(1140\cdot(8)^{0.5})$ = 0.402 hours (24.1 minutes)

The time-to-peak is estimated by Equation (11.13): $T_p = 0.67\cdot T_c$ = 0.67(0.402) = 0.269 hours (16.1 minutes)

The peak discharge from Eq'n (11.14) is: $q_p = (K_p\cdot A)/T_p$ = 484·[250 acres (1 sq.mi./640 acres)]/0.269 hrs = 703 cfs

The storm duration from Equation (11.11) is: $\Delta D = 0.133\cdot T_c$ = 0.133·0.402 = 0.0535 hrs (3.2 minutes)

The 3.2-min UH is displayed below using Table 11.8. The peak flow is **703 cfs** and occurs 16 minutes into the storm.

Time Ratios	Flow Ratios	Time	Flow
(t/t_p)	(q/q_p)	(min)	(cfs)
0.0	0.000	0.0	0
0.2	0.100	3.2	70
0.4	0.310	6.4	218
0.6	0.660	9.6	464
0.8	0.930	12.8	654
1.0	1.000	16.0	703
1.2	0.930	19.2	654
1.4	0.780	22.4	548

Time Ratios	Flow Ratios	Time	Flow
(t/t_p)	(q/q_p)	(min)	(cfs)
1.6	0.560	25.6	394
1.8	0.390	28.8	274
2.0	0.280	32.0	197
2.4	0.147	38.4	103
2.8	0.077	44.8	54
3.2	0.040	51.2	28
3.6	0.021	57.6	15
4.0	0.011	64.0	8

11.6.8

From Eq' (11.4): $S = (1000/CN) - 10 = (1000/84) - 10 = 1.90$ in. with CN found from Tables 11.3 and 11.4.

Now applying Equation (11.9): $T_c = [(5280)^{0.8} \cdot (1.90+1)^{0.7}]/(1140 \cdot (2)^{0.5}) = 1.24$ hours (74.4 minutes)

The time-to-peak is estimated by Equation (11.13): $T_p = 0.67 \cdot T_c = 0.67(1.24) = 0.831$ hours (50.0 minutes)

The peak discharge from Eq'n (11.14) is: $q_p = (K_p \cdot A)/T_p = 484 \cdot [400 \text{ acres } (1 \text{ sq.mi.}/640 \text{ acres})]/0.831 \text{ hrs} = 364$ cfs

The storm duration from Equation (11.11) is: $\Delta D = 0.133 \cdot T_c = 0.133 \cdot 1.24 = 0.165$ hrs ($\approx$ 10 minutes)

The 10-min UH is displayed below using Table 11.8. The peak flow is **364 cfs** and occurs 50 minutes into the storm.

Time Ratios	Flow Ratios	Time	Flow
(t/t_p)	(q/q_p)	(min)	(cfs)
0.0	0.000	0.0	0
0.2	0.100	10.0	36
0.4	0.310	20.0	113
0.6	0.660	30.0	240
0.8	0.930	40.0	339
1.0	1.000	50.0	364
1.2	0.930	60.0	339
1.4	0.780	70.0	284

Time Ratios	Flow Ratios	Time	Flow
(t/t_p)	(q/q_p)	(min)	(cfs)
1.6	0.560	80.0	204
1.8	0.390	90.0	142
2.0	0.280	100.0	102
2.4	0.147	120.0	54
2.8	0.077	140.0	28
3.2	0.040	160.0	15
3.6	0.021	180.0	8
4.0	0.011	200.0	4

11.6.9

From Eq' (11.4): $S = (1000/CN) - 10 = (1000/93) - 10 = 0.753$ in. with CN found from Tables 11.3 and 11.4.

Now applying Equation (11.9): $T_c = [(5280)^{0.8} \cdot (0.753+1)^{0.7}]/(1140 \cdot (2)^{0.5}) = 0.874$ hours (52.4 minutes)

The time-to-peak is estimated by Equation (11.13): $T_p = 0.67 \cdot T_c = 0.67(0.874) = 0.586$ hours (35.1 minutes)

The peak discharge from Eq'n (11.14) is: $q_p = (K_p \cdot A)/T_p = 484 \cdot [400 \text{ acres } (1 \text{ sq.mi.}/640 \text{ acres})]/0.586 \text{ hrs} = 516$ cfs

The storm duration from Equation (11.11) is: $\Delta D = 0.133 \cdot T_c = 0.133 \cdot 0.874 = 0.116$ hrs ($\approx$ 7 minutes)

The 7-min UH is displayed below using Table 11.8. The peak flow is **516 cfs** and occurs 35 minutes into the storm.

Time Ratios	Flow Ratios	Time	Flow
(t/t_p)	(q/q_p)	(min)	(cfs)
0.0	0.000	0.0	0
0.2	0.100	7.0	52
0.4	0.310	14.0	160
0.6	0.660	21.0	341
0.8	0.930	28.0	480
1.0	1.000	35.0	516
1.2	0.930	42.0	480
1.4	0.780	49.0	402

Time Ratios	Flow Ratios	Time	Flow
(t/t_p)	(q/q_p)	(min)	(cfs)
1.6	0.560	56.0	289
1.8	0.390	63.0	201
2.0	0.280	70.0	144
2.4	0.147	84.0	76
2.8	0.077	98.0	40
3.2	0.040	112.0	21
3.6	0.021	126.0	11
4.0	0.011	140.0	6

11.7.1

The *(2S/Δt)+O vs. O* relationship is tabulated below where Δt = 8 minutes = 480 sec.

Elevation, *h*	Outflow (*O*)	Storage (*S*)	*2S/Δt*	*(2S/Δt)+O*
(m)	(m^3/sec)	(m^3)	(m^3/sec)	(m^3/sec)
0.0	0.00	0	0.00	0.00
0.2	0.05	87	0.36	0.42
0.4	0.15	200	0.83	0.98
0.6	0.28	325	1.35	1.63
0.8	0.43	459	1.91	2.34
1.0	0.60	600	2.50	3.10

11.7.2

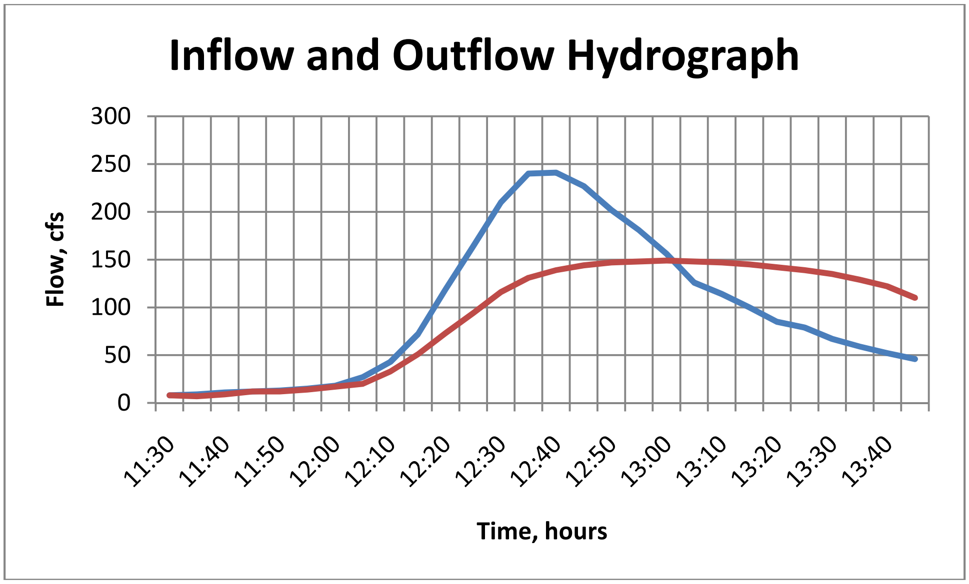
See the table below where $Q = Outflow(O) = C_d \cdot A \cdot (2gh)^{1/2} = 0.6\{(\pi/4)(2/12)^2\}(2gh)^{½}$, $S = 12 \cdot h$, and Δt = 10 sec.

Elevation, *h*	Outflow (*O*)	Storage (*S*)	*2S/Δt*	*(2S/Δt)+O*
(ft)	(ft^3/sec)	(ft^3)	(ft^3/sec)	(ft^3/sec)
0.0	0.00	0.0	0.00	0.00
0.5	0.07	6.0	1.20	1.27
1.0	0.11	12.0	2.40	2.51
1.5	0.13	18.0	3.60	3.73
2.0	0.15	24.0	4.80	4.95

11.7.3

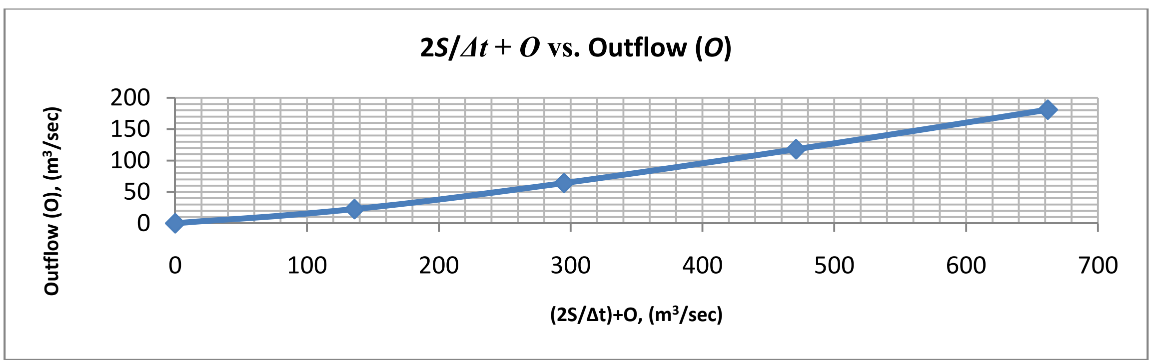
The rest of the routing table is seen below along with the inflow and outflow hydrographs. The peak elevation and storage is found by taking the peak outflow (149 cfs) back to the elevation-storage table and interpolating to determine the peak elevation and storage. $\mathbf{S_{peak}}$ **= 4.49 ac-ft** and $\mathbf{Elev_{peak}}$ **= 884.3 ft, MSL**

Time	Inflow (I_i) (cfs)	Inflow (I_j) (cfs)	*(2S/Δt)-O* (cfs)	*(2S/Δt)+O* (cfs)	Outflow, *O* (cfs)
13:20	85	79	897	1181	142
13:25	79	67	783	1061	139
13:30	67	59	659	929	135
13:35	59	52	527	785	129
13:40	52	46	394	638	122
13:45	46	40	272	492	110
13:50	40				



11.7.4

The [2S/Δt + O] vs. Outflow graph and the storage routing table are shown below. **Outflow peak = 125 m^3/sec.**

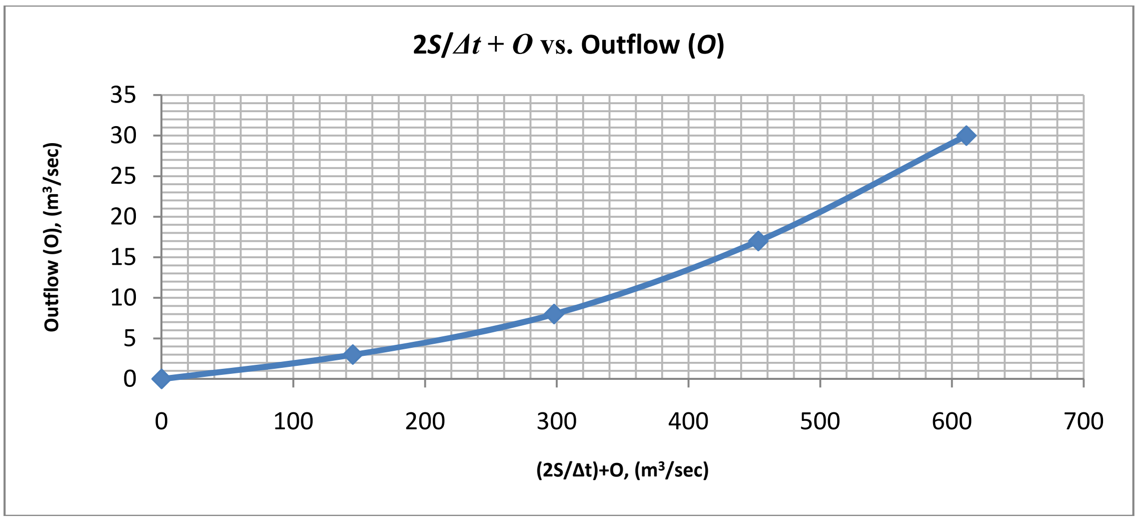


Time (hr)	Inflow (I_i) (m^3/sec)	Inflow (I_j) (m^3/sec)	$(2S/\Delta t)-O$ (m^3/sec)	$(2S/\Delta t)+O$ (m^3/sec)	Outflow, O (m^3/sec)
0:00	5	8	40		5
2:00	8	15	37	53	8
4:00	15	30	42	60	9
6:00	30	85	61	87	13
8:00	85	160	112	176	32
10:00	160	140	191	357	83
12:00	140	95	241	491	125
14:00	95	45	236	476	120
16:00	45	15	200	376	88

11.7.5

The [2S/Δt + O] vs. Outflow table is filled in below using the data available at given stages; either the outflow and storage to obtain *(2S/Δt)+O* or the outflow and *(2S/Δt)+O* to get the storage noting Δt = 10 min = 600 sec from the routing table. The [2S/Δt + O] vs. Outflow graph is then plotted to help perform the reservoir routing.

Stage (ft)	Outflow (*O*) (cfs)	Storage (*S*) (acre-ft)	*(2S/Δt)+O* (cfs)
0.0	0	0.0	0
0.5	3	1.0	148
1.0	8	2.0	298
1.5	17	3.0	453
2.0	30	4.0	611



The storage routing table is filled out below using normal calculation procedures as shown in Example 11.9 or using back calculations to fill in prior cells based on the information available in the table.

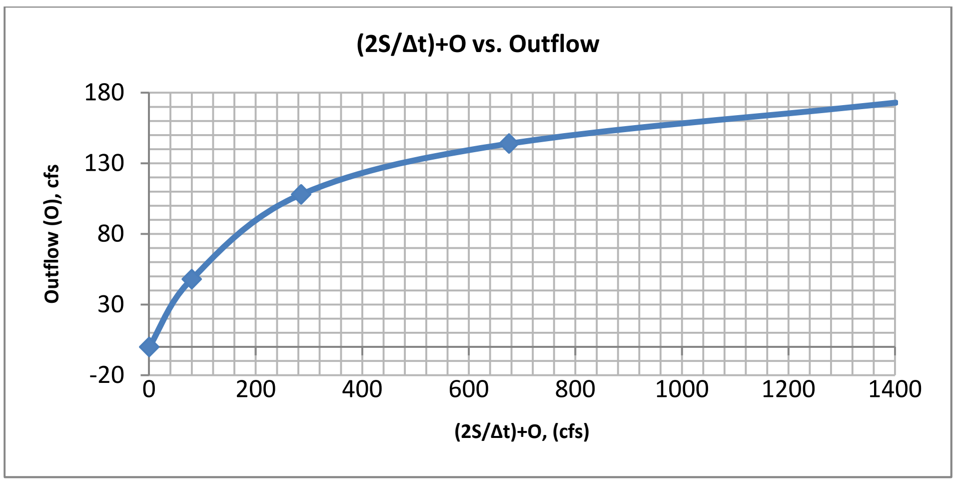
Time (min)	Inflow (I_i) (cfs)	Inflow (I_j) (cfs)	*(2S/Δt)-O* (cfs)	*(2S/Δt)+O* (cfs)	Outflow, *O* (cfs)
90	50	45	468	514	23
100	45	43	511	563	26
110	43	30	541	599	29
120	30	26	554	614	30

The peak outflow is 30 cfs, and using this flow yields a peak storage of 4.0 ac-ft and peak stage of 2.0 ft from the [2S/Δt + O] vs. Outflow table above.

11.7.6

The new [2S/Δt + O] vs. Outflow table is filled in below using the new time increment: *Δt* = 10 min = 600 sec from the routing table. The results should not change regardless of the time increment. However, if the time increment is too coarse, the peak may not be predicted accurately because the linearity assumption between flows is violated. The [2S/Δt + O] vs. Outflow graph is plotted below; the reservoir routing table gives **a new peak flow of 150 cfs.**

Elevation (ft, MSL)	Outflow (O) (cfs)	Storage (S) (acre-ft)	*2S/Δt* (cfs)	*(2S/Δt)+O* (cfs)
878	0	0	0	0
880	48	0.22	32	80
882	108	1.22	177	285
884	144	3.66	531	675
886	173	8.46	1228	1401

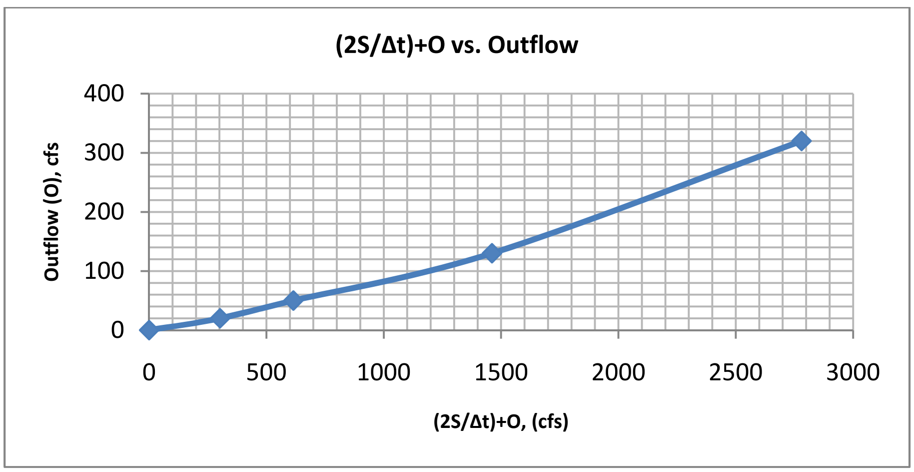


Time	Inflow (I_i) (cfs)	Inflow (I_j) (cfs)	*(2S/Δt)-O* (cfs)	*(2S/Δt)+O* (cfs)	Outflow, *O* (cfs)
11:30	8	11	**0**		**8**
11:40	11	13	-3	19	11
11:50	13	18	-3	21	12
12:00	18	43	-6	28	17
12:10	43	119	-11	55	33
12:20	119	210	7	151	72
12:30	210	241	104	336	116
12:40	241	202	279	555	138
12:50	202	156	430	722	146
13:00	156	114	488	788	150
13:10	114	85	462	758	148

11.7.7

The [2S/Δt + O] vs. Outflow table is filled in below using the time increment: Δt = 12 hours = 43,200 sec from the routing table. The [2S/Δt + O] vs. Outflow graph is plotted below and the reservoir routing calculations yields a **peak outflow of 262 cfs**. Taking this outflow to the [2S/Δt + O] vs. Outflow table and interpolating yields a **peak elevation of approximately 883.5 ft, MSL and a peak storage of approximately 1050 acre-ft.**

Elevation (ft, MSL)	Outflow (O) (cfs)	Storage (S) (acre-ft)	$2S/\Delta t$ (cfs)	$(2S/\Delta t)+O$ (cfs)
865	0	0	0	0
870	20	140	282	302
875	50	280	565	615
880	130	660	1331	1461
885	320	1220	2460	2780

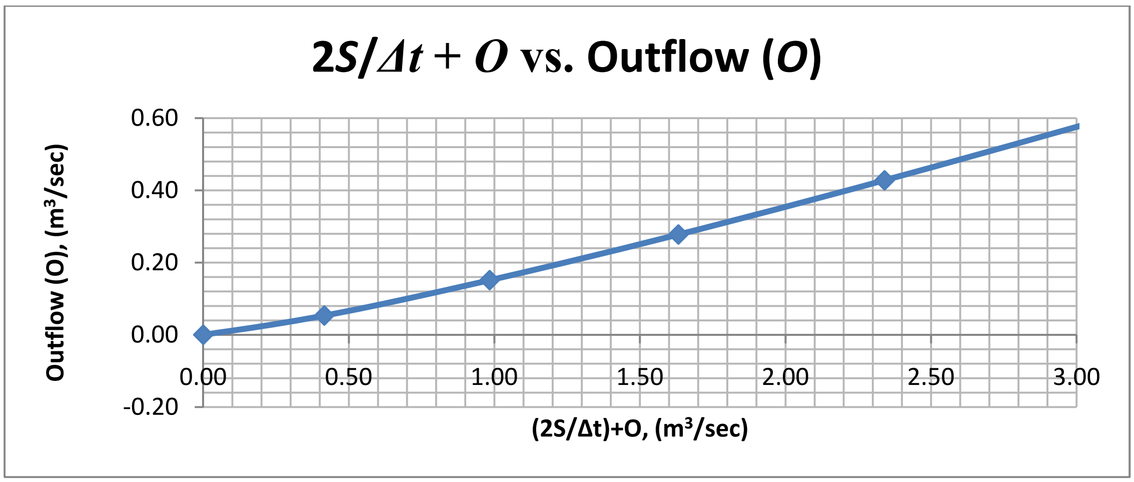


Day/Time	Inflow (I_i) (cfs)	Inflow (I_j) (cfs)	$(2S/\Delta t)-O$ (cfs)	$(2S/\Delta t)+O$ (cfs)	Outflow, O (cfs)
1 - noon	2	58	**0**		**2**
midnight	58	118	52	60	4
2 - noon	118	212	198	228	15
midnight	212	312	444	528	42
3 - noon	312	466	802	968	83
midnight	466	366	1286	1580	147
4 - noon	366	302	1668	2118	225
midnight	302	248	1824	2336	256
5 - noon	248	202	1850	2374	262
midnight	202	122	1798	2300	251
6 - noon	122	68	1672	2122	225
midnight	68				

11.7.8

The [2S/Δt + O] vs. Outflow table is shown below using the time increment: Δt = 8 min. = 480 sec from the routing table. The [2S/Δt + O] vs. Outflow graph is plotted below and the reservoir routing calculations yields a **peak outflow of 0.55 m^3/sec**. Taking this outflow to the [2S/Δt + O] vs. Outflow table and interpolating yields a **peak stage above the spillway of approximately 0.94 m.**

Stage, h (m)	Outflow (*O*) (m^3/sec)	Storage (*S*) (x10^3 m^3)	*(2S/Δt)* (m^3/sec)	*(2S/Δt)+O* (m^3/sec)
0.0	0.00	0	0.00	0.00
0.2	0.05	87	0.36	0.42
0.4	0.15	200	0.83	0.98
0.6	0.28	325	1.35	1.63
0.8	0.43	459	1.91	2.34
1.0	0.60	600	2.50	3.10

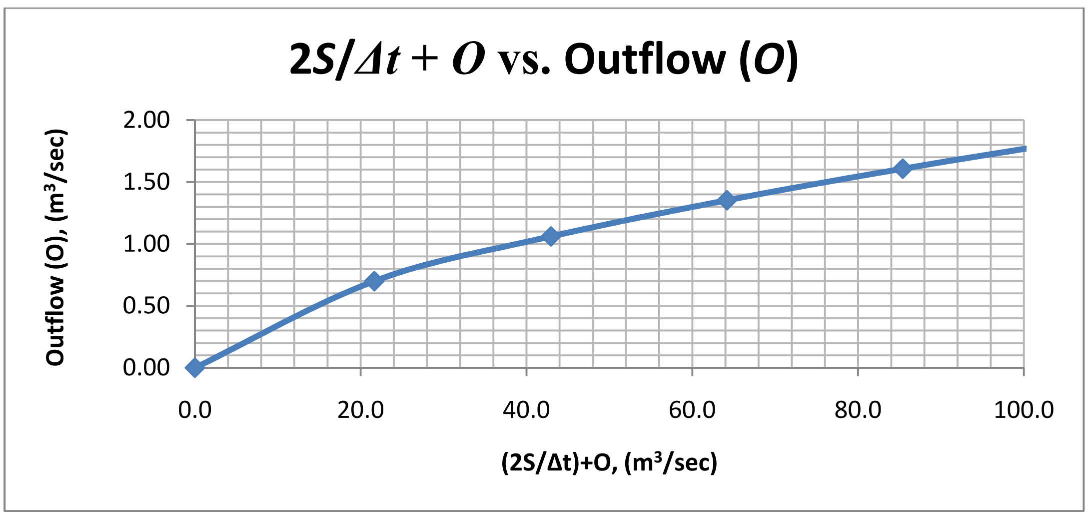


Time (min)	Inflow (I_i) (m^3/sec)	Inflow (I_j) (m^3/sec)	*(2S/Δt)-O* (m^3/sec)	*(2S/Δt)+O* (m^3/sec)	Outflow, *O* (m^3/sec)
0	0	0.15	0		0
8	0.15	0.38	0.11	0.15	0.02
16	0.38	0.79	0.46	0.64	0.09
24	0.79	0.71	1.07	1.63	0.28
32	0.71	0.58	1.61	2.57	0.48
40	0.58	0.46	1.80	2.90	0.55
48	0.46	0.35	1.76	2.84	0.54
56	0.35	0.21	1.61	2.57	0.48
64	0.21	0.1	1.39	2.17	0.39
72	0.1		1.12	1.70	0.29

11.7.9

The [2S/Δt + O] vs. Outflow table is shown below (Δt = 30 sec from the routing table). The [2S/Δt + O] vs. Outflow graph is plotted below and the reservoir routing calculations yields an outflow of 0.90 m^3/sec at t = 4 minutes. Taking this outflow to the [2S/Δt + O] vs. Outflow table and interpolating yields a **depth of about 1.55 m**. The **depth would be 1.91 meters if there was no leak** (h = Inflow Volume/tank area = 600 m^3/314 m^2).

Depth, h (m)	Outflow (*O*) (m^3/sec)	Storage (*S*) (x10^3 m^3)	*(2S/Δt)* (m^3/sec)	*(2S/Δt)+O* (m^3/sec)
0.0	0.00	0	0.0	0.0
1.0	0.70	314	20.9	21.6
2.0	1.06	628	41.9	42.9
3.0	1.35	942	62.8	64.2
4.0	1.61	1257	83.8	85.4
5.0	1.84	1571	104.7	106.6



Time (min)	Inflow (I_i) (m^3/sec)	Inflow (I_j) (m^3/sec)	*(2S/Δt)-O* (m^3/sec)	*(2S/Δt)+O* (m^3/sec)	Outflow, *O* (m^3/sec)
0.0	2.5	2.5	0.0		0.00
0.5	2.5	2.5	4.7	5.0	0.16
1.0	2.5	2.5	9.0	9.7	0.34
1.5	2.5	2.5	13.0	14.0	0.48
2.0	2.5	2.5	16.8	18.0	0.60
2.5	2.5	2.5	20.4	21.8	0.70
3.0	2.5	2.5	23.9	25.4	0.78
3.5	2.5	2.5	27.2	28.9	0.85
4.0	2.5	2.5	30.4	32.2	0.90
4.5	2.5		33.5	35.4	0.96

11.8.1

For **pre development conditions**, the SCS time-of-concentration can be computed. Applying Equation (11.4)

$$S = (1000/CN) - 10 = (1000/80) - 10 = 2.50 \text{ in.}$$

where the curve number is found in Table 11.4 (pasture land, clay or D soils). Substituting into Equation (11.9):

$$T_c = [L^{0.8}\cdot(S+1)^{0.7}]/(1140\cdot Y^{0.5}) = [(1200)^{0.8}\cdot(2.5+1)^{0.7}]/(1140\cdot(7)^{0.5}) = 0.232 \text{ hours (14 minutes)}$$

Applying Equation (11.19) for pre development conditions yields

$$\mathbf{Q_{10}} = C\cdot I\cdot A = (0.35)(5.2 \text{ in./hr})(20 \text{ acres}) \mathbf{= 36.4 \text{ cfs}}$$

where *C* is found using Table 11.10 and *I* is obtained from Figure 11.26 (with a storm duration equal to the 14 minute time of concentration). A *C* value for open space is used (high end of range since the soils are clay and the slopes are steep). Using lawns with clay soils steep soils (low end of steep range) would yield the same C.

For **post development conditions**, the SCS the maximum potential retention (*S*) from Equation (11.4) is

$$S = (1000/CN) - 10 = (1000/95) - 10 = 0.526 \text{ in.}$$ ➔ CN: Table 11.4 (commercial land, clay or D soils).

Substituting into Equation (11.9): $T_c = [(1200)^{0.8}\cdot(0.526+1)^{0.7}]/(1140\cdot(7)^{0.5}) = 0.130$ hours (7.8 minutes); thus

$$\mathbf{Q_{10}} = C\cdot I\cdot A = (0.95)(6.4 \text{ in./hr})(20 \text{ acres}) \mathbf{= 122 \text{ cfs}}$$

where *C* is found using Table 11.10 and *I* is obtained from Figure 11.26 (with a storm duration equal to the 7.8 minute time of concentration). A *C* value for commercial area is used (high end; clay soil and steep slopes).

11.8.2

The time of concentration is the travel time for the longest flow path, which in this case is the distance from the parking lot perimeter to the center. For the sheet flow travel time using Equation (11.5)

$$T_{t1} = [0.007\cdot(0.15\cdot 200)^{0.8}]/\{(2.4)^{0.5}\cdot(0.02)^{0.4}\} = 0.328 \text{ hrs } (\approx 20 \text{ min time of concentration})$$

Applying the rational equation (11.19) yields

$$\mathbf{Q_{10}} = C\cdot I\cdot A = (0.35)(4.3 \text{ in./hr})[\pi(200 \text{ ft})^2](1 \text{ acre}/43{,}560 \text{ ft}^2) \mathbf{= 4.34 \text{ cfs}}$$

where *C* is found using Table 11.10 (high end of range for flat lawns with clay soil and flat slopes since the slope is on the high end of the flat range) and *I* is obtained from Figure 11.26 (with a storm duration equal to 20 minutes).

11.8.3

Using the SCS sheet flow equation (11.5) and shallow concentrated flow equation (11.6) yields

$$T_{t1} = [0.007\cdot(0.011\cdot 300)^{0.8}]/\{(2.4)^{0.5}\cdot(0.005)^{0.4}\} = 0.0978 \text{ hrs (5.87 min)}$$

$$V = 20.3282(0.015)^{0.5} = 2.49 \text{ fps, and } T_{t2} = 600/[(2.49 \text{ ft/sec})(3600 \text{ sec/hr})] = 0.0669 \text{ hrs (4.01 min)}$$

Applying the rational equation (11.19) yields

$$\mathbf{Q_{10}} = C\cdot I\cdot A = (0.90)(6.0 \text{ in./hr})(300 \text{ ft})(600 \text{ ft})(1 \text{ acre}/43{,}560 \text{ ft}^2) \mathbf{= 22.3 \text{ cfs}}$$

where *C* is found using Table 11.10 (mid range) and *I* is obtained from Figure 11.26 (with a storm duration equal to the roughly 10 minute time of concentration; i.e., sheet flow time plus shallow concentrated flow).

11.8.4

Using the SCS sheet flow equation (11.5) and Manning's channel flow equation (11.8) yields

$T_{t1} = [0.007\,(n{\cdot}L)^{0.8}]/(P_2^{0.5}\cdot s^{0.4}) = [0.007{\cdot}(0.011{\cdot}270)^{0.8}]/\{(2.8)^{0.5}\cdot(0.005)^{0.4}\} = 0.083$ hrs (5.0 min)

$V = (1.49/n){\cdot}R_h^{2/3}{\cdot}S_e^{1/2} = (1.49/0.013){\cdot}(2/4)^{2/3}{\cdot}(0.005)^{1/2} = 5.1$ ft/sec

$T_{t3} = L/V = (600\text{ ft})/(5.1\text{ ft/sec}) = 118$ sec. = 2.0 min. Thus, $T_c = T_{t1} + T_{t3} = 5.0 + 2.0 = 7.0$ min.

Applying the rational equation (11.19) yields

$\mathbf{Q_5} = C{\cdot}I{\cdot}A = (0.90)(5.8\text{ in./hr})(270\text{ ft})(600\text{ ft})(1\text{ acre}/43{,}560\text{ ft}^2)$ **= 19.4 cfs**

where *C* is found using Table 11.10 (mid range) and *I* is obtained from Figure 11.26 (with a storm duration equal to the roughly 7 minute time of concentration; i.e., sheet flow time plus channel flow time).

11.8.5

Using the SCS sheet flow equation (11.5), the shallow concentrated flow equation (11.6), and Manning's channel flow equation (11.8) yields the time of concentration after development as

$T_{t1} = [0.007\,(n{\cdot}L)^{0.8}]/(P_2^{0.5}\cdot s^{0.4})$

$T_{t1} = [0.007{\cdot}(0.4{\cdot}100)^{0.8}]/\{(3.8)^{0.5}\cdot(0.02)^{0.4}\} = 0.33$ hrs (20 min)

$V = 16.1345(0.01)^{0.5} = 1.61$ fps, and

$T_{t2} = 1200/(1.61\text{ ft/sec}) = 745$ sec (12 min)

$V = (1.49/n){\cdot}R_h^{2/3}{\cdot}S_e^{1/2} = (1.49/0.035){\cdot}(18/24)^{2/3}{\cdot}(0.005)^{1/2} = 2.48$ ft/sec

$T_{t3} = L/V = (4300\text{ ft})/(2.48\text{ ft/sec}) = 1730$ sec. = 29 min.

Thus, $T_c = T_{t1} + T_{t2} + T_{t3} \approx 61$ min.

Applying the rational equation (11.19) for **pre-development conditions** yields

$\mathbf{Q_{25}} = C{\cdot}I{\cdot}A = (0.30)(2.1\text{ in./hr})(150\text{ ac})$ **= 94.5 cfs**

where *C* is found using Table 11.10 (mid range) and *I* is obtained from Figure 11.26 (with a storm duration equal to the 90 minute time of concentration). Applying the rational equation for **post-development conditions** yields

$\mathbf{Q_{25}} = C{\cdot}I{\cdot}A = (0.45)(2.7\text{ in./hr})(150\text{ ac})$ **= 182 cfs**

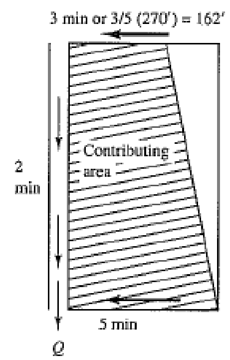
where $C = (60/150)(0.40) + (40/150)(0.70) + (50/150)(0.30) = 0.45$ (weighted average runoff coefficient) and *I* is obtained from Figure 11.26 (with a storm duration equal to the 61 minute time of concentration).

11.8.6

Applying the rational equation (11.19) yields

$$\mathbf{Q_p} = C \cdot I \cdot A = (0.90)(6.1 \text{ in./hr})[(162 \text{ ft} + 270 \text{ ft})/2](600 \text{ ft})(1 \text{ acre}/43{,}560 \text{ ft}^2) = \mathbf{16.3\ cfs}$$

where *C* is found using Table 11.10 (mid range) and *I* is obtained from Figure 11.26 (with a storm duration equal to 5 minutes), and the drainage area (*A*) contributing flow five minutes after the storm began based on flow travel times is depicted below. Note that the peak discharge of 16.3 cfs is lower than the 19.4 cfs peak discharge found in Problem 11.8.4 using the 7 minute (time of concentration) storm duration. Even though the rainfall intensity is less for the 7-minute duration storm, the full parking area is contributing runoff which yields a larger peak flow.



11.8.7

Referring to Example 11.11, the following logic can be applied to these changes taken one at a time:

a) For the 10-year design storm (was 5-year), the inlet would be located further to the east (or less than 220 feet down the street from the drainage divide) to accommodate the increased rainfall intensity.

b) For a spread of 8 feet (was 6 feet), the inlet would be located further to the west (or more than 220 feet down the street from the drainage divide) because more flow is allowed in the gutter before an inlet is required.

c) If the lawns were Bermuda grass, the inlet would be located further to the west (or more than 220 feet down the street from the drainage divide) because the time of concentration greater reducing the rainfall intensity.

d) If the longitudinal street slope is 3% (was 2.5%), the inlet would be located further to the west (or more than 220 feet down the street from the drainage divide) because the gutter capacity would be increased.

e) If the cross slope is 3/8-in. per foot (was ¼ -in.), the inlet would be located further to the west (or more than 220 feet down the street from the drainage divide) because the gutter capacity would be increased.

Even though the inlets on the north side of the street will accommodate runoff from residential lots based on the land slope, a few inlets will be required on the south side to accommodate rain that falls on the south side of the street.

11.8.8

Based on a cross slope of 3/8-inch per foot and a spread of 8 feet, the flow depth at the curb is 3.0 inches. Thus

$$Q = (1.49/n)\cdot A\cdot R_h^{2/3}\cdot S_e^{1/2} = (1.49/0.015)\cdot(1.00\ ft^2)\cdot(1.00/8.25)^{2/3}(0.025)^{1/2} = 3.85\ cfs$$

Applying Equation (11. 5) for sheet flow to determine the time of concentration yields

$$T_{t1} = [0.007\cdot(n\cdot L)^{0.8}]/(P_2^{0.5}\cdot s^{0.4}) = [0.007\cdot(0.41\cdot 100)^{0.8}]/[(3.2)^{0.5}\cdot(0.03)^{0.4}] = 0.310\ hrs = 18.6\ min.$$

Adding a little gutter flow time to the sheet flow time above gives us a time of concentration of roughly 20 minutes, which equals the storm duration for the rational method. Using Figure 11.26, and the rational equation results in

$$\mathbf{A} = Q/(C\cdot I) = 3.85\ cfs/[(0.35)(4.3\ in/hr)] = \mathbf{2.56\ acres}\ \ (\text{approximately } 111{,}000\ ft^2)$$

Since the drainage area on the north side of the street is roughly rectangular with a width of 100 feet, **the first inlet is placed about 1,100 feet down the street from the drainage divide.** (Note: Recheck T_c due to gutter length.)

11.8.9

Referring to Example 11.11, we see that the gutter flow at the first curb opening inlet is 0.914 cfs. The first inlet intercepts 75% of this flow, or 0.686 cfs. The gutter capacity at the second inlet will increase due to the increase in longitudinal slope. The flow depth at the curb is still 1.5 inches (i.e., no change in cross slope or spread). Thus

$$Q = (1.49/n)\cdot A\cdot R_h^{2/3}\cdot S_e^{1/2} = (1.49/0.015)\cdot(0.375\ ft^2)\cdot(0.375/6.13)^{2/3}(0.030)^{1/2} = 1.00\ cfs$$

The carryover flow from the first inlet (i.e., the flow not intercepted) is 0.228 cfs (i.e., 0.914 cfs – 0.686 cfs). Therefore, the drainage area for the second inlet can only contribute a portion of the 1.00 cfs flow capacity of the gutter, or 0.772 cfs (i.e., 1.00 cfs – 0.228 cfs). We will also assume that the time of concentration is the same for the second inlet as it was for the first inlet, or roughly 10 minutes, and the runoff coefficient remains the same. Using the IDF curve in Figure 11.26, a storm duration of 10 minutes produces an intensity of 5.2 in/hr for the 5-year storm. Substituting into the rational equation (Equation 11.21) results in

$$\mathbf{A} = Q/(C\cdot I) = 0.772\ cfs/[(0.35)(5.2\ in/hr)] = \mathbf{0.424\ acres}\ \ (\text{approximately } 18{,}500\ ft^2)$$

Since the drainage area on the north side of the street is roughly rectangular with a width of 100 feet, **the second inlet is placed about 185 feet down the street from the first inlet.**

NOTE: The timing of peak carryover flow from upstream inlets is not likely to correspond to the peak time of local runoff to individual inlets down the street. Therefore, carryover flow from upstream inlet may not be a factor as you move down the street; however, it is the conservative approach. Many local agencies have outlined very prescriptive procedures for inlet spacing including standards for time of concentration to inlets, street cross slopes, pavement encroachment or spread criteria, carryover flows, and inlet types. However, the general design procedure outlined here is likely to be followed with some minor variations.

11.8.10

Referring to Example 11.11, we see that longitudinal street slope is 2.5%. Applying Equaiton 11.22 yields

$\mathbf{D_r} = [n \cdot Q_p/\{0.463 \cdot (S_o)^{1/2}\}]^{3/8} = [0.013 \cdot 0.914/\{0.463 \cdot (0.025)^{1/2}\}]^{3/8}$ **= 0.506 ft (6.07 in.)**

The design pipe diameter is 12 inches (the minimum standard size). The full pipe area (Equation 11.23) is

$A_f = \pi D^2/4 = \pi(1.0)^2/4 = 0.785 \text{ ft}^2$

The full pipe hydraulic radius and the full pipe velocity, as computed from Equations 11.24 and 11.25 are

$R_f = D/4 = (1.0)/4 = 0.250$ ft; $V_f = (1.49/n) \cdot R_f^{2/3} \cdot S_o^{1/2} = (1.49/0.013) \cdot (0.250)^{2/3} \cdot (0.025)^{1/2} = 7.19$ ft/sec

Now, the full pipe flow (from continuity) and the flow ratios (Q_p/Q_f) can be determined.

$Q_f = (V_f)(A_f) = (7.19)(0.785) = 5.64$ cfs; $Q_p/Q_f = (0.914)/(5.64) = 0.162$

Using the flow ratio, we enter Figure 11.29 to obtain the depth of flow (y) and the actual flow velocity (V).

$y/D = 0.27$; $\mathbf{y} = 0.27(1.0 \text{ ft})$ **= 0.27 ft (3.2 in.)**; $V/V_f = 0.73$; $V = 0.73(7.19 \text{ ft/sec}) = 5.3$ ft/sec

Finally, the time required to travel through 100 ft of pipe is: **t** = L/V = 100 ft/(5.3 ft/sec) **= 19 sec.**

11.8.11

The revised design table is shown below.

Stormwater Pipe	1-2	2-3	3A-3	3-4
Length (ft)	350	300	350	250
Inlet Time, T_t (min)	15	15	15	15
Time of Concentration, $\boldsymbol{T_c}$ (min)	15	16	15	17.5
Runoff Coefficient, $\boldsymbol{C}$	0.4	0.5	0.5	0.48
R/F Intensity, $\boldsymbol{I}$ (in/hr)	5.0	4.9	5.0	4.6
Drainage Area, $\boldsymbol{A}$ (acres)	2.8	5.6	2.8	10.9
Peak Discharge, $\boldsymbol{Q_p}$ (cfs)	**5.6**	**13.7**	**7.0**	**24.1**
Slope (ft/ft)	0.01	0.02	0.006	0.03
Required Pipe Diameter, D_r (in.)	14.2	17.5	17.0	20.0
Design Pipe Diameter, $\boldsymbol{D}$ (in.)	**15**	**18**	**18**	**21**
Full Pipe Area, A_f (ft^2)	1.23	1.77	1.77	2.41
Full Pipe Velocity, V_f (ft/sec)	5.28	8.43	4.62	11.4
Full Pipe Flow, Q_f (cfs)	6.48	14.9	8.16	27.5
Q_p/Q_f (or Q/Q_f)	0.86	0.92	0.86	0.87
y/D	0.71	0.75	0.71	0.72
V/V_f	1.14	1.15	1.14	1.14
Flow Depth, y (in.)	10.7	13.5	12.8	15.1
Pipe Velocity, V (ft/sec)	6.02	9.69	5.3	13.0
Pipe Flow Time (min)	1.0	0.5	1.1	0.3

11.8.12

Stormwater Pipe	4A-4	4-5	5A-5	5-6
Length (ft)	350	320	250	100
Inlet Time, T_i (min)	14	12	12	10
Time of Concentration, $\boldsymbol{T_c}$ (min)	14	17.8	12	18.4
Runoff Coefficient, ***C***	0.6	0.50	0.4	0.52
R/F Intensity, ***I*** (in/hr)	4.4	3.9	4.8	3.8
Drainage Area, ***A*** (acres)	2.5	15.9	2.4	20.5
Peak Discharge, $\boldsymbol{Q_p}$ (cfs)	**6.6**	**31.0**	**4.6**	**40.5**
Slope (ft/ft)	0.004	0.01	0.02	0.015
Required Pipe Diameter, D_r (in.)	18.0	27.0	11.6	27.7
Design Pipe Diameter, *D* (in.)	**18**	**30**	**15**	**30**
Full Pipe Area, A_f (ft^2)	1.77	4.91	1.23	4.91
Full Pipe Velocity, V_f (ft/sec)	3.77	8.38	7.46	10.3
Full Pipe Flow, Q_f (cfs)	6.66	41.1	9.16	50.4
Q_p/Q_f (or Q/Q_f)	0.99	0.75	0.50	0.80
y/D	0.80	0.64	0.5	0.67
V/V_f	1.16	1.12	1.02	1.13
Flow Depth, y (in.)	14.4	19.2	7.5	20.1
Pipe Velocity, V (ft/sec)	4.37	9.38	7.6	11.6
Pipe Flow Time (min)	1.3	0.6	0.5	0.1

11.8.13

Stormwater Pipe	AB	CB	BD	DR
Length (ft)	200	300	300	200
Inlet Time, T_i (min)	12	10	13	10
Time of Concentration, $\boldsymbol{T_c}$ (min)	12	10	13	14.1
Runoff Coefficient, ***C***	0.3	0.4	0.33	0.36
R/F Intensity, ***I*** (in/hr)	5.2	5.7	5.0	4.8
Drainage Area, ***A*** (acres)	2.2	1.8	6.2	7.4
Peak Discharge, $\boldsymbol{Q_p}$ (cfs)	**3.4**	**4.1**	**10.2**	**12.8**
Slope (ft/ft)	0.01	0.005	0.003	0.002
Required Pipe Diameter, D_r (in.)	11.8	14.4	22.3	26.2
Design Pipe Diameter, *D* (in.)	**12**	**15**	**24**	**30**
Full Pipe Area, A_f (ft^2)	0.79	1.23	3.14	4.91
Full Pipe Velocity, V_f (ft/sec)	4.55	3.73	3.95	3.75
Full Pipe Flow, Q_f (cfs)	3.57	4.58	12.4	18.4
Q_p/Q_f (or Q/Q_f)	0.96	0.89	0.82	0.69
y/D	0.78	0.73	0.68	0.60
V/V_f	1.16	1.14	1.13	1.09
Flow Depth, y (in.)	9.4	11.0	16.3	18.0
Pipe Velocity, V (ft/sec)	5.28	4.25	4.5	4.1
Pipe Flow Time (min)	0.6	1.2	1.1	0.8

11.8.14

Stormwater Pipe	AB	CB	BD	DR
Length (ft)	200	300	300	200
Inlet Time, T_i (min)	14	10	10	10
Time of Concentration, $\boldsymbol{T_c}$ (min)	14	10	14.8	15.9
Runoff Coefficient, ***C***	0.3	0.4	0.33	0.36
R/F Intensity, ***I*** (in/hr)	4.8	5.7	4.7	4.5
Drainage Area, ***A*** (acres)	2.2	1.8	6.2	7.4
Peak Discharge, $\boldsymbol{Q_p}$ (cfs)	**3.2**	**4.1**	**9.6**	**12.0**
Slope (ft/ft)	0.005	0.005	0.003	0.002
Required Pipe Diameter, D_r (in.)	13.1	14.4	21.8	25.6
Design Pipe Diameter, *D* (in.)	**15**	**15**	**24**	**30**
Full Pipe Area, A_f (ft^2)	1.23	1.23	3.14	4.91
Full Pipe Velocity, V_f (ft/sec)	3.73	3.73	3.95	3.75
Full Pipe Flow, Q_f (cfs)	4.58	4.58	12.4	18.4
Q_p/Q_f (or Q/Q_f)	0.69	0.89	0.77	0.65
y/D	0.60	0.72	0.65	0.58
V/V_f	1.09	1.14	1.12	1.08
Flow Depth, y (in.)	9.0	10.8	15.6	17.4
Pipe Velocity, V (ft/sec)	4.07	4.25	4.4	4.0
Pipe Flow Time (min)	0.8	1.2	1.1	0.8

11.8.15

Stormwater Pipe	AB	CB	BD	DR
Length (ft)	200	300	300	200
Inlet Time, T_i (min)	12	14	13	10
Time of Concentration, $\boldsymbol{T_c}$ (min)	12	14	15.7	16.8
Runoff Coefficient, ***C***	0.3	0.4	0.33	0.36
R/F Intensity, ***I*** (in/hr)	5.2	4.8	4.5	4.4
Drainage Area, ***A*** (acres)	2.2	1.8	6.2	7.4
Peak Discharge, $\boldsymbol{Q_p}$ (cfs)	**3.4**	**3.5**	**9.3**	**11.7**
Slope (ft/ft)	0.01	0.002	0.003	0.0005*
Required Pipe Diameter, D_r (in.)	11.8	16.1	21.5	32.9
Design Pipe Diameter, *D* (in.)	**12**	**18**	**24**	**36**
Full Pipe Area, A_f (ft^2)	0.79	1.77	3.14	7.07
Full Pipe Velocity, V_f (ft/sec)	4.55	2.67	3.95	2.12
Full Pipe Flow, Q_f (cfs)	3.57	4.71	12.4	15.0
Q_p/Q_f (or Q/Q_f)	0.96	0.74	0.75	0.78
y/D	0.78	0.63	0.64	0.67
V/V_f	1.16	1.11	1.12	1.12
Flow Depth, y (in.)	9.4	11.3	15.4	24.1
Pipe Velocity, V (ft/sec)	5.28	2.96	4.4	2.4
Pipe Flow Time (min)	0.6	1.7	1.1	1.4

*Note: Slope was adjusted up from 0% (ground slope) to 0.05% to get a 2 ft/sec full pipe velocity.

Chapter 12 – Problem Solutions

12.2.1

Year	P_i	P_i - m	$(Q - m)^2$	$(Q - m)^3$
1989	44.2	4.20E+00	1.76E+01	7.41E+01
1990	47.6	7.60E+00	5.78E+01	4.39E+02
1991	38.5	-1.50E+00	2.25E+00	-3.38E+00
1992	35.8	-4.20E+00	1.76E+01	-7.41E+01
1993	40.2	2.00E-01	4.00E-02	8.00E-03
1994	41.2	1.20E+00	1.44E+00	1.73E+00
1995	38.8	-1.20E+00	1.44E+00	-1.73E+00
1996	39.7	-3.00E-01	9.00E-02	-2.70E-02
1997	40.5	5.00E-01	2.50E-01	1.25E-01
1998	42.5	2.50E+00	6.25E+00	1.56E+01
1999	39.2	-8.00E-01	6.40E-01	-5.12E-01
2000	38.3	-1.70E+00	2.89E+00	-4.91E+00
2001	46.1	6.10E+00	3.72E+01	2.27E+02
2002	33.1	-6.90E+00	4.76E+01	-3.29E+02
2003	35.0	-5.00E+00	2.50E+01	-1.25E+02
2004	39.3	-7.00E-01	4.90E-01	-3.43E-01
2005	42.0	2.00E+00	4.00E+00	8.00E+00
2006	41.7	1.70E+00	2.89E+00	4.91E+00
2007	37.7	-2.30E+00	5.29E+00	-1.22E+01
2008	38.6	-1.40E+00	1.96E+00	-2.74E+00
Sum	800	-1.28E-13	2.33E+02	2.17E+02

From Equations (12.1), (12.2), and (12.3):

$$m = \frac{1}{N}\sum_{i=1}^{N} P_i = (800)/20 = \mathbf{40.0\ in.}$$

$$s = \left[\frac{1}{N-1}\sum_{i=1}^{N}(P_i - m)^2\right]^{1/2} = [233/19]^{1/2} = \mathbf{3.50\ in.}$$

$$G = \frac{N\sum_{i=1}^{N}(P_i - m)^3}{(N-1)(N-2)\,s^3}$$

$\boldsymbol{G}$ = (20)(217)/[(19)(18)(3.50)^3] **= 0.296**

12.2.2

$log\ P_i$	$(\log P_i - m_l)$	$(log\ P_i - m_l)^2$	$(log\ P_i - m_l)^3$
1.65E+00	4.49E-02	2.02E-03	9.07E-05
1.68E+00	7.71E-02	5.95E-03	4.59E-04
1.59E+00	-1.50E-02	2.26E-04	-3.39E-06
1.55E+00	-4.66E-02	2.17E-03	-1.01E-04
1.60E+00	3.74E-03	1.40E-05	5.23E-08
1.61E+00	1.44E-02	2.08E-04	2.99E-06
1.59E+00	-1.17E-02	1.36E-04	-1.58E-06
1.60E+00	-1.70E-03	2.88E-06	-4.89E-09
1.61E+00	6.97E-03	4.85E-05	3.38E-07
1.63E+00	2.79E-02	7.78E-04	2.17E-05
1.59E+00	-7.20E-03	5.19E-05	-3.73E-07
1.58E+00	-1.73E-02	2.99E-04	-5.17E-06
1.66E+00	6.32E-02	4.00E-03	2.53E-04
1.52E+00	-8.07E-02	6.51E-03	-5.25E-04
1.54E+00	-5.64E-02	3.18E-03	-1.80E-04
1.59E+00	-6.09E-03	3.71E-05	-2.26E-07
1.62E+00	2.28E-02	5.18E-04	1.18E-05
1.62E+00	1.96E-02	3.86E-04	7.59E-06
1.58E+00	-2.41E-02	5.83E-04	-1.41E-05
1.59E+00	-1.39E-02	1.93E-04	-2.69E-06
3.20E+01	-2.44E-15	2.73E-02	1.34E-05

From Equations (12.4), (12.5), and (12.6):

$$m_l = \frac{1}{N}\sum_{i=1}^{N}\log P_i = (32.0)/20 = \mathbf{1.60}$$

P (log mean) = 10$^{1.60}$ **= 39.8 in.**

$$s_l = \left[\frac{1}{N-1}\sum_{i=1}^{N}(\log P_i - m_l)^2\right]^{1/2} = \mathbf{0.0379}$$

$$G_l = \frac{N\sum_{i=1}^{N}(\log P_i - m_l)^3}{(N-1)(N-2)\,s_l^{\,3}} = \mathbf{0.0144}$$

12.2.3

Year	Q_i	Q_i - m	$(Q_i - m)^2$	$(Q_i - m)^3$
1950	114	-1.00E+02	1.00E+04	-1.00E+06
1951	198	-1.60E+01	2.56E+02	-4.10E+03
1952	297	8.30E+01	6.89E+03	5.72E+05
1953	430	2.16E+02	4.67E+04	1.01E+07
1954	294	8.00E+01	6.40E+03	5.12E+05
1955	113	-1.01E+02	1.02E+04	-1.03E+06
1956	165	-4.90E+01	2.40E+03	-1.18E+05
1957	211	-3.00E+00	9.00E+00	-2.70E+01
1958	94.0	-1.20E+02	1.44E+04	-1.73E+06
1959	91.0	-1.23E+02	1.51E+04	-1.86E+06
1960	222	8.00E+00	6.40E+01	5.12E+02
1961	376	1.62E+02	2.62E+04	4.25E+06
1962	215	1.00E+00	1.00E+00	1.00E+00
1963	250	3.60E+01	1.30E+03	4.67E+04
1964	218	4.00E+00	1.60E+01	6.40E+01
1965	98.0	-1.16E+02	1.35E+04	-1.56E+06
1966	283	6.90E+01	4.76E+03	3.29E+05
1967	147	-6.70E+01	4.49E+03	-3.01E+05
1968	289	7.50E+01	5.63E+03	4.22E+05
1969	175	-3.90E+01	1.52E+03	-5.93E+04
Sum	4280	0.00E+00	1.70E+05	8.55E+06

From Equations (12.1), (12.2), and (12.3):

$$m = \frac{1}{N}\sum_{i=1}^{N} Q_i = (4{,}280)/20 = \mathbf{214\ \ m^3/sec}$$

$$s = \left[\frac{1}{N-1}\sum_{i=1}^{N}(Q_i - m)^2\right]^{1/2}$$

$\boldsymbol{s}$ = $[170{,}000/19]^{1/2}$ = **94.6 $\mathbf{m^3}$/sec**

$$G = \frac{N\sum_{i=1}^{N}(Q_i - m)^3}{(N-1)(N-2)\,s^3}$$

$\boldsymbol{G}$ = (20)(8.55E+06)/[(19)(18)$(94.6)^3$] = **0.591**

12.2.4

$log\ Q_i$	$(logQ_i\text{-}m_l)$	$(logQ_i\text{-}m_l)^2$	$(logQ_i\text{-}m_l)^3$
2.06E+00	-2.31E-01	5.34E-02	-1.23E-02
2.30E+00	8.68E-03	7.53E-05	6.53E-07
2.47E+00	1.85E-01	3.41E-02	6.31E-03
2.63E+00	3.45E-01	1.19E-01	4.12E-02
2.47E+00	1.80E-01	3.25E-02	5.87E-03
2.05E+00	-2.35E-01	5.52E-02	-1.30E-02
2.22E+00	-7.05E-02	4.97E-03	-3.50E-04
2.32E+00	3.63E-02	1.32E-03	4.78E-05
1.97E+00	-3.15E-01	9.91E-02	-3.12E-02
1.96E+00	-3.29E-01	1.08E-01	-3.56E-02
2.35E+00	5.84E-02	3.41E-03	1.99E-04
2.58E+00	2.87E-01	8.25E-02	2.37E-02
2.33E+00	4.44E-02	1.98E-03	8.78E-05
2.40E+00	1.10E-01	1.21E-02	1.33E-03
2.34E+00	5.05E-02	2.55E-03	1.29E-04
1.99E+00	-2.97E-01	8.81E-02	-2.61E-02
2.45E+00	1.64E-01	2.68E-02	4.39E-03
2.17E+00	-1.21E-01	1.46E-02	-1.76E-03
2.46E+00	1.73E-01	2.99E-02	5.17E-03
2.24E+00	-4.50E-02	2.02E-03	-9.08E-05
4.58E+01	4.22E-15	7.72E-01	-3.20E-02

From Equations (12.4), (12.5), and (12.6):

$$m_l = \frac{1}{N}\sum_{i=1}^{N}\log Q_i = (45.8)/20 = \mathbf{2.29}$$

P (log mean) = $10^{2.29}$ = **195 $\mathbf{m^3}$/sec**

$$s_l = \left[\frac{1}{N-1}\sum_{i=1}^{N}(\log Q_i - m_l)^2\right]^{1/2} = \mathbf{0.202}$$

$$G_l = \frac{N\sum_{i=1}^{N}(\log Q_i - m_l)^3}{(N-1)(N-2)\,s_l^{\,3}} = \mathbf{-0.227}$$

12.3.1

From Example 12.1, m = 9,820 cfs and s = 4,660 cfs.

Assuming a normal probability density function,

$$f_X(x) = \frac{1}{4660\sqrt{2\pi}}\, exp\left[-\frac{(x - 9820)^2}{2(4660)^2} \right]$$

From Example 12.2, m_l = 3.95 and s = 0.197.

Assuming a log normal probability density function,

$$f_X(x) = \frac{1}{(0.197x)\sqrt{2\pi}}\, exp\left[\frac{-\ (\log x - 3.95)^2}{2(0.197)^2} \right]$$

12.3.2

From Example 12.1, m = 9,820 cfs and s = 4,660 cfs.

Assuming a Gumbel probability density function,

$f_X(x) = y \cdot exp[-\,y\,(x - u) - exp[-\,y\,(x - u)]]$, where

$y = \pi/[s(6)^{0.5}] = \pi/[4660(6)^{0.5}] = 0.000275$

$u = m - 0.45 \cdot s = 9820 - [0.45(4660)] = 7720$

therefore, $f_X(x) = (0.000275) \cdot exp[z - exp[z]]$

where $z = -\,0.000275\ (x - 7720\ )$

12.3.3

From Example 12.2: m_l = 3.95 and s_l = 0.197.

From Ex 12.3: g = 0.389 (weighted skew) = Gl

Hence, from Equations 12.14, 12.15, and 12.16:

$b = 4/G_l^2 = 4/g^2 = 4/(0.389)^2 = 26.4$

$v = s_l/b^{0.5} = 0.197/(26.4)^{0.5} = 0.0383$

$r = m_l - s_l(b)^{0.5} = 3.95 - 0.197(26.4)^{0.5} = 2.94$

These values should be substituted into the following log-Pearson type III probability density function:

$$f_X(x) = \frac{v^b\,(\log x - r)^{b-1}\, exp\,[-v\,(\log x - r)]}{x\,\Gamma\,(b)}$$

12.4.1

The probability of being flooded next year (Eq'n 12.22),

p = 1/T = 1/5 = 0.20 **= 20.0 %**

The probability of being flooded at least once in the next three years (Equation 12.23) is,

R = $1 - (1 - 0.20)^3 = 0.488$ **= 48.8 %**

12.4.2

Applying Equation 12.23 with R = 0.20 (20%) yields

$R = 0.20 = 1 - (1 - p)^{25}$; p = 0.00889

The return interval for this probability (Eq'n 12.22) is,

T = 1/0.00889 = **112 yr (> 100-yr)**

12.4.3

a) The probability of the small reservoir being used at least once in the next two years (Equation 12.23) is,

$R = 1 - (1 - 0.70)^2 = 0.910$ = **91.0 %**

b) The probability of not having to rely on the reservoir in the next two years is,

p = 1 – R = 1 – 0.91 = 0.09 = **9.0 %**

c) The probability of having to utilize the reservoir during each (or both) of the next two years is,

p = (0.70)(0.70) = 0.490 = **49.0 %**

since the probability of using it in any one year is 70%.

d) The probability of having to utilize the reservoir exactly once during the next two years is,

p = (0.7)(0.3) + (0.3)(0.7) = 0.420 **= 42.0 %**

since the probability of using it in the first year is 70% and not in the second is 30% plus the opposite order. Alternatively, using it exactly once in two years can be found by subtracting from 100% the sum of using it in both years and not using it in either year. Therefore,

100% - (49% + 9.0%) **= 42%**

12.4.4

a) The probability of having to pipe in water at least once in the next two years (Equation 12.23) is,

$$R = 1 - (1 - 0.20)^2 = 0.360 = \mathbf{36.0\ \%}$$

b) The probability of not having to pipe in water in the next two years is,

$$p = 1 - R = 1 - 0.36 = 0.64 = \mathbf{64.0\ \%}$$

c) The probability of having to pipe in water during each (or both) of the next two years is,

$$p = (0.20)(0.20) = 0.04 = \mathbf{4.0\ \%}$$

since the probability of piping in any one year is 20%.

d) The probability of having to pipe in water exactly once during the next two years is,

$$p = (0.2)(0.8) + (0.8)(0.2) = 0.320 \mathbf{= 32.0\ \%}$$

since the probability of piping in the first year is 20% and not in the second is 80% plus the opposite order. Alternatively, piping exactly once in two years can be found by subtracting from 100% the sum of piping it in both years and not piping it in either year. Therefore,

$$100\% - (4\% + 64\%) \mathbf{= 32\%}$$

12.4.5

a) The probability of flooding the first year (or any year) based on Equation 12.22 is

$$\mathbf{p} = 1/T = 1/10 = 0.10 \mathbf{= 10\%}$$

b) The risk of flooding during the four -year construction period based on Equation 12.23 is

$$\mathbf{R} = 1 - (1 - 0.10)^4 = 0.344 = \mathbf{34.4\ \%}$$

c) The probability of not being flooded in four years:

$$\mathbf{p} = 1 - R = 1 - 0.344 = 0.656 = \mathbf{65.6\ \%}$$

d) Reducing the risk of construction during the four-year period to 25% (from 34.4%) requires a higher levee or a faster construction time. Based on Equation 12.23:

$$R = 0.250 = 1 - (1 - 0.10)^n; \ \ \mathbf{n = 2.73\ years}$$

12.5.1

a) Normal: $K_{10} = 1.282$ from Table 12.2 for p = 0.10 (i.e. T = 10 years). Then from Equation (12.24)

$$\mathbf{P_{10}} = m + K_{10}(s) = 40.0 + 1.282(3.50) \mathbf{= 44.5\ in.}$$

b) Gumbel: $K_{10} = 1.305$ from Table 12.2 for p = 0.10 (i.e. T = 10 years). Then from Equation (12.24)

$$\mathbf{P_{10}} = m + K_{10}(s) = 40.0 + 1.305(3.50) \mathbf{= 44.6\ in.}$$

The 10-year precipitation depth of 44.5 in. (Normal) was **exceeded twice in the 20-year record.**

12.5.2

a) Log Normal: $K_{10} = 1.282$ from Table 12.2 for p = 0.10 (i.e. T = 10 years). Then from Equation (12.25)

$$\log P_{10} = m_l + K_{10}(s_l) = 1.60 + 1.282(0.0379) = 1.65$$

Then, taking the antilog of 1.65, we obtain

$\mathbf{P_{10} = 44.7\ in.}$

b) Log-Pearson Type III: Use the equation sequence; Equations (12.31a), (12.29a), (12.28), and (12.30).

$$k = G_l/6 = 0.0144/6 = 0.00240$$

$$w = [\ln T^2]^{1/2} = [\ln 10^2]^{1/2} = 2.15$$

$$z = 2.15 - \frac{2.515517 + 0.802853 \cdot 2.15 + 0.010328 \cdot (2.15)^2}{1 + 1.432788 \cdot 2.15 + 0.189269 \cdot (2.15)^2 + 0.001308 \cdot (2.15)^3}$$

$z = 1.29$

$$K_T = z + (z^2 - 1)k + (z^3 - 6z)(k^2/3) - (z^2 - 1)k^3 + zk^4 + k^5/3$$

$$K_T = 1.29 + (1.29^2 - 1)0.0024 + (1.29^3 - 6 \cdot 1.29)(0.0024^2/3) - (1.29^2 - 1)0.0024^3 + 1.29 \cdot 0.0024^4 + 0.0024^5/3$$

K_T = 1.29; Now from Equation (12.25)

$$\log P_{10} = m_l + K_T(s_l) = 1.60 + 1.29(0.0379) = 1.65$$

Taking the antilog of 4.32, $\mathbf{P_{10} = 44.7\ in.}$

The 10-year precipitation depth of 44.7 in. (Log-Normal) was **exceeded twice in the 20-year record.**

12.5.3

a) Normal: Use Eq'n (12.24) with $Q_T = 430$ m^3/s

$Q_T = m + K_T(s)$; $430 = 214 + K_T\,(94.6)$;

$K_T = 2.28$; from Table 12.2, we can see that this is a little less than a 100-year flood, **T ≈ 90 years.**

b) Gumbel: Use Eq'n (12.24) with $Q_T = 430$ m^3/s

$Q_T = m + K_T(s)$; $430 = 214 + K_T\,(94.6)$;

$K_T = 2.28$; from Table 12.2, we can see that this is a little less than a 40-year flood, **T ≈ 35 years.**

Using the appropriate distribution is critical for analysis.

12.5.4

a) Log-Normal: Use Eq'n (12.25) with $Q_T = 430$ m^3/s

$\log Q_T = m_l + K_T(s_l)$; $\log(430) = 2.29 + K_T\,(0.202)$;

$K_T = 1.70$; from Table 12.2, we can see that this is a little less than a 25-year flood, **T ≈ 23 years.**

b) Log-Pearson Type III: Use the equation sequence; (12.25), (12.31a), (12.30), (12.28), and (12.29a).

$\log Q_T = m_l + K_T(s_l)$; $\log(430) = 2.29 + K_T\,(0.202)$;

$K_T = 1.70$; now, $k = G_l/6 = -0.227/6 = -0.0378$, and

$$K_T = z + (z^2 - 1)k + (z^3 - 6z)(k^2/3) - (z^2 - 1)k^3 + zk^4 + k^5/3$$

$$1.70 = z + (z^2 - 1)(-0.0378) + (z^3 - 6z)\{(-0.0378)^2/3) - (z^2 - 1)(-0.0378)^3 + z(-0.0378)^4 + (-0.0378)^5/3$$

Solving the implicit equation yields: $z = 1.78$

$$z = 1.78 = w - \frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3}$$

Solving the implicit equation yields: $w = 2.56$

$w = 2.56 = [\ln T^2]^{1/2}$; solving yields **T = 26.5 yrs**

or a little greater than the 25-year flood.

12.5.5

In Example 12.6, $\chi^2 = 0.872$ (the calculated value). From Table 12.4 we obtain $\chi_{0.05}{}^2 = 5.99$ and $\chi_{0.50}{}^2 = 1.39$. The calculated value is smaller than the table value in both cases. Therefore, we can conclude that the data fit the log-normal distribution at these significance levels.

It is interesting to note that the data pass the chi-square test with a larger margin for smaller values of α. The reason is that α represents the probability that, based on this test, we will reject a distribution that in reality fits the data well.

12.5.6

Three additional flows would need to appear in the first class interval and **three less in the middle** for the chi-square test to fail ($\chi^2 > \chi_\alpha{}^2 = 4.61$) at α = 0.010 as shown.

E_i	O_i	$(O_i\text{-}E_i)^2/E_i$
7.8	12	2.262
7.8	9	0.185
7.8	4	1.851
7.8	6	0.415
7.8	8	0.005
39	39	4.718

Four additional flows would need to appear in the first class interval and **four less in the fifth** for the chi-square test to fail ($\chi^2 > \chi_\alpha{}^2 = 4.61$) at α = 0.010 as shown.

E_i	O_i	$(O_i\text{-}E_i)^2/E_i$
7.8	13	3.467
7.8	9	0.185
7.8	7	0.082
7.8	6	0.415
7.8	4	1.851
39	39	6.000

12.5.7

The class intervals for the five equal probability increment is (1.0 – 0.0) / 5 = 0.20 are provided below. The exceedence probabilities of p = 0.8, 0.6, 0.4, and 0.2 are provided below along with the corresponding frequency factors obtained from Table 12.2. The corresponding discharges are determined, by using Equation (12.24) with m = 9,820 cfs and s = 4,660 cfs. The rest of the solution table is filled out as in Example 12.6. Also, $v = 5 - 2 - 1 = 2$. Then from Table 12.4 for $\alpha = 0.10$, we obtain $\chi_\alpha^2 = 4.61$. **Since $\chi^2 < \chi^2_\alpha$ (i.e., since 4.462 < 4.61), we conclude that the Normal distribution does indeed adequately fit the annual maximum discharge data series of the Meherrin River, but just barely.**

p =	0.8	0.6	0.4	0.2	m =	9820
K_T =	-0.841	-0.253	0.253	0.841	s =	4660

Class Interval	Exceedence Probability Limits		Discharge Limits (cfs)		E_i	O_i	$(O_i - E_i)^2/E_i$
I	Higher	Lower	Lower	Upper			
1	1	0.8	0	5,900	7.8	9	0.185
2	0.8	0.6	5,900	8,640	7.8	12	2.262
3	0.6	0.4	8,640	11,000	7.8	5	1.005
4	0.4	0.2	11,000	13,700	7.8	5	1.005
5	0.2	0	13,700	Infinity	7.8	8	0.005
				Totals	39	39	4.462

12.5.8

The class intervals for the five equal probability increment is (1.0 – 0.0) / 5 = 0.20 are provided below. The exceedence probabilities of p = 0.8, 0.6, 0.4, and 0.2 are provided below along with the corresponding frequency factors obtained from Table 12.2. The corresponding discharges are determined, by using Equation (12.24) with m = 9,820 cfs and s = 4,660 cfs. The rest of the solution table is filled out as in Example 12.6. Also, $v = 5 - 2 - 1 = 2$. Then from Table 12.4 for $\alpha = 0.50$, we obtain $\chi_\alpha^2 = 1.39$. **Since $\chi^2 < \chi^2_\alpha$ (i.e., since 1.385 < 1.39), we conclude that the Gumbel distribution does indeed adequately fit the annual maximum discharge data series of the Meherrin River, but just barely.**

p =	0.8	0.6	0.4	0.2	m =	9820
K_T =	-0.821	-0.382	0.074	0.719	s =	4660

Class Interval	Exceedence Probability Limits		Discharge Limits (cfs)		E_i	O_i	$(O_i - E_i)^2/E_i$
I	Higher	Lower	Lower	Upper			
1	1	0.8	0	5,990	7.8	9	0.185
2	0.8	0.6	5,990	8,040	7.8	9	0.185
3	0.6	0.4	8,040	10,200	7.8	8	0.005
4	0.4	0.2	10,200	13,200	7.8	5	1.005
5	0.2	0	13,200	Infinity	7.8	8	0.005
				Totals	39	39	1.385

12.5.9

Using Table 12.2, K_{10} = 1.282 (normal distribution) and K_{10} = 1.305 (Gumbel distribution). Also, when a 90-percent confidence level is used, $z = 1.645$ for $\beta = 0.90$. From Equation (12.40) and Equation (12.41)

$$a = 1 - \frac{z^2}{2(N-1)} = 1 - \frac{1.645^2}{2(20-1)} = 0.9288;$$

$$b = K_T^2 - \frac{z^2}{N} = 1.282^2 - \frac{1.645^2}{20} = 1.508$$

Next, from Equations (12.38) and (12.39)

$$K_{10U} = \frac{K_T + \sqrt{K_T^2 - ab}}{a} = \frac{1.282 + \sqrt{1.282^2 - (0.9288)(1.508)}}{0.9288} = 1.91$$

$$K_{10L} = \frac{K_T - \sqrt{K_T^2 - ab}}{a} = \frac{1.282 - \sqrt{1.282^2 - (0.9288)(1.508)}}{0.9288} = 0.850$$

Then, for the **Normal distribution** using Equations (12.34) and (12.35), the confidence limits are

$\boldsymbol{U_{10}} = m + K_{10U}(s) = 40.0 + 1.91(3.50)$ **= 46.7 in.**

$\boldsymbol{L_{10}} = m + K_{10L}(s) = 40.0 + 0.850(3.50)$ **= 43.0 in.**

For the Gumbel distribution; $a = 0.9288$ (same as normal) and from Equation (12.41)

$$b = K_T^2 - \frac{z^2}{N} = 1.305^2 - \frac{1.645^2}{20} = 1.568$$

and from Equations (12.38) and (12.39)

$$K_{10U} = \frac{K_T + \sqrt{K_T^2 - ab}}{a} = \frac{1.305 + \sqrt{1.305^2 - (0.9288)(1.568)}}{0.9288} = 1.94$$

$$K_{10L} = \frac{K_T - \sqrt{K_T^2 - ab}}{a} = \frac{1.305 - \sqrt{1.305^2 - (0.9288)(1.568)}}{0.9288} = 0.870$$

Then, for the **Gumbel distribution** using Equations (12.34) and (12.35), the confidence limits are

$\boldsymbol{U_{10}} = m + K_{10U}(s) = 40.0 + 1.94(3.50)$ **= 46.8 in.**

$\boldsymbol{L_{10}} = m + K_{10L}(s) = 40.0 + 0.870(3.50)$ **= 43.0 in.**

12.5.10

Using Table 12.2, K_{10} = 1.282 (normal distribution). Also, when a 90-percent confidence level is used, z = 1.645 for β = 0.90. From Equation (12.40) and Equation (12.41)

$$a = 1 - \frac{z^2}{2(N-1)} = 1 - \frac{1.645^2}{2(20-1)} = 0.9288; \qquad b = K_T^2 - \frac{z^2}{N} = 1.282^2 - \frac{1.645^2}{20} = 1.508$$

Next, from Equations (12.38) and (12.39)

$$K_{10U} = \frac{K_T + \sqrt{K_T^2 - ab}}{a} = \frac{1.282 + \sqrt{1.282^2 - (0.9288)(1.508)}}{0.9288} = 1.91$$

$$K_{10L} = \frac{K_T - \sqrt{K_T^2 - ab}}{a} = \frac{1.282 - \sqrt{1.282^2 - (0.9288)(1.508)}}{0.9288} = 0.850$$

For the **Log-Normal distribution**, the confidence limits using Equations (12.36) and (12.37) are

$\log U_{10} = m_l + K_{25U}(s_l) = 1.60 + 1.91(0.0379) = 1.67$ (**U_{10} = 46.8 in.)**

$\log L_{10} = m_l + K_{25L}(s_l) = 1.60 + 0.850(0.0379) = 1.63$ (**L_{10} = 42.7 in.)**

To solve for K_{10} in the Log-Pearson Type III distribution, we will solve Eq'ns (12.31a), (12.29a), (12.28), and (12.30).

$k = G_l/6 = 0.0144/6 = 0.0024; \qquad w = [\ln T^2]^{1/2} = [\ln 10^2]^{1/2} = 2.15$

$$z = w - \frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3} = 2.15 - \frac{2.515517 + 0.802853 \cdot 2.15 + 0.010328 \cdot (2.15)^2}{1 + 1.432788 \cdot 2.15 + 0.189269 \cdot (2.15)^2 + 0.001308 \cdot (2.15)^3} = 1.29$$

$K_T = z + (z^2 - 1)k + (z^3 - 6z)(k^2/3) - (z^2 - 1)k^3 + zk^4 + k^5/3$

$K_T = 1.29 + (1.29^2 - 1)0.0024 + (1.29^3 - 6 \cdot 1.29)(0.0024^2/3) - (1.29^2 - 1)0.0024^3 + 1.29 \cdot 0.0024^4 + 0.0024^5/3 = 1.29$

From Equation (12.40) and Equation (12.41)

$$a = 1 - \frac{z^2}{2(N-1)} = 1 - \frac{1.645^2}{2(20-1)} = 0.9288; \qquad b = K_T^2 - \frac{z^2}{N} = 1.29^2 - \frac{1.645^2}{20} = 1.529$$

and from Equations (12.38) and (12.39)

$$K_{10U} = \frac{K_T + \sqrt{K_T^2 - ab}}{a} = \frac{1.29 + \sqrt{1.29^2 - (0.9288)(1.529)}}{0.9288} = 1.92$$

$$K_{10L} = \frac{K_T - \sqrt{K_T^2 - ab}}{a} = \frac{1.29 - \sqrt{1.29^2 - (0.9288)(1.529)}}{0.9288} = 0.857$$

For the **Log-Pearson Type III distribution**, the confidence limits using Equations (12.36) and (12.37) are

$\log U_{10} = m_l + K_{10U}(s_l) = 1.60 + 1.92(0.0379) = 1.67$ (**U_{10} = 46.8 in.)**

$\log L_{10} = m_l + K_{10L}(s_l) = 1.60 + 0.857(0.0379) = 1.63$ (**L_{10} = 42.7 in.)**

12.5.11

The solution is presented in the table below follows the same procedure as Example 12.8. The values in column 2 are obtained from Table 12.2, and Equation (12.24) is used to determine the entries in column 3. Equations (12.40) and (12.41) are used to calculate the values in columns 4 and 5, respectively. Likewise, Equations (12.38) and (12.39) are used to determine the entries in columns 6 and 7. The upper and lower confidence limits listed in columns 8 and 9 are obtained by using Equations (12.34) and (12.35), respectively.

1	2	3	4	5	6	7	8	9
T	K_T	Q_T	a	b	K_{TU}	K_{TL}	U_T	L_T
1.25	-0.841	**5.90E+03**	0.964	0.638	-0.557	-1.187	**7.22E+03**	**4.29E+03**
2	0	**9.82E+03**	0.964	-0.069	0.268	-0.268	**1.11E+04**	**8.57E+03**
10	1.282	**1.58E+04**	0.964	1.574	1.697	0.962	**1.77E+04**	**1.43E+04**
25	1.751	**1.80E+04**	0.964	2.997	2.251	1.381	**2.03E+04**	**1.63E+04**
50	2.054	**1.94E+04**	0.964	4.150	2.613	1.647	**2.20E+04**	**1.75E+04**
100	2.327	**2.07E+04**	0.964	5.346	2.941	1.884	**2.35E+04**	**1.86E+04**
200	2.576	**2.18E+04**	0.964	6.566	3.242	2.100	**2.49E+04**	**1.96E+04**

12.5.12

The solution is presented in the table below follows the same procedure as Example 12.8. The values in column 2 are obtained from Table 12.2, and Equation (12.24) is used to determine the entries in column 3. Equations (12.40) and (12.41) are used to calculate the values in columns 4 and 5, respectively. Likewise, Equations (12.38) and (12.39) are used to determine the entries in columns 6 and 7. The upper and lower confidence limits listed in columns 8 and 9 are obtained by using Equations (12.34) and (12.35), respectively.

1	2	3	4	5	6	7	8	9
T	K_T	Q_T	a	b	K_{TU}	K_{TL}	U_T	L_T
1.25	-0.821	**5.99E+03**	0.964	0.605	-0.539	-1.164	**7.31E+03**	**4.40E+03**
2	-0.164	**9.06E+03**	0.964	-0.042	0.100	-0.440	**1.03E+04**	**7.77E+03**
10	1.305	**1.59E+04**	0.964	1.634	1.724	0.983	**1.79E+04**	**1.44E+04**
25	2.044	**1.93E+04**	0.964	4.109	2.601	1.638	**2.19E+04**	**1.75E+04**
50	2.592	**2.19E+04**	0.964	6.649	3.261	2.114	**2.50E+04**	**1.97E+04**
100	3.137	**2.44E+04**	0.964	9.771	3.923	2.583	**2.81E+04**	**2.19E+04**
200	3.679	**2.70E+04**	0.964	13.466	4.583	3.047	**3.12E+04**	**2.40E+04**

12.6.1

For Q = 21,100 cfs, from the theoretical straight line of Figure 12.3, we obtain p = 3% or 0.03.

Then **T =** 1/0.03 **= 33 years**. Alternatively, using Equation (12.24) with $Q_T = 430\ m^3/s$

$\log Q_T = m_l + K_T(s_l)$; $\log(21{,}100) = 3.95 + K_T(0.197)$; $K_T = 1.90$; from Table 12.2, **T ≈ 35 years.**

12.6.2

For Q = 25,500 cfs, from the theoretical straight line of Figure 12.3 we obtain p = 0.01 and T = 100 years. Therefore, the probability of exceedence in any one year is p = 0.01= 1%. The risk of flooding during the 50-year service life can be determined from Equation (12.23): **R** $= 1 - (1 - 0.01)^{50} = 0.395 =$ **39.5 %**

12.6.3

The table below lists the annual precipitation depths in decreasing order for the twenty year record. A rank, r = 1 to 20 is entered in the first column. The exceedence probability, p, and the return period, T, for each precipitation depth are calculated using Equations (12.43) and (12.44) and are tabulated in the third and forth columns. This information is then plotted on Normal probability paper along with the theoretical probability distribution.

Rank, r	Rainfall (in.)	Plotting position p	Plotting position T (years)
1	47.6	0.048	21.0
2	46.1	0.095	10.5
3	44.2	0.143	7.00
4	42.5	0.190	5.25
5	42.0	0.238	4.20
6	41.7	0.286	3.50
7	41.2	0.333	3.00
8	40.5	0.381	2.63
9	40.2	0.429	2.33
10	39.7	0.476	2.10
11	39.3	0.524	1.91
12	39.2	0.571	1.75
13	38.8	0.619	1.62
14	38.6	0.667	1.50
15	38.5	0.714	1.40
16	38.3	0.762	1.31
17	37.7	0.810	1.24
18	35.8	0.857	1.17
19	35.0	0.905	1.11
20	33.1	0.952	1.05

The theoretical probability distribution is found (three points) using Equation (12.24) and Table 12.2:

P_2 $= m + K_2(s) = 40.0 + 0.00(3.50)$ **= 40.0 in.**

P_{10} $= m + K_{10}(s) = 40.0 + 1.282(3.50)$ **= 44.5 in.**

P_{25} $= m + K_{25}(s) = 40.0 + 1.751(3.50)$ **= 46.1 in.**

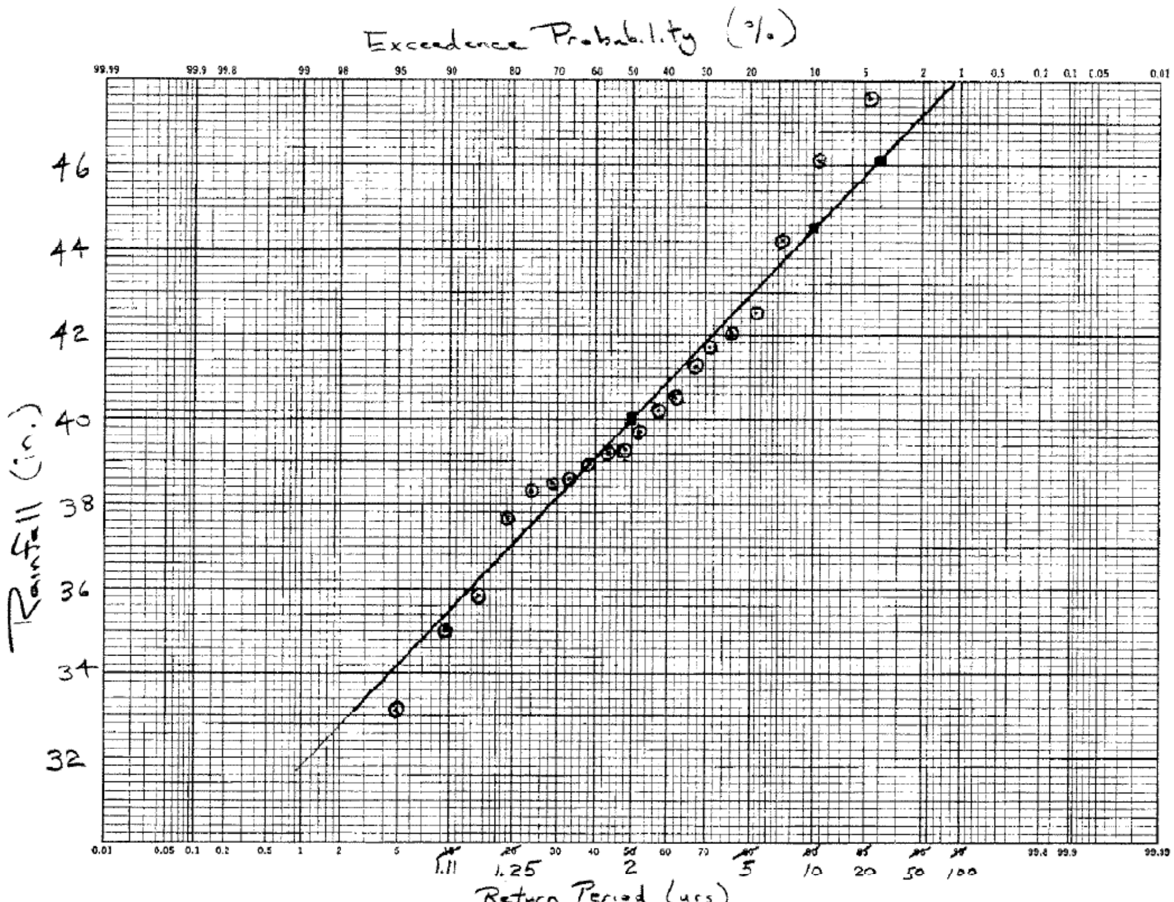


Figure 12.2 Probability (Normal distribution) graph paper.