Planejamento de Rotas - Parte III

SSC5955

Slides adaptados de Masahiro Ono - MIT

Sumário

- Aspectos de Modelagem
- Fixed Risk
  - Fixed Risk Relaxation
  - Fixed Risk Tightening
  - Customized Approach
- Métodos
- Alguns resultados computacionais

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- Aspectos de Modelagem
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MILP Formulation

- **Objective Function**

$$\min g(\cdot) \qquad (1)$$

- **Dynamic Equation**

$$\mu_t = A^t \hat{x}_0 + \sum_{s=0}^{t-1} A^{t-s-1} B u_s \qquad \forall(t) \qquad (2)$$

- **Goal Position**

$$\mu_T = x_{goal} \qquad (3)$$

MILP Formulation

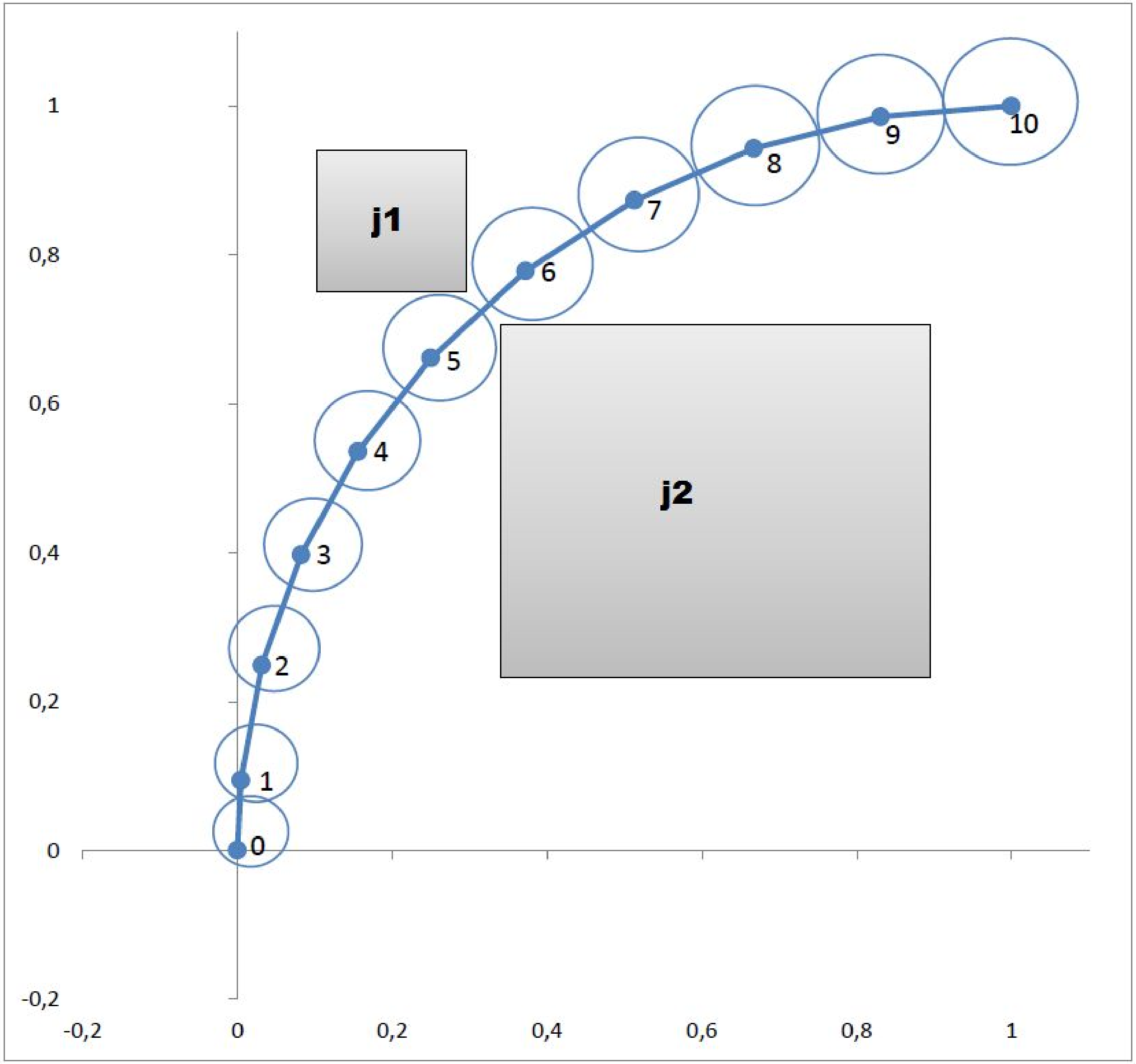
- **Obstacle Avoidance Constraints O(j,t)**

$$c_{t,i}(\delta_{jt}) + b_{ji} - a_{ji}^T \mu_t \leq M(1 - Z_{jti}) \qquad \forall(j,t,i) \qquad (4)$$

$$c_{t,i}(\delta_{jt}) = \boldsymbol{err^{-1}}(\mathbf{1} - \mathbf{2}\boldsymbol{\delta}_{jt})\sqrt{a_{ji}^T \sum\nolimits_{\boldsymbol{t}} a_{ji}} \quad \forall(j,t,i) \qquad (5)$$

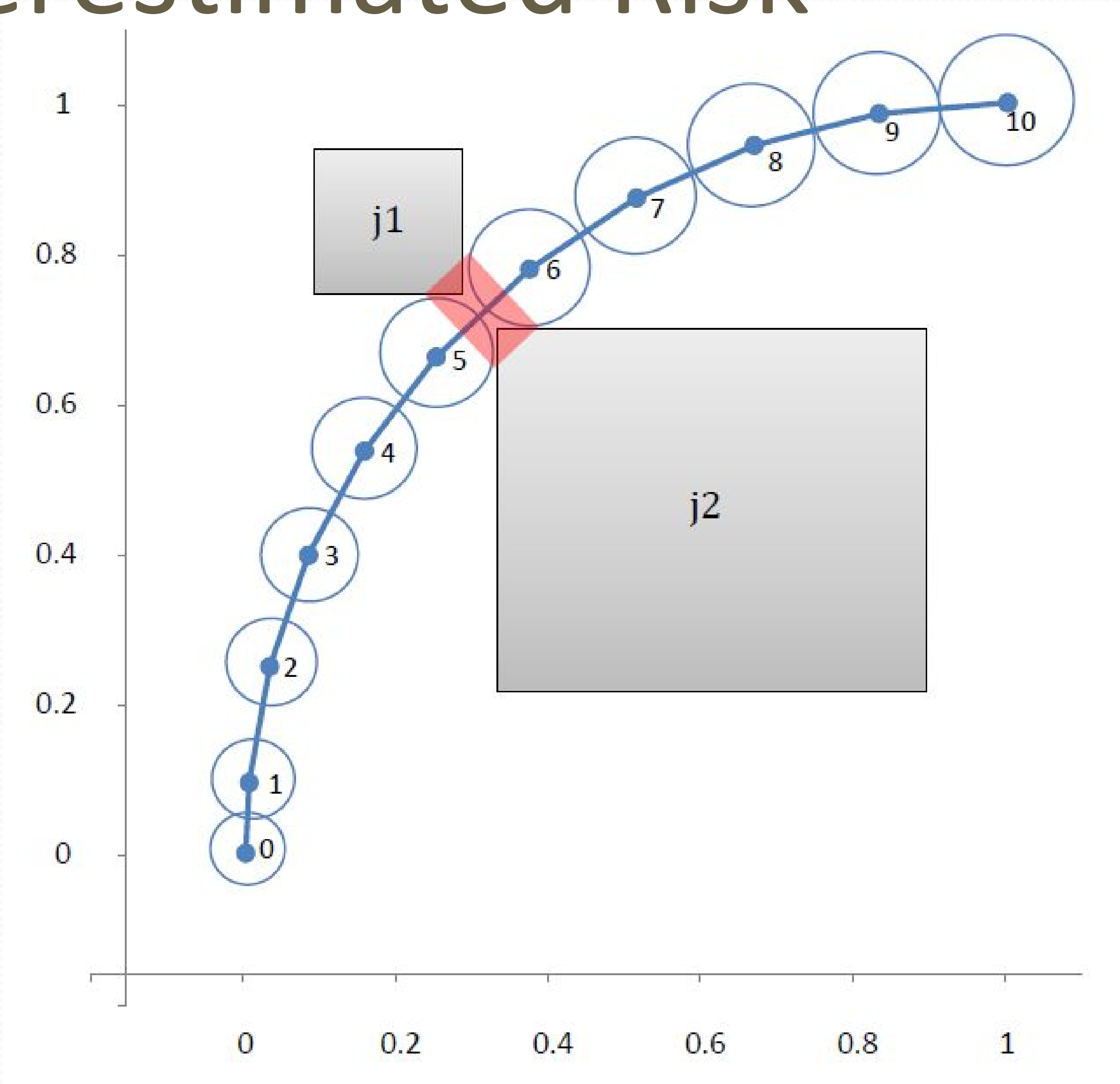
$$\sum_j \sum_t \delta_{jt} \leq \Delta \qquad (6)$$

$$\sum_{i \in G_j} Z_{jti} \geq 1 \qquad \forall(j,t) \qquad (7)$$

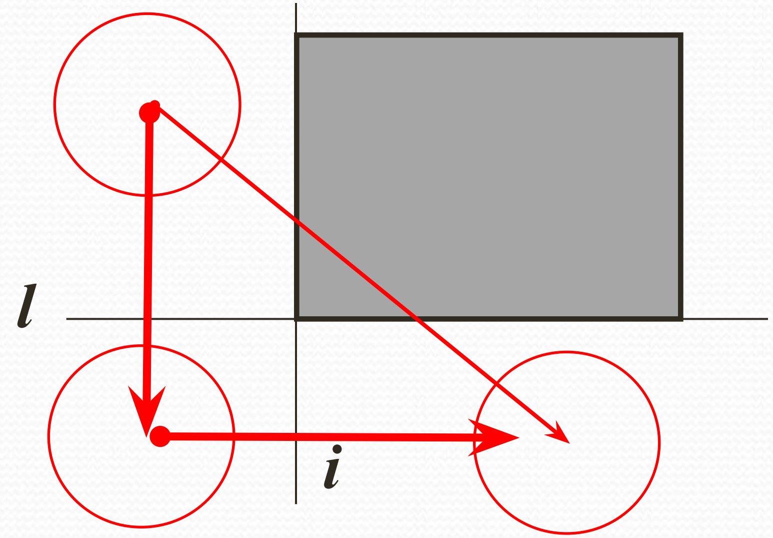


j1
j2
0
1
2
3
4
5
6
7
8
9
10
1
0,8
0,6
0,4
0,2
0
-0,2
-0,2
0
0,2
0,4
0,6
0,8
1

Underestimated Risk



Underestimated Risk



- **Proposed New Constraints N(j,t):**

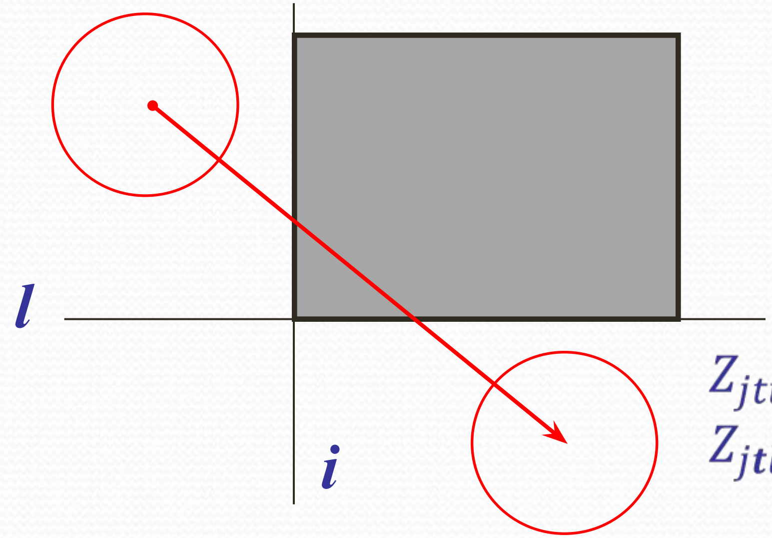
$$\sum_{i} Z_{tji} \geq 1 + \sum_{i \in G_j} \sum_{l \in G_j} \alpha_{jtil} \qquad \forall (j,t) \qquad \textbf{(8)}$$

$$\alpha_{jtil} \geq \left| Z_{jti} - Z_{j(t-1)i} \right| + \left| Z_{jtl} - Z_{j(t-1)l} \right| - 1 \qquad \forall (j,t,i,l)$$

(9)

Underestimated Risk

$Z_{j(t-1)i} = \mathbf{1}$
$Z_{j(t-1)l} = \mathbf{0}$



$Z_{jti} = \mathbf{0}$
$Z_{jtl} = \mathbf{1}$

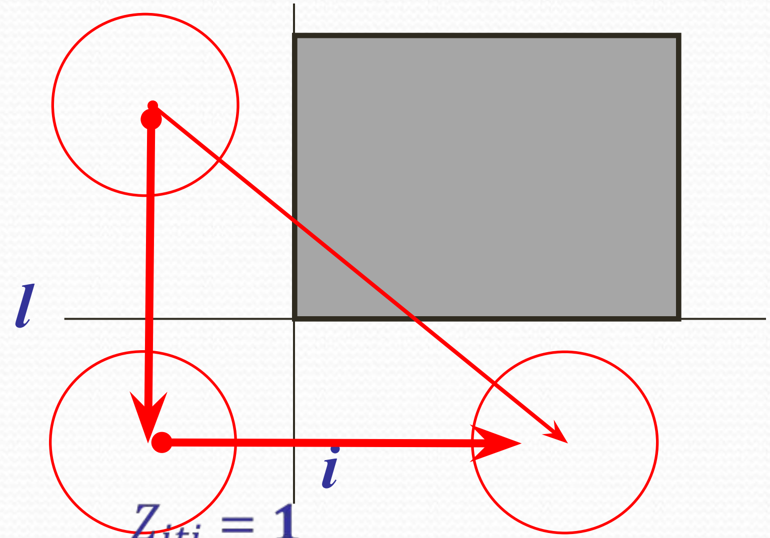
$$\alpha_{jtil} \geq \left|Z_{jti} - Z_{j(t-1)i}\right| + \left|Z_{jtl} - Z_{j(t-1)l}\right| - 1 \qquad \forall(j,t,i,l) \quad \mathbf{(9)}$$

$$\alpha_{jtil} \geq |\mathbf{0} - \mathbf{1}| + |\mathbf{1} - \mathbf{0}| - 1 = \mathbf{1}$$

$$\mathbf{1} = \sum_{i} Z_{jti} \geq 1 + \sum_{i \in G_j} \sum_{l \in G_j} \alpha_{jtil} = \mathbf{2} \; \forall(j,t) \qquad \mathbf{(8)}$$

Underestimated Risk

$Z_{j(t-1)i} = \mathbf{1}$
$Z_{j(t-1)l} = \mathbf{0}$



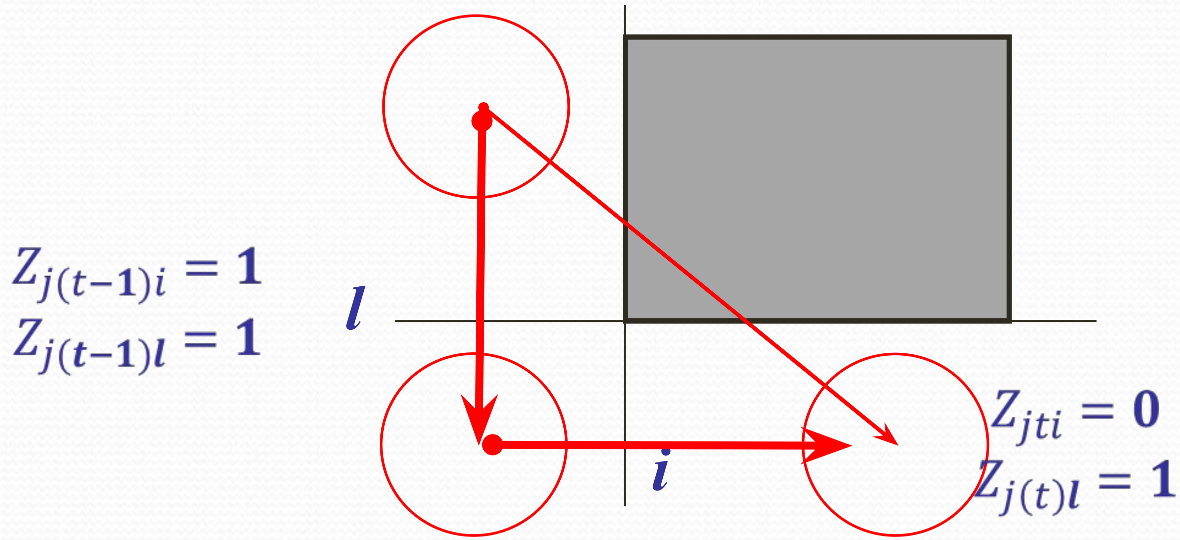
$Z_{jti} = \mathbf{1}$
$Z_{jtl} = \mathbf{1}$

$$\alpha_{jtil} \geq \left|Z_{jti} - Z_{j(t-1)i}\right| + \left|Z_{jtl} - Z_{j(t-1)l}\right| - 1 \qquad \forall (j,t,i,l) \quad \mathbf{(9)}$$

$$\alpha_{jtil} \geq |\mathbf{1} - \mathbf{1}| + |\mathbf{1} - \mathbf{0}| - 1 = \mathbf{0}$$

$$\mathbf{2} = \sum_{i} Z_{jti} \geq 1 + \sum_{i \in G_j} \sum_{l \in G_j} \alpha_{jtil} = \mathbf{1} \; \forall (j,t) \qquad \mathbf{(8)}$$

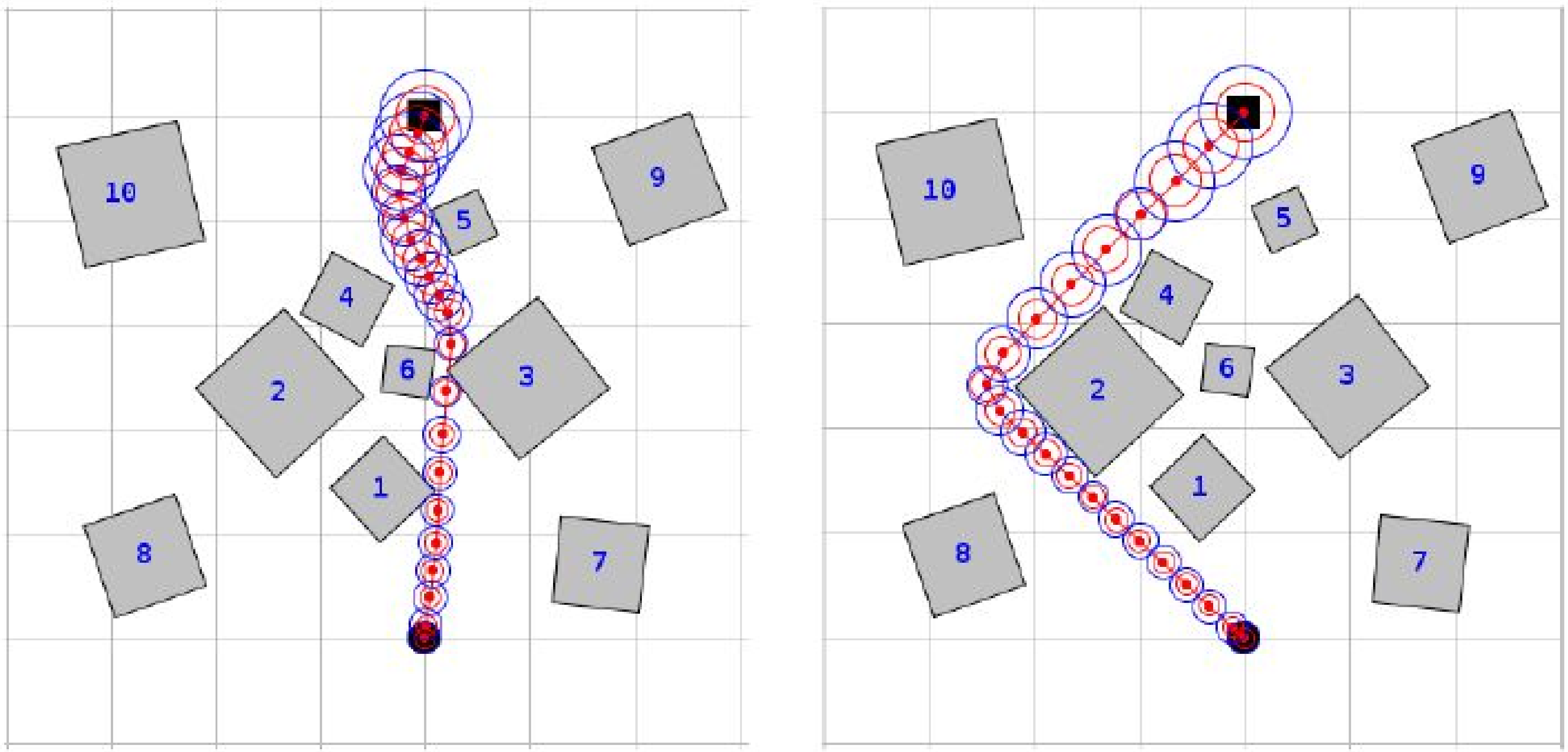
Underestimated Risk



$$\alpha_{jtil} \geq \left|Z_{jti} - Z_{j(t-1)i}\right| + \left|Z_{jtl} - Z_{j(t-1)l}\right| - 1 \qquad \forall(j,t,i,l) \quad \mathbf{(9)}$$

$$\alpha_{jtil} \geq |\mathbf{0} - \mathbf{1}| + |\mathbf{1} - \mathbf{1}| - 1 = \mathbf{0}$$

$$\mathbf{1} = \sum_{i} Z_{jti} \geq 1 + \sum_{i \in G_j} \sum_{l \in G_j} \alpha_{jtil} = \mathbf{1} \ \forall(j,t) \qquad \mathbf{(8)}$$



(a) Model-1: 16.35*sec* | 10.32*m*

(b) Model-2: 3600.69*sec* | 14.09*m*

Underestimated Risk

- **Proposed New New Constraints – Not used yet!!**

$$\left[\bigvee_{i\in G_j}\left(Z_{jt,i}\ \wedge\ Z_{j,t-1,i}\right)\right] = 1 \qquad \forall(j, t > 0)$$

$$\Leftrightarrow$$

$$\left[\sum_{i\in G_j}\left(Z_{jt,i}\ \wedge\ Z_{j,t-1,i}\right)\right] \geq 1 \qquad \forall(j, t > 0)$$

$$\Leftrightarrow$$

$$\left[\sum_{i\in G_j} p_i\right] \geq 1 \qquad \forall(j, t > 0)$$

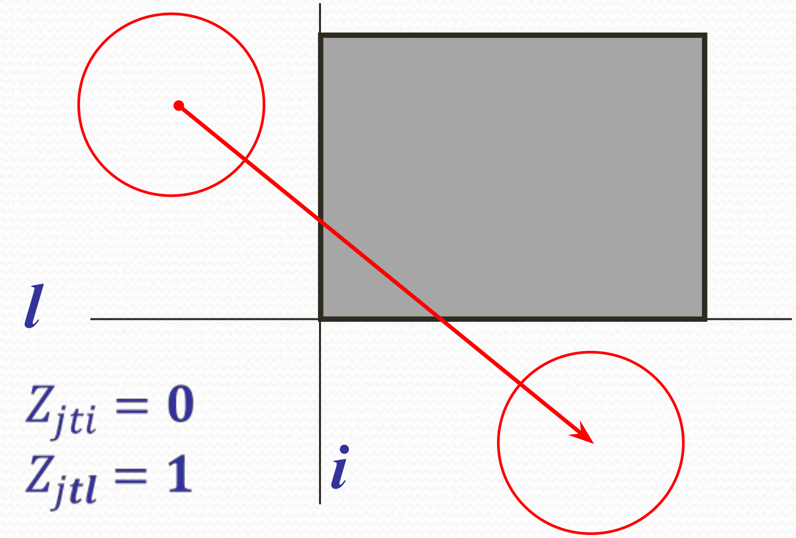
$$p_i = Z_{jt,i}\ \wedge\ Z_{j,t-1,i} \quad \Leftrightarrow \quad p_i \geq Z_{jt,i} + Z_{j,t-1,i} - 1$$

$$p_i \leq Z_{jt,i}$$

$$p_i \leq Z_{j,t-1,i}$$

Underestimated Risk

$$Z_{j(t-1)i} = 1$$
$$Z_{j(t-1)\boldsymbol{l}} = 0$$



$$Z_{jti} = 0$$
$$Z_{jt\boldsymbol{l}} = 1$$

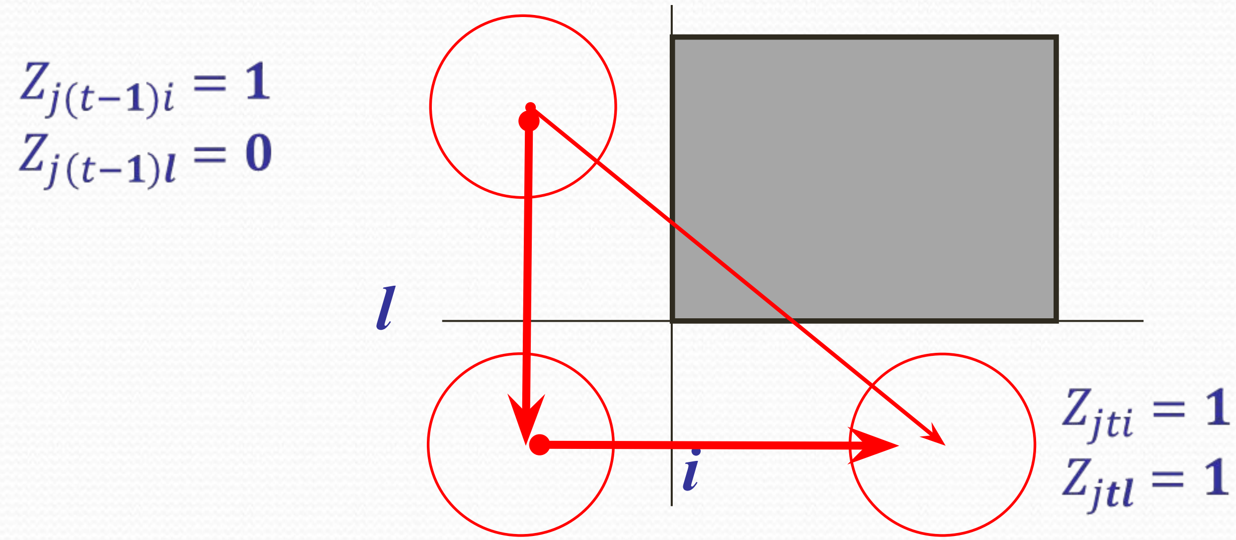
$$p_i \geq Z_{jt,i} + Z_{j,t-1,i} - 1 = 0 + 1 - 1 = 0$$
$$p_{\boldsymbol{l}} \geq Z_{jt,\boldsymbol{l}} + Z_{j,t-1,\boldsymbol{l}} - 1 = 1 - 0 - 1 = 0$$

$$0 = p_i \leq Z_{jt,i} = 0$$
$$0 = p_i \leq Z_{j,t-1,i} = 1$$

$$0 = p_{\boldsymbol{l}} \leq Z_{jt,\boldsymbol{l}} = 1$$
$$0 = p_{\boldsymbol{l}} \leq Z_{j,t-1,\boldsymbol{l}} = 0$$

$$0 = \left[\sum_{i \in G_j} p_i\right] \geq 1 \qquad \forall (j, t > 0)$$

Underestimated Risk



$$p_i \geq Z_{jt,i} + Z_{j,t-1,i} - 1 = 1 + 1 - 1 = 1$$
$$p_l \geq Z_{jt,l} + Z_{j,t-1,l} - 1 = 1 - 0 - 1 = 0$$

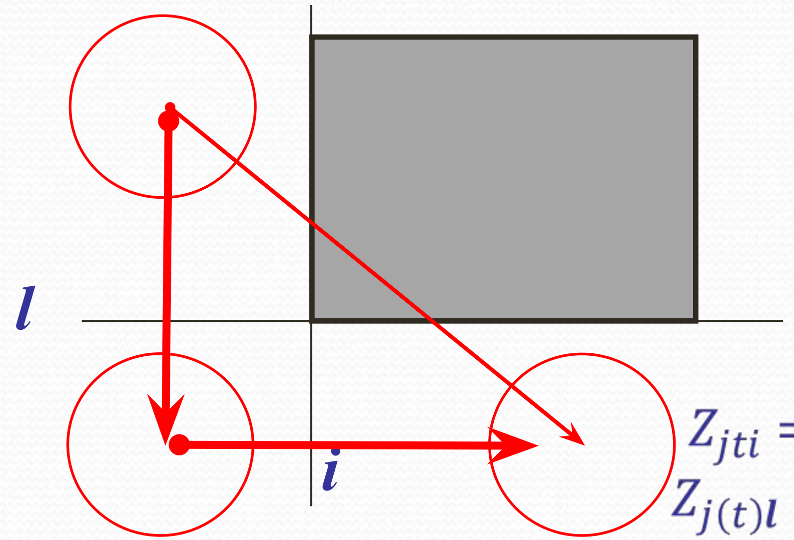
$$1 = p_i \leq Z_{jt,i} = 1$$
$$1 \leq Z_{j,t-1,i} = 1$$

$$0 = p_l \leq Z_{jt,l} = 1$$
$$0 = p_l \leq Z_{j,t-1,l} = 0$$

$$1 + 0 = \left[\sum_{i \in G_j} p_i\right] \geq 1 \qquad \forall (j, t > 0)$$

Underestimated Risk

$Z_{j(t-1)i} = \mathbf{1}$
$Z_{j(t-1)\boldsymbol{l}} = \mathbf{1}$



$Z_{jti} = \mathbf{0}$
$Z_{j(t)\boldsymbol{l}} = \mathbf{1}$

$$p_i \geq Z_{jt,i} + Z_{j,t-1,i} - \mathbf{1} = \mathbf{0} + \mathbf{1} - \mathbf{1} = \mathbf{0}$$
$$p_{\boldsymbol{l}} \geq Z_{jt,\boldsymbol{l}} + Z_{j,t-1,\boldsymbol{l}} - \mathbf{1} = \mathbf{1} + \mathbf{1} - \mathbf{1} = \mathbf{1}$$

$$\mathbf{0} = p_i \leq Z_{jt,i} = \mathbf{0}$$
$$\mathbf{0} \leq Z_{j,t-1,i} = \mathbf{1}$$

$$\mathbf{1} = p_{\boldsymbol{l}} \leq Z_{jt,\boldsymbol{l}} = \mathbf{1}$$
$$\mathbf{1} = p_{\boldsymbol{l}} \leq Z_{j,t-1,\boldsymbol{l}} = \mathbf{1}$$

$$\mathbf{1} + \mathbf{0} = \left[\sum_{i \in G_j} p_i\right] \geq 1 \qquad \forall (j, t > 0)$$

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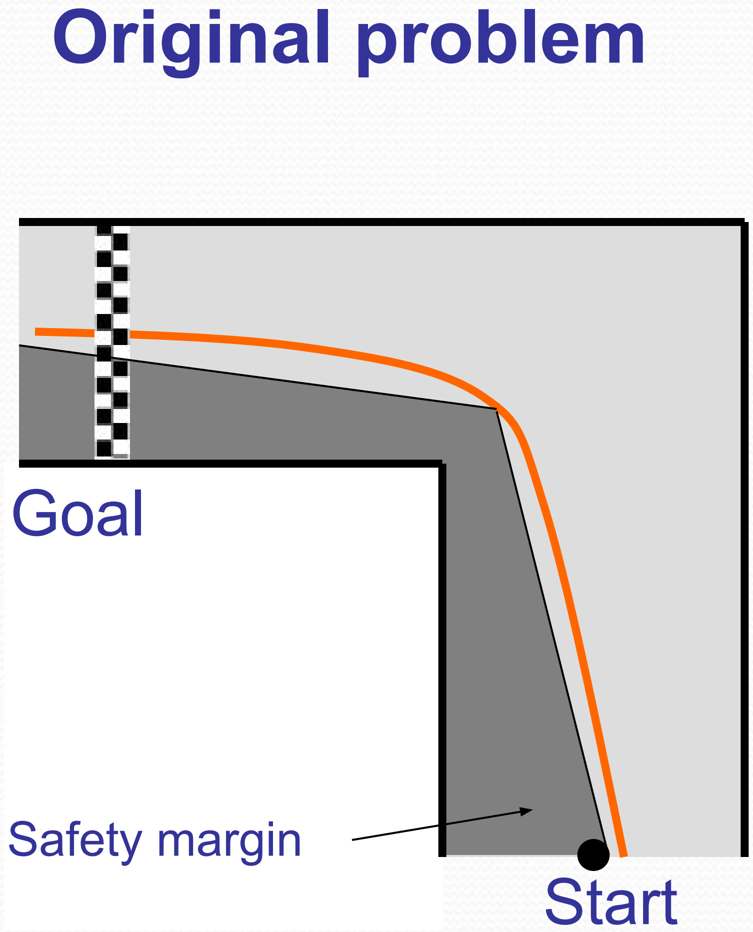
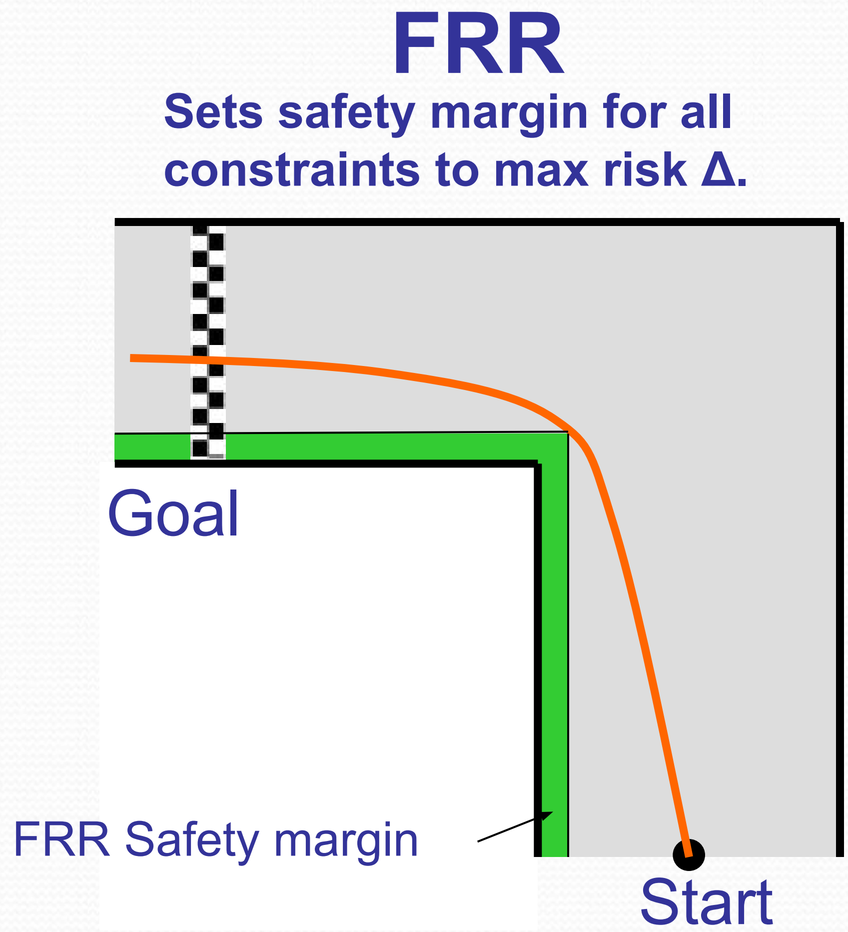
Sumário

- Aspectos de Modelagem
- Fixed Risk
  - Fixed Risk Relaxation
  - Fixed Risk Tightening
  - Customized Approach
- Métodos
- Alguns resultados computacionais

Fixed Risk Relaxation: Intuição





- Results in an **infeasible** solution to the original problem.
- Gives **lower bound** for the cost of the original problem.

Fixed Risk Relaxation (FRR)

$$\delta_{jt} = \Delta$$

- **Obstacle Avoidance Constraints O(j,t)**

$$c_{t,i}(\delta_{jt}) + b_{ji} - a_{ji}^{T}\mu_t \leq M(1 - Z_{jti}) \quad \forall(j,t,i) \qquad (4)$$

$$c_{t,i}(\delta_{jt}) = \boldsymbol{err}^{-1}(\mathbf{1} - \mathbf{2}\boldsymbol{\delta}_{jt})\sqrt{a_{ji}^{T}\sum\nolimits_{\boldsymbol{t}} a_{ji}} \quad \forall(j,t,i) \qquad (5)$$

$$\sum_{j}\sum_{t}\delta_{jt} \leq \Delta \qquad (6)$$

Fixed Risk Relaxation (FRR)

$$\delta_{jt} = \Delta$$

- **Obstacle Avoidance Constraints O(j,t)**

$$c_{t,i}(\Delta) + b_{ji} - a_{ji}^T \mu_t \leq M\left(1 - Z_{jti}\right) \quad \forall (j,t,i) \qquad (4)$$

$$c_{t,i}(\Delta) = \boldsymbol{err}^{-1}(\mathbf{1} - \mathbf{2}\Delta)\sqrt{a_{ji}^T \sum\nolimits_{\boldsymbol{t}} a_{ji}} \quad \forall (j,t,i) \qquad (5)$$

$$\cancel{\sum_j \sum_t \delta_{jt} \leq \Delta \qquad (6)}$$

Risk Allocation Approach (RAA)

- Define risk allocation from solution retuned by FRR or FRT.
  - Variables Zijt are fixed.

- Constraints (5) are non-linear
  - Non-linear solver
  - MILP solver using Piece-wise linear approximation.

Fixed Risk Tightening (FRT)

$$\delta_{jt} = \frac{\Delta}{\boldsymbol{T.J}}$$

- **Obstacle Avoidance Constraints O(j,t)**

$$c_{t,i}(\delta_{jt}) + b_{ji} - a_{ji}^{T}\mu_t \leq M\left(1 - Z_{jti}\right) \quad \forall (j,t,i) \quad (4)$$

$$c_{t,i}(\delta_{jt}) = \boldsymbol{err^{-1}(1 - 2\delta_{jt})}\sqrt{a_{ji}^{T}\sum\nolimits_{\boldsymbol{t}} a_{ji}} \quad \forall (j,t,i) \quad (5)$$

$$\sum_{j}\sum_{t}\delta_{jt} \leq \Delta \quad (6)$$

Fixed Risk Tightening (FRT)

$$\delta_{jt} = \frac{\Delta}{\boldsymbol{T.J}}$$

- **Obstacle Avoidance Constraints O(j,t)**

$$c_{t,i}\left(\frac{\Delta}{\boldsymbol{T.J}}\right) + b_{ji} - a_{ji}^{T}\mu_t \le M\left(1 - Z_{jti}\right) \qquad \forall(j,t,i) \qquad (4)$$

$$c_{t,i}\left(\frac{\Delta}{\boldsymbol{T.J}}\right) = \boldsymbol{err^{-1}(1-2}\frac{\Delta}{\boldsymbol{T.J}})\sqrt{a_{ji}^{T}\sum\nolimits_{\boldsymbol{t}} a_{ji}} \quad \forall(j,t,i) \qquad (5)$$

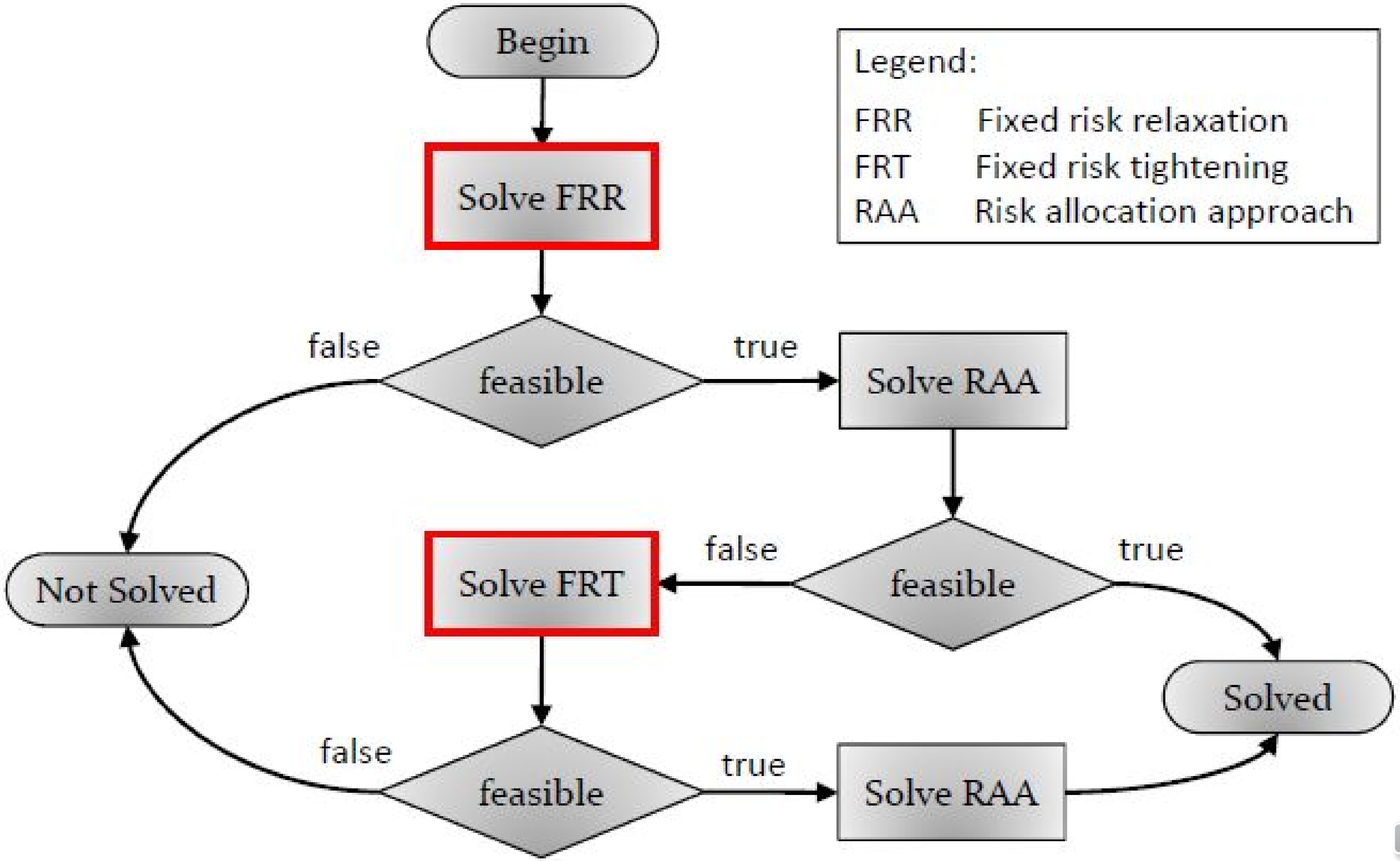
$$\cancel{\sum_{j}\sum_{t}\delta_{jt} \le \Delta \qquad (6)}$$

Sumário

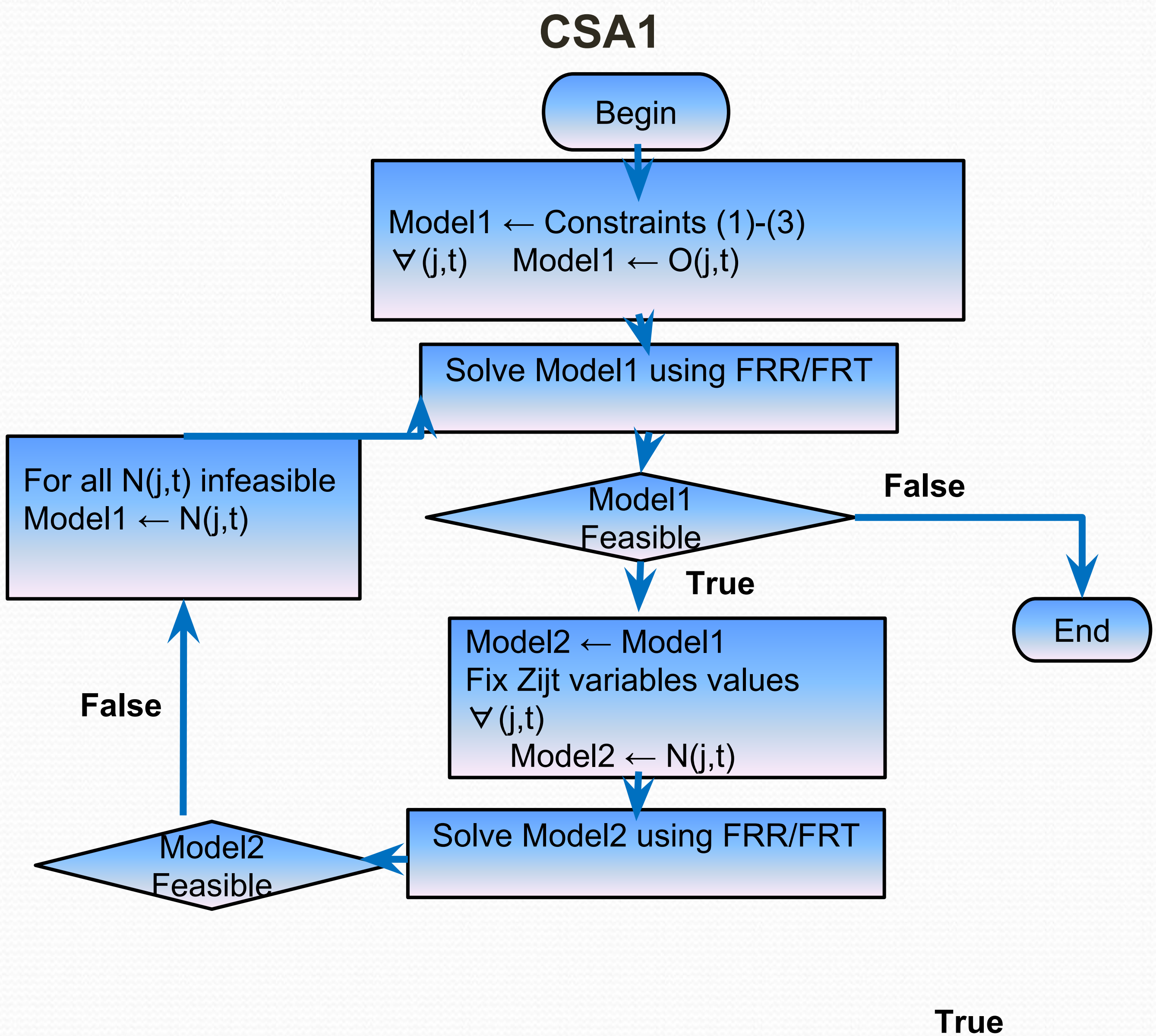
- Aspectos de Modelagem
- Fixed Risk
- Métodos
  - Customized Solution Approach - CSA
  - CSA1, CSA2 and CSA3.
- Alguns resultados computacionais

Solution Approaches

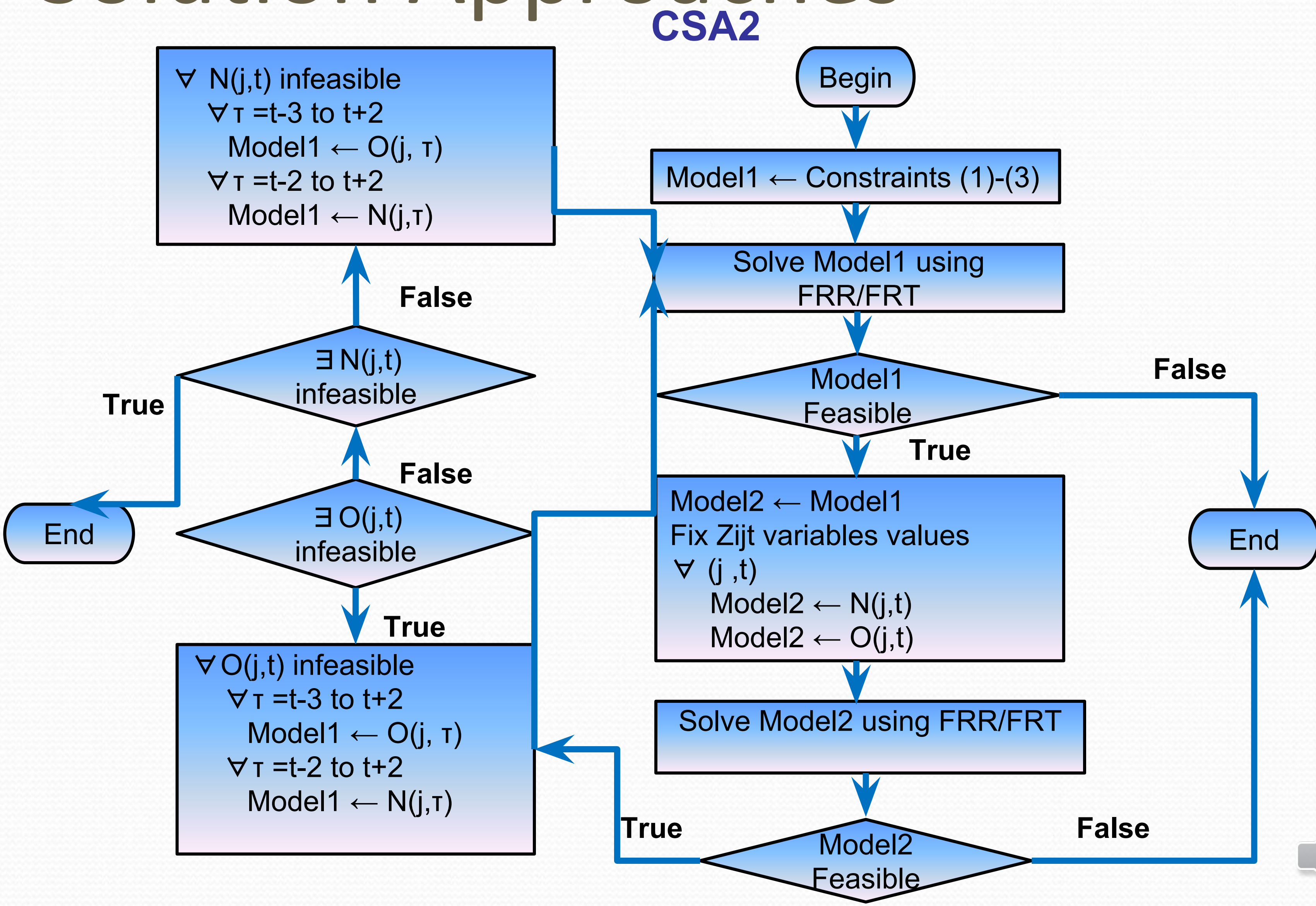
Customized Solution Approach (CSA)



Solution Approaches



Solution Approaches



Outline

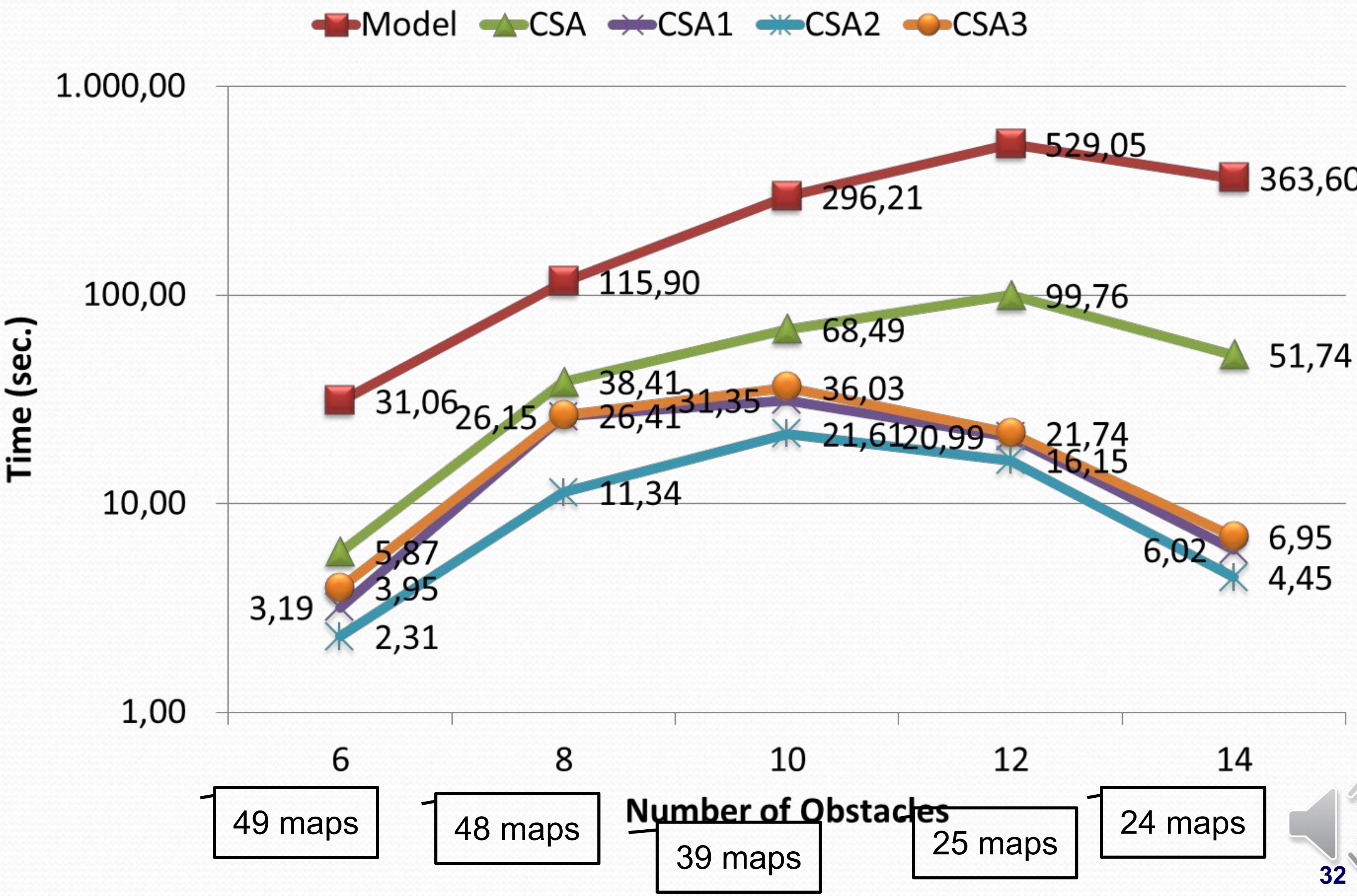
- Aspectos de Modelagem
- Fixed Risk
- Métodos
- Alguns resultados computacionais
  - Increasing number of obstacles
  - Increasing number of time steps

Computational Results

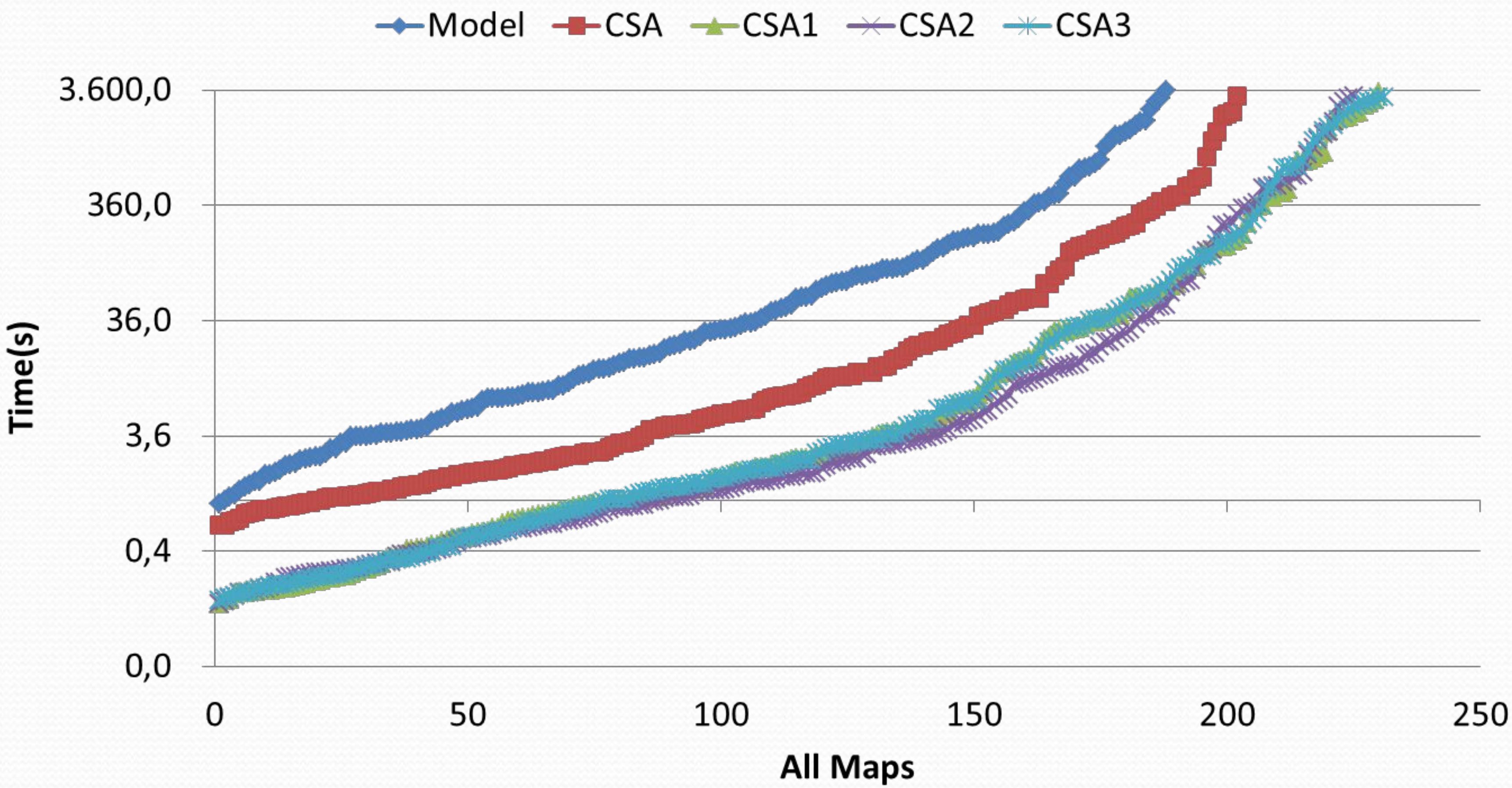
- Random map generator based on one described in Blackmore et al 2011.
  - Time limit 1 hour, 500 maps evaluated;
  - 50 maps for $J = 6; 8; 10; 12; 14$ with $T = 20$ and $\Delta = 0{:}001$.
  - 50 maps for $T = 10; 15; 20; 25; 30$ with $J = 10$ and $\Delta = 0{:}001$.

Method	Number of Obstacles				
	6	8	10	12	14
Model	49	48	40	27	24
CSA	50	50	44	29	29
CSA1	50	50	46	43	41
CSA2	50	50	45	42	38
CSA3	50	50	46	43	42

Computational Results

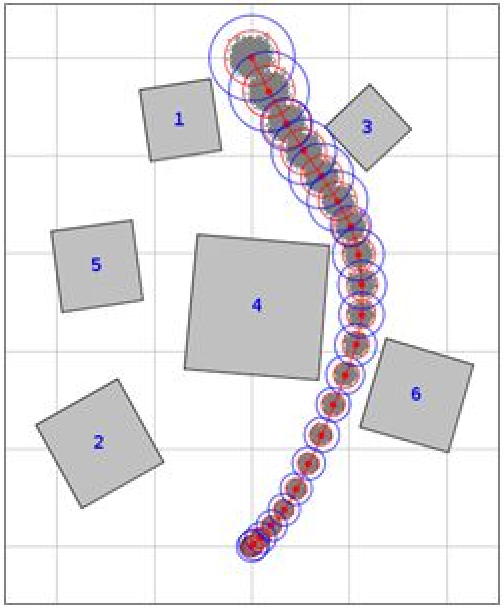


Computational Results

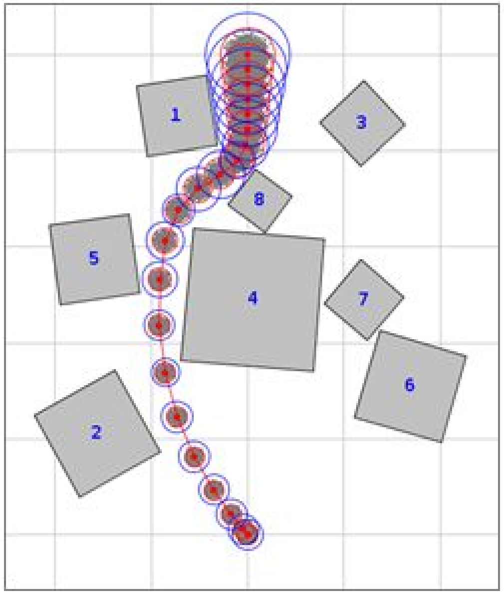


Computational Results

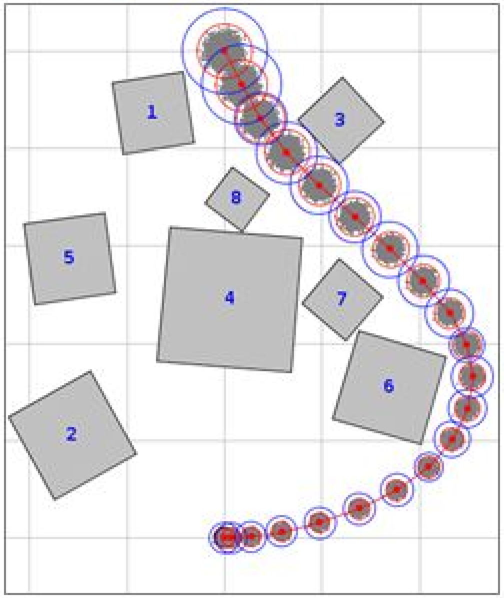
(a)	J = 6	
Method	**Time(s)**	**Dist.(m)**
Model2	26.31	11.166
C&A	2.50	11.166
C&Ai1	1.68	11.181
C&Ai2	1.29	11.181
C&Ai3	1.49	11.181



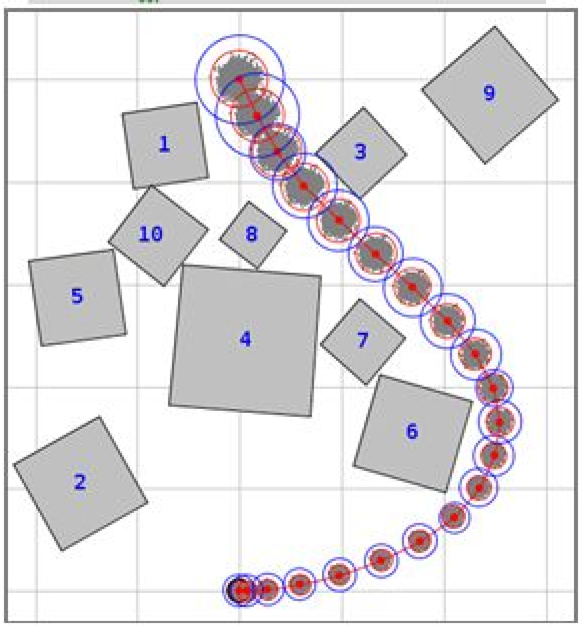
(b)	J = 8	
Method	**Time(s)**	**Dist.(m)**
Model2	26580	11.138
-	-	-
-	-	-
-	-	-
-	-	-



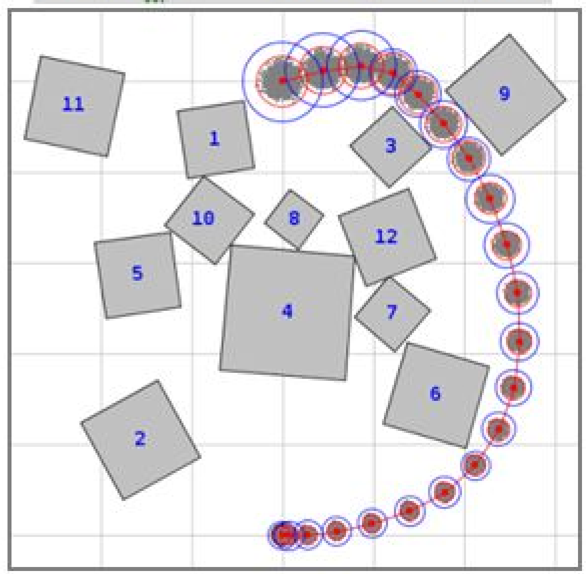
(c)	J = 8	
Method	**Time(s)**	**Dist.(m)**
-	-	-
C&A	3156	15.202
C&Ai1	1364	15.202
C&Ai2	3068	15.212
C&Ai3	2210	15.213



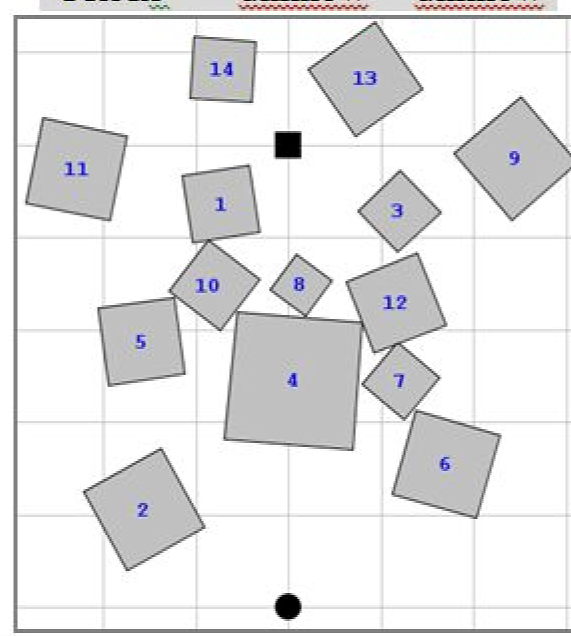
(d)	J = 10	
Method	Time(s)	Dist.(m)
Model2	unknow	unknow
C&A	4964	15.190
C&Ai1	8264	15.210
C&Ai2	5387	15.216
C&Ai3	3011	15.215



(e)	J = 12	
Method	Time(s)	Dist.(m)
Model2	unknow	unknow
C&A	unknow	unknow
C&Ai1	14207	16.689
C&Ai2	16521	16.855
C&Ai3	18190	16.673

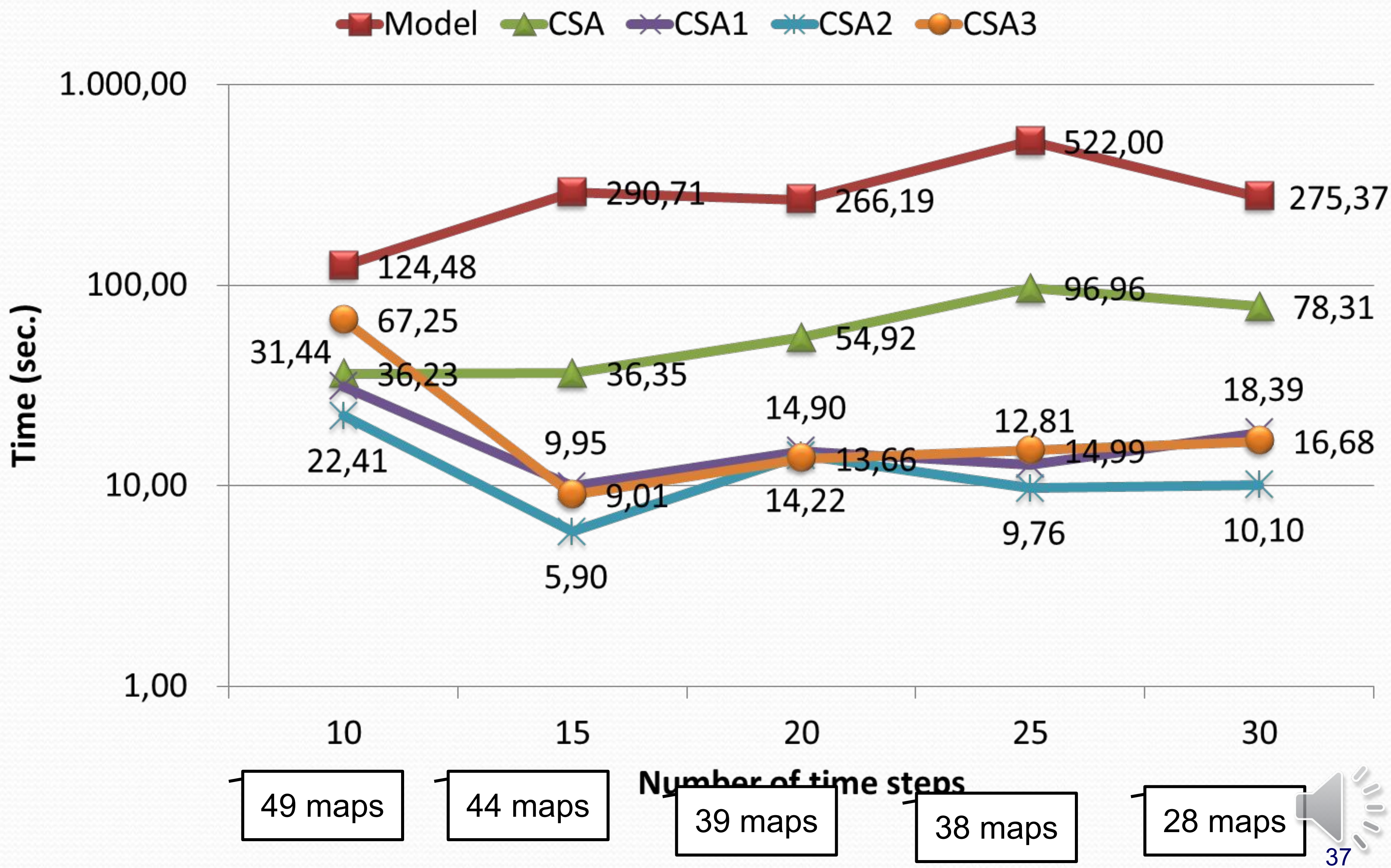


(f)	J = 14	
Method	Time(s)	Dist.(m)
Model2	unknow	unknow
C&A	unknow	unknow
C&Ai1	unknow	unknow
C&Ai2	unknow	unknow
C&Ai3	unknow	unknow

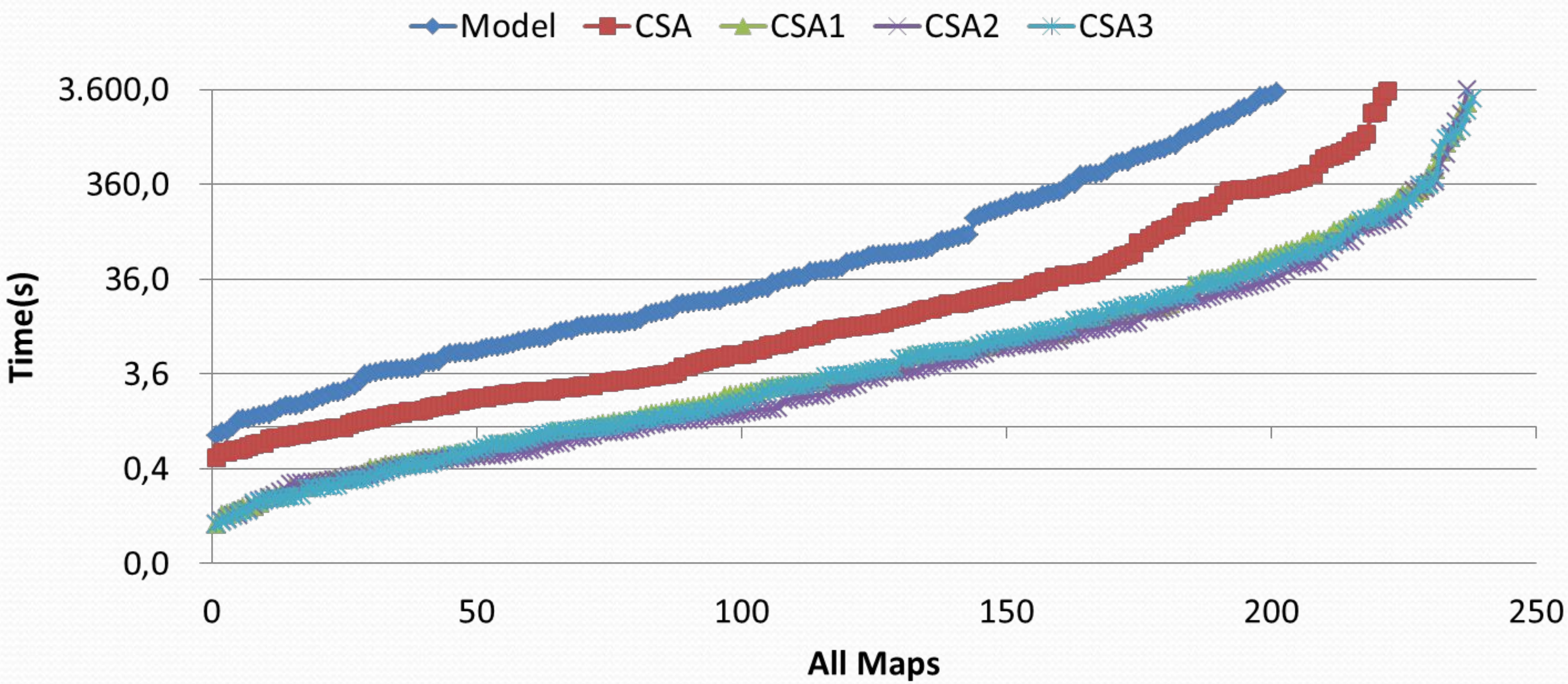


Method	Time Steps				
	10	15	20	25	30
Model	49	45	40	38	29
CSA	50	46	44	44	38
CSA1	50	50	48	46	43
CSA2	50	49	47	47	44
CSA3	50	50	48	46	44

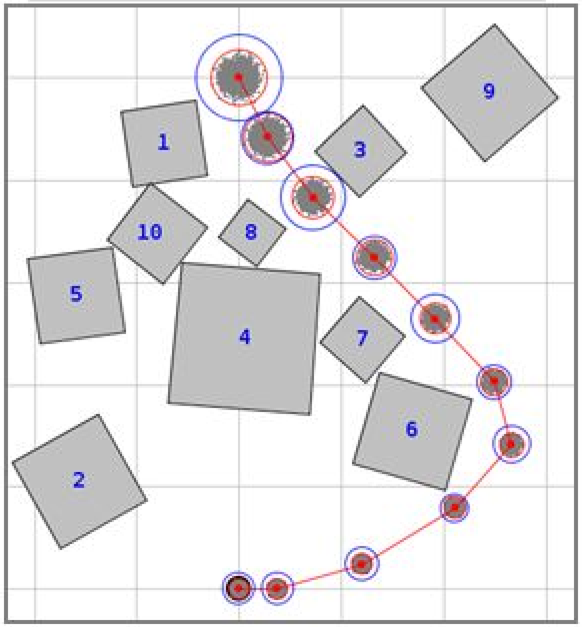
Computational Results



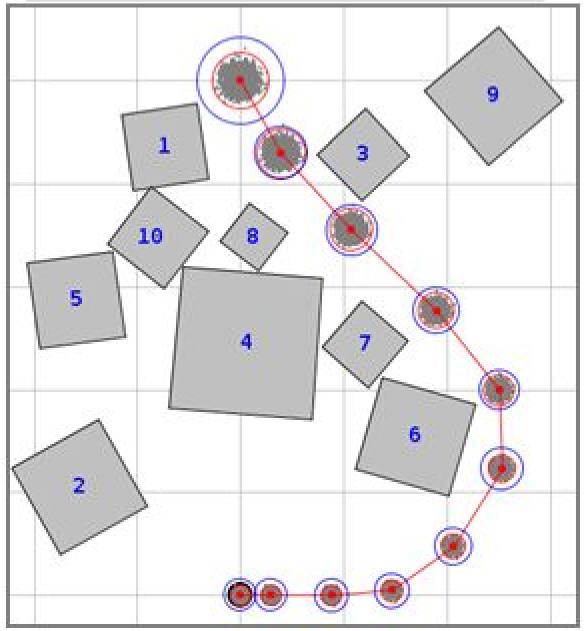
Computational Results



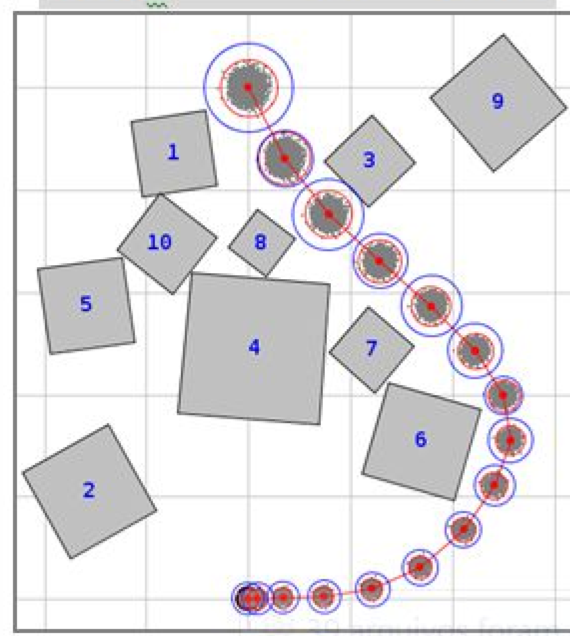
(a)	T = 10	
Method	Time(s)	Dist.(m)
Model2	625	15.352
-	-	-
-	-	-
-	-	-
-	-	-



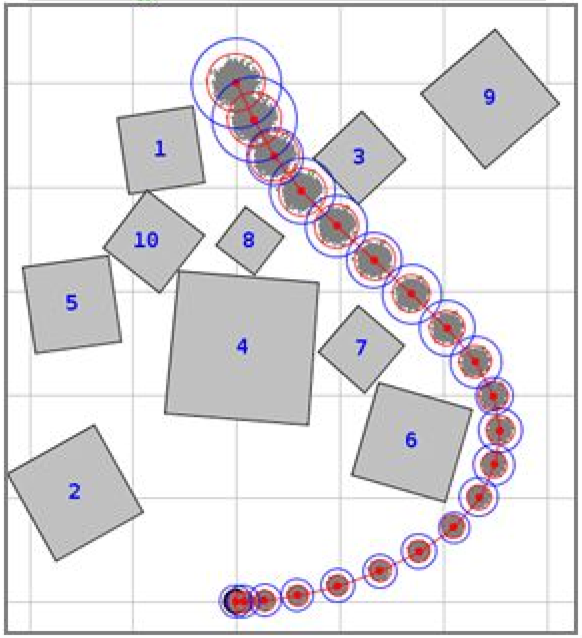
(b)	T = 10	
Method	Time(s)	Dist.(m)
-	-	-
C&A	52.6	15.587
C&Ai1	30.3	15.561
C&Ai2	66.9	15.561
C&Ai3	44.0	15.678



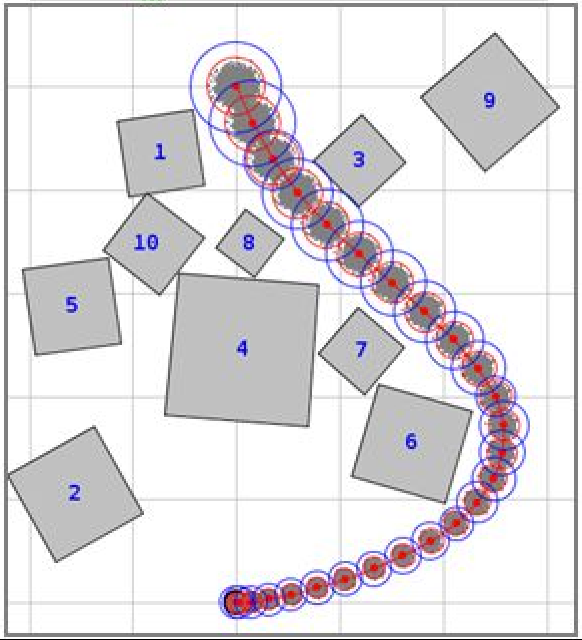
(c)	T = 15	
Method	Time(s)	Dist.(m)
Model2	7100	15.380
C&A	589	15.257
C&Ai1	288	15.258
C&Ai2	1188	15.414
C&Ai3	290	15.386



(d)	T = 20	
Method	**Time(s)**	**Dist.(m)**
Model2	unknow	unknow
C&A	4964	15.190
C&Ai1	8264	15.210
C&Ai2	5387	15.216
C&Ai3	3011	15.215



(e)	T = 25	
Method	**Time(s)**	**Dist.(m)**
Model2	unknow	unknow
C&A	57409	15.296
C&Ai1	1156	15.257
C&Ai2	1230	15.254
C&Ai3	1098	15.250



(f)	T = 30	
Method	**Time(s)**	**Dist.(m)**
Model2	unknow	unknow
C&A	unknow	unknow
C&Ai1	4161	15.287
C&Ai2	2122	15.312
C&Ai3	9307	15.313