LETTERS TO THE EDITOR

ON THE THEORY OF SUPERFLUIDITY OF HELIUM II

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(Received November 15, 1946)

The velocity of the "second sound" in helium II has been measured by V. Peshkov ([1]) with a great precision. His results give an opportunity to perform a quantitative comparison of the theory developed by the author ([2]) with the experiment. Such a comparison gives full support to the general picture given by the theory, but at the same time it reveals a noticeable discrepancy between the calculated and observed values of the velocity (*e. g.* 25 m/sec. calculated and 19 m/sec. observed at the temperature of 1.6°K). Although this discrepancy is not very large, it is too large to be attributed to the inaccuracy of the experimental data on the thermodynamic quantities of helium II.

For calculating the velocity of the second sound the formulae were used for the thermodynamic quantities (entropy, specific heat), derived in ([2]) under the assumption of the energy spectrum of the liquid to consist of two branches — the phonon and roton ones. The direction of the observed discrepancy indicates in what way these assumptions must be altered. Using the experimental data, one can formally compute the roton mass μ according to formulae

$$\rho_n = N\mu, \quad F_r = -NkT. \tag{1}$$

Here N is the number of rotons per unit volume, F_r—the "roton part" of the free energy (*i. e.* the free energy without the vibrational part), ρ_n—the density of the "normal part" of the liquid (the phonon part in ρ_n is negligible as compared with the roton part). The mass μ calculated in this way appears to be approximately inversely proportional to the temperature (in temperature interval 1.3—1.7°K), instead of being constant. It is, however, to be noted, that although the very fact of the variation of μ is apparent, the quantitative law of its variation can be established only in a very approximate way (owing mainly to the scarcity of experimental data on the specific heat of helium II).

If one does not make the assumption $\varepsilon = \Delta + p^2/2\mu$ for the dependence of the energy ε of a roton on its momentum p, but considers the general dependence $\varepsilon(p)$, then the calculation according to the general formulae derived in ([2]) shows, that in the formula $\rho_n = N\mu$ the quantity $\overline{p^2}/3kT$ enters instead of μ ($\overline{p^2}$ is the mean square of the momentum). If this quantity is inversely proportional to the temperature, then $\overline{p^2}$=const., *i. e.* the values of the roton momenta lie mainly in the neighbourhood of a certain p_0. At the first glance this fact appears to be very strange, but it can be explained in a natural manner by assuming, that the energy spectrum of helium II is of the type shown in Fig. 1. For small momenta p of an

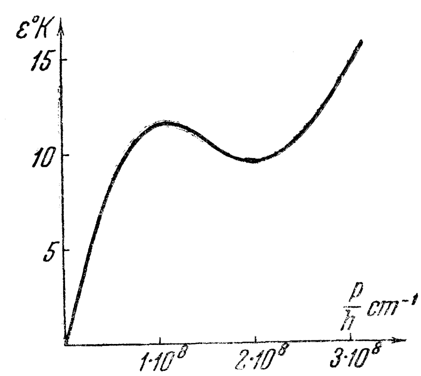

Fig. 1

elementary excitation its energy ε increases linearly (phonons), then reaches a maximum, begins to decrease and at a certain value $p = p_0$ the function $\varepsilon(p)$ has a minimum. In the neighbourhood of the latter we can write

$$\varepsilon = \Delta + \frac{(p - p_0)^2}{2\mu}. \tag{2}$$

μ being a constant. With such a spectrum it is of course impossible to speak strictly of rotons and phonons as of qualitatively different types of elementary excitations. It would be more correct to speak simply of the long wave (small p) and short wave (p in the neighbourhood of p_0) excitations. It is to be stressed, that all the conclusions concerning the superfluidity and the entire macroscopical hydrodynamics of helium II, developed in ([2]), maintain their validity also with the spectrum proposed here.

Only the formulae for the thermodynamic quantities must be changed. Instead of formulae (3), (4)—(7) in [2] we have for the "roton" parts of the free energy, entropy, specific heat (per unit mass) and the density of the "normal liquid":

$$F_r = -\frac{2\mu^{1/2}(kT)^{3/2}p_0^2}{(2\pi)^{3/2}\rho\hbar^3}e^{-\Delta/kT}; \qquad (3)$$

$$S_r = \frac{2(k\mu)^{1/2}p_0^2\Delta}{(2\pi)^{3/2}\rho T^{1/2}\hbar^3}\left(1+\frac{3kT}{2\Delta}\right)e^{-\Delta/kT}; \quad (4)$$

$$C_r = \frac{2\mu^{1/2}p_0^2\Delta^2}{(2\pi)^{3/2}\rho k^{1/2}T^{3/2}\hbar^3}\times \times\left[1+\frac{kT}{\Delta}+\frac{3}{4}\left(\frac{kT}{\Delta}\right)^2\right]e^{-\Delta/kT}; \qquad (5)$$

$$\frac{(\rho_n)_r}{\rho} = \frac{2\mu^{1/2}p_0^4}{3(2\pi)^{3/2}\rho(kT)^{1/2}\hbar^3}e^{-\Delta/kT}. \qquad (6)$$

In such a form the theory contains three constants: Δ, p_0 and μ. It is, therefore, difficult to check it on the basis of the experimental data which are now available. For the values of Δ, p_0 and μ one gets:

$$\frac{\Delta}{k}=9.6^\circ, \quad \frac{p_0}{\hbar}=1.95\cdot 10^8\ \mathrm{cm}^{-1}, \quad \mu=0.77\,m_{\mathrm{He}}. \quad (7)$$

Note that μ is of the order of the mass m_{He} of the helium atom and $\hbar/p_0$ is even less than the atomic dimensions. The values (7) have been used in drawing the curve in Fig. 1.

1 V. Peshkov, Journ. of Phys., **10**, 389 (1946).
2 L. Landau, Journ. of Phys., 5, 71 (1941).

OBSERVATION OF REFRACTIONAL STRUCTURES

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(Received July 6, 1946)

As it is well known, transparent objects which possess only refractional structure, *i. e.* which do not change practically the amplitude of a light wave passing through them but only its phase, cannot be observed visually or photographed. Many biological micropreparations, thermal flows, stresses in glasses, etc. are examples. For the microscopic observation of such structures apart from the methods connected with an influence upon the preparation itself (staining) purely optical methods are widely employed (as, for instance, the method of the darkened field of vision). For structures of large dimensions Toepler's method plays a similar rôle [1].

In 1934 Zernike showed that the usual methods of influence upon the light beam *e. g.*, as shutting off the direct beam or cutting off half of the diffraction pattern in the principal focal plane of the objective, do not remove the direct beam which, as it was assumed before, disguises the image of the structure, but simply influence the secondary sources so that the light becomes distributed in the image plane with alternating intensity. Zernike also demonstrated that the contrastness and brightness of the image increase if the phase of the central beam be turned by 90° instead of shutting off the beam. Zernike applied his considerations to the problem of detecting deviations of the surface of a concave spherical mirror [2].

In 1935 Zernike described the application of the phase method to the microscopical observation of refractional structures [3]. The method hat received some development in this direction and as present the phase microscopy is rather widely employed [4].

In about 1942 Prof. L. Mandelstam had drew the attention of one of us to the fact that the phase method may be successfully used not only in microscopy and in particular problem considered by Zernike, but also, in general, in all the cases of observation of refractional structures. In accordance with this remark we have undertaken an experimental investigation of images obtained from the refractional structures in an optical device which is a modification of Toepler's method. The object is illuminated by a parallel central beam while either ordinary diaphragms, used in Toepler's method, or transparent plates with etched portions on the surface, which change the optical length of the direct ray, are placed in the principal focal plane of a lens located behind the object.

The theoretical treatment shows that the method of the darkened field of vision not only cannot detect weak refractional structures but sometimes even those with very strong phase modulation. In all these cases an image of the object can be obtained by applying the phase method. Besides, use of the method of the darkened field of vision may lead to the doubling of the structure (in the image of periodical structures), *i. e.* may give an image which is dissimilar to the object itself. This defect is absent in the phase method. Finally, the phase method allows one to reduce the time of exposition (as much as up to 40 times) as compared to that necessary in Toepler's method.

The data of preliminary experiments are in good agreement with the calculations.

Fig. 1 represents the image of a part of a mirror glass: *a*—the ordinary photograph shows only the surface defects and contamination; *b*—the method of the darkened field of vision shows the internal inhomogeneities in a section of the glass and a number of the surface defects. The image appears as a light picture against a dark background; *c*—the phase method reveals a large number of internal inhomogeneities in all the parts of the glass plate. Surface defects, which were dark on photograph *a*, become bright here.

Fig. 2 shows the image of a grating etched on the plane plate of the optical glass by means of hydrofluoric acid (the upper part of the grating is etched three times as deep as the lower one): *a*—is an ordinary photograph; *b*—the method of the darkened field of vision reveals satisfactorily only strongly etched parts of the grating. The doubling of the structure is clearly visible, ghost lines having appeared between the true ones; *c*—the phase method gives a clear image of the grating with the correct periodicity.

1 A. Toepler, Pogg. Ann., **131**, 33 (1867).
2 F. Zernike, Physica, **1**, 689 (1934); Monthly Notes, **94**, 377 (1934).
3 F. Zernike, Phys. ZS., **36**, 848 (1935).
4 A. Koehler u. W. Loos, Naturwiss., **29**, 49 (1941); E. Linfoot, Nature, **155**, 76 (1945); A. Cox, Nature, **155**, 425 (1945).