Extensive Form Games

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January 2002

1 Introduction

The models we have described so far can capture a wide range of strategic environments, but they have an important limitation. In each game we have looked at, each player moves once and strategies are chosen simultaneously. This misses a common feature of many economic settings (and many classic "games" such as chess or poker). Consider just a few examples of economic environments where the timing of strategic decisions is important:

- Compensation and Incentives. Firms first sign workers to a contract, then workers decide how to behave, and frequently firms later decide what bonuses to pay and/or which employees to promote.

- Research and Development. Firms make choices about what technologies to invest in given the set of available technologies and their forecasts about the future market.

- Monetary Policy. A central bank makes decisions about the money supply in response to inflation and growth, which are themselves determined by individuals acting in response to monetary policy, and so on.

- Entry into Markets. Prior to competing in a product market, firms must make decisions about whether to enter markets and what products to introduce. These decisions may strongly influence later competition.

These notes take up the problem of representing and analyzing these dynamic strategic environments.

2 The Extensive Form

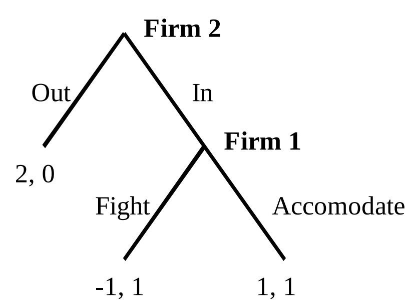
The extensive form of a game is a complete description of:

1. The set of players
2. Who moves when and what their choices are
3. What players know when they move
4. The players' payoffs as a function of the choices that are made.

2.1 Examples

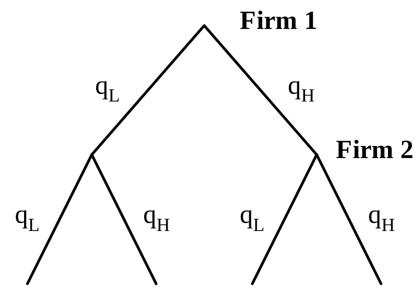
We start with a few examples.

An Entry Model Firm 1 is an incumbent monopolist. A second firm, Firm 2, has the opportunity to enter. After Firm 2 enters, Firm 1 will have to choose how to compete: either aggressively (Fight), or by ceding some market share (Accomodate). The strategic situation can be represented as follows.

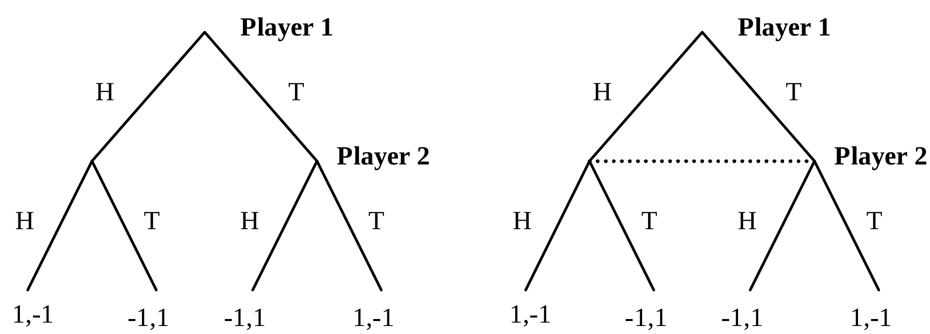


Stackleberg Competition An alternative to the Bertrand or Cournot models of imperfect competition is to assume that one firm is the market *leader*, while the other firm (or firms) are *followers*. In the Stackleberg model, we think of Firm 1 as moving first, and setting a quantity q_1, and Firm 2 as moving second, and setting a quantity q_2, *after having observed* q_1. The price is then determined by

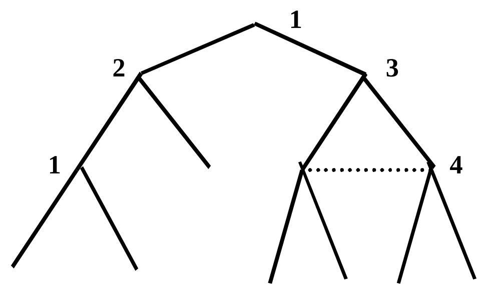
$P(q_1 + q_2) = 1 - (q_1 + q_2)$. Let's assume that the two firms have constant marginal costs, $c = 0$. To keep the picture simple, we think of q_i as taking only two values, Low and High.



Matching Pennies Or consider two variants of the matching pennies game. In the first variant, player one moves first, and then player two moves second *after having observed player one's action.* In the second, player two does not observe player one's action.



Example We can also have more complicated games, where players move multiple times, or select which player will move.



2.2 Formal Definitions

Formally, a finite extensive form game consists of:

- A finite set of players $i = 1, ..., I$.
- A finite set X of nodes that form a tree, with $Z \subset X$ being the terminal nodes.
- A set of functions that describe for each $x \notin Z$,
  - The player $i(x)$ who moves at x.
  - The set $A(x)$ of possible actions at x.
  - The successor node $n(x, a)$ resulting from action a.
- Payoff functions $u_i : Z \to \mathbb{R}$ assigning payoffs to players as a function of the terminal node reached.
- An information partition: for each x, let $h(x)$ denote the set of nodes that are possible given what player $i(x)$ knows. Thus, if $x' \in h(x)$, then $i(x') = i(x)$, $A(x') = A(x)$ and $h(x') = h(x)$.

We will sometimes use the notation $i(h)$ or $A(h)$ to denote the player who moves at information set h and his set of possible actions.

Matching Pennies, cont. Let's revisit the two versions of matching pennies above. In both, we have seven nodes, three of which are non-terminal. The key difference is the information partition. In the first version, each $h(x) = \{x\}$ for each x. In the second, for the two middle nodes we have $h(x) = h(x') = \{x, x'\}$.

In an extensive form game, write H_i for the set of information sets at which player i moves.

$$H_i = \{S \subset X : S = h(x) \text{ for some } x \in X \text{ with } i(x) = i\}$$

Write A_i for the set of actions available to i at any of his information sets.

2.3 Strategies

Definition 1 *A pure strategy for player i in an extensive form game is a function* $s_i : H_i \to A_i$ *such that* $s_i(h) \in A(h)$ *for each* $h \in H_i$.

A strategy is a complete contingent plan explaining what a player will do in every situation. Let S_i denote the set of pure strategies available to player i, and $S = S_1 \times ... \times S_I$ denote the set of pure strategy profiles. As before, we will let $s = (s_1, ..., s_I)$ denote a strategy profile, and s_{-i} the strategies of i's opponents.

Matching Pennies, cont. In the first version of matching pennies, $S_1 = \{H, T\}$ and $S_2 = \{HH, HT, TH, TT\}$. In the second version, $S_1 = S_2 = \{H, T\}$.

There are two ways to represent mixed strategies in extensive form games.

Definition 2 *A mixed strategy for player i in an extensive form game is a probability distribution over pure strategies, i.e. some* $\sigma_i \in \Delta(S_i)$.

Definition 3 *A behavioral strategy for player i in an extensive form game is a function* $\sigma_i : H_i \to \Delta(A_i)$ *such that support*$(\sigma_i(h)) \subset A(h)$ *for all* $h \in H_i$.

A famous theorem in game theory, *Kuhn's Theorem*, says that in games of *perfect recall* (these are games where *(i)* a player never forgets a decision he or she took in the past, and *(ii)* never forgets information she had when making a past decision — see Kreps, p. 374 for formalities), mixed and behavioral strategies are equivalent, in the sense that for any mixed strategy there is an equivalent behavioral strategy and vice versa. Since essentially all the games we will consider have perfect recall, we will use mixed and behavioral strategies interchangeably.

3 The Normal Form and Nash Equilibrium

Any extensive form game can also be represented in the normal form. If we adopt a normal form representation, we can solve for the Nash equilibrium.

Matching Pennies, cont. For our two versions of Matching Pennies, the normal forms are:

	HH	HT	TH	TT
H	$1,-1$	$1,-1$	$-1,1$	$-1,1$
T	$-1,1$	$1,-1$	$-1,1$	$1,-1$

	H	T
H	$1,-1$	$-1,1$
T	$-1,1$	$1,-1$

In the first version, Player two has a winning strategy in the sense that she can always create a mismatch if she adopts the strategy TH. Any strategy for player one, coupled with this strategy for player two is a Nash equilibrium. In the second version, the Nash equilibrium is for both players to mix $\frac{1}{2}H + \frac{1}{2}T$.

Entry Game, cont. For the entry game above, the normal form is:

	Out	In
F	$2,0$	$-1,1$
A	$2,0$	$1,1$

There are several Nash equilibria: (A, In), (F, Out) and $(\alpha F + (1-\alpha)A, Out)$ for any $\alpha \geq 1/2$.

Note that in the entry game, some of the Nash equilibria seem distinctly less intuitive that others. For instance, in the (F, Out) equilibrium, it is the threat of $Fight$ that keeps Firm 2 from entering. However, if Firm 2 *were* to enter, is it reasonable to think that Firm 1 will *actually* fight? At this point, it is not in Firm 1's interest to fight, since it does better by accomodating.

Consider another example, where this problem of incredible threats arises in way that is potentially even more objectionable.

Stackleberg Competition In the Stackleberg model, for any $q_1' \in [0,1]$, there is a Nash equilibrium in which Firm 1 chooses quantity q_1'. To see this, consider the strategies:

$$s_1 = q_1'$$

and

$$s_2 = \begin{cases} \frac{1-q_1'}{2} & \text{if } q_1 = q_1' \\ 1 - q_1' & \text{if } q_1 \neq q_1' \end{cases}.$$

Let's check that these are all equilibria. First, given Firm 2's strategy, Firm 1 can either set $q_1 \neq q_1'$, or $q_1 = q_1'$. If it does the former, the eventual price will be zero, and Firm 1 will make zero profits. If it does the latter, then Firm 1 will make profits:

$$q_1' \left(1 - q_1' - \frac{1 - q_1'}{2}\right) = \frac{1}{2} q_1' \left(1 - q_1'\right) \geq 0.$$

Now, consider Firm 2. Does it have a strategy that yields strictly higher payoff. Clearly, changing its strategy in response to $s_1 \neq q_1'$ will have no effect on its payoff given Firm 1's strategy. On the other hand, in response to $s_1 = q_1'$, its best response solves:

$$\max_{q_2} q_2 \left(1 - q_1' - q_2\right).$$

The solution to this problem is $(1 - q_1')/2$, so Firm 2 is playing a best response.

As in the previous case, many of the Nash equilibria in the Stackelberg model seem unreasonable. If Firm 1 sets $q_1 \neq q_1'$, then Firm 2 typically has options that give a positive payoff. However, it chooses to flood the market and drive the price to zero. Thus, off the equilibrium path, unreasonable things are happening. And not only is Firm 2 being allowed to make incredible threats, we have a huge multiplicity of equilibria.

4 Subgame Perfect Equilibrium

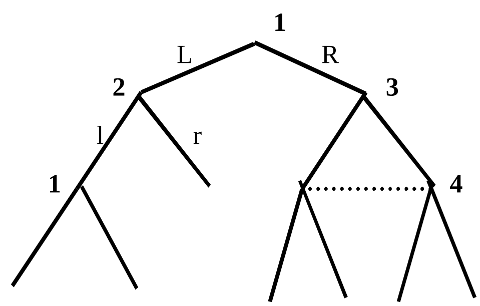
In response to the problems of credibility we have seen in the last two examples, we now introduce the idea of a *subgame perfect equilibrium*. Subgame perfection tries to rule out incredible threats by assuming that once something has happened, players will always optimize going forward.

4.1 Subgame Perfection

Definition 4 *Let G be an extensive form game, a subgame G' of G consists of (i) a subset Y of the nodes X consisting of a single non-terminal node x and all of its successors, which has the property that if $y \in Y$, $y' \in h(y)$ then $y' \in Y$, and (ii) information sets, feasible moves, and payofs at terminal nodes as in G.*

Entry Game, cont. In the entry game, there are two subgames. The entire game (which is always a subgame) and the subgame after Firm 2 has entered the market.

Example In game below, there are four subgames: (1) The entire game, (2) the game after player one chooses R, (3) the game after player one choose L, and (4) the game after Player 1 chooses L and player 2 chooses l.



Definition 5 *A strategy profile s is a **subgame perfect equilibrium** of G if it induces a Nash equilibrium in every subgame of G.*

Note that since the entire game is always a subgame, any SPE must also be a NE.

Entry Game, cont. In the entry game, only (A, In) is subgame perfect.

4.2 Application: Stackleberg Competition

Consider the model of Stackleberg Competition where Firm 1 moves first and chooses quantity q_1, and then Firm 2 moves second and chooses quantity q_2. Once both firms have chosen quantities, the price is determined by: $P(Q) = 1 - Q$, where $Q = q_1 + q_2$. So that we can compare this model to Bertrand and Cournot competition, let's assume that both firms have constant marginal cost equal to $0 \leq c < 1$.

To solve for the subgame perfect equilibrium, we work backward. Suppose that Firm 1 has set some quantity q_1. Then Firm 2's best response solves:

$$\max_{q_2} q_2 \left(1 - q_1 - q_2 - c\right)$$

The first-order condition for this problem is:

$$0 = 1 - q_1 - c - 2q_2,$$

which gives a best response:

$$q_2^*(q_1) = \max\left\{0, \frac{1 - q_1 - c}{2}\right\}.$$

Now consider the problem facing Firm 1, knowing that if it chooses q_1, Firm 2 will respond with a quantity $q_2^*(q_1)$. It solves

$$\max_{q_1} q_1 \left(1 - q_1 - q_2^*(q_1) - c\right)$$

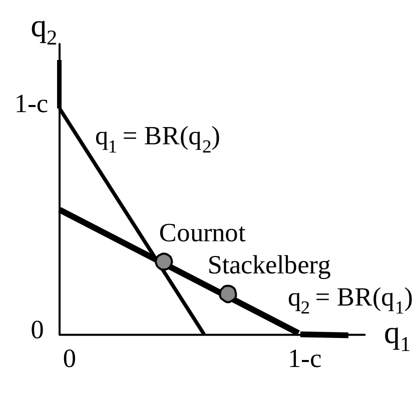
The first-order condition for this problem is:

$$0 = 1 - q_2 - c - 2q_1 - q_1 \frac{dq_2^*}{dq_1}.$$

Solving this out yields:

$$q_1 = \frac{1 - c}{2} \qquad \text{and} \qquad q_2 = \frac{1 - c}{4}.$$

The total quantity is $\frac{3}{4}(1 - c)$ and the price is $p^S = (1 + 3c)/4$. In comparison, under Cournot competition, both firms set identical quantity $\frac{1-c}{3}$, so total quantity is $\frac{2}{3}(1 - c)$ and the price is $p^C = (1 + 2c)/3$.



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Stackelberg Competition

Relative to Cournot, in Stackelberg Competition, the Leader (Firm 1) can choose any point on the Follower's (Firm 2's) best-response curve. Note that this game has a first-mover advantage in the sense that there is an advantage to being the leader.

4.3 Backward Induction

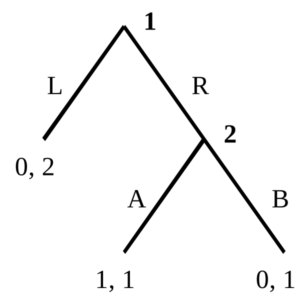
As the previous examples illustrate, a common technique for identifying subgame perfect equilibria is to start at the end of the game and work back to the front. This process is called *backward induction.*

Definition 6 *An extensive form game is said to have* ***perfect information*** *if each information set contains a single node.*

Proposition 7 *(Zermelo's Theorem) Any finite game of perfect information has a pure strategy subgame perfect equilibrium. For generic payoffs, there is a unique SPE.*

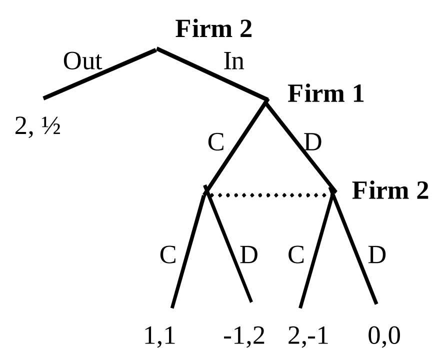
Proof. (very informal) In a finite game of perfect information, we can label each node as belonging to stage 1, stage 2, ..., stage K. To find a pure strategy SPE (and for generic payoffs, the unique SPE), we first consider all nodes x in stage K. Each node starts a subgame, so by NE the player who moves must maximize his or her expected payoff. Identify the optimal choice (generically, it will be unique). Now move back to stage $K-1$, and consider the problem facing a player who moves here. This player can assume that at stage K, play will follow the choices just identified. Since each node starts a subgame, we look for the payoff-maximizing choice facing a player who gets to move. Once these choices have been identified, we move back to stage $K-2$. This process of continues until we reach the beginning of the game, at which point we will have at least one (and typically no more than one). *Q.E.D.*

Example Here is a non-generic game where backward induction reveals three pure strategy subgame perfect equilibria: (R, A), (R, B) and (L, B).



We can use backward induction even in games without perfect information as the next example demonstrates.

Example To use backward induction in the game below, we first solve for the subgame perfect equilibrium after Firm 2 has entered. We see that the unique equilibrium in this subgame is for both Firm 1 and Firm 2 to play D. (Note that this subgame is a prisoners' dilemma. Hence Firm 2 will choose not to enter at the first node.

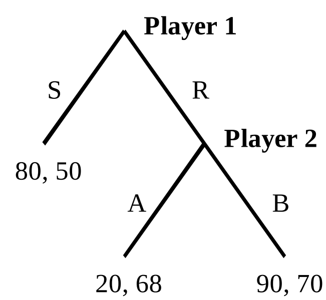


Another Entry Game

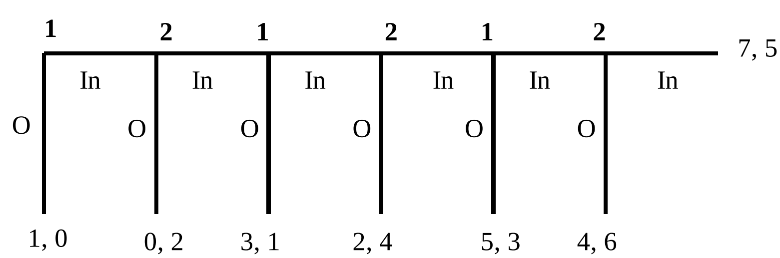
4.4 Criticisms of Subgame Perfection

We motivated Subgame Perfection as an attempt to eliminate equilibria that involved incredible threats. As we go on to consider applications, we will use SPE regularly as a solution concept. Before we do this, however, it is worth pausing momentarily to ask whether SPE might be *over*-zealous in eliminating equilibria.

Example: Trusting Someone to be Rational Here the unique SPE is for Player 1 to choose R and Player 2 to choose B. However, Goeree and Holt (2001) report an experiment in which more than 50% of player ones play S.



The Centipede Game In games with many stages, backward induction *greatly* stresses the assumption of rationality (and common knowledge of rationality). A famous example due to Rosenthal (1981) is the centipede game. The unique SPE is for Player 1 to start by moving Out, but in practice people do not seem to play the game this way.



5 Stackelberg (Leader-Follower) Games

Above, we considered Stackelberg competition in quantities. We now consider Stackelberg competition in prices, then fit both models into a more general framework.

5.1 Stackelberg Price Competition

In the Stackelberg version of price competition, Firm 1 moves first and commits to a price p_1. Firm 2 observes p_1 and responds with a price p_2. Sales for Firm i are then given by $Q_i(p_1, p_2)$. Supposing that the two firms have constant marginal costs equal to c, firm i's profits can be written as:

$$\pi_i(p_1, p_2) = (p_i - c)\, Q_i(p_i, p_j).$$

Homogeneous Products. If the firms' products are heteregenous, then firm that sets a lower price gets demand $Q(p)$ (where $p = \min\{p_1, p_2\}$), and the firm that sets a higher price gets no demand. Suppose that if $p_1 = p_2$ the consumers split equally between the two firms. In this case, if Firm 1 chooses $p_1 > c$, Firm 2 would like to choose the highest price less than p_1. Unfortunately, such a price does not exist! So there are subgames in which Nash equilibria do not exist and no SPE.

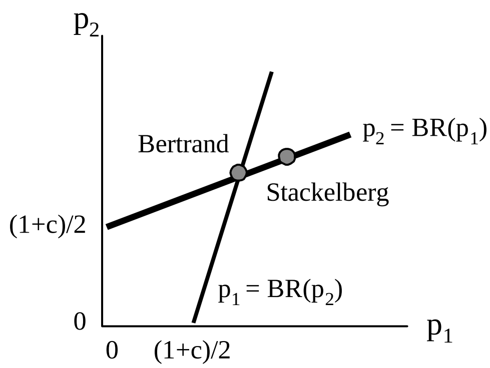
To resolve this, assume that if $p_1 = p_2$, then all consumers purchase from Firm 2. Then after observing $p_1 \geq c$, Firm 2 responds by taking the entire market — either by choosing $p_2 = p_1$ if $p_1 \leq p^m$ (the monopoly price), or by choosing $p_2 = p^m$ if $p_1 > p^m$. It follows that Firm 2's best response function is given by:

$$p_2^*(p_1) = \begin{cases} p^m & \text{if } p_1 > p^m \\ p_1 & \text{if } p_1 \in [c, p^m] \\ c & \text{if } p_1 < c \end{cases}.$$

Thus, for any $p_1 < c$, Firm 1 gets the entire market but loses money, which for any $p_1 \geq c$, Firm 1 gets no demand. It follows that any pair $(p_1, p_2^*(p_1))$ with $p_1 \geq c$ is an SPE.

Note that compared to the Bertrand (simultaneous move) equilibrium, the price may be higher. In particular, both Firms 1 and 2 set (weakly) higher prices. Moreover, Firm 2 (the follower) does better than in the simultaneous game, while Firm 1 does the same. Thus we say the game has a *second mover advantage.*

Heterogeneous Products. With heterogeneous products, the situation is similar to Stackelberg quantity competition, except with Bertrand best-responses rather than Cournot!



Stackelberg Price Competition

5.2 General Leader-Follower Games

The Stackelberg models of imperfect competition are examples of what Gibbons calls "Leader-Follower" games. These games have the following structure:

1. Player 1 moves first and chooses an action $a_1 \in A_1$.
2. Player 2 observes a_1 and chooses an action $a_2 \in A_2$.

There is a simple algorithm to identify the subgame perfect equilibria of this sort of game. We just apply backwards induction. We first define player 2's best response to any action by Player 1:

$$a_2^*(a_1) = \arg\max_{a_2 \in A_2} \pi_2(a_1, a_2).$$

We then identity what player one should do, assuming that player two will best respond. To do this, define:

$$a_1^* = \arg\max_{a_1 \in A_1} \pi_1\left(a_1, a_2^*(a_1)\right).$$

A subgame perfect equilibrium is a pair $(a_1^*, a_2^*(a_1^*))$.

From our examples, we can make several observations about leader-follower games.

1. The Leader always does (weakly) better than in a simultaneous move pure strategy equilibrium setting (note that this is not true for mixed strategy equilibria — think about matching pennies).

2. The Leader tends to distort his behavior relative to the simultaneous move game (how he does so depends on Firm 2's best response function and on what sort of action he prefers Firm 2 to choose).

3. Whether the Follower does better than in the simultaneous game depends on both Firm 2's best response function and the interdependence in payoffs (how i's action affects j's payoff and vice-versa).

6 Strategic Pre-Commitment

We now turn to a class of problems that arise frequently in industrial organization. These problems have the following structure. First, one player (Firm 1) has the opportunity to take some sort of action or investment — for instance, installing capacity, investing in a cost-saving technology or in product development, signing an exclusive contract with a key upstream suppliers, building a relationship with customers, or so on. Then Firm 2 decides whether or not to enter the market. Finally, firms compete with Firm 1 either operating as a monopoly or the two firms competing as a duopoly.

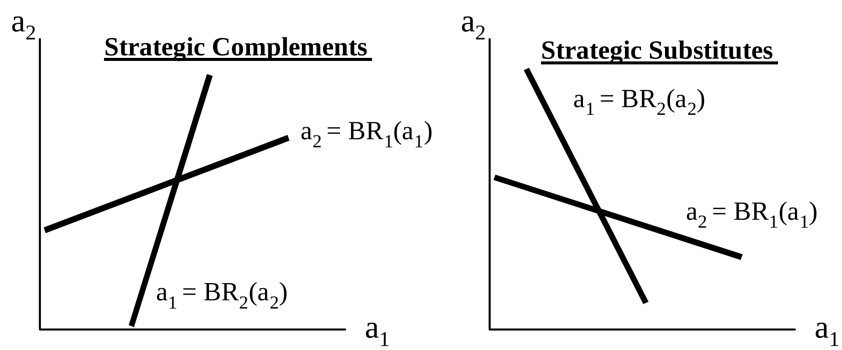
By taking an action in advance of competition, Firm 1 has the opportunity to *strategically pre-commit* (just as the Stackelberg leader pre-commits to a price or quantity). It turns out that it is possible to give a very intuitive analysis of the economics of pre-commitment by using the idea of strategic complements and substitutes.[1] This analysis can then be used to shed light on a range of problems in industrial organization and other fields.

6.1 Strategic Complements and Substitutes

Let G be a simultaneous move game in which the players 1 and 2 takes actions $a_1, a_2 \in \mathbb{R}$, and have payoffs $\pi_1(a_1, a_2)$, $\pi_2(a_1, a_2)$. Let $BR_1(a_2)$ and $BR_2(a_1)$ denote the best-response functions (assume best responses are unique).

Definition 8 *The players' actions are* ***strategic complements*** *if* $BR_i'(\cdot) \geq 0$*. The actions are* ***strategic substitutes*** *if* $BR_i'(\cdot) \leq 0$*.*

[1] This section is drawn from Tirole (1988, Chapter 8). See Fudenberg and Tirole (1984, AER) or Bulow, Geanokoplos and Klemperer (1985, JPE) for the original analyses.



We have already seen that in Cournot Competition, quantities are strategic substitutes, while in differentiated products Bertrand Competition, prices are strategic complements.

Proposition 9 *Suppose for* $i = 1, 2,.$ π_i *is twice continuously differentiable. Then* G *has strategic complements if* $\frac{\partial^2 \pi_i}{\partial a_i \partial a_j} \geq 0$ *and strategic substitutes if* $\frac{\partial^2 \pi_i}{\partial a_i \partial a_j} \leq 0$.

For those of you who took Econ 202, these conditions should look very familiar from our study of Monotone Comparative Statics. A good exercise is to try to prove this result using Topkis' Theorem.

6.2 Strategic Pre-Commitment

We consider the following model:

- Firm 1 moves first and chooses and investment k.
- Firm 2 observes k and decides whether or not to compete (enter).
- Firms 1 and 2 choose actions $a_1 \in A_1$, $a_2 \in A_2$.
- Payoffs are given by $\pi_1(k, a_1, a_2)$ and $\pi_2(k, a_1, a_2)$.[2]

[2] Note that Leader-Follower games are a special case of this setting. To get the Leader-Follower case, let A_1 be a singleton, and think of k as Firm 1's action and a_2 as Firm 2's action.

We assume that for any choice k, the competition subgame has a unique Nash Equilibrium, which we can denote $a_1^c(k), a_2^c(k)$. Payoffs in this game are given by:

$$\pi_i\left(k, a_1^c\left(k\right), a_2^c\left(k\right)\right) \text{ for } i = 1, 2.$$

Thus given a choice of k, Firm 2 will choose to enter if

$$\pi_2\left(k, a_1^c(k), a_2^c(k)\right) > 0.$$

If Firm 2 does not enter, then Firm 1 sets chooses the monopoly strategy a_1^m. Payoffs are

$$\pi_1^m\left(k, a_1^m(k)\right)$$

for Firm 1 and zero for Firm 2. Let k^* denote the subgame perfect level of investment.

We say that:

- Entry is *deterred* if $\pi_2\left(k^*, a_1^c(k^*), a_2^c(k^*)\right) \leq 0$.
- Entry is *accomodated* if $\pi_2\left(k^*, a_1^c(k^*), a_2^c(k^*)\right) > 0$,

If entry is deterred, then the SPE involves Firm 1 choosing $a_1^m(k^*)$, and achieves profits $\pi_1^m\left(k^*, a_1^m(k^*)\right)$. If entry is accomodated, Firms choose $a_1^c(k^*), a_2^c(k^*)$.

An alternative to this model would be a case without pre-commitment where Firm 1 chooses k at the same time as a_1, a_2 (or chooses k in advance but without it being observed). Let's assume that this game also has a unique Nash Equilibrium, denoted $(k^{nc}, a_1^{nc}, a_2^{nc})$. We will say that:

- Entry is *blockaded* if $\pi_2\left(k^{nc}, a_1^{nc}, a_2^{nc}\right) \leq 0$.

In what follows, we assume that entry is not blockaded. In addition, we will assume that π_1^m and π_1 are both concave in k.

6.3 Entry Deterrence

Let's first consider SPE in which entry is deterred. In this case, Firm 1 need to choose a level of k that makes Firm 2 less profitable. Indeed, it will choose an investment k^* such that:[3]

$$\pi_2\left(k^*, a_1^c(k^*), a_2^c(k^*)\right) = 0.$$

[3] The fact that Firm 1 will choose k^* to make Firm 2's competition profits *exactly* zero follows from the assumption that entry is not blockaded and that payoffs functions are concave.

Let's consider what sort of strategy by Firm 1 works to make Firm 2 unprofitable and hence deter entry. From second period optimization:

$$\frac{\partial \pi_2}{\partial a_2}\left(k^*, a_1^c(k^*), a_2^c(k^*)\right) = 0$$

Hence:

$$\frac{d\pi_2}{dk} = \underbrace{\frac{\partial \pi_2}{\partial k}}_{\text{direct effect}} + \underbrace{\frac{\partial \pi_2}{\partial a_1}\frac{\partial a_1}{\partial k}}_{\text{strategic effect}} .$$

To deter entry, Firm 1 wants to choose an investment that will make Firm 2 *less profitable*. It has two ways to do this. It may be able to invest in a way that makes Firm 2 *directly less profitable*. It make also be able to change the nature of competition — for example, if k is investment in capacity, k has no direct effect, but only a *strategic effect*.

To classify Firm 1's strategies, we adopt the terminology of Fudenberg and Tirole (1984).

Definition 10 *Investment makes Firm 1* ***tough*** *if* $\frac{d\pi_2}{dk} < 0$. *Investment makes Firm 1* ***soft*** *if* $\frac{d\pi_2}{dk} > 0$.

Fudenberg and Tirole suggest the following typology of strategies for investment.

- *Top Dog*: Be big (invest a lot) to look tough.
- *Puppy Dog:* Be small (invest only a little) to look soft.
- *Lean and Hungry Look:* Be small to look tough.
- *Fat Cat:* Be big to look soft.

Example: Reducing Own Costs Suppose that Firm 1 has the opportunity to invest to lower its marginal costs. If Firm 2 enters, competition will be Cournot. There is no direct on Firm 2. But there is a strategic effect. Investment will shift out Firm 1's best-response function, and lead to a competitive outcome where Firm 1 produces higher quantity and Firm 2 lower quantity. This *top dog* strategy may deter entry.

Example: Building a Customer Base Suppose that Firm 1 has the opportunity to invest in customer relations, building up a loyal customer base. The direct effect of this is to limit the potential market for Firm

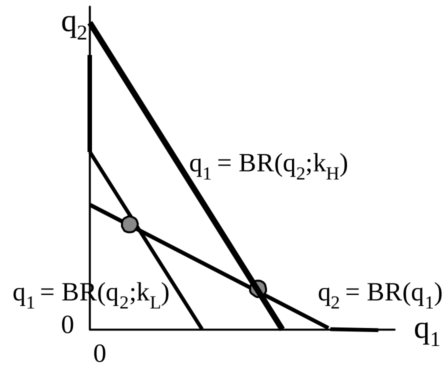


Figure 1:

2, making Firm 1 tough. However, there may be a second effect. If Firm 1 cannot price discriminate, it may be tempted to set a high price to take advantage of its locked in customers. This strategic effect can work to make Firm 1 soft. The overall effect is ambiguous. Thus either a "top dog" strategy or a "lean and hungry look" might work to deter entry depending on the specifics.

6.4 Accomodation: Changing the Nature of Competition

Suppose now that Firm 1 finds it too costly to deter entry. How should it behave in order to make more profits once Firm 2 enters? In this case, Firm 1 is interested in choosing its investment to maximize:

$$\pi_1\left(k, a_1^c\left(k\right), a_2^c\left(k\right)\right).$$

The total derivative is given by (using the envelope theorem):

$$\frac{d\pi_1}{dk} = \underbrace{\frac{\partial \pi_1}{\partial k}}_{\text{direct effect}} + \underbrace{\frac{\partial \pi_1}{\partial a_2}\frac{da_2}{dk}}_{\text{strategic effect}}.$$

That is, the first term is the *direct* effect. This would exist even if investment was not observed by Firm 2. The *strategic* effect results from the fact that Firm 1's investment can change the way Firm 2 will compete.

Let's investigate the strategic effect further. To do this, let's assume that the second-period actions of the two firms have the same nature in the sense that $\partial\pi_1/\partial a_2$ has the same sign as $\partial\pi_2/\partial a_1$. For instance, either both firms choose quantities or both choose prices. Then:

$$\frac{da_2^c}{dk} = \frac{da_2^c}{da_1}\frac{da_1}{dk} = \frac{dBR_2(a_1)}{da_1}\frac{da_1}{dk}$$

Thus,

$$\text{sign}\left(\frac{\partial\pi_1}{\partial a_2}\frac{da_2}{dk}\right) = \text{sign}\left(\frac{\partial\pi_1}{\partial a_2}\frac{da_2}{dk}\right)\cdot\text{sign}\left(\frac{dBR_2(a_1)}{da_1}\right)$$

Assuming that $\partial\pi_2/\partial k = 0$, we can identify the sign of the strategic effect with two things: (1) whether investment makes Firm 1 Tough or Soft (the first term) and (2) whether there are strategic substitutes or complements (the second term). We have four cases:

- If investment makes Firm 1 tough and reaction curve slope down, investment by Firm 1 softens Firm 2's action — thus Firm 1 should overinvest (top dog).
- If investment makes Firm 1 tough and reaction curves slope up, Firm 1 should overinvest so as not to trigger an aggressive response by Firm 2 (puppy dog).
- If invesmtnent makes Firm 1 soft and reaction curves slope down, Firm 1 should stay lean and hungry.
- If invesment makes Firm 1 soft and reaction curves slope up, Firm 1 should overinvest to become a fat cat.

To summarize,

	Investment makes Firm 1 Tough	Investment makes Firm 1 Soft
Strategic Complements	Puppy Dog	Fat Cat
Strategic Substitutes	Top Dog	Lean and Hungry

Example: Reducing Costs Suppose Firm 1 can invest to reduce its costs before competing. With Cournot competition, the strategic effect to make Firm 1 more aggressive. The equilibrium changes so that Firm 2 ends up producing less. Thus Firm 1 wants to be a Top Dog and invest heavily. On the other hand, if Firm 1 can reduce its costs before price competition, the strategic effect is to make Firm 1 more aggressive so that in equilibrium Firm 2 ends up pricing more aggressively as well. Thus, Firm 2 might want to be a Puppy Dog to soften price competition.

6.5 Applications in Industrial Organization

1. **Product Differentiation**. Suppose Firm 1 can choose its product's "location" prior to competing in price with Firm 2. Producing a product that is "close" to Firm 2's will tend to make price competition more intense, lowering prices and profits. To deter entry or to drive Firm 2 from the market, Firm 1 might want to adopt a "Top Dog" strategy. But to change the nature of competition in a favorable way, Firm 1 might want to adopt a "Puppy Dog" ploy and differentiate its offering from Firm 2's.

2. **Most-Favored Customer Clause**. With price competition, if Firm 1 wants to accomodate Firm 2, it wants to look inoffensive so as to keep Firm 2 from cutting price. In particular, it would like to commit itself to charging high prices (a "Puppy Dog" ploy). One way to do this is most-favored customer clauses. Firm 1 can write contracts with its customers promising that if it ever offers a low price to another customer, the original customer can get the new low price. This makes it *very* costly for Firm 1 to drop prices, effectively committing itself to be unaggressive in competing with Firm 2.

3. **Advertising**. Suppose Firm 1 can invest in advertising that makes customer more excited not just about its own product, but about the whole market. This kind of advertising makes Firm 1 soft. To deter entry, Firm 1 should not do much of this sort of advertising — rather it should run advertisements that increase the demand for its own product and decrease the demand for other firms' product. But what if Firm 2 surely plans to enter. If competition is in prices, then Firm 1 will want to advertise in a way that increases demand (direct effect) and in a way that softens price competition (for example by establishing separate niches for different products in the market). This is a fat cat approach to advertising.

4. **Leverage and Tying**. An old story in IO is that a firm with monopoly power in one market can leverage this power to monopolize a second. (Think Microsoft and browsers.) One way to do this is to tie the two products together. Suppose that there are two markets, that Firm 1 has a monopoly in the first, and that Firms 1 and 2 may compete in prices in the second. Firm 1 must decide whether to bundle or tie its two products. The question is how this action will effect its pricing behavior. This depends on how we model demand, but in many cases,

bundling will make demand more elastic. This leads Firm 1 to price *more aggressively* in response to Firm 2's prices. It follows that from a strategic point of view, bundling is a top dog strategy that works to deter entry. But if entry is to be accomodated, it may be better to use the puppy dog ploy of not bundling.