

Álgebra de Chaveamento

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- Ao final do estudo deste tópico você saberá:
 - Os conceitos de Álgebra Booleana e Álgebra de Chaveamento
 - Os axiomas e Teoremas da Álgebra de Chaveamento
 - Os Teoremas de DeMorgan
 - Demonstração de Teoremas por Indução Finita
 - As portas lógicas Inversora, AND e OR
 - O Diagrama Lógico
 - O Princípio da Dualidade
 - A Tabela Verdade
 - Os conceitos de Literal, Termo Produto, Soma de Produtos, Termo Soma, Produto de Somas, Termo Normal, Mintermo e Maxtermo
 - A Soma e o Produto Canônico

Teoremas de 1 variável

(T1)	$X + 0 = X$	(T1')	$X \cdot 1 = X$	(Identities)
(T2)	$X + 1 = 1$	(T2')	$X \cdot 0 = 0$	(Null elements)
(T3)	$X + X = X$	(T3')	$X \cdot X = X$	(Idempotency)
(T4)	$(X')' = X$			(Involution)
(T5)	$X + X' = 1$	(T5')	$X \cdot X' = 0$	(Complements)

Fonte das figuras: Wakerly - Digital Design

Teoremas de 2 ou 3 variáveis

$$(T6) \quad X + Y = Y + X$$

$$(T7) \quad (X + Y) + Z = X + (Y + Z)$$

$$(T8) \quad X \cdot Y + X \cdot Z = X \cdot (Y + Z)$$

$$(T9) \quad X + X \cdot Y = X$$

$$(T10) \quad X \cdot Y + X \cdot Y' = X$$

$$(T11) \quad X \cdot Y + X' \cdot Z + Y \cdot Z = X \cdot Y + X' \cdot Z$$

$$(T11') \quad (X + Y) \cdot (X' + Z) \cdot (Y + Z) = (X + Y) \cdot (X' + Z)$$

$$(T6') \quad X \cdot Y = Y \cdot X \quad (\text{Commutativity})$$

$$(T7') \quad (X \cdot Y) \cdot Z = X \cdot (Y \cdot Z) \quad (\text{Associativity})$$

$$(T8') \quad (X + Y) \cdot (X + Z) = X + Y \cdot Z \quad (\text{Distributivity})$$

$$(T9') \quad X \cdot (X + Y) = X \quad (\text{Covering})$$

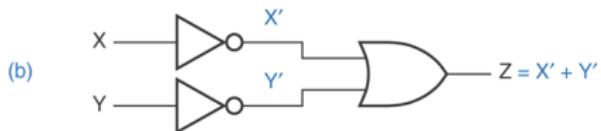
$$(T10') \quad (X + Y) \cdot (X + Y') = X \quad (\text{Combining})$$

$$(\text{Consensus})$$

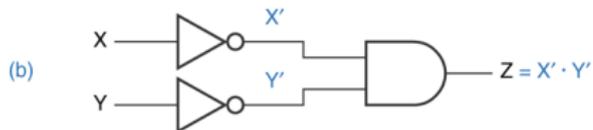
Teoremas de n variáveis

- (T12) $X + X + \dots + X = X$ (Generalized idempotency)
- (T12') $X \cdot X \cdot \dots \cdot X = X$
- (T13) $(X_1 \cdot X_2 \cdot \dots \cdot X_n)' = X_1' + X_2' + \dots + X_n'$ (DeMorgan's theorems)
- (T13') $(X_1 + X_2 + \dots + X_n)' = X_1' \cdot X_2' \cdot \dots \cdot X_n'$
- (T14) $[F(X_1, X_2, \dots, X_n, \cdot, \cdot)]' = F(X_1', X_2', \dots, X_n', \cdot, \cdot)$ (Generalized DeMorgan's theorem)
- (T15) $F(X_1, X_2, \dots, X_n) = X_1 \cdot F(1, X_2, \dots, X_n) + X_1' \cdot F(0, X_2, \dots, X_n)$ (Shannon's expansion theorems)
- (T15') $F(X_1, X_2, \dots, X_n) = [X_1 + F(0, X_2, \dots, X_n)] \cdot [X_1' + F(1, X_2, \dots, X_n)]$

DeMorgan: Teorema T13



DeMorgan: Teorema T13'



Circuito Lógico "Tipo 1"



X	Y	Z
LOW	LOW	LOW
LOW	HIGH	LOW
HIGH	LOW	LOW
HIGH	HIGH	HIGH



X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1



X	Y	Z
1	1	1
1	0	1
0	1	1
0	0	0

Circuito Lógico "Tipo 2"



X	Y	Z
LOW	LOW	LOW
LOW	HIGH	HIGH
HIGH	LOW	HIGH
HIGH	HIGH	HIGH

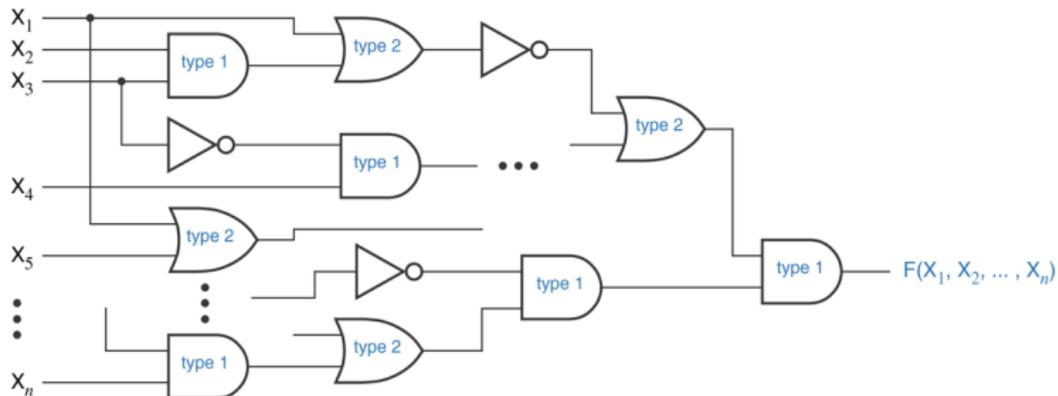


X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1



X	Y	Z
1	1	1
1	0	0
0	1	0
0	0	0

Circuito de Lógica Positiva



Circuito de Lógica Negativa

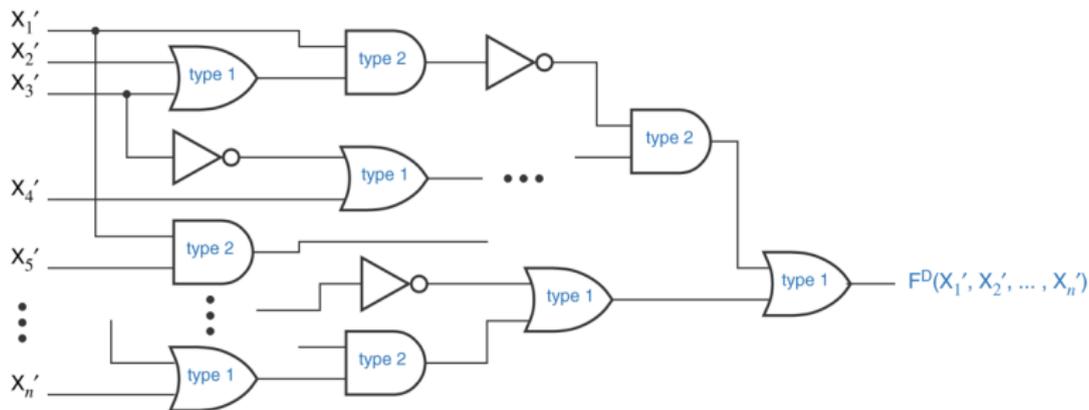


Tabela Verdade

Row	X	Y	Z	F	Minterm	Maxterm
0	0	0	0	F(0,0,0)	$X' \cdot Y' \cdot Z'$	$X + Y + Z$
1	0	0	1	F(0,0,1)	$X' \cdot Y' \cdot Z$	$X + Y + Z'$
2	0	1	0	F(0,1,0)	$X' \cdot Y \cdot Z'$	$X + Y' + Z$
3	0	1	1	F(0,1,1)	$X' \cdot Y \cdot Z$	$X + Y' + Z'$
4	1	0	0	F(1,0,0)	$X \cdot Y' \cdot Z'$	$X' + Y + Z$
5	1	0	1	F(1,0,1)	$X \cdot Y' \cdot Z$	$X' + Y + Z'$
6	1	1	0	F(1,1,0)	$X \cdot Y \cdot Z'$	$X' + Y' + Z$
7	1	1	1	F(1,1,1)	$X \cdot Y \cdot Z$	$X' + Y' + Z'$