1) $\omega_c = \frac{\pi}{2}$ rad/am , $N = \cancel{11}\ 20 \rightarrow L = \cancel{8} \rightarrow M = \cancel{2}\ 5$

a) $h_d(n) = \dfrac{\sin\left[\frac{\pi}{2}(n-\cancel{2}\,10)\right]}{\pi(n-\cancel{2}\,10)} = \frac{1}{2}\,\mathrm{sinc}\left[\frac{1}{2}(n-\cancel{2}\,10)\right]$

b) $\delta_r = -53\,dB \longrightarrow \delta_r = 0{,}0022 = 2{,}2 \times 10^{-3}$

$\delta_p = 0{,}0022$

c) $\Delta\omega = \dfrac{8\pi}{L-1}$

$= \dfrac{8\pi}{\cancel{11}\,20} \rightarrow \Delta\omega = \cancel{0{,}8\pi}\ 0{,}4\pi$

d) $w(n) = 0{,}54 - 0{,}46\cos\left(2\pi\frac{(n-M)}{L-1}\right)$, $n = 0, 1, \dots, L-1$

$= 0{,}54 - 0{,}46\cos\left(\frac{\pi}{10}(n-10)\right)$

$h(n) = h_d(n) \cdot w(n)$

e) $\varepsilon_{2exc} = \sum_{n=0}^{L-1}\left[h(n) - h_d(n)\right]^2 \rightarrow \varepsilon_{2exc} = \sum_{n=0}^{20}\left[h(n) - h_d(n)\right]^2$

c) $\omega_p = \omega_c - \dfrac{\Delta\omega}{2} \rightarrow \omega_p = 0{,}3\pi$

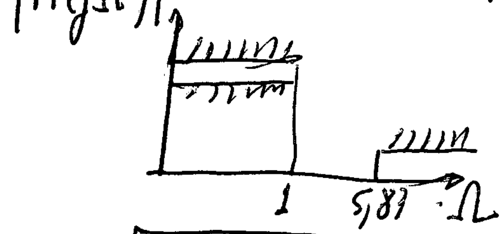
$\omega_r = \omega_c + \dfrac{\Delta\omega}{2} \rightarrow \omega_r = 0{,}7\pi$

2)

a) Com pré-distorção $\begin{cases} \Omega_r = \operatorname{tg}\left(\frac{\omega_r}{2}\right) \rightarrow \Omega_r = 0{,}4142 \text{ rad/s} \\ \Omega_p = \operatorname{tg}\left(\frac{\omega_p}{2}\right) \rightarrow \Omega_p = 2{,}4142 \text{ rad/s} \end{cases}$

Com a transformação pb-pa

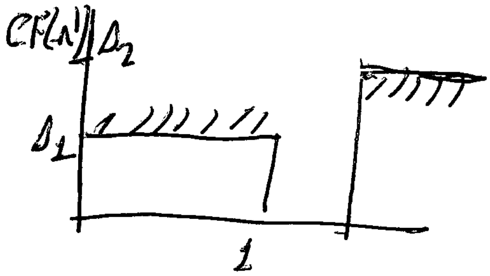
$$\Omega' = \frac{\Omega'_p \Omega_p}{\Omega} \rightarrow \begin{cases} \Omega'_p = 1 \text{ rad/s} \\ \Omega'_r = 5{,}8284 \text{ rad/s} \end{cases}$$



b)

$$\Delta_1 = \frac{\sqrt{2\delta_p - \delta_p^2}}{1-\delta_p} \rightarrow \Delta_1 = 0{,}4843$$

$$\Delta_2 = \frac{\sqrt{1-\delta_r^2}}{\delta_r} \rightarrow \Delta_2 = 9{,}9499$$



c) $C_e = \Delta_2$

$$C_d = \Delta_1 \cosh\left(N \cosh^{-1}\left(\frac{\Omega'_r}{\Omega'_p}\right)\right)$$

$$N \geq \frac{\cosh^{-1}(\Delta_2/\Delta_1)}{\cosh^{-1}(\Omega'_r/\Omega'_p)} = 1{,}5173$$

Escolhemos $N = 2$

assim

$C_e = 9{,}9499$

$C_d = 32{,}4211$

Escolhemos $C = \frac{C_e + C_d}{2}$

$C = 21{,}1855$

d) $b_0 = \frac{1}{N}\sinh^{-1}(\varepsilon)$

$= \frac{1}{2}\sinh^{-1}(21{,}1855) \longrightarrow \boxed{b_0 = 1{,}8735}$

Polos do protótipo passa-baixas

$$P_k' = \frac{-\Omega_r}{-\sin\left(\frac{2k-1}{2N}\pi\right)\sinh(b_0) + j\cos\left(\frac{2k-1}{2N}\pi\right)\cosh(b_0)}$$

ou

$P_1' = -1{,}2354 - 1{,}2951j$ ou $P_1' = 1{,}7898 \angle -0{,}7425\pi$

$P_2' = -1{,}2354 + 1{,}2951j$ $\quad P_2' = 1{,}7898 \angle 0{,}7425\pi$

Zeros do protótipo passa-baixas

$$z_k' = \frac{j\,\Omega_r}{\cos\left(\frac{2k-1}{2N}\pi\right)}$$

ou $z_1' = 8{,}2426j$ e $z_2' = -8{,}2426j$

Polos do passa-altas em TC

$$P_k = \frac{\Omega_p \Omega_p'}{P_k'}$$

ou $P_1 = -0{,}9310 + 0{,}9760j$ ou $P_1 = 1{,}3489 \angle 0{,}7425\pi$

$P_2 = -0{,}9310 - 0{,}9760j$ $\quad P_2 = 1{,}3486 \angle -0{,}7425\pi$

Zeros do passa-altas em TC

$$z_k = \frac{\Omega_p \Omega_p'}{z_k'}$$

ou $z_1 = -0{,}2929j$ e $z_2 = 0{,}2929j$

e) <u>Polos do passa-altas em TD</u>

$$p_k = \frac{1+P_k}{1-P_k} \longrightarrow \begin{cases} p_1 = -0{,}1750 + j0{,}4170 \\ p_2 = -0{,}1750 - j0{,}4170 \end{cases}$$

ou $$\begin{cases} p_1 = 0{,}4522 \angle 0{,}6265\pi \\ p_2 = 0{,}4522 \angle -0{,}6265\pi \end{cases}$$

<u>Zeros do passa-altas em TD</u>

$$z_k = \frac{1+Z_k}{1-Z_k} \longrightarrow \begin{cases} z_1 = 0{,}8420 - j0{,}5395 \\ z_2 = 0{,}8420 + j0{,}5395 \end{cases}$$

ou $$\begin{cases} z_1 = 1 \angle -0{,}1814\pi \\ z_2 = 1 \angle 0{,}1814\pi \end{cases}$$

f) $$H(z) = K \frac{(1-z_1 z^{-1})(1-z_2 z^{-1})}{(1-p_1 z^{-1})(1-p_2 z^{-1})}$$

$$= K \frac{1 - 1{,}6840 z^{-1} + z^{-2}}{1 + 0{,}3501 z^{-1} + 0{,}2045 z^{-2}}$$

Com $H(-1) = 1$, temos

$$K = \frac{1 - 0{,}3501 + 0{,}2045}{1 + 1{,}6840 + 1} \longrightarrow \boxed{K = 0{,}2319}$$

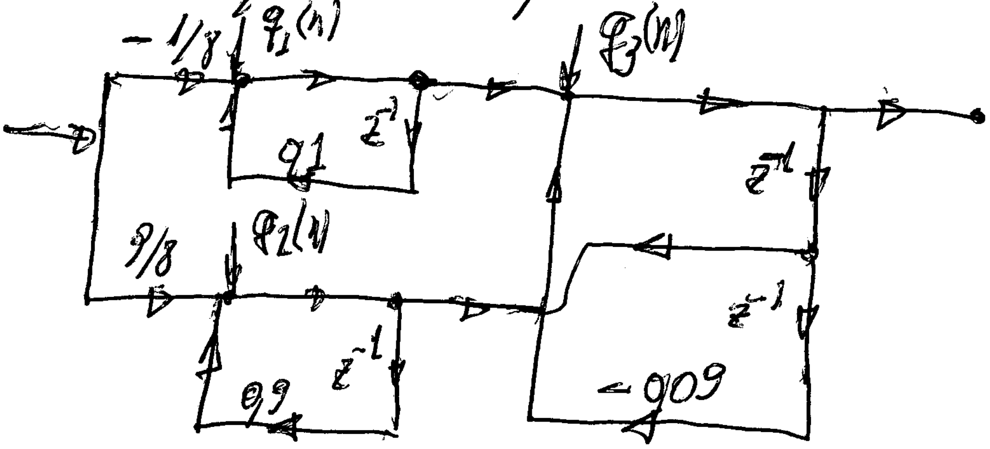
g) $$H(z) = 0{,}2319 \frac{1 - 1{,}6840 z^{-1} + z^{-2}}{1 + 0{,}3501 z^{-1} + 0{,}2045 z^{-2}}$$

a) $H_1(z) = H_2(z) = \frac{1}{1-z^{-1}+0{,}09z^{-1}} = \frac{A_1}{1-0{,}1z^{-1}} + \frac{A_2}{1-0{,}9z^{-1}}$

com

$A_1 = -\frac{1}{8}, \quad A_2 = \frac{9}{8}$

$= -0{,}125 \qquad = 1{,}125$



$$H_{11}(z) = \frac{A_1}{1-0{,}1z^{-1}} \cdot \frac{1}{1-z^{-2}+0{,}09z^{-1}}$$

$$H_{22}(z) = \frac{A_2}{1-0{,}9z^{-1}} \cdot \frac{1}{1-z^{-2}+0{,}09z^{-1}}$$

b)

$$H_{1e}(z) = \frac{1}{1-0{,}1z^{-1}}\left[\frac{A_1}{1-0{,}1z^{-1}} + \frac{A_2}{1-0{,}9z^{-1}}\right]$$

$$H_{2e}(z) = \frac{1}{1-0{,}9z^{-1}}\left[\frac{A_1}{1-0{,}1z^{-1}} + \frac{A_2}{1-0{,}9z^{-1}}\right]$$

c) $\mu_{q_1} = \mu_{q_2} = \mu_{q_3} = 0$

$\sigma_{q_1}^2 = \sigma_{q_2}^2 = \sigma_{q_3}^2 = \frac{2^{-2B}}{12}, B=7 \rightarrow \sigma_q^2 = 5{,}0863 \times 10^{-6}$

$= \sigma_q^2 \qquad \sigma_q^2\big|_{dB} = -52{,}94\,dB$

$$H_{1e}(z) = \frac{-1/8}{(1-0{,}1z^{-1})^2} + \frac{9/8}{(1-0{,}1z^{-1})(1-0{,}9z^{-1})} \qquad H_{3e}(z) = \frac{-1/8}{1-0{,}1z^{-1}} + \frac{9/8}{1-0{,}9z^{-1}}$$

$$H_{2e}(z) = \frac{-1/8}{(1-0{,}1z^{-1})(1-0{,}9z^{-1})} + \frac{9/8}{(1-0{,}9z^{-1})^2}$$

d) Calculamos o ganho de $\sigma^2_{q_3}$ à saída:

$$G_3 = \sum_{n=0}^{\infty} h^2_{e_3}(n) = A_1^2 G_1 + A_2^2 G_2$$

com

$$G_1 = 1{,}0101 = \frac{1}{1-p_1^2}$$

$$G_2 = 5{,}2632 = \frac{1}{1-p_2^2}$$

ou

$$G_3 = 6{,}3679$$

A parcela devida a $q_3(n)$ na saída tem variância

$$\sigma^2_{e_3} = G_3\, \sigma_q^2 \longrightarrow \begin{cases} \sigma^2_{e_3} = 32{,}39 \times 10^{-6} \\ \sigma^2_{e_3}\big|_{dB} = -44{,}90\,\text{dB} \end{cases}$$