Lista 4 de EAE0205 — Microeconomia II

Exercícios de Jogos

Não haverá gabarito para todas as questões.

Questão 1. Consider the game described by the payoff matrix below

		COL L	COL C	COL R
ROW	U	(0,1)	(2,5)	(-1,0)
ROW	M	(3,2)	(1,1)	(2,0)
ROW	D	(1,3)	(6,2)	(1,4)

(a) Does ROW have a dominant strategy? Does COL have a dominant strategy?

(b) Is there a dominated strategy for ROW? For COL?

(c) Which actions can be ruled out by iteractive elimination of dominated strategies?

(d) What is the relationship between *dominant strategy* and *dominated strategy*? Can you have one without the other?

(e) What is the unique Nash Equilibrium of this game?

Questão 2. Consider the game represent by the matrix:

		COL betray	COL cooperate
ROW	betray	(0,0)	(g,l)
ROW	cooperate	(l,g)	(c,c)

Assume that $g > c > 0 > l$.

(a) Suppose this game is played once, and the choices are made simultaneously. What is the Nash equilibrium?

For the game played multiple times, the players value payoffs streams according to the function

$$U = \sum_{t=0}^{N} \delta^t \, x_t,$$

where x_t is the payoff obtained in period t, and δ is the discount rate.

(b) What happens if this game is played twice? What if it is played N times, with N finite and known by the players?

Questão 3. Two players, Amy and Beth, take turns choosing numbers; Amy goes first. On her turn, a player may choose any number between 1 and 10, inclusive, and this number is added to a running total. When the running total of both players' choices reaches 100, the game ends. Consider two alternative endings: (*i*) the player whose choice of number takes the total to **exactly** 100 is the winner, and (*ii*) the player whose choice of number causes this total to **equal or exceed** 100 is the loser. For each case, answer the following questions:

(a) Who will win the game?

(b) What are the optimal strategies (complete plans of action) for each player?

Questão 4. Two roommates are planning to clean their room on a Sunday. They each have an hour to spend and they can spend it either by watching sports on TV or by cleaning the room. We denote the amount of time contributed by person i by c_i, where $0 \leq c_i \leq 1$; that person spends her remaining time $1 - c_i$ watching TV. Each person cares about both how clean the room is and the amount of time they get to watch TV. In particular, we assume that the payoff function of individual i ($i = 1, 2$) is given by

$$u_i(c_1, c_2) = b_i \ln(c_1 + c_2) + 1 - c_i,$$

where $b_i \ln(c_1 + c_2)$ represents the utility that the individual i derives from having a clean room, and $1 - c_i$ represents the utility of watching TV. You can interpret b_i as the relative value individual i attaches to having a clean room. Find the Nash equilibria of this game when (*i*) $b_1 > b_2$; and (*ii*) $b_1 = b_2$.

Questão 5. Country 1 must decide whether to attack Country 2, which is occupying an island between the two countries. In the event of an attack, Country 2 may fight or retreat over a bridge to its mainland. Each country prefers to occupy the island than not to occupy it; a fight is the worst outcome for both armies. Model this situation as an extensive form game and show that country 2 can increase its payoff by burning the bridge to the mainland, eliminating its option to retreat if attacked.

Questão 6. Two drivers, player 1 and player 2, are simultaneously approaching an intersection from different directions. They may choose to stop (S) or continue (C) at the intersection. If they both stop they prevent a crash and each receives a payoff of 1. If they both continue they crash and each receives a payoff of 0. If only player 2 stops, then player 1 gets a payoff of 2 whereas player 2 gets a payoff of $1 - c$, where c reflects the dislike of the player being the only one to stop. Conversely, if only player 1 stops, then she gets $1 - c$ and player 2 gets 2. Assume $0 < c < 1$.

(a) Formulate this situation as a strategic form game.

(b) Find all the Nash equilibria (in pure and mixed strategies). Find the expected payoff of each player at the mixed strategy equilibrium.

Questão 7. Two ice cream vendors independently choose a location on the boardwalk. Each can locate at any point between the beginning, which we label 0, and the end, which we label 1. Therefore, the action spaces of vendor 1 and vendor 2 are $A_1 = A_2 = [0, 1]$.

There is a *continuum* of customers, with mass equal to 1, distributed uniformly over the line $[0,1]$. Each vendor will attract the customers closest to it, and they will share equally customers who are equidistant between the two. Thus, for instance, if vendor 1 chooses x and the vendor 2 chooses $y \geq x$; then the first will get a share $x + (y - x)/2$ and the second will get a share $(1 - y) + (y - x)/2$ of the customers (draw a picture to help you see why). If they locate at the same point, assume that they share the total customer base equally and hence get 1/2 each.) Each customer contributes \$1 in profits to the vendor s/he visits.

(a) Formulate this situation as a strategic form game.

(b) Find its unique (pure strategy) Nash equilibrium.

Questão 8. A firm's revenue function when it uses L units of labor is given by

$$R(L) = L^{1/2}.$$

A union that represents workers presents a wage offer – w – to the firm. The firm, after observing the offer, either accepts or rejects. If the firm accepts the offer, it chooses the number L of workers to employ (which you should take to be a continuous variable, not an integer). If it rejects the offer, no one is hired and the firm's revenue is zero. The firm's payoff function is given by

$$\pi(w, L) = R(L) - wL$$

and the union's payoff function is given by

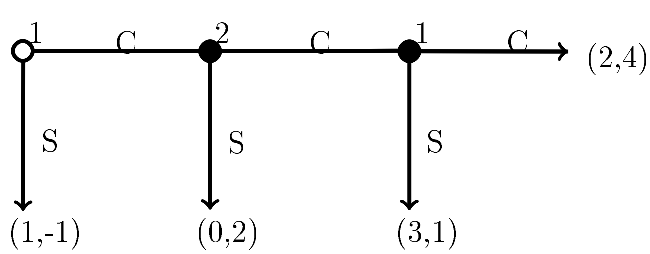
$$u(w, L) = (w - 1)\ L.$$

They are both zero if the firm rejects the wage offer.

(a) Find the subgame perfect equilibrium (equilibria?) of this game.

(b) Is there an outcome of the game that both parties prefer to any subgame perfect equilibrium outcome? (♠)

Questão 9. Consider the following extensive form game.



(a) Write down the strategic form of this game and find all of its pure strategy Nash equilibria. (♠)

(b) Find the set of pure strategy subgame perfect equilibria of the game.

Questão 10. Dois partidos – A e B – competem por votos utlizando gastos em publicidade. O objetivo é maximizar a fração de votos obtidos. O partido i, $i = A, B$, gasta R\$ x_i milhões na campanha, e a fração de votos obtida é proporcional aos gastos. Assim, os *payoffs* são

$$V_A = \frac{x_A}{x_A + k\, x_B} - x_A \quad \text{e} \quad V_B = \frac{k\, x_B}{x_A + k\, x_B} - c\, x_B. \tag{1}$$

(a) Interprete o significado dos parâmetros k e c.

(b) Encontre as funções de resposta dos partidos.

(c) Encontre o equilíbrio de Nash.

(d) Qual o efeito de variações em k e c nos gastos de campanha e no *payoff* dos partidos?

Questão 12. The Internal Revenue Service (IRS) must decide whether to audit a certain tax return to discover whether it is accurate or not. Suppose that the IRS and the tax payer simultaneously choose whether to audit (A) or not (NA) and cheat (C) or not (NC), respectively. The true amount the tax payer owes the IRS is 1. He either reports the true amount or cheats and reports 0. If the IRS audits the tax return the truth is always discovered and 1 is collected. Auditing costs c, where $c < 1$, and if the tax payer is caught cheating, a fine of amount $f > 0$ is collected from the tax payer. The goal of the IRS is to maximize expected revenues (*i.e.*, tax collection minus auditing costs, if any, plus fines, if any) whereas the taxpayer wants to minimize expected payments (*i.e.*, tax payment minus fines, if any).

(a) Draw a 2×2 payoff matrix for this interaction, clearly labelling the players, their strategies, and the payoffs.

(b) Find all the (pure or mixed strategy) Nash equilibria. (♠)

Questão 13. In Stanley Kubrick's dark comedy *Dr. Strangelove – or How I Learned to Stop Worrying and Love the Bomb*, the rationality of Cold War nuclear policy was called into question. An American general goes insane and orders a nuclear attack against the Soviet Union, that the President's staff, gathered in The War Room, tries to stop. In sequential games, the possibility to commit to some course of action in advance can provide some strategic advantage to players. Sometimes it may be "rational to be irrational", especially when deterrence requires both harsh punishment and strong credibility that one is willing to punish, since it is unnatractive to do so *ex post*. "The Doomsday Machine"serves that purpose, but leaves little room for error...
Below it is the script of a scene from the movie, which takes place in the The War Room.
Scene Cast:
Muffley: U.S. President
Alexiy DeSadeski: USSR Ambassador
Dr. Strangelove: Science Advisor to the U.S. President
Turgidson: Military Advisor to the U.S. President.

Muffley: *What... what is it, what?*

DeSadeski: *The fools... the mad fools.*

Muffley: *What's happened?*

DeSadeski: *The Doomsday Machine.*

Muffley: *The Doomsday Machine? What is that?*

DeSadeski: *A device which will destroy all human and animal life on earth.*

Muffley: *All human and animal life? ... I'm afraid I don't understand something, Alexiy. Is the Premier threatening to explode this if our planes carry out their attack?*

DeSadeski: *No sir. It is not a thing a sane man would do. The Doomsday Machine is designed to to trigger itself automatically!*

Muffley: *But surely you can disarm it somehow.*

DeSadeski: *No. It is designed to explode if any attempt is ever made to untrigger it.*

Muffley: *Automatically? ... But, how is it possible for this thing to be triggered automatically, and at the same time impossible to untrigger?*

Strangelove: *Mr. President, it is not only possible, it is essential. That is the whole idea of this machine, you know. Deterrence is the art of producing in the mind of the enemy... the fear to attack. And so, because of the automated and irrevocable decision making process which rules out human meddling, the Doomsday Machine is terrifying. It's simple to understand. And completely credible, and convincing.*

Turgidson: *Gee, I wish we had one of the Doomsday Machine, Stainsy.*

Muffley: *But this is fantastic, Strangelove. How can it be triggered automatically?*

Strangelove: *Well, it's remarkably simple to do that. When you merely wish to bury bombs, there is no limit to the size. After that they are connected to a gigantic complex of computers. Now then, a specific and clearly defined set of circumstances, under which the bombs are to be exploded, is programmed into a tape memory bank. ... Yes, but the... whole point of the Doomsday Machine... is lost... if you keep it a secret! Why didn't you tell the world, eh?*

DeSadeski: *It was to be announced at the Party Congress on Monday. As you know, the Premier loves surprises.*

Consider the game played between two countries: the USSR and the USA. The USA moves first and choose to attack or not attack the USSR. In case of an attack, the USSR can either counterattack, starting a global nuclear war, or appease. If the USA does not launch an attack, the *status quo* is maintained, and both countries get a zero payoff. A global nuclear war would be catastrophic, causing both countries to lose 1000. On the other hand, if the USSR appeases, the USA gets 200 (a major damaged was inflicted on the adversary), and the USSR loses 300. Write this game in strategic form (draw the game tree). What is the subgame-perfect Nash Equilibrium?

Now suppose a modified game. Before the game begns, the USSR can turn on the Doomsday Machine. Once it is turned on, it cannot be turned off. This machine is capable of detecting any attack to USSR and automatically counterattack, by detonating powerful A-bombs buried in its territory, launching dust and radiation throughout the Earth atmosphere. This would cause a long lasting and severe cold weather – the Nuclear Winter – which would annihilate mankind. In this case, both countries would lose 1000. Write this game in strategic form (draw the game tree). What is the subgame-perfect Nash Equilibrium?

Compare and interpret the results.