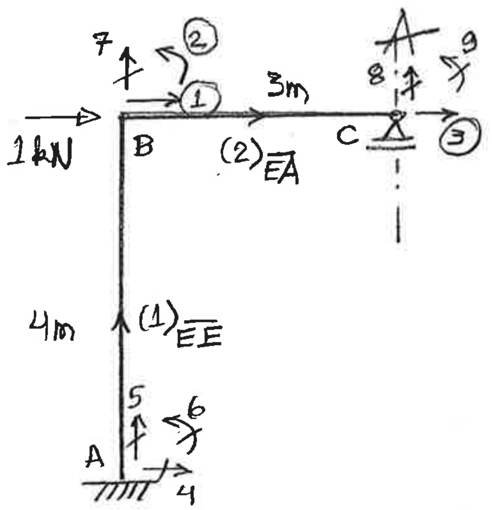
$\boxed{\text{fE08}}$



$E = 4{,}32 \times 10^7 \ \frac{kN}{m^2}$

livres

$LM_1 = [4 \ 5 \ 6 \ | \ 1 \ 7 \ 2]$

$LM_2 = [1 \ 7 \ 2 \ | \ 3 \ 8 \ 9]$

$EA = 4{,}32 \times 10^6 \ kN \qquad EI = 4{,}32 \times 10^3 \ kN\,m^2$

a) Matrizes das barras

- Barra 1 (EE) $\quad \ell_1 = 4m \qquad \frac{EA}{\ell_1} = 1{,}08 \times 10^6$

$\frac{2EI}{\ell_1} = 2160 \qquad \frac{6EI}{\ell_1^2} = 1620 \qquad \frac{12EI}{\ell_1^3} = 810$

$\times 2 \rightarrow 4320$

$$\underline{T}_1 = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & & & \\ -1 & 0 & 0 & & \underline{0} & \\ 0 & 0 & 1 & & & \\ \hline & & & 0 & 1 & 0 \\ & \underline{0} & & -1 & 0 & 0 \\ & & & 0 & 0 & 1 \end{array}\right]$$

$$\underline{\bar{k}}_1 = \left[\begin{array}{ccc|ccc} \vdots & \vdots & \vdots & -1{,}08\times10^6 & 0 & 0 \\ & & & 0 & -810 & 1620 \\ & & & 0 & -1620 & 2160 \\ \hline & & & 1{,}08\times10^6 & 0 & 0 \\ & & & 0 & 810 & -1620 \\ & & & 0 & -1620 & 4320 \end{array}\right]$$

$\underbrace{\qquad\qquad}_{\bar{k}_1^*}$

$$\underline{t}_1^T(\underline{\bar{k}}_1^* \underline{t}_1) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1{,}08\times10^6 & 0 \\ -810 & 0 & -1620 \\ 1620 & 0 & 4320 \end{bmatrix} = \begin{bmatrix} 810 & 0 & 1620 \\ 0 & 1{,}08\times10^6 & 0 \\ 1620 & 0 & 4320 \end{bmatrix}$$

$$\underline{k}_1 = \begin{array}{c} \begin{array}{cccccc} 4 & 5 & 6 & 1 & 7 & 2 \end{array} \\ \left[\begin{array}{ccc|ccc} \vdots & \vdots & \vdots & & & \\ & & & & & \\ & & & & & \\ \hline & & & 810 & 0 & 1620 \\ & & & 0 & 1{,}08\times10^6 & 0 \\ & & & 1620 & 0 & 4320 \end{array}\right] \begin{array}{c} 4 \\ 5 \\ 6 \\ 1 \\ 7 \\ 2 \end{array} \end{array}$$

- Barra 2 (EA) $\quad \ell_2 = 3\,m \qquad \frac{EA}{\ell_2} = 1{,}44\times10^6 \qquad \frac{3EI}{\ell} = 4320 \qquad \frac{3EI}{\ell^2} = 1440 \qquad \frac{3EI}{\ell^3} = 480$

$$\underline{k}_2 = \underline{\bar{k}}_2 = \begin{array}{c} \begin{array}{cccccc} 1 & 7 & 2 & 3 & 8 & 9 \end{array} \\ \left[\begin{array}{ccc|ccc} 1{,}44\times10^6 & 0 & 0 & -1{,}44\times10^6 & 0 & 0 \\ & 480 & 1440 & 0 & -480 & 0 \\ & & 4320 & 0 & -1440 & 0 \\ \hline & & & 1{,}44\times10^6 & 0 & 0 \\ & & & & 480 & 0 \\ & & & & & 0 \end{array}\right] \begin{array}{c} 1 \\ 7 \\ 2 \\ 3 \\ 8 \\ 9 \end{array} \end{array}$$

b) Matrizes da estrutura

$$\underset{\sim}{F_a} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

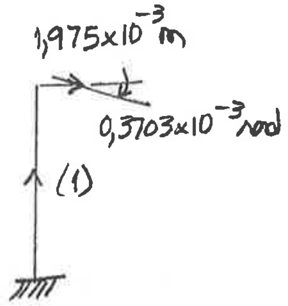
$$
\underset{\sim}{K_{aa}} = \begin{array}{c} \\ \\ \\ \end{array}
\begin{array}{ccc} 1 & 2 & 3 \\ \end{array}
$$

$$
\underset{\sim}{K_{aa}} = \left[\begin{array}{c|c|c} 810 + 1{,}44\times10^{6} & 1620 + 0 & -1{,}44\times10^{6} \\ \hline & 4320 + 4320 & 0 \\ \hline & & 1{,}44\times10^{6} \end{array}\right]\begin{array}{c} 1 \\ 2 \\ 3 \end{array} = \begin{bmatrix} 1440810 & 1620 & -1{,}44\times10^{6} \\ & 8640 & 0 \\ & & 1{,}44\times10^{6} \end{bmatrix}
$$

Resolvendo o sistema: $\underset{\sim}{K_{aa}} \; \underset{\sim}{U_a} = \underset{\sim}{F_a}$, obtemos: $\underset{\sim}{U_a} = \begin{bmatrix} 1{,}975\times10^{-3}\,m \\ -0{,}3703\times10^{-3}\,rad \\ 1{,}975\times10^{-3}\,m \end{bmatrix}$

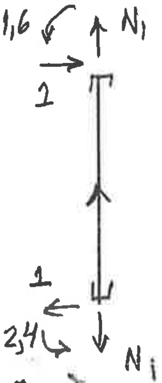
c) Deslocamentos e esforços de extremidade das barras

$$
\underset{\sim}{\bar{u}_1} = \underset{\sim}{T_1}\,\underset{\sim}{u_1} = 10^{-3}\,\underset{\sim}{T_1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 1{,}975 \\ 0 \\ -0{,}3703 \end{bmatrix}\begin{array}{c} 4 \\ 5 \\ 6 \\ 1 \\ 7 \\ 2 \end{array} = 10^{-3}\begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ -1{,}975 \\ -0{,}3703 \end{bmatrix}
$$

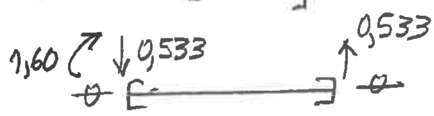


N_1 deve ser obtida por equilíbrio!

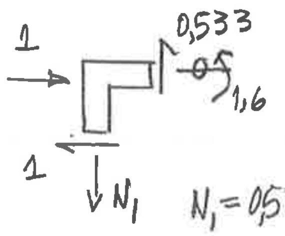
$$
\underset{\sim}{\bar{f}_1} = \underset{\sim}{\bar{k}_1}\,\underset{\sim}{\bar{u}_1} = 10^{-3}\left[\begin{array}{c|ccc} & -1{,}08\times10^{6} & 0 & 0 \\ & 0 & -810 & 1620 \\ & 0 & -1620 & 2160 \\ \hline & 1{,}08\times10^{6} & 0 & 0 \\ & & 810 & -1620 \\ & & -1620 & 4320 \end{array}\right]\begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ -1{,}975 \\ -0{,}3703 \end{bmatrix} = \begin{bmatrix} \boxed{0} \\ 1 \\ 2{,}40 \\ \hline \boxed{0} \\ -1 \\ 1{,}60 \end{bmatrix}
$$



$$
\underset{\sim}{\bar{f}_2} = \underset{\sim}{f_2} = \underset{\sim}{k_2}\,\underset{\sim}{u_2} = 10^{-3}\left[\begin{array}{ccc|ccc} 1{,}44\times10^{6} & 0 & 0 & -1{,}44\times10^{6} & 0 & 0 \\ 0 & 480 & 1440 & 0 & -480 & 0 \\ 0 & 1440 & 4320 & 0 & -1440 & 0 \\ \hline -1{,}44\times10^{6} & 0 & 0 & 1{,}44\times10^{6} & 0 & 0 \\ 0 & -480 & -1440 & 0 & 480 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]\begin{bmatrix} 1{,}975 \\ 0 \\ -0{,}3703 \\ \hline 1{,}975 \\ 0 \\ 0 \end{bmatrix}\begin{array}{c} 1 \\ 7 \\ 2 \\ 3 \\ 8 \\ 9 \end{array} = \begin{bmatrix} 0 \\ -0{,}533 \\ -1{,}60 \\ \hline 0 \\ 0{,}533 \\ 0 \end{bmatrix}
$$

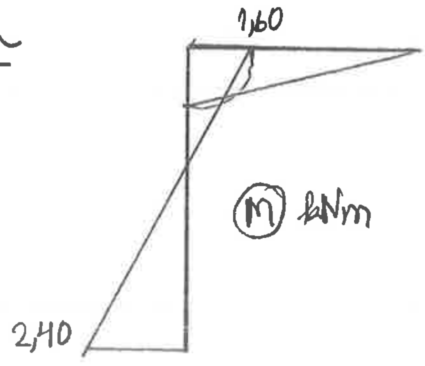


Equilíbrio do nó B



$N_1 = 0{,}533\,kN$
(tração)

d) Diagrama



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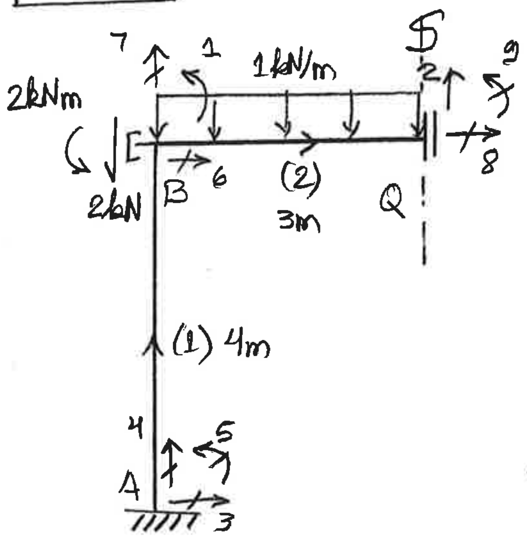
FE09



$$E = 4{,}32 \times 10^7 \ \frac{kN}{m^2}$$

$$LM_1 = [\,3 \ 4 \ 5 \mid 6 \ 7 \ 1\,]$$

$$LM_2 = [\,6 \ 7 \ 1 \mid 8 \ 2 \ 9\,]$$

$$EA = 4{,}32 \times 10^6 \ kN \qquad EI = 4{,}32 \times 10^3 \ kNm^2$$

a) Matrizes das barras

• Barra 1 (EE) $\ell_1 = 4m$ $\quad \frac{EA}{\ell_1} = 1{,}08 \times 10^6$

$$\frac{2EI}{\ell_1} = 2160 \qquad \frac{6EI}{\ell^2} = 1620 \qquad \frac{12EI}{\ell^3} = 810$$

$$\underline{T}_1 = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & & & \\ -1 & 0 & 0 & & \underline{0} & \\ 0 & 0 & 1 & & & \\ \hline & & & 0 & 1 & 0 \\ & \underline{0} & & -1 & 0 & 0 \\ & & & 0 & 0 & 1 \end{array}\right]$$

$$\underline{t}_1^T \underline{k}_1 \underline{t}_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1{,}08 \times 10^6 & 0 \\ -810 & 0 & -1620 \\ 1620 & 0 & 4320 \end{bmatrix} = \begin{bmatrix} 810 & 0 & 1620 \\ 0 & 1{,}08 \times 10^6 & 0 \\ 1620 & 0 & 4320 \end{bmatrix}$$

$$\underline{\bar{k}}_1 = \left[\begin{array}{ccc|ccc} | & | & | & -1{,}08 \times 10^6 & 0 & 0 \\ | & | & | & 0 & -810 & 1620 \\ & & & 0 & -1620 & 2160 \\ \hline & & & 1{,}08 \times 10^6 & 0 & 0 \\ & & & 0 & 810 & -1620 \\ & & & 0 & -1620 & 4320 \end{array}\right]$$

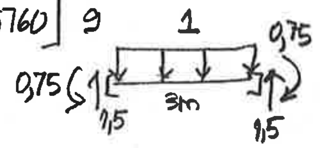
$$\underbrace{\qquad\qquad}_{\bar{k}_1^*}$$

$$\underline{k}_1 = \begin{array}{c} \begin{array}{cccccc} 3 & 4 & 5 & 6 & 7 & 1 \end{array} \\ \left[\begin{array}{ccc|ccc} & & & & & \\ & & & & & \\ & & & & & \\ \hline & & & 810 & 0 & 1620 \\ & & & 0 & 1{,}08 \times 10^6 & 0 \\ & & & 1620 & 0 & 4320 \end{array}\right] \end{array} \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 1 \end{array}$$

• Barra 2 (EE) $\ell_2 = 3m$ $\quad \frac{EA}{\ell_2} = 1{,}44 \times 10^6 \quad \frac{2EI}{\ell_2} = 2880 \quad \frac{6EI}{\ell_2^2} = 2880 \quad \frac{12EI}{\ell^3} = 1920$

$$\underline{k}_2 = \underline{\bar{k}}_2 = \begin{array}{c} \begin{array}{cccccc} 6 & 7 & 1 & 8 & 2 & 9 \end{array} \\ \left[\begin{array}{ccc|ccc} 1{,}44 \times 10^6 & 0 & 0 & -1{,}44 \times 10^6 & 0 & 0 \\ & 1920 & 2880 & 0 & -1920 & 2880 \\ & & 5760 & 0 & -2880 & 2880 \\ \hline & & & 1{,}44 \times 10^6 & 0 & 0 \\ & & & & 1920 & -2880 \\ & & & & & 5760 \end{array}\right] \end{array} \begin{array}{c} 6 \\ 7 \\ 1 \\ 8 \\ 2 \\ 9 \end{array}$$

$$\underset{\sim}{f_2^0} = \underset{\sim}{\bar{f}_2^0} = \left[\begin{array}{c} 0 \\ \frac{1 \times 3}{2} \\ + \frac{1 \times 3^2}{12} \\ \hline 0 \\ \frac{1 \times 3}{2} \\ -\frac{1 \times 3^2}{12} \end{array}\right] = \left[\begin{array}{c} 0 \\ 1{,}5 \\ 0{,}75 \\ \hline 0 \\ 1{,}5 \\ -0{,}75 \end{array}\right] \begin{array}{c} 6 \\ 7 \\ 1 \\ 8 \\ 2 \\ 9 \end{array}$$



b) Matrizes da estrutura

$$\underset{\sim}{F_a} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \end{array} \qquad \underset{\sim}{F_a^0} = \begin{bmatrix} 0{,}75 \\ 1{,}5 \end{bmatrix} \begin{array}{c} 1 \\ 2 \end{array}$$

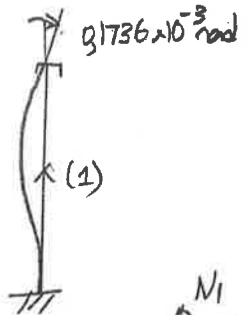
$$\underline{K_{aa}} = \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \left[\begin{array}{c|c} 4320 + 5760 & -2880 \\ \hline & 1920 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \end{array} = \begin{bmatrix} 10080 & -2880 \\ -2880 & 1920 \end{bmatrix}$$

Resolvendo o sistema: $\underline{K_{aa}}\,\underset{\sim}{U_a} = \underset{\sim}{F_a} - \underset{\sim}{F_a^0} = \begin{bmatrix} 1{,}25 \\ -1{,}5 \end{bmatrix}$ obtemos,

$$\underset{\sim}{U_a} = \begin{bmatrix} -0{,}1736\times10^{-3}\ \text{rad} \\ -1{,}042\times10^{-3}\ \text{m} \end{bmatrix}$$

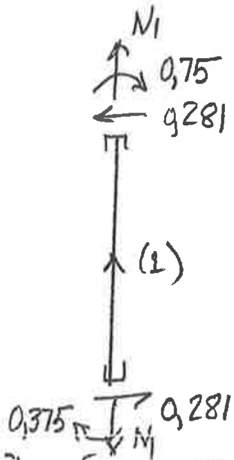
c) Deslocamentos e esforços de extremidade das barras

$$\underset{\sim}{\bar{u}_1} = \underline{T_1}\,\underset{\sim}{u_1} = 10^{-3}\,\underline{T_1} \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ -0{,}1736 \end{array}\right] \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 1 \end{array} = 10^{-3} \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ -0{,}1736 \end{array}\right]$$

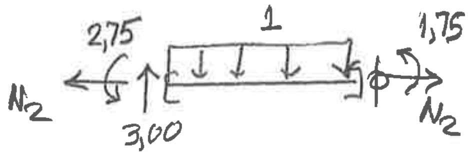


equilíbrio

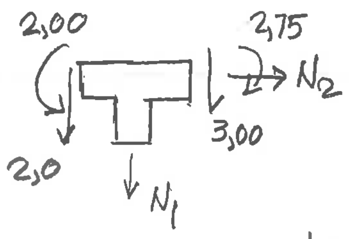
$$\underset{\sim}{\bar{f}_1} = \underline{\bar{k}_1}\,\underset{\sim}{\bar{u}_1} + \cancel{\underset{\sim}{f_1^0}}^{\,\underset{\sim}{0}} = 10^{-3} \left[\begin{array}{ccc|ccc} & & & & & 0 \\ & & & & & 1620 \\ & & & & & 2160 \\ \hline & & & & & 0 \\ & & & & & -1620 \\ & & & & & 4320 \end{array}\right] \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ -0{,}1736 \end{array}\right] = \left[\begin{array}{c} \boxed{N_1} \\ -0{,}281 \\ -0{,}375 \\ \hline \boxed{N_1} \\ 0{,}281 \\ -0{,}750 \end{array}\right]$$



$$\underset{\sim}{\bar{f}_2} = \underset{\sim}{f_2} = \underline{k_2}\,\underset{\sim}{u_2} + \underset{\sim}{f_2^0} = 10^{-3} \left[\begin{array}{ccc|ccc} & & 0 & & 0 & \\ & & 2880 & & -1920 & \\ & & 5760 & & -2880 & \\ \hline & & 0 & & 0 & \\ & & -2880 & & 1920 & \\ & & 2880 & & -2880 & \end{array}\right] \left[\begin{array}{c} 0 \\ 0 \\ -0{,}1736 \\ \hline 0 \\ -1{,}042 \\ 0 \end{array}\right] \begin{array}{c} 6 \\ 7 \\ 1 \\ 8 \\ 2 \\ 9 \end{array} + \left[\begin{array}{c} 0 \\ 1{,}5 \\ 0{,}75 \\ \hline 0 \\ 1{,}5 \\ -0{,}75 \end{array}\right] = \left[\begin{array}{c} 0+0 \\ 1{,}50+1{,}50 \\ 2{,}00+0{,}75 \\ \hline 0+0 \\ -1{,}50+1{,}50 \\ 2{,}50-0{,}75 \end{array}\right] = \left[\begin{array}{c} N_2 \\ 3{,}00 \\ 2{,}75 \\ \hline N_2 \\ 0 \\ 1{,}75 \end{array}\right]$$

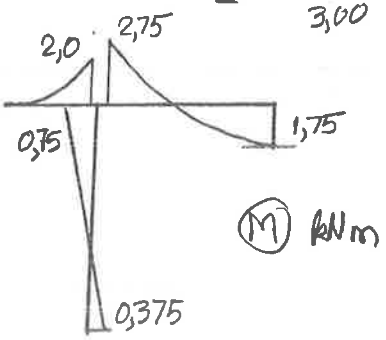


Equilíbrio do nó B



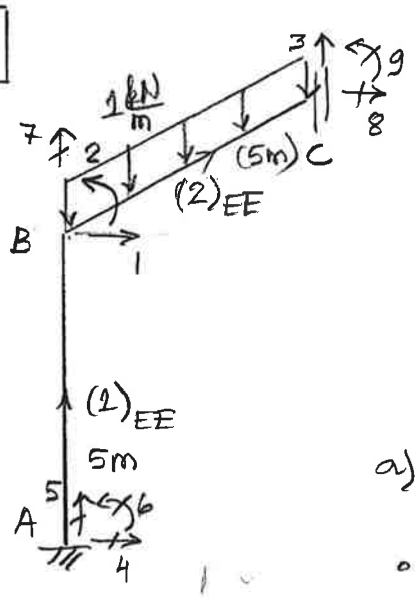
$N_1 = -5{,}00\,\text{kN}$ (compr.)

$N_2 = 0$



d) Diagrama

FE10



$$E = 1\times 10^7 \ \frac{kN}{m^2}$$

$$LM_1 = [\,4 \ 5 \ 6 \mid 1 \ 7 \ 2\,]$$

$$LM_2 = [\,1 \ 7 \ 2 \mid 8 \ 3 \ 9\,]$$

$$EA = 1\times 10^6 \ kN \qquad EI = 1\times 10^3 \ kNm^2$$

a) Matrizes das barras

- Barra 1 (EE) $\quad l_1 = 5m \qquad \frac{EA}{l_1} = 2\times 10^5$

$$\frac{2EI}{l_1} = 400 \qquad \frac{6EI}{l_1^2} = 240 \qquad \frac{12EI}{l_1^3} = 96$$

$$\underline{T_1} = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & & & \\ -1 & 0 & 0 & & \underline{0} & \\ 0 & 0 & 1 & & & \\ \hline & & & 0 & 1 & 0 \\ & \underline{0} & & -1 & 0 & 0 \\ & & & 0 & 0 & 1 \end{array}\right]$$

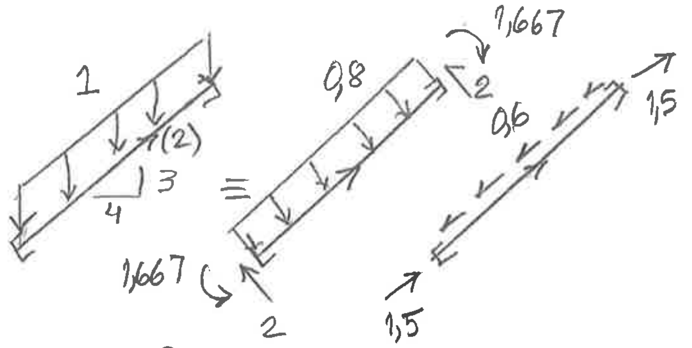
$$\underline{\bar{k}_1} = \left[\begin{array}{ccc|ccc} 2\times10^5 & 0 & 0 & -2\times10^5 & 0 & 0 \\ 0 & 96 & 240 & 0 & -96 & 240 \\ 0 & 240 & 800 & 0 & -240 & 400 \\ \hline -2\times10^5 & 0 & 0 & 2\times10^5 & 0 & 0 \\ 0 & -96 & -240 & 0 & 96 & -240 \\ 0 & 240 & 400 & 0 & -240 & 800 \end{array}\right] \left.\vphantom{\begin{array}{c}0\\0\\0\end{array}}\right\} \underline{\bar{k}^*}$$

$$\underline{k_1} = \begin{array}{c} \begin{array}{cccccc} 4 & 5 & 6 & 1 & 7 & 2 \end{array} \\ \left[\begin{array}{ccc|ccc} \vdots & \vdots & \vdots & & & \\ & & & & & \\ & & & & & \\ \hline & & & 96 & 0 & 240 \\ & & & 0 & 2\times10^5 & 0 \\ & & & 240 & 0 & 800 \end{array}\right] \end{array} \begin{array}{c} 4 \\ 5 \\ 6 \\ 1 \\ 7 \\ 2 \end{array}$$

- Barra 2 (EE) $\quad l_2 = l_1 = 5m \qquad \boxed{\underline{\bar{k}_2} = \underline{\bar{k}_1}}$

$$\underline{T_2} = \left[\begin{array}{ccc|ccc} 0{,}8 & 0{,}6 & 0 & & & \\ -0{,}6 & 0{,}8 & 0 & & \underline{0} & \\ 0 & 0 & 1 & & & \\ \hline & & & 0{,}8 & 0{,}6 & 0 \\ & \underline{0} & & -0{,}6 & 0{,}8 & 0 \\ & & & 0 & 0 & 1 \end{array}\right]$$

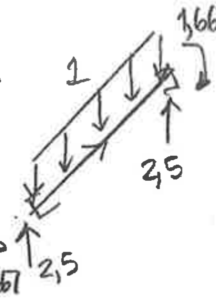
$$\underline{k_2} = \underline{T_2}^T \, \underline{\bar{k}_2} \, \underline{T_2} = \begin{array}{c} \begin{array}{cccccc} 1 & 7 & 2 & 8 & 3 & 9 \end{array} \\ \left[\begin{array}{ccc|ccc} 1{,}28035\times10^5 & 95954 & -144 & -1{,}28035\times10^5 & -95954 & -144 \\ & 72061 & 192 & -95954 & -72061 & 192 \\ & & 800 & 144 & -192 & 400 \\ \hline & & & 1{,}28035\times10^5 & 95954 & 144 \\ & \text{sim.} & & & 72061 & -192 \\ & & & & & 800 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 7 \\ 2 \\ 8 \\ 3 \\ 9 \end{array}$$



$$\bar{f}_2^{\,0} = \begin{bmatrix} 0 & +1{,}5 \\ 2 & 0 \\ +1{,}667 & 0 \\ \hline 0 & +1{,}5 \\ 2 & 0 \\ -1{,}667 & 0 \end{bmatrix} = \begin{bmatrix} 1{,}5 \\ 2 \\ +1{,}667 \\ \hline 1{,}5 \\ 2 \\ -1{,}667 \end{bmatrix}$$

$$\frac{pl^2}{12} = \frac{0{,}8 \times 5^2}{12} = 1{,}667$$

$$f_2^{\,0} = T^T \bar{f}_2^{\,0} = \left[\begin{array}{ccc|ccc} 0{,}8 & -0{,}6 & 0 & & & \\ 0{,}6 & 0{,}8 & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ \hline & & & 0{,}8 & -0{,}6 & 0 \\ & 0 & & 0{,}6 & 0{,}8 & 0 \\ & & & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} 1{,}5 \\ 2 \\ +1{,}667 \\ \hline 1{,}5 \\ 2 \\ -1{,}667 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 0 \\ 2{,}5 \\ +1{,}667 \\ \hline 0 \\ 2{,}5 \\ -1{,}667 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 7 \\ 2 \\ 8 \\ 3 \\ 9 \end{array}$$



b) Matrizes da estrutura

$$F_a = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad F_a^{\,0} = \begin{bmatrix} 0 \\ +1{,}667 \\ 2{,}5 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

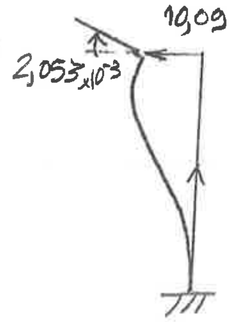
$$K_{aa} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 96 + 1{,}28035 \times 10^5 & 240 - 144 & -95954 \\ & 800 + 800 & -192 \\ & & 72061 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} = \begin{bmatrix} 128131 & 96 & -95954 \\ & 1600 & -192 \\ & & 72061 \end{bmatrix}$$

Resolvendo o sistema: $K_{aa} U_a = F_a - F_a^{\,0} = \begin{bmatrix} 0 \\ -1{,}667 \\ -2{,}5 \end{bmatrix}$

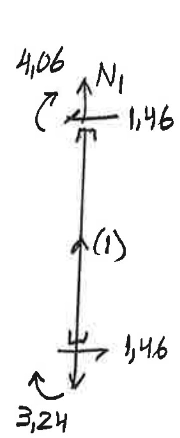
$$U_a = \begin{bmatrix} -10{,}09 \\ -2{,}053 \\ -13{,}47 \end{bmatrix} \times 10^{-3} \begin{array}{c} m \\ rad \\ m \end{array}$$

c) Deslocamentos e esforços nas extremidades das barras

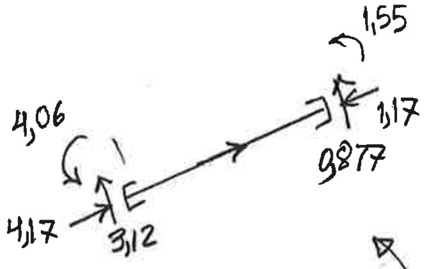
$$\bar{u}_1 = T_1 u_1 = 10^{-3} T_1 \begin{bmatrix} \cancel{0} \\ \cancel{0} \\ \cancel{0} \\ \hline -10{,}09 \\ 0 \\ -2{,}053 \end{bmatrix} \begin{array}{c} 4 \\ 5 \\ 6 \\ 1 \\ 7 \\ 2 \end{array} = 10^{-3} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ 10{,}09 \\ -2{,}053 \end{bmatrix}$$



$$\underset{\sim}{\bar{f}_1} = \bar{k}_1 \underset{\sim}{\bar{u}_1} + \cancel{\underset{\sim}{\bar{f}_1^{0}}}^{\,0} = 10^{-3}\left[\begin{array}{cccc|cc} \vdots & \vdots & \vdots & \vdots & 0 & 0 \\ & & & & -96 & 240 \\ & & & & -240 & 400 \\ \hline \vdots & \vdots & \vdots & \vdots & 0 & 0 \\ & & & & 96 & -240 \\ & & & & -240 & 800 \end{array}\right]\left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \hline 0 \\ 19{,}09 \\ -2{,}053 \end{array}\right] = \overset{\text{equil.}}{\left[\begin{array}{c} \boxed{0} \\ -1{,}46 \\ -3{,}24 \\ \hline \boxed{0} \\ 1{,}46 \\ -4{,}06 \end{array}\right]}$$



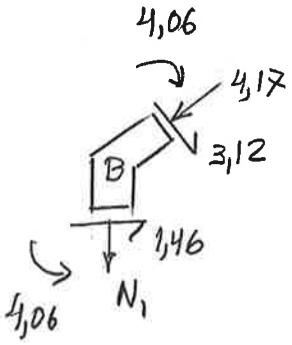
$$\underset{\sim}{\bar{u}_2} = \underline{T}_2\, \underset{\sim}{u_2} = 10^{-3}\, \underline{T}_2 \begin{array}{c} \left[\begin{array}{c} -19{,}09 \\ 0 \\ -2{,}053 \\ \hline 0 \\ -13{,}47 \\ 0 \end{array}\right] \end{array}\begin{array}{c} 1 \\ 7 \\ 2 \\ 8 \\ 3 \\ 9 \end{array} = \left[\begin{array}{c} 8{,}071 \\ 6{,}054 \\ -2{,}053 \\ \hline -8{,}085 \\ -19{,}780 \\ 0 \end{array}\right] 10^{-3}$$



$$\bar{k}_1$$
$$\downarrow$$

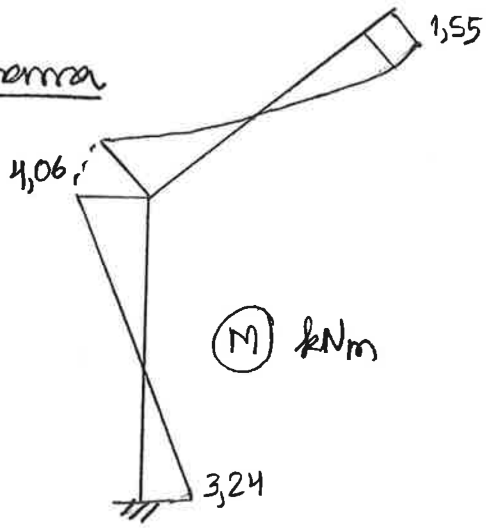
$$\underset{\sim}{\bar{f}_2} = \bar{k}_2\, \underset{\sim}{\bar{u}_2} + \underset{\sim}{\bar{f}_2^{0}} = 10^{-3}\left[\begin{array}{ccc|ccc} 2\times10^5 & 0 & 0 & -2\times10^5 & 0 & 0 \\ 0 & 96 & 240 & 0 & -96 & 240 \\ 0 & 240 & 800 & 0 & -240 & 400 \\ \hline -2\times10^5 & 0 & 0 & 2\times10^5 & 0 & 0 \\ 0 & -96 & -240 & 0 & 96 & -240 \\ 0 & 240 & 400 & 0 & -240 & 800 \end{array}\right]\left[\begin{array}{c} 8{,}071 \\ 6{,}054 \\ -2{,}053 \\ \hline -8{,}085 \\ -10{,}78 \\ 0 \end{array}\right] + \left[\begin{array}{c} 1{,}5 \\ 2 \\ 1{,}667 \\ 1{,}5 \\ 2 \\ -1{,}667 \end{array}\right] = \left[\begin{array}{c} 4{,}17 \\ 3{,}12 \\ 4{,}06 \\ \hline -1{,}17 \\ 0{,}877 \\ 1{,}55 \end{array}\right]$$

Equilíbrio do nó B

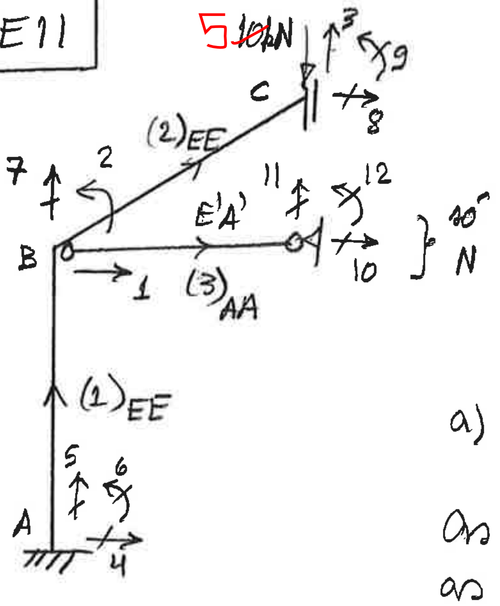


$$N_1 = -4{,}17 \times 0{,}6 - 3{,}12 \times 0{,}8$$
$$= -5{,}0 \text{ kN}$$
(compressão)

d) Diagrama



$\boxed{\text{FE11}}$



$$E = 1 \times 10^7 \ \frac{kN}{m^2}$$

$$LM_1 = [4 \ 5 \ 6 \ | \ 1 \ 7 \ 2]$$

$$LM_2 = [1 \ 7 \ 2 \ | \ 8 \ 3 \ 9]$$

$$LM_3 = [1 \ 7 \ 2 \ | \ 10 \ 11 \ 12]$$

a) Matrizes das barras

As matrizes das barras (1) e (2), incluindo as matrizes LM_1 e LM_2 são iguais ao do problema anterior.

$$\frac{E'A'}{\ell_3} = \frac{20 \times 10^3}{4} = 5000$$

$$f_1^0 = 0 \qquad f_2^0 = 0 \qquad f_3^0 = 0$$

$$k_3 = \bar{k}_3 = \begin{array}{c} \begin{array}{cccccc} 1 & 7 & 2 & 10 & 11 & 12 \end{array} \\ \left[\begin{array}{ccc|ccc} 5\times10^3 & 0 & 0 & -5\times10^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -5\times10^3 & 0 & 0 & 5\times10^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 7 \\ 2 \\ 10 \\ 11 \\ 12 \end{array}$$

b) Matrizes da estrutura.

$$F_a = \begin{bmatrix} 0 \\ 0 \\ -\cancel{10}5 \end{bmatrix} \qquad F_a^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$K_{aa} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{array}{c|c|c} 128131 + 5000 & 96 & -95954 \\ \hline & 1600 & -192 \\ \hline & & 72061 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} = \begin{bmatrix} 133131 & 96 & -95954 \\ & 1600 & -192 \\ & & 72061 \end{bmatrix}$$

Resolvendo o sistema: $K_{aa} \, u_a = F_a$

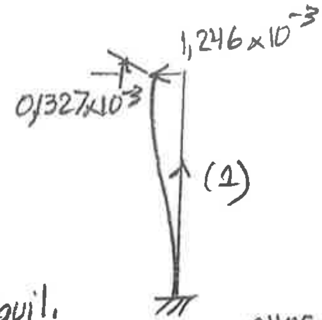
$$u_a = \begin{bmatrix} -1{,}246 \\ -0{,}1327 \\ -1{,}728 \end{bmatrix} \times 10^{-3} \begin{array}{c} m \\ rad \\ m \end{array}$$

considerando a flexibilidade axial da barra (1) obtemos

$$u_a = \begin{bmatrix} -1{,}246 \times 10^{-3} \\ -0{,}1327 \times 10^{-3} \\ -1{,}753 \times 10^{-3} \end{bmatrix} \begin{array}{c} m \\ rad \\ m \end{array}$$

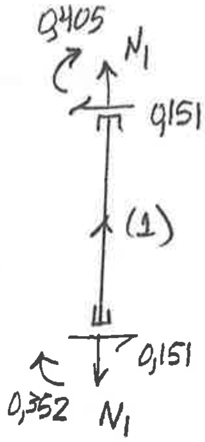
c) Deslocamentos e esforços nas extremidades das barras

$$\bar{\underset{\sim}{u}}_1 = \underline{T}_1 \underset{\sim}{u}_1 = 10^{-3}\, \underline{T}_1 \begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline -1{,}246 \\ 0 \\ -0{,}1327 \end{bmatrix} \end{array} \begin{array}{c} 4 \\ 5 \\ 6 \\ 1 \\ 7 \\ 2 \end{array} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ +1{,}246 \\ -0{,}1327 \end{bmatrix} 10^{-3}$$

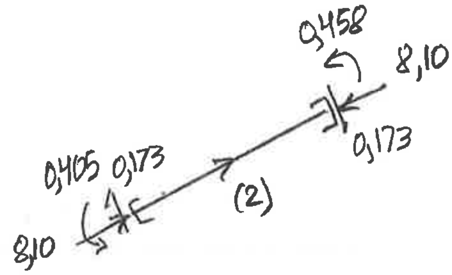


$$\bar{\underset{\sim}{f}}_1 = \bar{\underline{k}}_1 \bar{\underline{u}}_1 = 10^{-3} \left[\begin{array}{ccc|ccc} \vdots & \vdots & \vdots & \vdots & 0 & 0 \\ & & & & -96 & 240 \\ & & & & -240 & 400 \\ \hline \vdots & \vdots & \vdots & \vdots & 0 & 0 \\ & & & & 96 & -240 \\ & & & & -240 & 800 \end{array}\right] \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ 1{,}246 \\ -0{,}1327 \end{bmatrix} = \begin{bmatrix} \boxed{0} \\ -0{,}151 \\ -0{,}352 \\ \hline \boxed{0} \\ 0{,}151 \\ -0{,}405 \end{bmatrix}$$

Equil.



$$\bar{\underset{\sim}{u}}_2 = \underline{T}_2 \underset{\sim}{u}_2 = 10^{-3}\, \underline{T}_2 \begin{bmatrix} -1{,}246 \\ 0 \\ -0{,}1327 \\ \hline 0 \\ -1{,}728 \\ 0 \end{bmatrix} \begin{array}{c} 1 \\ 7 \\ 2 \\ 8 \\ 3 \\ 9 \end{array} = \begin{bmatrix} -0{,}9964 \\ 0{,}7473 \\ -0{,}1327 \\ \hline -1{,}037 \\ -1{,}383 \\ 0 \end{bmatrix} 10^{-3}$$



$$\bar{\underset{\sim}{f}}_2 = \bar{\underline{k}}_2 \underset{\sim}{u}_2 = 10^{-3} \left[\begin{array}{ccc|ccc} 2\times10^5 & 0 & 0 & -2\times10^5 & 0 & \vdots \\ 0 & 96 & 240 & 0 & -96 & \\ 0 & 240 & 800 & 0 & -240 & \\ \hline -2\times10^5 & 0 & 0 & 2\times10^5 & 0 & \vdots \\ 0 & -96 & -240 & 0 & 96 & \\ 0 & 240 & 400 & 0 & -240 & \end{array}\right] \begin{bmatrix} -0{,}9964 \\ 0{,}7473 \\ -0{,}1327 \\ \hline -1{,}037 \\ -1{,}383 \\ 0 \end{bmatrix} = \begin{bmatrix} 8{,}10 \\ 0{,}173 \\ 0{,}405 \\ \hline -8{,}10 \\ -0{,}173 \\ 0{,}458 \end{bmatrix}$$

$$\bar{\underset{\sim}{u}}_3 = \underset{\sim}{u}_3 = \begin{bmatrix} -1{,}246 \\ 0 \\ -0{,}1327 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix} 10^{-3} \begin{array}{c} 1 \\ 7 \\ 2 \\ 10 \\ 11 \\ 12 \end{array}$$

$$N_3 = \frac{EA}{\ell_3}\left[u_4 - u_1\right] = 5\times10^{3}\left[0 - (-1{,}246\times10^{-3})\right]$$
$$= +\,6{,}23 \text{ kN (tração)}$$

d) Diagrama de M