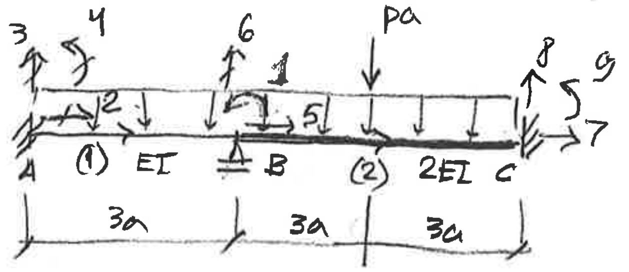
$\boxed{bE02}$

(a)



$LM_1 = [2\ 3\ 4 \mid 5\ 6\ 1]$

$LM_2 = [5\ 6\ 1 \mid 7\ 8\ 9]$

a resolução simplifica-se se admitirmos valores unitários na resolução.

$p = 1\ kN/m$; $a = 1m$; $EI = 1\ kNm^2$

os deslocamentos obtidos devem ser multiplicados por $\frac{pa^4}{EI}$ e as rotações por $\frac{pa^3}{EI}$.

a) Matrizes das barras

• Barra 1 $\quad l_1 = 3a \quad \frac{EA}{3a} \quad \frac{2EI}{l} = \frac{2}{3} \quad \frac{6EI}{(3a)^2} = \frac{2}{3} \quad \frac{12EI}{(3a)^3} = \frac{4}{9}$

$$\underline{k_1} = \underline{\bar{k}_1} = \begin{array}{c} \begin{array}{cccccc} 2 & 3 & 4 & 5 & 6 & 1 \end{array} \\ \left[\begin{array}{ccc|ccc} \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ & & & & & 2/3 \\ & & & & & 2/3 \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ & & & & & -2/3 \\ & & & & & 4/3 \end{array}\right] \end{array} \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 1 \end{array} \qquad \underset{\sim}{f_1^0} = \underset{\sim}{\bar{f}_1^0} = \left[\begin{array}{c} 0 \\ 1{,}5 \\ 0{,}75 \\ \hline 0 \\ 1{,}5 \\ -0{,}75 \end{array}\right] \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 1 \end{array}$$

$\frac{pl^2}{12} = \frac{p(3a)^2}{12}$

$$\underline{k_2} = \underline{\bar{k}_2} = \begin{array}{c} \begin{array}{cccccc} 5 & 6 & 1 & 7 & 8 & 9 \end{array} \\ \left[\begin{array}{ccc|ccc} \vdots & \vdots & 0 & \vdots & \vdots & \vdots \\ & & 1/3 & & & \\ & & 4/3 & & & \\ \hline \vdots & \vdots & 0 & \vdots & \vdots & \vdots \\ & & -1/3 & & & \\ & & 2/3 & & & \end{array}\right] \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 1 \\ 7 \\ 8 \\ 9 \end{array} \qquad \underset{\sim}{f_2^0} = \underset{\sim}{\bar{f}_2^0} = \left[\begin{array}{c} 0 + 0 \\ 3 + 0{,}5 \\ 3 + 0{,}75 \\ \hline 0 + 0 \\ 3 + 0{,}5 \\ -3 - 0{,}75 \end{array}\right] \begin{array}{c} 5 \\ 6 \\ 1 \\ 7 \\ 8 \\ 9 \end{array} = \left[\begin{array}{c} 0 \\ 3{,}5 \\ 3{,}75 \\ \hline 0 \\ 3{,}5 \\ -3{,}75 \end{array}\right]$$

• Barra 2 $\quad l_2 = 6a \quad \frac{EA}{6a} \quad \frac{2(2EI)}{6a} = \frac{2}{3} \quad \frac{6(2EI)}{(6a)^2} = \frac{1}{3} \quad \frac{12(2EI)}{(6a)^3} = \frac{2}{9}$

b) Matrizes da estrutura

$K_{aa} = \frac{4}{3} + \frac{4}{3} = \frac{8}{3} \qquad F_a = 0 \qquad F_a^0 = -0{,}75 + 3{,}75 = 3$

c) Resolução : $K_{aa} U_a = F_a - F_a^0$

$$\frac{8}{3} U_1 = 0 - 3 \quad \Rightarrow \quad U_1 = -\frac{9}{8}\frac{pa^3}{EI} = -1{,}125\frac{pa^3}{EI}$$

d) <u>Deslocamentos e esforços de extremidade das barras.</u>

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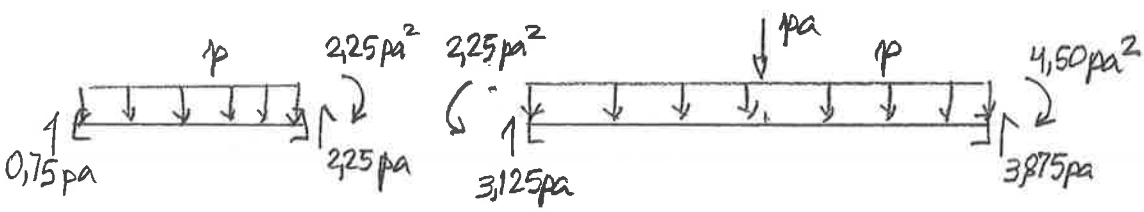
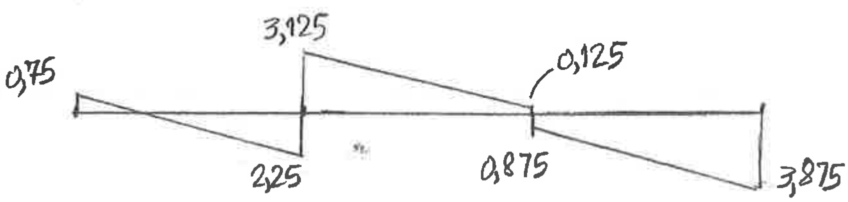
$$\underset{\sim}{\bar{f}_1} = \underset{-}{\bar{k}_1}\,\underset{\sim}{\bar{u}_1} + \underset{\sim}{\bar{f}_1^0} = \underset{-}{\bar{k}_1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ -\frac{9}{8} \end{bmatrix} \begin{matrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 1 \end{matrix} + \begin{bmatrix} 0 \\ 1{,}5 \\ 0{,}75 \\ \hline 0 \\ 1{,}5 \\ -0{,}75 \end{bmatrix} = \begin{bmatrix} 0 \\ -0{,}75 \\ -0{,}75 \\ \hline 0 \\ 0{,}75 \\ -1{,}5 \end{bmatrix} + \begin{bmatrix} 0 \\ 1{,}5 \\ 0{,}75 \\ \hline 0 \\ 1{,}5 \\ -0{,}75 \end{bmatrix} = \begin{bmatrix} 0 \\ 0{,}75 \\ 0 \\ \hline 0 \\ 2{,}25 \\ -2{,}25 \end{bmatrix}$$

$$\underset{\sim}{\bar{f}_2} = \underset{-}{\bar{k}_2}\,\underset{\sim}{\bar{u}_2} + \underset{\sim}{\bar{f}_2^0} = \underset{-}{\bar{k}_2} \begin{bmatrix} 0 \\ 0 \\ -\frac{9}{8} \\ \hline 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 1 \\ 7 \\ 8 \\ 9 \end{matrix} + \begin{bmatrix} 0 \\ 3{,}5 \\ 3{,}75 \\ \hline 0 \\ 3{,}5 \\ -3{,}75 \end{bmatrix} = \begin{bmatrix} 0 \\ -0{,}375 \\ -1{,}50 \\ \hline 0 \\ 0{,}375 \\ -0{,}75 \end{bmatrix} + \begin{bmatrix} 0 \\ 3{,}5 \\ 3{,}75 \\ \hline 0 \\ 3{,}5 \\ -3{,}75 \end{bmatrix} = \begin{bmatrix} 0 \\ 3{,}125 \\ 2{,}25 \\ \hline 0 \\ 3{,}875 \\ -4{,}50 \end{bmatrix}$$





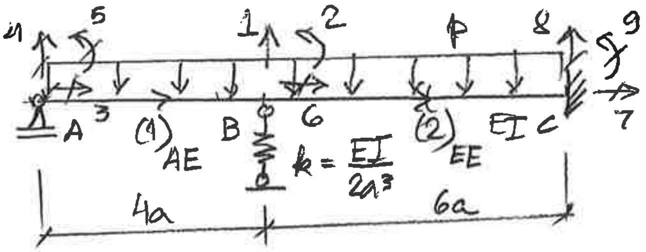
(V) pa

$$\frac{pl^2}{8} = \frac{p(3a)^2}{8} = \frac{9}{8}pa^2$$

(M) pa²

$\boxed{bE04}$

(c)



$LM_1 = [3 \; 4 \; \boxed{5} \,|\, 6 \; 1 \; 2]$

$LM_2 = [6 \; 1 \; 2 \,|\, 7 \; 8 \; 9]$

a) Matrizes das barras

• Barra 1 $\quad l_1 = 4a \qquad \frac{EA}{4a} \qquad \frac{3EI}{l^2} = \frac{3EI}{16a^2} \qquad \frac{3EI}{64a^3} \qquad \frac{3EI}{4a}$

$$k_1 = \bar{k}_1 = \begin{array}{c} \begin{array}{cccccc} 3 & 4 & 5 & 6 & 1 & 2 \end{array} \\ \left[\begin{array}{ccc|ccc} \vdots & \vdots & \vdots & \vdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & -\frac{3EI}{64a^3} & \frac{3EI}{16a^2} \\ & & & & 0 & 0 \\ \hline \vdots & \vdots & \vdots & \vdots & 0 & 0 \\ & & & & \frac{3EI}{64a^3} & -\frac{3EI}{16a^2} \\ & & & & -\frac{3EI}{16a^2} & \frac{3EI}{4a} \end{array}\right] \end{array} \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \\ 1 \\ 2 \end{array}$$

$$f_1^0 = \bar{f}_1^0 = \left[\begin{array}{c} 0 \\ \frac{3p(4a)}{8} \\ 0 \\ \hline 0 \\ \frac{5p(4a)}{8} \\ \frac{p(4a)^2}{8} \end{array}\right] = \left[\begin{array}{c} 0 \\ \frac{3pa}{2} \\ 0 \\ \hline 0 \\ \frac{5pa}{2} \\ -2pa^2 \end{array}\right] \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \\ 1 \\ 2 \end{array}$$

• Barra 2 $\quad l_1 = 6a \qquad \frac{EA}{6a} \qquad \frac{2EI}{l} = \frac{EI}{3a} \qquad \frac{6EI}{l^2} = \frac{EI}{6a^2} \qquad \frac{12EI}{l^3} = \frac{EI}{18a^3}$

$$k_2 = \bar{k}_2 = \begin{array}{c} \begin{array}{cccccc} 6 & 1 & 2 & 7 & 8 & 9 \end{array} \\ \left[\begin{array}{ccc|ccc} \vdots & 0 & 0 & \vdots & \vdots & \vdots \\ \vdots & \frac{EI}{18a^3} & \frac{EI}{6a^2} & & & \\ & \frac{EI}{6a^2} & \frac{2EI}{3a} & & & \\ \hline & 0 & 0 & & & \\ & -\frac{EI}{18a^3} & -\frac{EI}{6a^2} & & & \\ & \frac{EI}{6a^2} & \frac{EI}{3a} & & & \end{array}\right] \end{array} \begin{array}{c} 6 \\ 1 \\ 2 \\ 7 \\ 8 \\ 9 \end{array}$$

$$f_2^0 = \bar{f}_2^0 = \left[\begin{array}{c} 0 \\ 3pa \\ 3pa^2 \\ \hline 0 \\ 3pa \\ -3pa^2 \end{array}\right] \begin{array}{c} 6 \\ 1 \\ 2 \\ 7 \\ 8 \\ 9 \end{array}$$

b) Matrizes da estrutura (mola no GL 1)

$$K_{aa} = \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \left[\begin{array}{c|c} \frac{3EI}{64a^3} + \frac{EI}{18a^3} + \frac{EI}{2a^3} & -\frac{3EI}{16a^2} + \frac{EI}{6a^2} \\ \hline & \frac{3EI}{4a} + \frac{2EI}{3a} \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \end{array} = \left[\begin{array}{cc} \frac{59}{576}\frac{EI}{a^3} + \frac{EI}{2a^3} & -\frac{1}{48}\frac{EI}{a^2} \\ -\frac{1}{48}\frac{EI}{a^2} & \frac{17}{12}\frac{EI}{a} \end{array}\right]$$

$$= \frac{EI}{a^3}\left[\begin{array}{cc} 0{,}6024 & -0{,}02083a \\ -0{,}02083a & 1{,}4166a^2 \end{array}\right]$$

$$\underset{\sim}{F_a} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \underset{\sim}{F_a^0} = \begin{bmatrix} \frac{5}{2}pa + 3pa \\ 2pa^2 + \frac{pa^2}{3} \end{bmatrix} = \begin{bmatrix} 5{,}5\,pa \\ 1\,\cancel{\tfrac{7}{3}}\,pa^2 \end{bmatrix}$$

c) Resolução do sistema $\underline{K_{aa}}\,\underset{\sim}{U_a} = \underset{\sim}{F_a} - \underset{\sim}{F_a^0}$

Dividindo a segunda equação por a e adotando U2*a como incógnita temos uma matriz de coeficientes numérica.

$$\frac{EI}{a^3}\begin{bmatrix} 0{,}6024 & -0{,}02083 \\ -0{,}02083\cancel{a} & 1{,}4166\cancel{a} \end{bmatrix}\begin{bmatrix} U_1 \\ U_2 a \end{bmatrix} = \begin{bmatrix} -5{,}5 \\ -\cancel{1}\tfrac{7}{3}\cancel{a} \end{bmatrix} pa \qquad \begin{bmatrix} U_1 \\ U_2 a \end{bmatrix} = \begin{bmatrix} -9{,}159 \\ -0{,}8406 \end{bmatrix}\frac{pa^4}{EI}$$

d) Deslocamentos e esforços de extremidade

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$$\underset{\sim}{\bar{f}_1} = \underset{\sim}{f_1} = \underline{\bar{k}_1}\,\underset{\sim}{\bar{u}_1} + \underset{\sim}{\bar{f}_1} = \underline{\bar{k}_1}\begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ -9{,}159a \\ -0{,}8406 \end{bmatrix}\frac{pa^3}{EI}\begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \\ 1 \\ 2 \end{array} + \begin{bmatrix} 0 \\ 1{,}5pa \\ 0 \\ \hline 0 \\ 2{,}5pa \\ 2pa^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1{,}772 \\ 0 \\ \hline 0 \\ 2{,}228 \\ -0{,}913a \end{bmatrix} pa$$

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$$\underset{\sim}{\bar{f}_2} = \underset{\sim}{f_2} = \underline{\bar{k}_2}\,\underset{\sim}{u_2} + \underset{\sim}{\bar{f}_2} = \underline{\bar{k}_2}\begin{bmatrix} 0 \\ -9{,}159a \\ -0{,}8406 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}\frac{pa^3}{EI}\begin{array}{c} 6 \\ 1 \\ 2 \\ 7 \\ 8 \\ 9 \end{array} + \begin{bmatrix} 0 \\ 3pa \\ \frac{pa^2}{3} \\ \hline 0 \\ 3pa \\ -\frac{pa^2}{3} \end{bmatrix} = \begin{bmatrix} 0 \\ 2{,}351 \\ 0{,}913a \\ \hline 0 \\ 3{,}649 \\ -4{,}807a \end{bmatrix} pa$$

e) Diagramas

$$\frac{p(4a)^2}{8} = 2pa^2$$

$$\frac{p(6a)^2}{8} = 4{,}5pa^2$$