Gabarito

Exemplo 10 (97PSQ1) Trace os diagrama de estado da grelha da figura. Observe que os apoios A e D impedem apenas os deslocamentos na direção vertical e considere $GI_T = EI$.

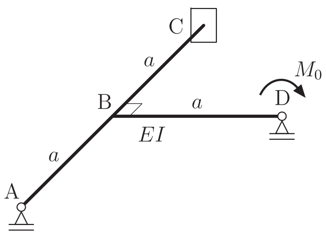
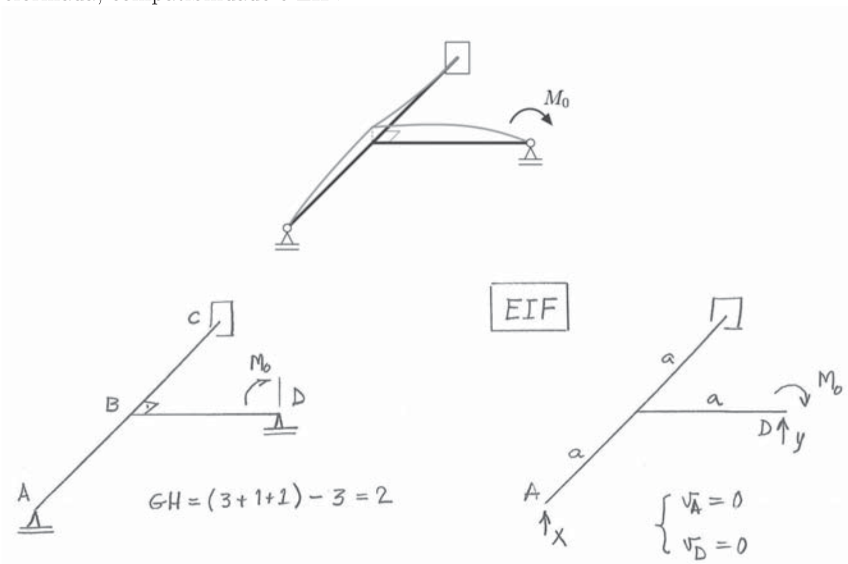


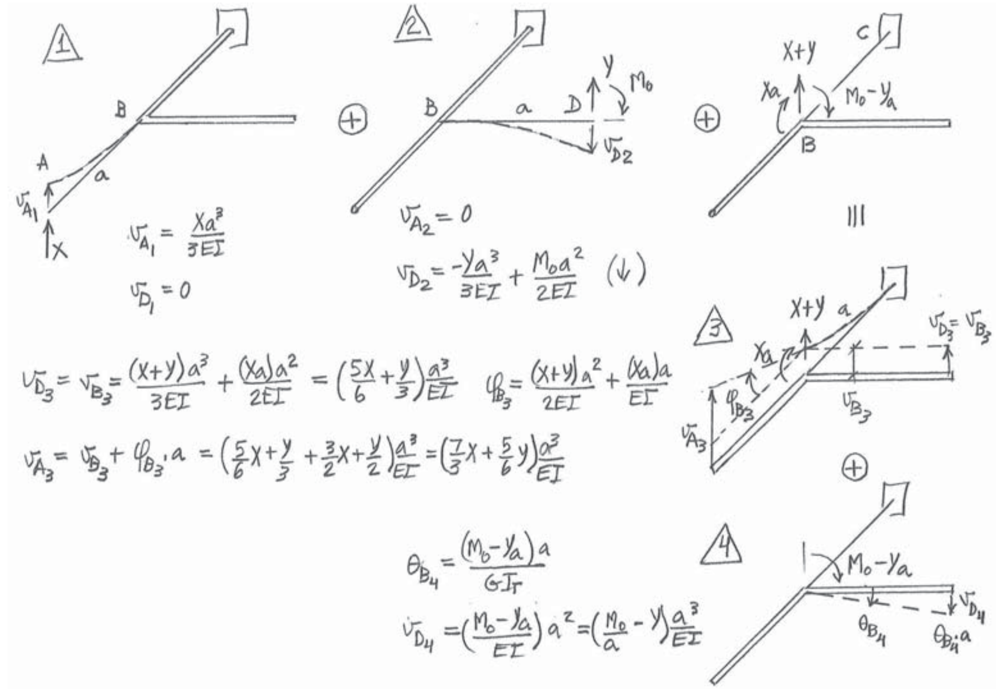
Fig. E 10: 97PSQ1.

Solução

Deformada, compatibilidade e EIF:



Cálculo dos deslocamentos:



Compatibilid.

$v_A = 0 \Rightarrow v_A = v_{A_1} + v_{A_3} = \left(\frac{X}{3} + \frac{7X}{3} + \frac{5}{6}y\right)\frac{a^3}{EI} = 0 \Rightarrow X = -\frac{5}{16}y$

$v_D = 0 \Rightarrow v_D = -v_{D_2} + v_{D_3} - v_{D_4} = \left(\frac{y}{3} - \frac{M_0}{2a} + \frac{5X}{6} + \frac{y}{3} - \frac{M_0}{a} + y\right)\frac{a^3}{EI} \Rightarrow \frac{5X}{6} + \frac{5y}{3} = \frac{3}{2}\frac{M_0}{a}$

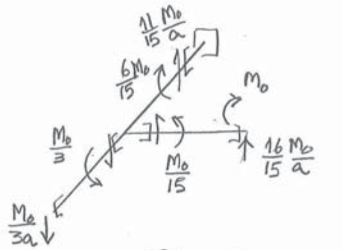
$$\begin{cases} X = -\frac{5}{16}y \\ 5X + 10y = 9\frac{M_0}{a} \end{cases} \Rightarrow -\frac{25}{16}y + 10y = \frac{9M_0}{a}$$

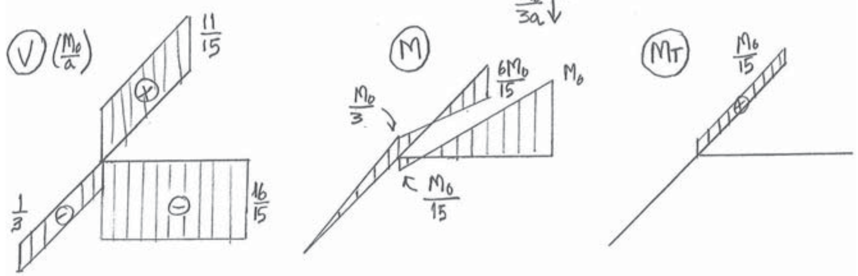
$$\frac{160-25}{16}y = \frac{9M_0}{a} \Rightarrow \begin{cases} y = \frac{16}{15}\frac{M_0}{a} \\ X = -\frac{1}{3}\frac{M_0}{a} \end{cases}$$

Esforços solicitantes

$M = -\frac{M_0}{3a} \times 2a + \frac{16}{15}M_0 = \frac{6}{15}M_0$

$V = \frac{16}{15}\frac{M_0}{a} - \frac{M_0}{3a} = \frac{16-5}{15}\frac{M_0}{a} = \frac{11}{15}\frac{M_0}{a}$





Exemplo 11 (12P1Q3) Na estrutura da figura, a viga horizontal tem produto de rigidez axial EI e o tirante vertical tem produto de rigidez axial $E'A' = 16EI/a^2$ e é constituído de material com coeficiente de dilatação térmica α. Determine a força normal no tirante quando ele, e apenas ele, é submetido a um resfriamento Δt. Trace os diagramas de momento fletor e momento de torção da estrutura.

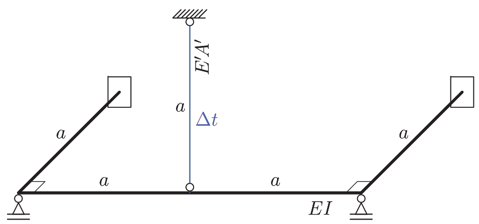
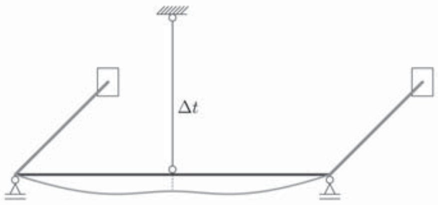


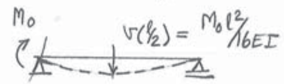
Fig. E11: 12P1Q3.

Solução

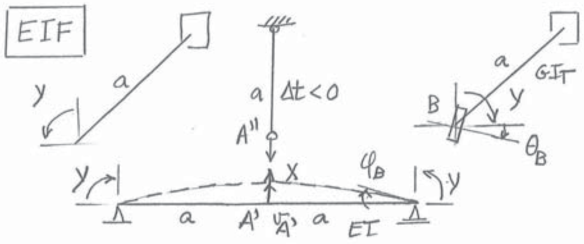


$EI = GI_T$

$EA = \frac{16EI}{a^2}$



GH = 2



• Compatibilidade:

$$\begin{cases} v_{A'} = v_{A''} & (1) \\ \varphi_B = \theta_B & (2) \end{cases}$$

$v_{A'} = \frac{X(2a)^3}{48EI} - \frac{2y(2a)^2}{16EI} = \frac{a^2}{EI}\left(\frac{Xa}{6} - \frac{y}{2}\right)$

$v_{A''} = \alpha a\Delta t - \frac{Xa}{EA} = \alpha a\Delta t - \frac{Xa^3}{16EI}$

$(1) \Rightarrow \frac{Xa}{6} - \frac{y}{2} = \frac{\alpha\Delta t EI}{a} - \frac{Xa}{16}$

$11Xa - 24y = 48\frac{\alpha\Delta t EI}{a}$ (3)

$\varphi_B = \frac{X(2a)^2}{16EI} - \frac{y(2a)}{6EI} - \frac{y(2a)}{3EI} = \frac{a}{EI}\left(\frac{Xa}{4} - y\right)$

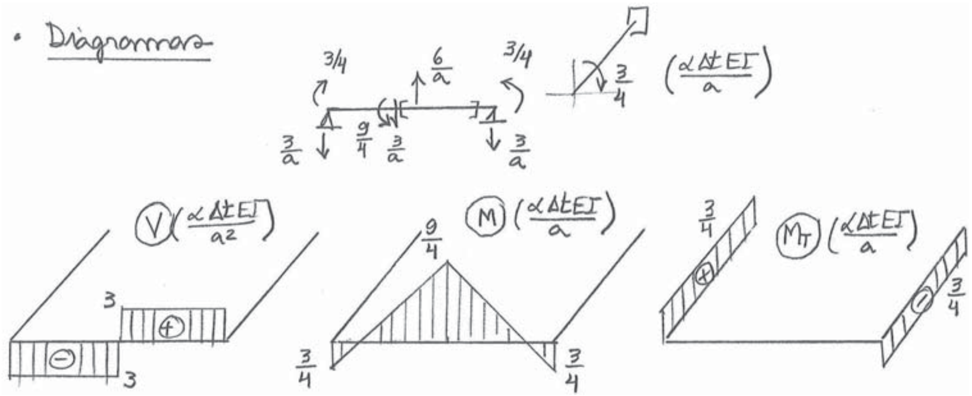
$\theta_B = \frac{ya}{GI_T} = \frac{ya}{EI}$

$(2) \Rightarrow \frac{Xa}{4} = 2y$

$X = \frac{8y}{a}$

Introduzindo X em (3): $88y - 24y = 48\frac{\alpha\Delta t EI}{a} \Rightarrow y = \frac{3}{4}\frac{\alpha\Delta t EI}{a}$

$X = 6\frac{\alpha\Delta t EI}{a^2}$



Exemplo 12 Trace os diagramas de momento fletor e de momento de torção da estrutura da figura. A estrutura está num plano horizontal e a força P é perpendicular a esse plano. O produto de rigidez EI é o mesmo em todas as barras e $GI_\mathrm{T} = EI$.

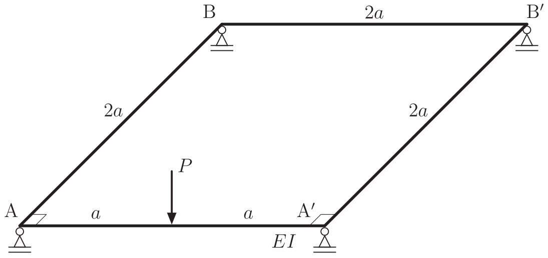
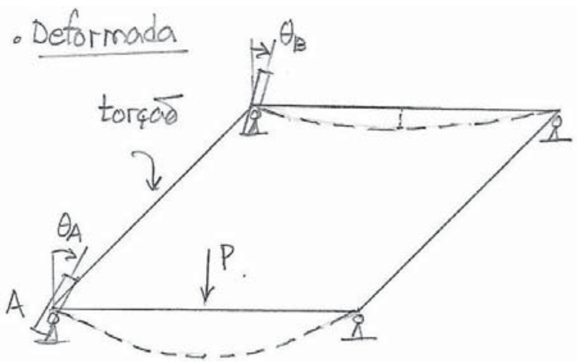


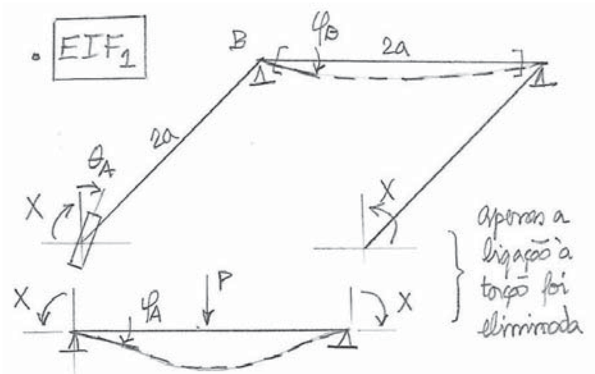
Fig. E 12: Grelha fechada.

Solução



EIF$_1$: simetria usada na escolha das incógnitas hiperestáticas

EIF$_2$: simetria usada para obter a estrutura aux.



apenas a ligação à torção foi eliminada

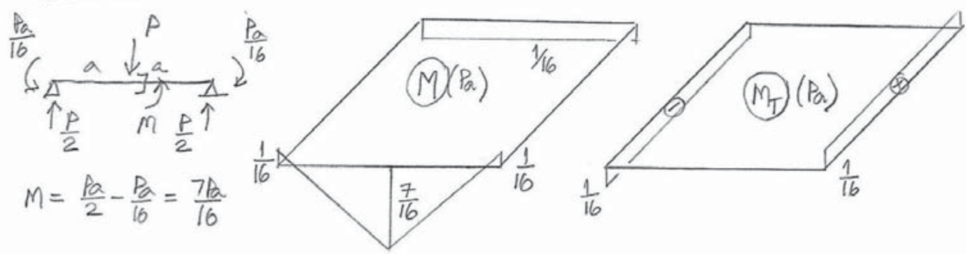
Compatib. $\varphi_A = \theta_A$

$$\varphi_A = \frac{P(2a)^2}{16EI} - \frac{X(2a)}{3EI} - \frac{X(2a)}{6EI} = \frac{a}{EI}\left(\frac{Pa}{4} - X\right)$$

$$\theta_A = \varphi_B + \Delta\theta_{AB} = \left(\frac{X(2a)}{3EI} + \frac{X(2a)}{6EI}\right) + \frac{X(2a)}{GI_T} = \frac{a}{EI}\left[X + 2X\right] = \frac{a}{EI}(3X)$$

Eq. de compatib. $\frac{a}{EI}\left(\frac{Pa}{4} - X\right) = \frac{a}{EI}(3X) \Rightarrow 4X = \frac{Pa}{4} \Rightarrow X = \frac{Pa}{16}$

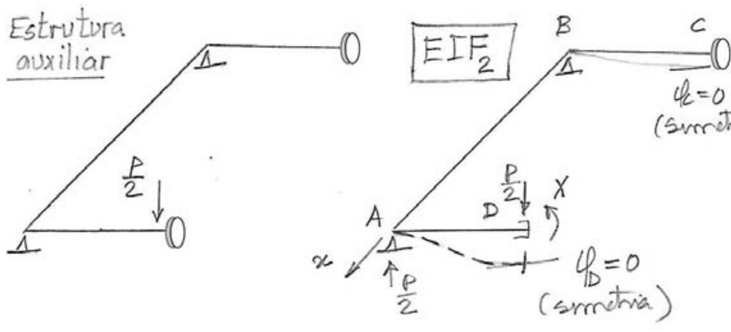
• Diagramas



$$M = \frac{Pa}{2} - \frac{Pa}{16} = \frac{7Pa}{16}$$

Estrutura auxiliar

$\boxed{EIF_2}$

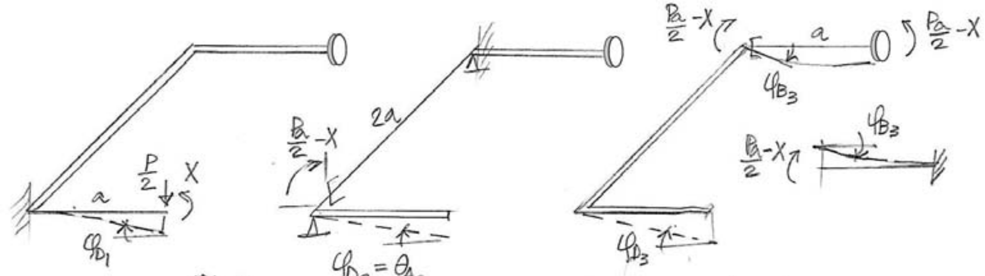


$\left\{ \frac{Pa}{2} - X - M = 0 \right.$

$M = \frac{Pa}{2} - X$

Compatibilidade:

$\boxed{\varphi_D = 0}$



$\varphi_{D_2} = \theta_{A_2}$

$$\varphi_{D_1} = \frac{-Xa}{EI} + \frac{\left(\frac{P}{2}\right)a^2}{2EI} = \frac{a}{EI}\left(-X + \frac{Pa}{4}\right)$$

$$\varphi_{D_2} = \frac{\left(\frac{Pa}{2} - X\right)2a}{GI_T} = \frac{a}{EI}(Pa - 2X)$$

$$\varphi_{D_3} = \varphi_{B_3} = \frac{\left(\frac{Pa}{2} - X\right)a}{EI} = \frac{a}{EI}\left(\frac{Pa}{2} - X\right)$$

Eq. de compatibilidade:

$$\varphi_D = \varphi_{D_1} + \varphi_{D_2} + \varphi_{D_3} = \frac{a}{EI}\left(-X + \frac{Pa}{4} + Pa - 2X + \frac{Pa}{2} - X\right) = 0 \Rightarrow -4X + \frac{7}{4}Pa = 0 \Rightarrow X = \frac{7Pa}{16}$$

Exemplo 13 A estrutura da figura está sujeita a uma força P que varia no intervalo de 0 a $2EI\delta/a^3$. Considerando $GI_T = EI$, trace a curva do deslocamento vertical do ponto D em função da carga P para esse intervalo.

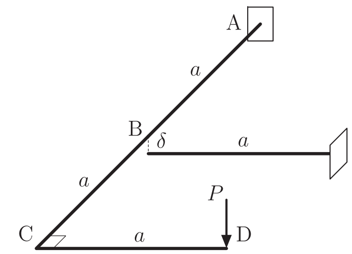
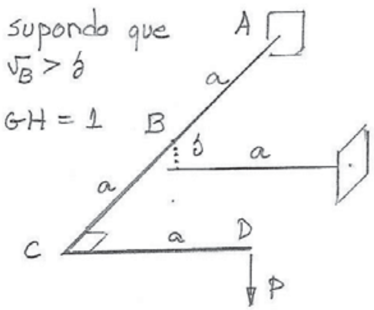


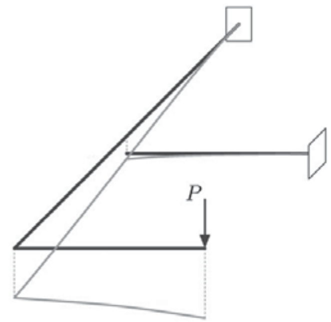
Fig. E 13: Folga δ.

Solução

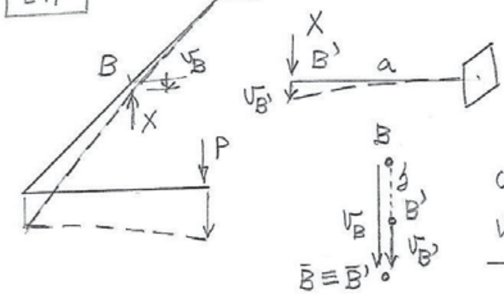
supondo que $v_B > \delta$

GH = 1





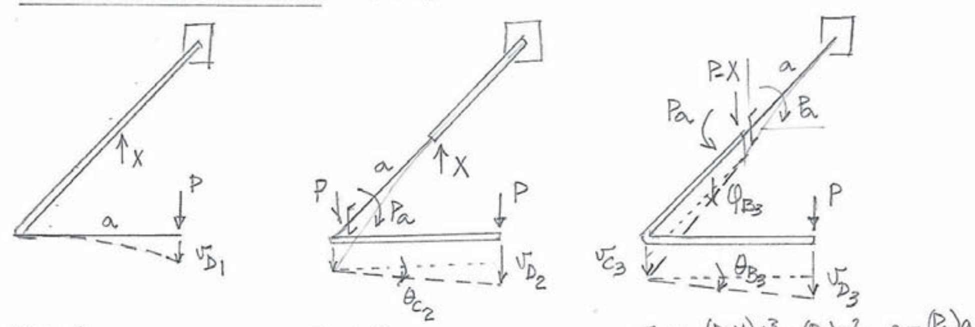
$\boxed{EIF}$



compatib.

$v_B = v_{B'} + \delta$

• Cálculo dos deslocamentos ABCD



$v_{B_1} = 0$

$$v_{D_1} = \frac{Pa^3}{3EI}$$

$v_{B_2} = 0$

$$v_{D_2} = \frac{Pa^3}{3EI} + \frac{(Pa)a}{GI_T}\cdot a = \frac{a^3}{EI}\left(\frac{4P}{3}\right)$$

$$v_{B_3} = \frac{(P-X)a^3}{3EI} + \frac{(Pa)a^2}{2EI} \qquad \theta_{B_3} = \frac{(Pa)a}{GI_T}$$

$$\varphi_{B_3} = \frac{(P-X)a^2}{2EI} + \frac{(Pa)a}{EI} = \frac{a^2}{EI}\left(\frac{3P}{2} - \frac{X}{2}\right)$$

$$v_{D_3} = v_{B_3} + \varphi_{B_3}\cdot a + \theta_{B_3}\cdot a = \frac{a^3}{EI}\left(\frac{P-X}{3} + \frac{P}{2} + \frac{3P}{2} - \frac{X}{2} + P\right) = \frac{a^3}{EI}\left(\frac{10P}{3} - \frac{5X}{6}\right)$$

Equação de compatibili// $\quad v_B = \delta + v_B'$

$$\frac{a^3}{EI}\left(\frac{P-X}{3} + \frac{P}{2}\right) = \delta + \frac{Xa^3}{3EI}$$

$$\frac{a^3}{EI}\left(\frac{5P}{6} - \frac{2X}{3} - \frac{\delta EI}{a^3}\right) = 0$$

$$X = \frac{1}{4}\left(5P - \frac{6EI\delta}{a^3}\right)$$

para $X \geq 0$

$$P \geq \frac{6}{5}\frac{EI\delta}{a^3}$$

$$v_D = v_{D_1} + v_{D_2} + v_{D_3} = \frac{a^3}{EI}\left(\frac{P}{3} + \frac{4P}{3} + \frac{10P}{3} - \frac{5X}{6}\right) = \frac{a^3}{EI}\left(5P - \frac{5X}{6}\right)$$

$$X = \frac{5P}{4} - \frac{3EI\delta}{2a^3}$$

$$v_D = \frac{a^3}{EI}\left(\frac{95}{24}P + \frac{5}{4}\frac{EI\delta}{a^3}\right)$$

Carregamento:

$0 \leq P < \frac{6}{5}\frac{EI\delta}{a^3}$	$X = 0$	$v_D = \frac{5Pa^3}{EI}$
$P = \frac{6}{5}\frac{EI\delta}{a^3}$	$X = 0$	$v_D = \frac{5a^3}{EI}\left(\frac{6}{5}\frac{EI\delta}{a^3}\right) = 6\delta$
$\frac{6}{5}\frac{EI\delta}{a^3} < P \leq \frac{2EI\delta}{a^3}$	$X = \frac{5P}{4} - \frac{3}{2}\frac{EI\delta}{a^3}$	$v_D = \frac{a^3}{EI}\left(\frac{95}{24}P + \frac{5}{4}\frac{EI\delta}{a^3}\right)$
$P = \frac{2EI\delta}{a^3}$	$X = \frac{EI\delta}{a^3}$	$v_D = \delta\left(\frac{95}{12} + \frac{5}{4}\right) = \frac{55}{6}\delta$

gráfico