PSI3262 – Fundamentos de Circuitos Eletrônicos Digitais e Analógicos

Solução da Lista 7: Potência e energia em Regime Permanente Senoidal

1 –

Cargas	P(kW)	Q(kVAr)	$\lvert P_{ap} \rvert$ (kVA)	cos ψ
C1	8	6	10	0,8 atr.
C2	12	-16	20	0,6 ad.
C3	5	10	11,18	0,45 atr.
	25	0		

Carga 1: $P = \lvert P_{ap} \rvert \cos\psi \rightarrow \lvert P_{ap} \rvert = 10$ kVA
$Q = \lvert P_{ap} \rvert \operatorname{sen}\psi \rightarrow Q = 6$ kVAr (> 0 ind.)

Carga 2: $P = \lvert P_{ap} \rvert \cos\psi = 12$ kW
$Q = \lvert P_{ap} \rvert \operatorname{sen}\psi = -16$ kVAr

Carga 3: $Y = \dfrac{1}{Z} = \dfrac{1}{2{,}5 + j5} = 0{,}08 - j0{,}16$

$$P_{ap} = G\lvert V \rvert^2 - j B \lvert V \rvert^2 = 5 + j10$$

$$P_{ap} \text{ total} = (25 + j0) \text{ kVA}$$

$$\lvert P_{ap} \rvert = \lvert V \rvert \lvert I_L \rvert \rightarrow \lvert I_L \rvert = 25000/250 = 100 \text{ Aef}$$

Como $Qt = 0 \rightarrow \hat{I}_L = 100 \angle 0°$

$$\hat{V}_p = 100(0{,}05 + j0{,}5) = 5 + j50$$

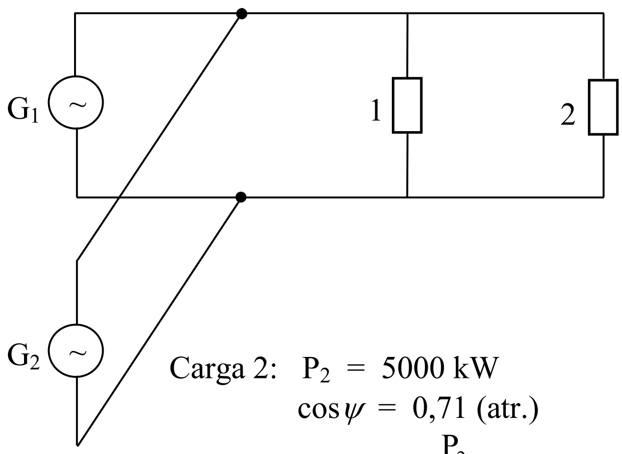
$$\hat{V}_s = \hat{V}_p + 250 \angle 0° = 255 + j50 = 259{,}86 \angle 11{,}09°$$

$$v_s(t) = 367{,}49 \cos(377t + 11{,}09°) \quad \text{(V,s)}$$

Potência real nas cargas: $P1 = 8$ kW
$P2 = 12$ kW
$P3 = 5$ kW

Potência real na linha: $0{,}05 \cdot \lvert I_L \rvert^2 = 0{,}5$ kW

2 –



Carga 1: $P_1 = 3000$ kW
$\cos\psi = 1$
$Q_1 = 0$

Carga 2: $P_2 = 5000$ kW
$\cos\psi = 0{,}71$ (atr.)

$$Q_2 = \frac{P_2}{\cos\psi}\,\mathrm{sen}\,\psi = 4959{,}2 \text{ kVAr}$$

Gerador $G_1 \rightarrow P_{G1} = 5000$ kW
$\cos\psi = 0{,}8$ (atr.)
(convenção de gerador)

$$Q_{G1} = \frac{P_{G1}}{0{,}8}\,\mathrm{sen}\psi = 3750 \text{ kVAr}$$

Teorema da conservação das potências:

$$P_1 + P_2 = P_{G1} + P_{G2} \rightarrow P_{G2} = 3000 \text{ kW}$$
$$\underbrace{Q_1 + Q_2}_{\substack{\text{conv.}\\ \text{receptor}}} = \underbrace{Q_{G1} + Q_{G2}}_{\substack{\text{conv.}\\ \text{gerador}}} \rightarrow Q_{G2} = 1209{,}2 \text{ kVAr}$$

$$\mathrm{tg}\ \psi_2 = \frac{1209{,}2}{3000} = \frac{Q_{G2}}{P_{G2}} = 0{,}40$$

$\cos\psi_2 = 0{,}93$ (atr.) $\leftarrow$ conv. gerador !

3 – a) $P_{ap\ total} = 300 + j100 = P_{ap1} + P_{ap2}$

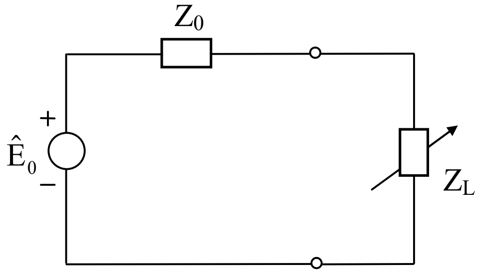
$$\left|P_{ap\ total}\right| = \sqrt{(300)^2 + (100)^2} = 316{,}23 = |V|\,|I| \rightarrow |I| = 1{,}58 \text{ Aef}$$

$$P_{total} = R(\omega)\,|I|^2 \rightarrow R(\omega) = 300/(1{,}58)^2 = 120\ \Omega$$
$$Q_{total} = X(\omega)\,|I|^2 \rightarrow X(\omega) = 100/(1{,}58)^2 = 40\ \Omega \rightarrow Z = 120 + j40\ \Omega$$

b) $P_1 = R_1|I|^2 \rightarrow R_1 = 40\ \Omega$
$Q_1 = X_1|I|^2 \rightarrow X_1 = 80\ \Omega$
$Z_1 = 40 + j80\ \Omega$

$P_2 = R_2|I|^2 \rightarrow R_2 = 80\ \Omega$
$Q_2 = X_2|I|^2 \rightarrow X_2 = -40\ \Omega$
$Z_2 = 80 - j\,40\ \Omega$

4 – Gerador de Thévenin equivalente:



Tensão em aberto: Divisor de tensão :

$$\hat{E}_0 = \frac{100\angle 0^\circ \;.\; j3}{25 + j10 + j3} = \frac{300\angle 90^\circ}{28{,}18 \;\; 27{,}5^\circ}$$

$$\hat{E}_0 = 10{,}65\angle 62{,}5^\circ$$

$$\text{Impedância : } Z_0 = \left(25 + j10\right) // \; j3 = \frac{\left(25 + j10\right) j3}{25 + j13}$$

$$Z_0 = 2{,}87\angle 84{,}3 = 0{,}28 + j2{,}85 \; \Omega$$

Condição de máxima transferência de potência ativa à carga Z_L :

a) $Z_L = Z_0^* = 0{,}28 - j\,2{,}85 \; \Omega$

b) $P_{máx} = \dfrac{|E_0|^2}{4R} = \dfrac{(10{,}65)^2}{4 \,.\, 0{,}28} = 101{,}3 \text{ W}$

5 – a)

	P (kW)	Q(kVAr)	$\cos\psi$	$\lvert P_{ap}\rvert$/(kVA)
A	5	0	1	5
B	5	6,67	0,6 atr.	8,33
C	3,72	5,58	0,55 atr.	6,71
	13,72	12,25		

$$\text{Carga C:} \quad Y = \frac{1}{4 + j6} = 0{,}0769 - j0{,}1154 \;\; S$$

$P_{ap} = 0{,}0769\,|V|^2 + 0{,}1154\,|V|^2 = 3{,}72 + j5{,}58 \;\; \text{kVA}$
$P_{ap\ total} = 13{,}72 + j12{,}25 \;\; \text{kVA}$
$P_{total} = 13{,}72 \text{ kW}$ $\qquad Q_{total} = 12{,}25 \text{ kVAr}$

b) $\hat{I}_A = \dfrac{5000}{110}\angle 0^\circ = 45{,}45\angle 0^\circ \quad \text{Aef}$

$$\hat{I}_B = \frac{8333}{110}\angle{-53{,}13^\circ} = 75{,}75\angle{-53{,}13^\circ} \quad \text{Aef}$$

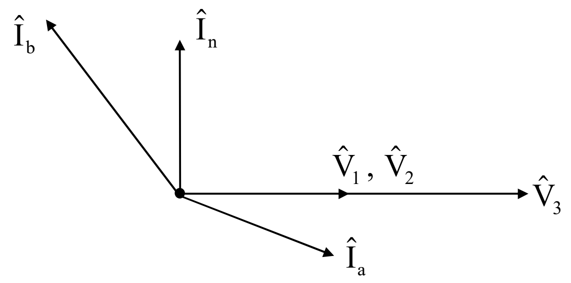
$$\hat{I}_C = \frac{220\angle 0^\circ}{4 + j6} = 30{,}51 \angle -56{,}31^\circ \quad \text{Aef}$$

$$\hat{I}_a = \hat{I}_A + \hat{I}_C = 67{,}34 \angle -22{,}15^\circ \quad \text{Aef}$$

$$\hat{I}_b = -\left(\hat{I}_B + \hat{I}_C\right) = 106{,}23 \angle 125{,}96^\circ \quad \text{Aef}$$

$$\hat{I}_n = \hat{I}_A - \hat{I}_B = 60{,}6 \angle 90^\circ \quad \text{Aef}$$

c)



6 –

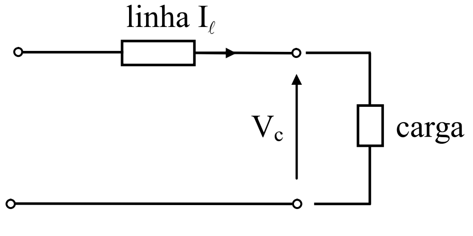
Cargas	P (kW)	Q (kVAr)	cos φ	$\lvert P_{ap} \rvert$ (kVA)
i	250	0	1	250
ii	1500	726,48	0,9 at.	1666,67
iii	1000	750	0,8 at.	1250
iv	700	-339,02	0,9 ad.	777,78

$\Sigma P = 3450$ kW $\qquad \Sigma Q = 1137{,}45$ kVAr

a) $P_{ap\,t} = 3450 + j\,1137{,}45$ kVA

$$\cos\varphi t = \frac{P_t}{\left|P_{ap\,t}\right|} = \frac{3450}{3632{,}67} = 0{,}95 \text{ atrasado}$$

b)



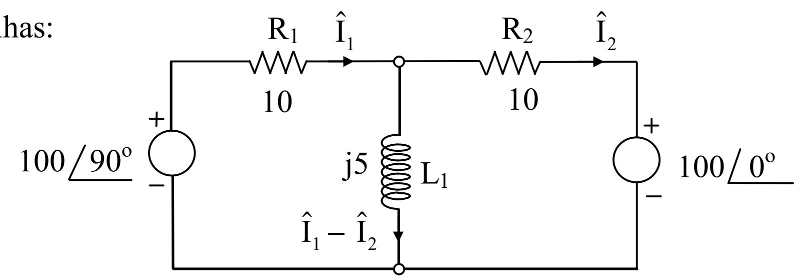
Mesmo aquecimento $\longrightarrow$ mesma $\lvert I_\ell \rvert$

$P_t = V_c\, I_\ell \cos\varphi_t = 3450$ kW

Para $\cos\varphi_t' = 1$, mantendo fixos V_c e I_ℓ $\rightarrow$

$$\longrightarrow P_t' = V_c\, I_\ell = \frac{3450}{0{,}95} = 3632{,}67 \text{ kW}$$

7 – Por análise de malhas:



2^a LK :

$10\,\hat{I}_1 + j5\left(\hat{I}_1 - \hat{I}_2\right) = 100\angle 90^o$

$j5\left(\hat{I}_1 - \hat{I}_2\right) - 10\,\hat{I}_2 = 100\angle 0^o$ que fornece o sistema :

$$\begin{bmatrix} 10 + j5 & -j5 \\ j5 & -10 - j5 \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 100\angle 90^\circ \\ 100 \end{bmatrix}$$

Resolvendo :

$\hat{I}_1 = 5\angle 90^o$ Aef $\qquad \hat{I}_2 = 11{,}18\angle\text{-}206{,}56^o$ Aef

$P_{ap\,g1} = 100\angle 90^o \,.\, \hat{I}_1^* = 500 + j\,0$ VA fornecida

$P_{ap\,g2} = 100\angle 0^o \,.\, \left(-\hat{I}_2^*\right) = 1000 + j\,500$ VA fornecida

$P_{R1} = R_1 \,.\, \left|\hat{I}_1\right|^2 = 10 \,.\, 25 = 250$ W

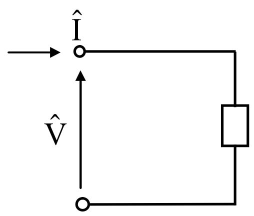
$P_{R2} = R_2 \,.\, \left|\hat{I}_2\right|^2 = 10 \,.\, (11{,}18)^2 = 1250$ W

$Q_{L1} = 5 \,.\, \left(\left|\hat{I}_1 - \hat{I}_2\right|\right)^2 = 500$ Var

Portanto: $P_{g1} + P_{g2} = P_{R1} + P_{R2} = 1500$ W

$Q_{g1} + Q_{g2} = Q_{L1} = 500$ Var

8 –



a) Sabe-se que: $P_{ap} = \hat{V} . \hat{I}^* = P + j\,Q \longrightarrow$

$\longrightarrow \hat{I}^* = \dfrac{P + jQ}{\hat{V}}$

$\longrightarrow \hat{I} = \dfrac{P - jQ}{\hat{V}^*} = \dfrac{P}{\hat{V}^*} - j\,\dfrac{Q}{\hat{V}^*}$

(aplicando-se propriedades dos números complexos)

b) $\hat{I}_a = \hat{I}_1 + \hat{I}_2$ $\qquad$ Carga 1: P = 12 kW $\qquad \hat{V} = \hat{E}_1 = 127\angle 0^o$

$Q = \sqrt{P_{ap}^2 - P^2} = 21.931{,}71$ Var

$\longrightarrow \hat{I}_1 = \dfrac{12.000}{127} - \dfrac{j\,21.931{,}71}{127}$ (item a)

$\hat{I}_1 = 196{,}85\angle\text{-}61{,}31^o$ Aef

Carga 2: $Q = -12.000 \text{ kVAr}$ $\quad P = \sqrt{P_{ap}^2 - Q^2} = 48.538,64 \text{ W}$

$\hat{V} = \hat{E}_1 + \hat{E}_2 = 127\angle 60^o$ Vef

$$\hat{I}_2 = \frac{48.538,64}{127\angle -60^\circ} + \frac{j\ 12.000}{127\angle -60^\circ} = 393,69\angle 73,89^o \text{ Aef}$$

$$\hat{I}_a = \hat{I}_1 + \hat{I}_2 = 289,41\angle 45,25^o \text{ Aef}$$

c) $P_{ap\ g1} = 25,88 - j\ 26,10$ kVA

Carga 1 $\longrightarrow$ $P_1 = 12$ kW $\quad Q_1 = 21,93$ kVAr

Carga 1' $\longrightarrow$ $P_1' = 12$ kW $\quad Q_1' = 21,93$ kVAr

Carga 2 $\longrightarrow$ $P_2 = 48,54$ kW $\quad Q_2 = -12$ kVAr

Pela conservação das potências:

$P_{g2} = P_1 + P_1' + P_2 - 25,88 = 46,66$ kW

$Q_{g2} = Q_1 + Q_1' + Q_2 + 26,10 = 57,96$ kVAr

Portanto:

$P_{ap\ g2} = 46,66 + j\ 57,96$ kVA fornecida.