Escola Politécnica
Universidade de São Paulo

PSI.2211

Circuitos Elétricos I

Bloco 8

Potência em RPS: Potências ativa, reativa e complexa
Fator de potência

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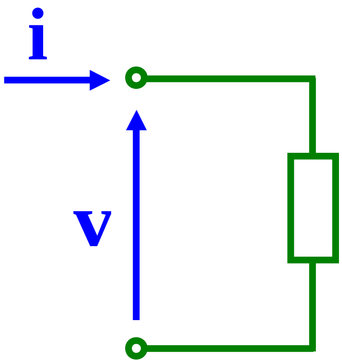
POTÊNCIA



$$p(t) = v(t)\, i(t) \qquad (\,W\,)$$

$$w(t) = \int_{t_1}^{t_2} v(t)\, i(t)\, dt \qquad (\,J\,)$$

$$P = \frac{1}{T} \int_0^T p(t)\, dt$$

RPS $\longrightarrow$ $v(t) = V_m \cos \omega t$

$i(t) = I_m \cos(\omega t - \varphi)$

$\varphi > 0$ indutivo

$\varphi < 0$ capacitivo

$$p(t) = \underbrace{\frac{1}{2} V_m I_m \cos\varphi}_{\text{potência média } P} + \underbrace{\frac{1}{2} V_m I_m \cos(2\omega t - \varphi)}_{\text{potência flutuante}}$$

Valores eficazes: V, I

$P = V\, I \cos\varphi$ (W, kW)

Valor Eficaz

$$P = R\,I^2 = R\,I_{ef}^2$$

Potência média $\rightarrow$

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\int_0^T i^2(t)\,R\,dt = \frac{R}{T}\int_0^T i^2(t)\,dt}$$

$$\boxed{I_{ef}^2 = \frac{1}{T}\int_0^T i^2(t)\,dt}$$

$$\boxed{I_{ef} = \left[\frac{1}{T}\int_0^T i^2(t)\,dt\right]^{1/2}}$$

Para senóide :

$$I_{ef} = \left[\frac{1}{T}\int_0^T \left(I_m \cos(\omega t + \theta)\right)^2 dt\right]^{1/2}$$

$$I_{ef} = I_m/\sqrt{2}$$

VALOR EFICAZ E FASORES

O *valor eficaz* de uma *f(t)* periódica, com período *T*, é

$$F_{ef} = \left[\frac{1}{T}\int_T f^2(t)dt\right]^{1/2}$$

Se a *f (t)* for senoidal,

$$F_{ef} = F_{max} / \sqrt{2}$$

Fasores da função

$$\mathbf{v(t) = V_m \cos(\omega\, t + \Phi\ ):}$$

Podem ser *fasores*

de valor máximo: $\hat{V}_m = V_m\, e^{j\,\Phi}$

de valor eficaz: $\hat{V}_{ef} = (V_m/\sqrt{2})\, e^{j\Phi}$

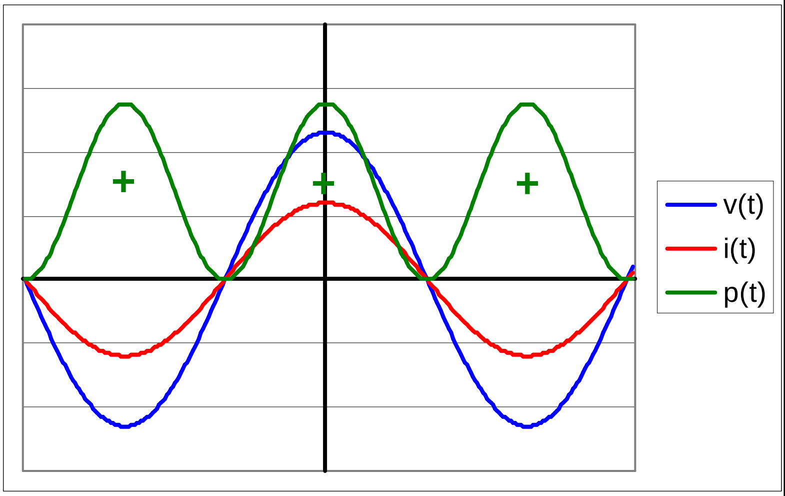
R ideal

$\varphi = 0 \rightarrow$

$\cos\varphi = 1$

$P = V.I$

$p(t) = V.I + V.I.\cos 2\omega.t$



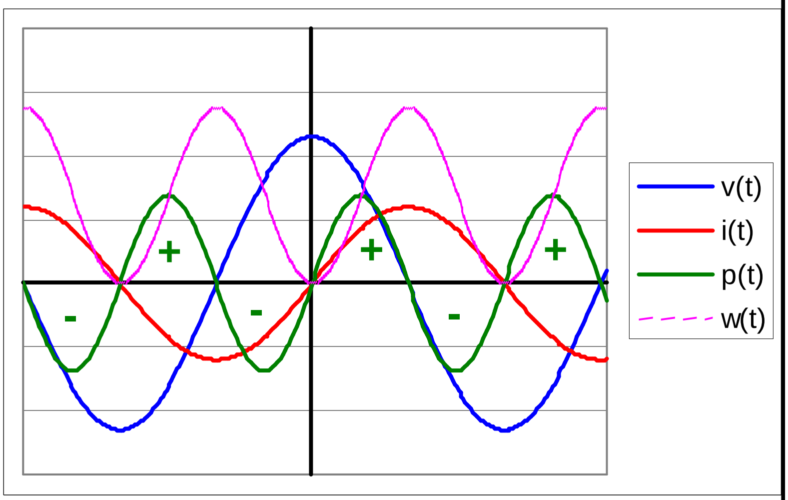
L ideal

$\varphi = \pi/2 \rightarrow$

$\cos\varphi = 0$

$P = 0$

$p(t) = V.I.\cos(2\omega.t - \pi/2)$



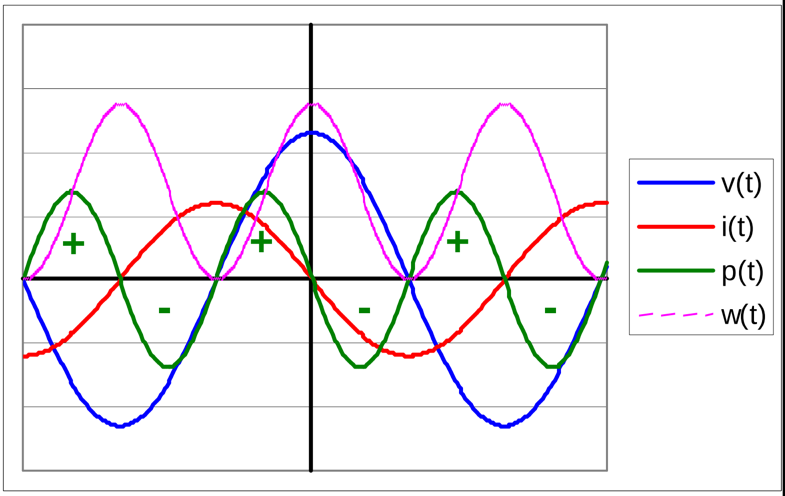
C ideal

$\varphi = -\pi/2 \rightarrow$

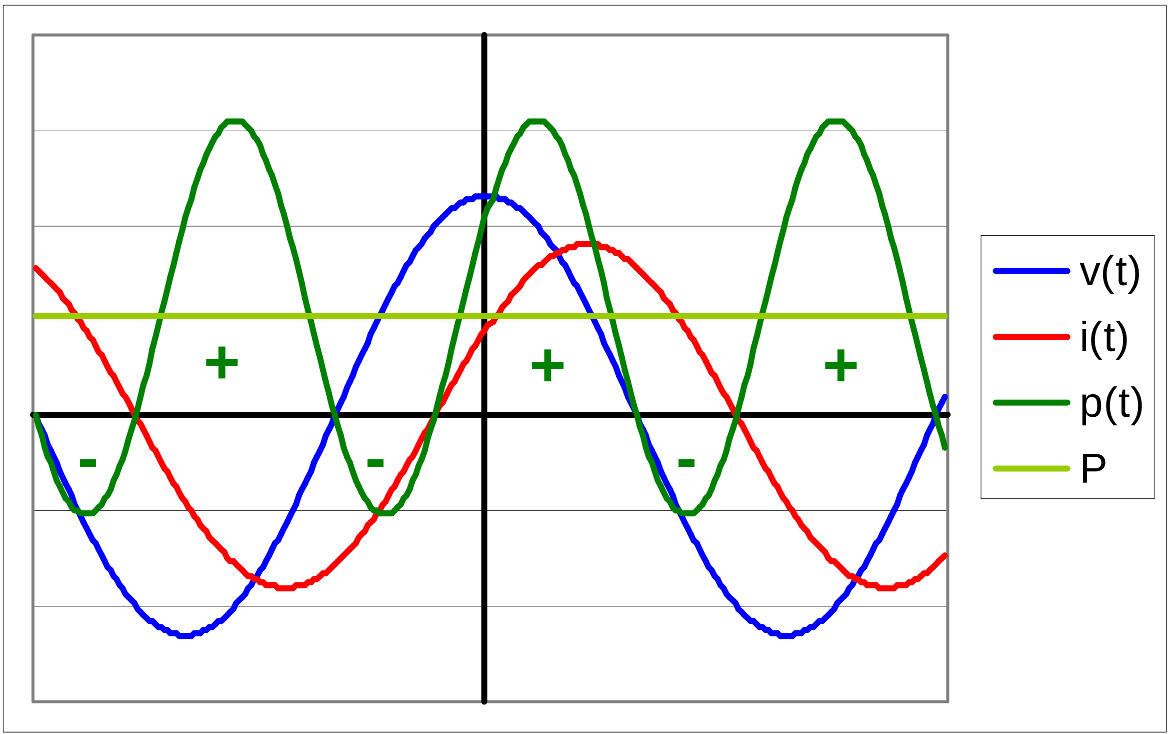
$\cos\varphi = 0$

$P = 0$

$p(t) = V.I.\cos(2\omega.t + \pi/2)$



Potência em RPS - I



Carga Genérica

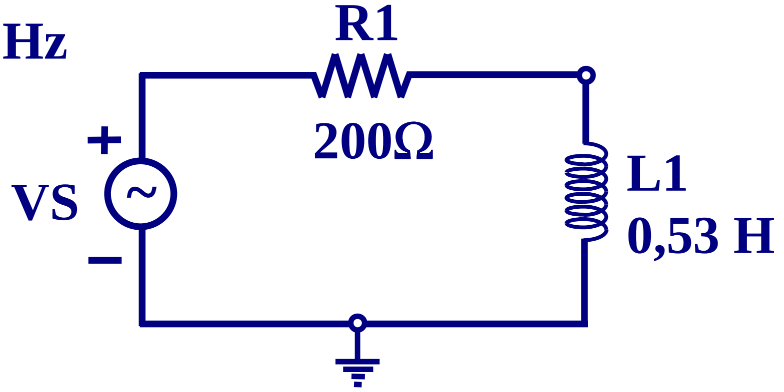
$$p(t) = VI\cos\varphi + VI\cos(2\omega t - \varphi)$$

$$-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$$

$$P = VI\cos\varphi > 0 \begin{cases} \text{potência ativa ou} \\ \text{potência real} \end{cases}$$

EXEMPLO

Amplitude = 200 V
Frequência = 60 Hz



$$V_{ef} = 200/\sqrt{2}$$

$$I_{ef} = 0{,}5$$

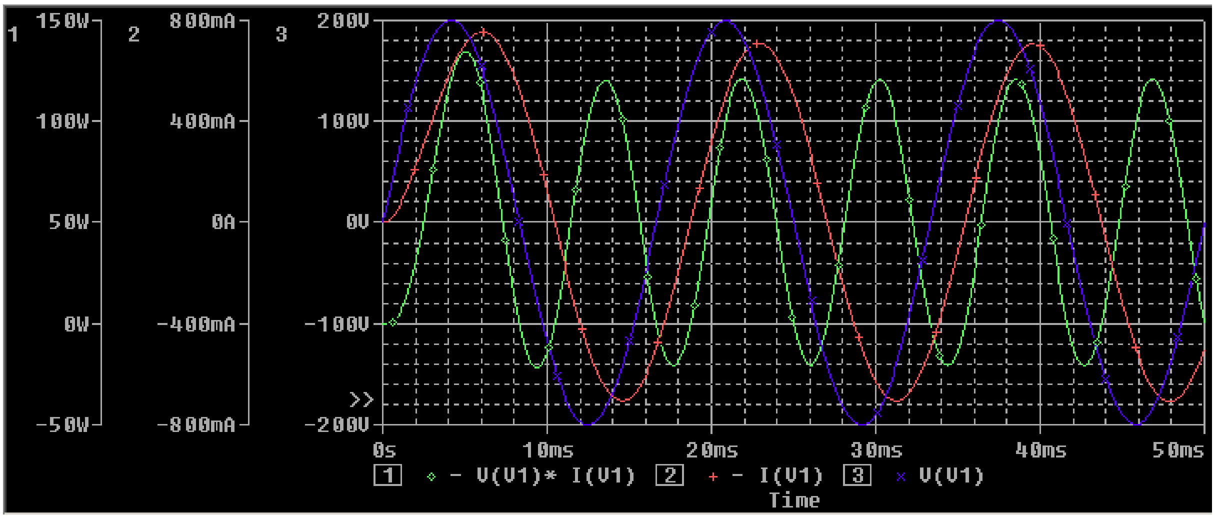
$$Z = 200 + j\,200\ \Omega$$

$$P = VI\cos\varphi = 50\ W$$

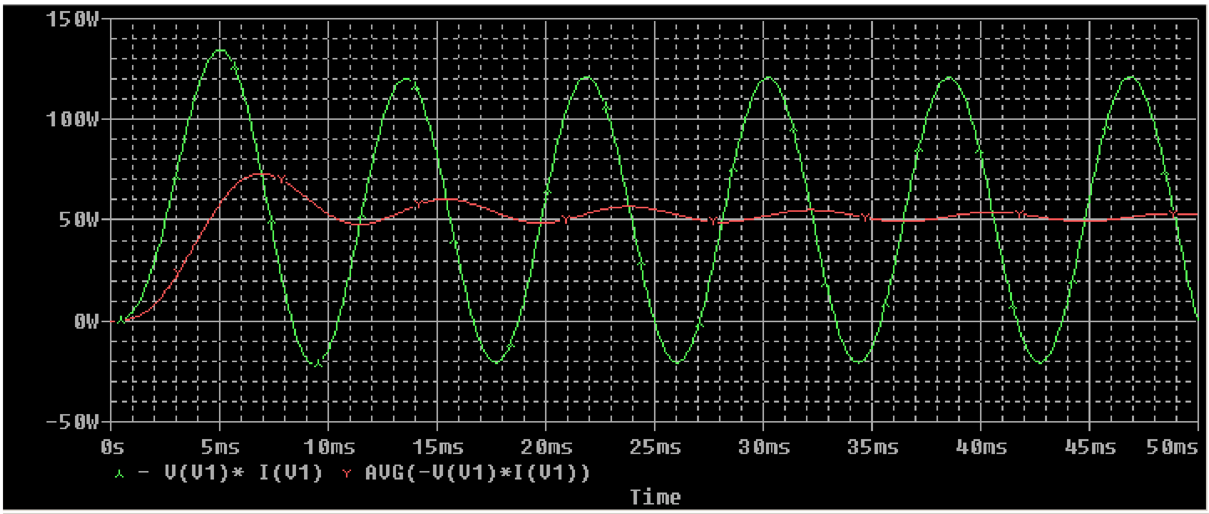
$$VI\cos\varphi - VI < p(t) < VI\cos\varphi + VI$$

$$-20{,}71\ W < p(t) < 120{,}71\ W$$

EXEMPLO



x v(t) + i(t) ◇ p(t)



- 20,71 W < p(t) < 120,71 W

$$P = \frac{1}{t}\int_0^t p(\tau)d\tau = \text{AVG (v(t) * i(t))} = 50\ \text{W}$$

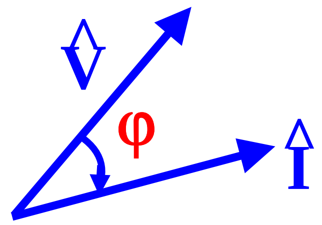
Potência aparente

$$Pap = VI \qquad (\,VA,\ kVA\,)$$

Fator de Potência :

– **Caso geral :** $\quad f.\,p. = \dfrac{P}{VI} = \dfrac{P}{Pap}$

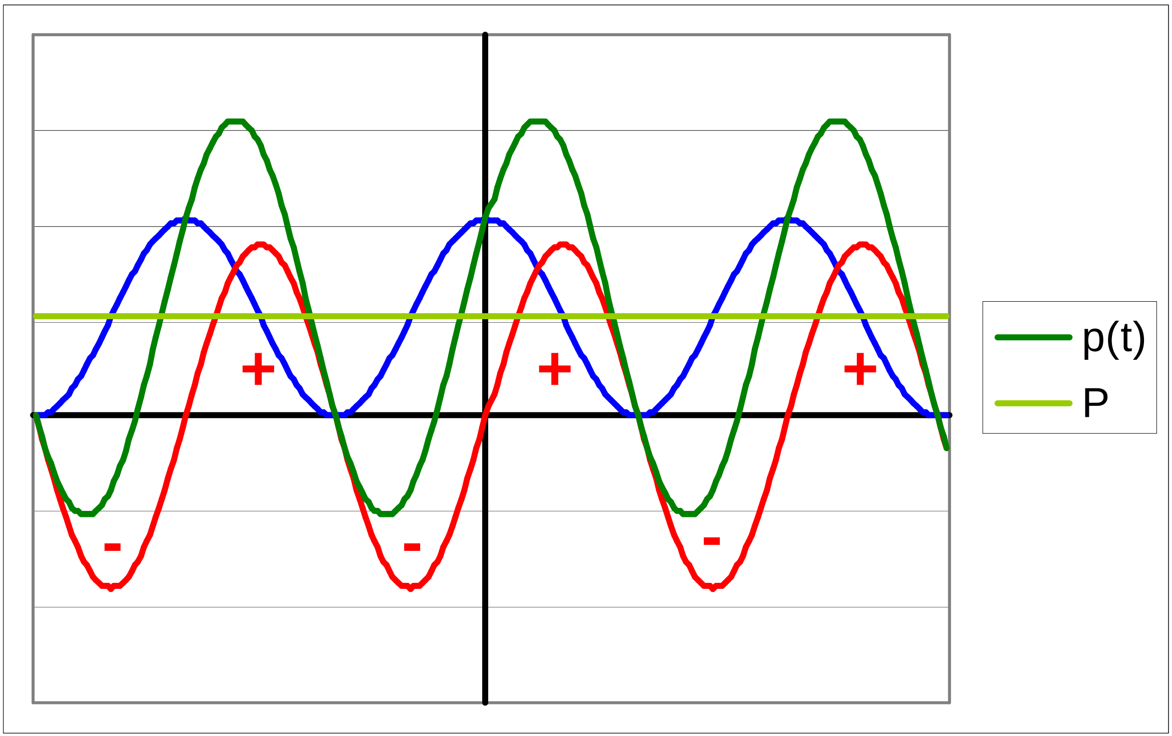
– **RPS :** $\quad f.\,p. = \cos\varphi = \dfrac{P}{VI}$



$$\begin{cases} \text{atrasado :} & 0 < \varphi < \pi/2 \\ \text{adiantado:} & -\dfrac{\pi}{2} < \varphi < 0 \end{cases}$$

Carga resistiva →	**f.p. = 1**
Carga puramente reativa →	**f.p. = 0**
Bipolo receptor →	$0 \leq f.p. \leq 1$

Potência em RPS - II



p(t)

VIcosφ (1 + cos 2ωt)

VIsenφ sen 2ωt

$$p(t) = VI\cos\varphi\,(1 + \cos 2\omega t) + VI\,\text{sen}\varphi\ \text{sen}\,2\omega t$$

Potência Reativa

$Q \triangleq VI \sin \varphi$ (VAr , kVAr)

$Q > 0 \rightarrow$ indutivo
$Q < 0 \rightarrow$ capacitivo

$$Pap = VI = \sqrt{P^2 + Q^2}$$

Convenção do receptor

	X	φ	Q	I
Bip.Indutivo	> 0	> 0	> 0	atras.
Bip.Capacitivo	< 0	< 0	< 0	adiant.

Teorema da Conservação das Potências

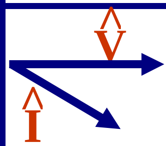
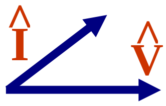
– Rede passiva linear
– Geradores sincronizados
– RPS
– Relações $\hat{V}_k$ e $\hat{J}_k$ $\rightarrow$ convenção do receptor

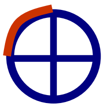
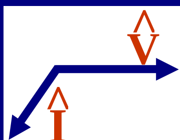
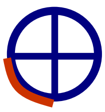
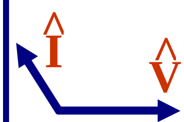
• Soma complexa Pap nos bipolos = soma complexa Pap fornecidas pelos geradores

• Soma P recebidas pelos bipolos = soma P fornecidas pelos geradores

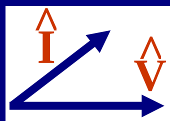
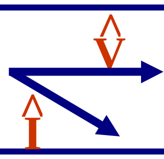
• Soma algébrica Q nos bipolos = soma algébrica Q fornecidas pelos geradores

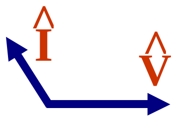
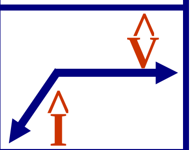
Bipolo em Convenção de Receptor

Recebe Potência	φ	cosφ	P	Q	I	
Indutivo		> 0	> 0	> 0	atras.	
Capacitivo		> 0	> 0	< 0	adiant.	

Fornece Potência	φ	cosφ	P	Q	I	
Indutivo		< 0	< 0	> 0	atras.	
Capacitivo		< 0	< 0	< 0	adiant.	

Bipolo em Convenção de Gerador

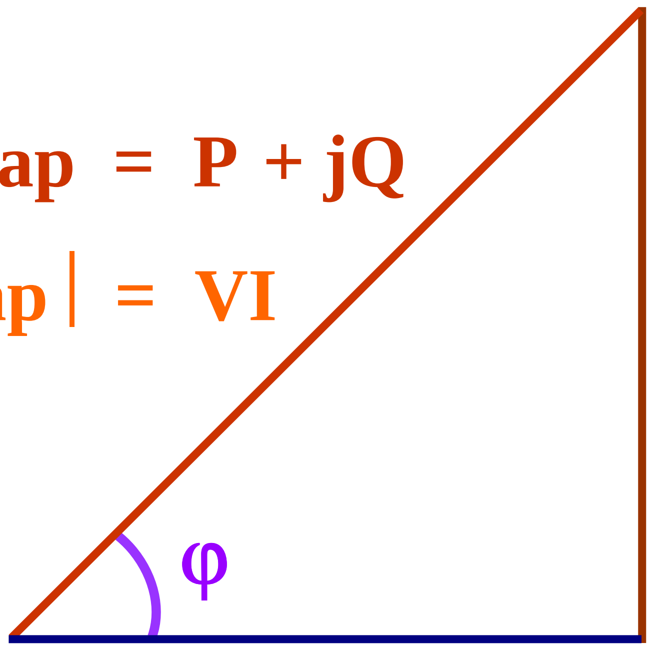
Fornece Potência	φ	cosφ	P	Q	I	
Indutivo		> 0	> 0	< 0	adiant.	
Capacitivo		> 0	> 0	> 0	atras.	

Recebe Potência	φ	cosφ	P	Q	I	
Indutivo		< 0	< 0	< 0	adiant.	
Capacitivo		< 0	< 0	> 0	atras.	

Triângulo de Potência

$$Pap = P + jQ$$

$$|Pap| = VI$$



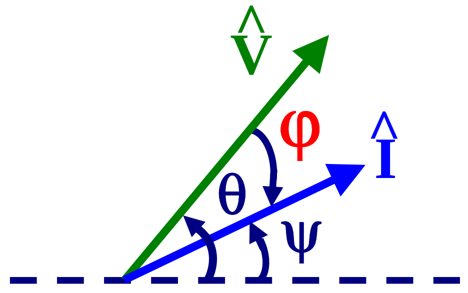
$$Q = VI \sin\varphi$$

$$P = VI \cos\varphi$$

$$tg\,\varphi = \frac{Q}{P}$$

Pap → Potência Aparente Complexa

Potência e Fasores



$$\hat{V} = V e^{j\theta}$$

$$\hat{I} = I e^{j\psi}$$

$$\varphi = \theta - \psi$$

$$\hat{V} . \hat{I}^{*} = VI\, e^{j(\theta - \psi)}$$

$$= VI \cos\varphi + j\, VI \,\mathrm{sen}\, \varphi$$

ou seja :

$$\boxed{\hat{V} . \hat{I}^{*}} = P + jQ = VI\, e^{j\varphi}$$

Potência aparente complexa → Pap

P → potência ativa

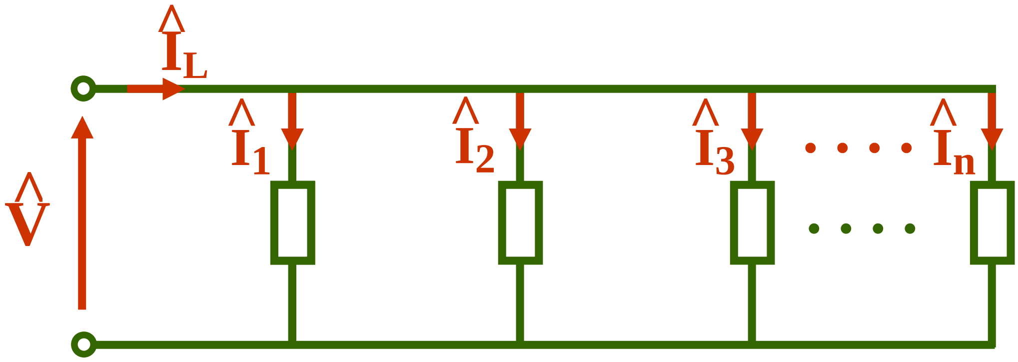
Q → potência reativa

$$|Pap| = \sqrt{P^2 + Q^2}$$

$$\mathrm{tg}\, \varphi = \frac{Q}{P}$$

$$\cos\varphi = \frac{P}{|Pap|} = \text{f.p.}$$

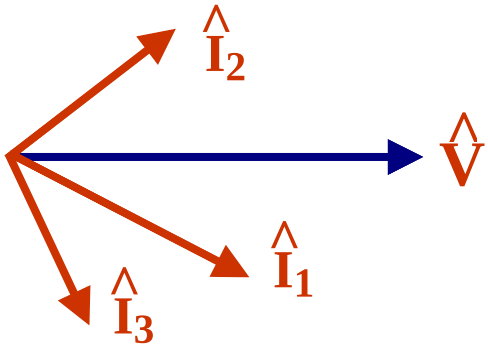
Circuito de Distribuição Monofásico



Corrente de linha ⇨ $$\hat{I}_L = \sum_{k=1}^{n} \hat{I}_k$$

Potência aparente :

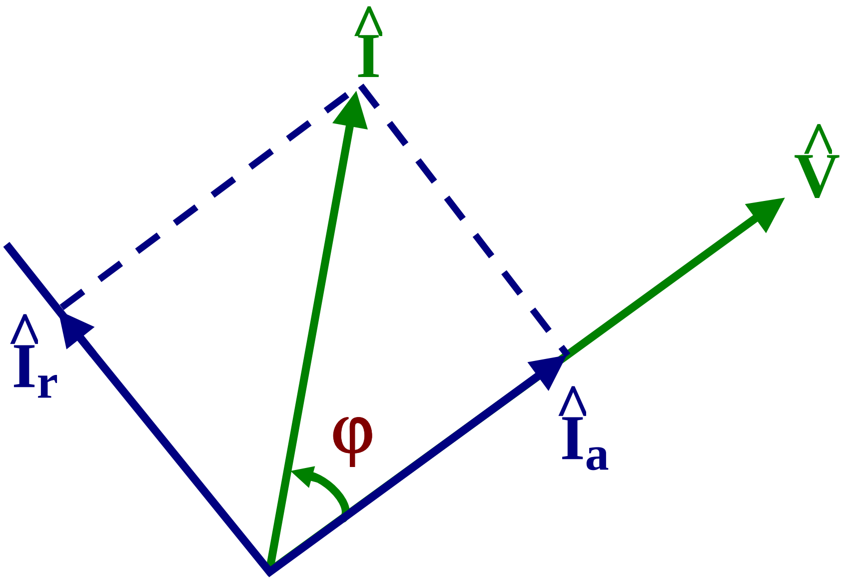
$$Pap = \hat{V} \,.\, \hat{I}_L^* = \sum_{k=1}^{n} \hat{V} \,.\, \hat{I}_k^* = \sum_{k=1}^{n} Pap_k$$



$$Pap = \sum_{k=1}^{n} P_k + j \sum_{k=1}^{n} Q_k$$

$$|\hat{I}_L| = \frac{|Pap|}{|\hat{V}|} \qquad \cos\varphi_L = \frac{\sum P_k}{|Pap|}$$

Corrente Complexa



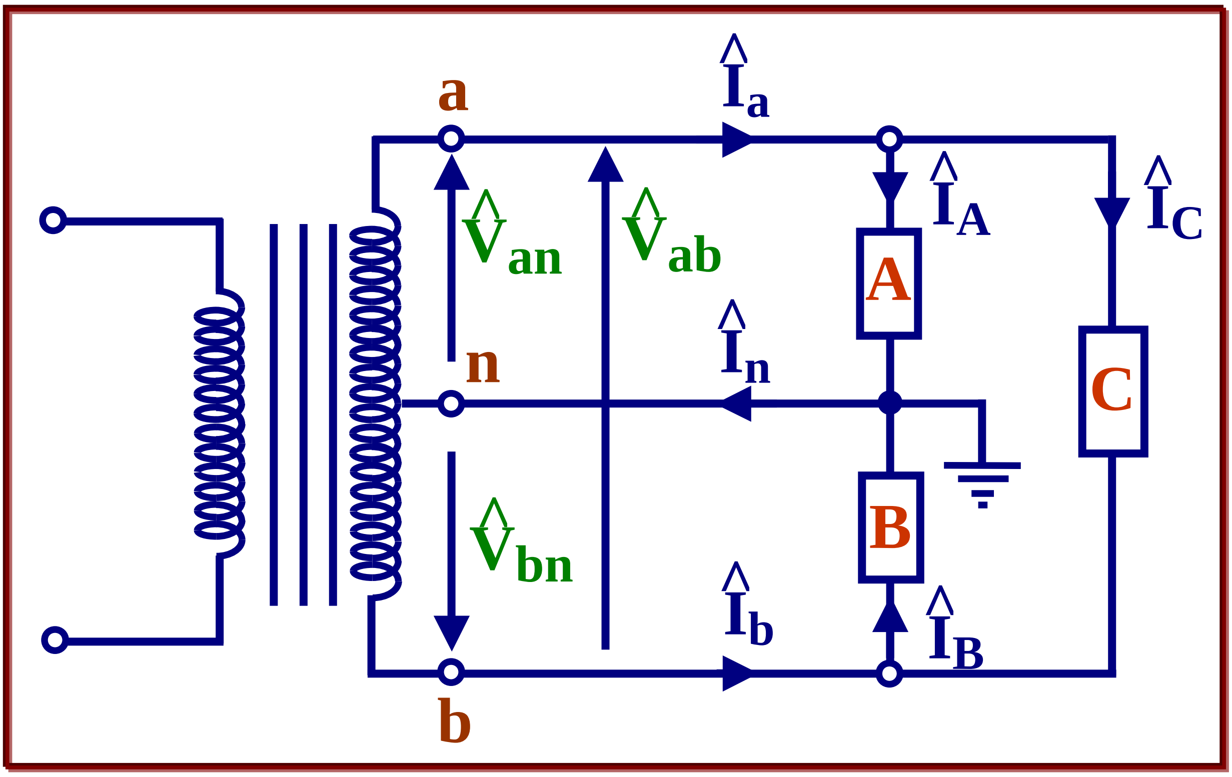
$$\text{Pap} = \underbrace{VI\cos\varphi}_{VI_a} + j\,\underbrace{VI\,\text{sen}\varphi}_{VI_r}$$

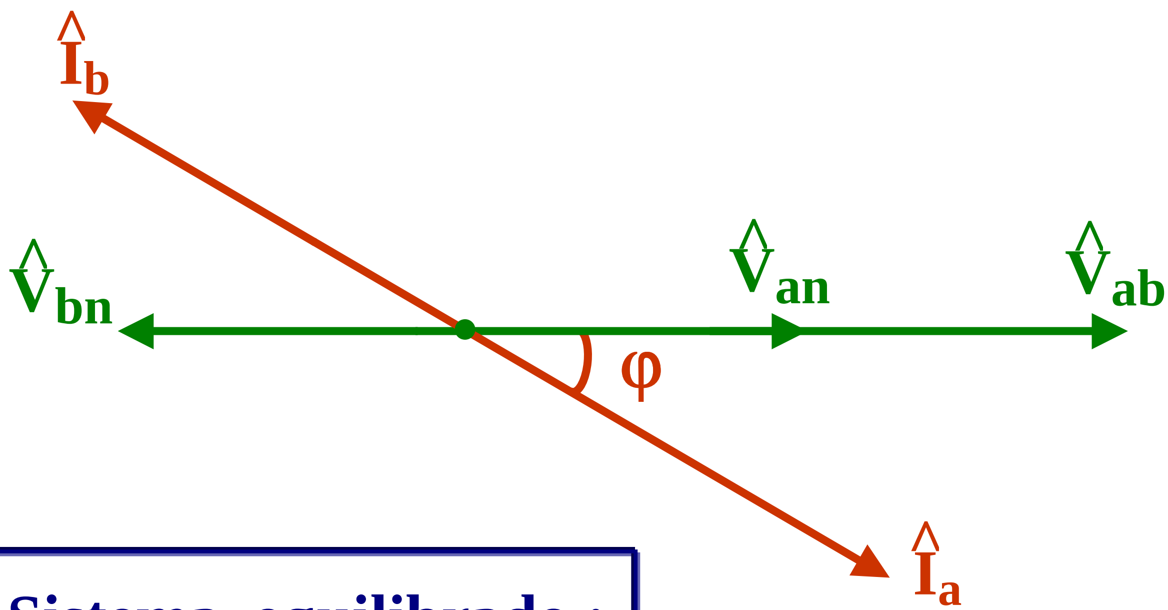
$VI_a \rightarrow$ **P** $\rightarrow$ **Potência "real"**

$VI_r \rightarrow$ **Q** $\rightarrow$ **Potência em quadratura**

Monofásico a 3 fios

(110 / 220 V)





Sistema equilibrado :

$\hat{I}_n = 0$

Cálculo da Potência Ativa no caso não - senoidal

$$v(t) = V_0 + \sum_{k=1}^{N} \sqrt{2}\, V_k \cos\left(k\,\omega_0\, t + \theta_k\right)$$

$$i(t) = I_0 + \sum_{k=1}^{N} \sqrt{2}\, I_k \cos\left(k\,\omega_0\, t + \psi_k\right)$$

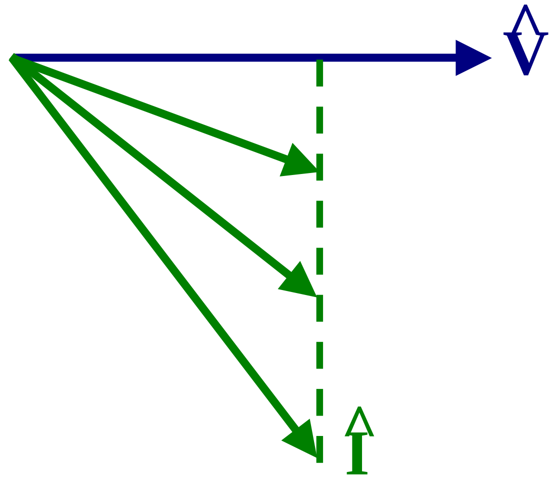
$$P = \frac{1}{T_0} \int_{T_0} v(t)\, i(t)\, dt$$

$$P = P_0 + P_1 + P_2 + \ldots + P_N$$

$$P = V_0 I_0 + \sum_{k=1}^{N} V_k I_k \cos \varphi_k$$

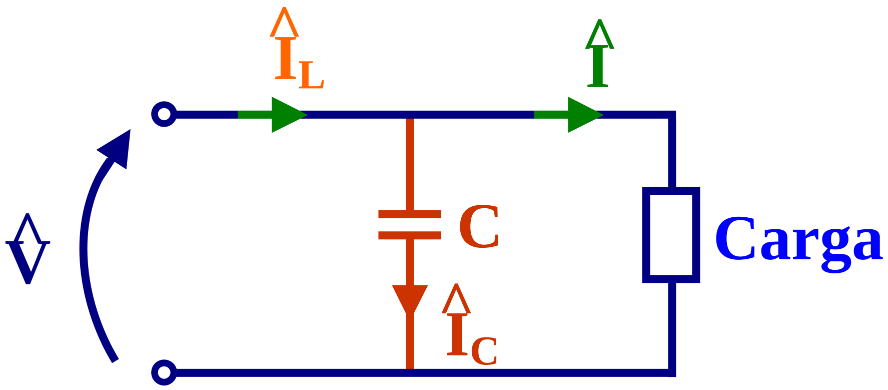
$$\varphi_k = \theta_k - \psi_k$$

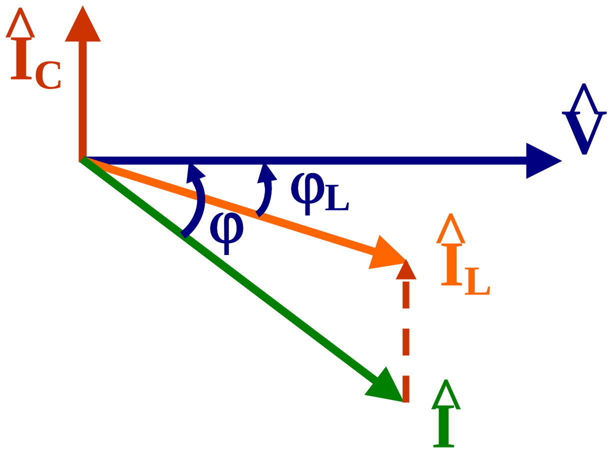
Correção de Fator de Potência



f.p. deve ser
> 0,92 atrasado

Manter P





$$Pap = P + jQ + jQ_C$$

$$C = \frac{P\,(\,tg\,\varphi - tg\,\varphi_L\,)}{\omega V^2}$$

Fator de Potência e Energia

Energia Ativa : $E_a = \int_{\Delta t} P\,dt$

Energia reativa : $E_r = \int_{\Delta t} Q\,dt$

$\Delta t = 1$ **hora, p. ex.**

Fator de Potência Médio em Δt :

$$F.P. = \frac{E_A}{\sqrt{E_A^2 + E_R^2}}$$

Correção do F.P. em Δt :

$$Pap\ cap = \hat{V} \cdot \hat{I}^* = \hat{V}\,(\hat{V} \cdot j\omega C)^*$$

$$= |\hat{V}|^2 \cdot (-j\omega C)$$

$$Q\ cap = -\omega C\, V^2$$

Energia Reativa no Capacitor em Δt:

$$E\ cap = -\omega C\, V^2 \cdot \Delta t$$

Potência & Impedância / Admitância

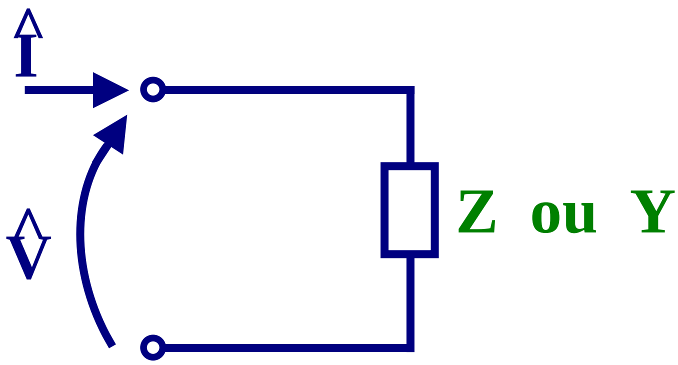
→ Generalização da Lei de Joule

$$Z(j\omega) = R(\omega) + jX(\omega)$$

$$Pap = \underbrace{R(\omega)\,|\hat{I}|^2}_{P} + j\,\underbrace{X(\omega)\,|\hat{I}|^2}_{Q}$$

$$Y(j\omega) = G(\omega) + jB(\omega)$$

$$Pap = \underbrace{G(\omega)\,|\hat{V}|^2}_{P} + j\,\underbrace{(-B(\omega))\,|\hat{V}|^2}_{Q}$$

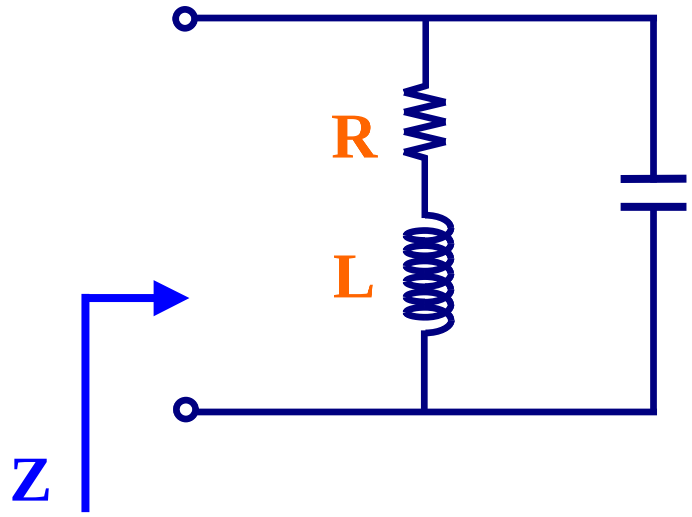


$$Pap = \hat{V}.\hat{I}^*$$

$$\hat{V} = Z\,\hat{I}$$

$$\hat{I} = Y\,\hat{V}$$

Exemplo



$$X_L = \omega L$$

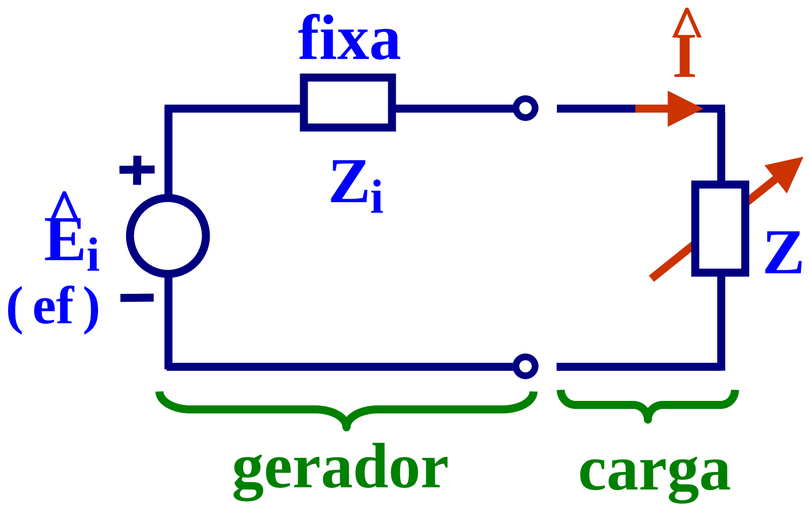
$$X_C = \frac{-1}{\omega C}$$

$$Z = \frac{(R + j X_L)(j X_C)}{R + j(X_L + X_C)}$$

$$Z = \underbrace{\frac{R X_C^2}{R^2 + (X_L + X_C)^2}}_{R(\omega)} + j\underbrace{\frac{X_C\left[R^2 + X_L(X_L + X_C)\right]}{R^2 + (X_L + X_C)^2}}_{X(\omega)}$$

$$Y = \frac{1}{Z} = \underbrace{\frac{R X_C^2}{X_C^2(R^2 + X_L^2)}}_{G(\omega)} - j\underbrace{\frac{X_C\left[R^2 + X_L(X_L + X_C)\right]}{X_C^2(R^2 + X_L^2)}}_{B(\omega)}$$

Transferência de Potência em RPS



$Z_i = R_i + j\,X_i$

$Z = R + j\,X$

Potência ativa : $P = R\,|\hat{I}\,|^2$

$$P = \frac{R}{(R + R_i)^2 + (X + X_i)^2}\,|E_i|^2$$

Condição de máximo :

$$P_{máx} = \frac{|E_i|^2}{4R} \qquad \text{para}$$

$\eta = 50\ \%$ $\qquad \boxed{Z = Z_i^*}$

Casamento de Impedâncias !

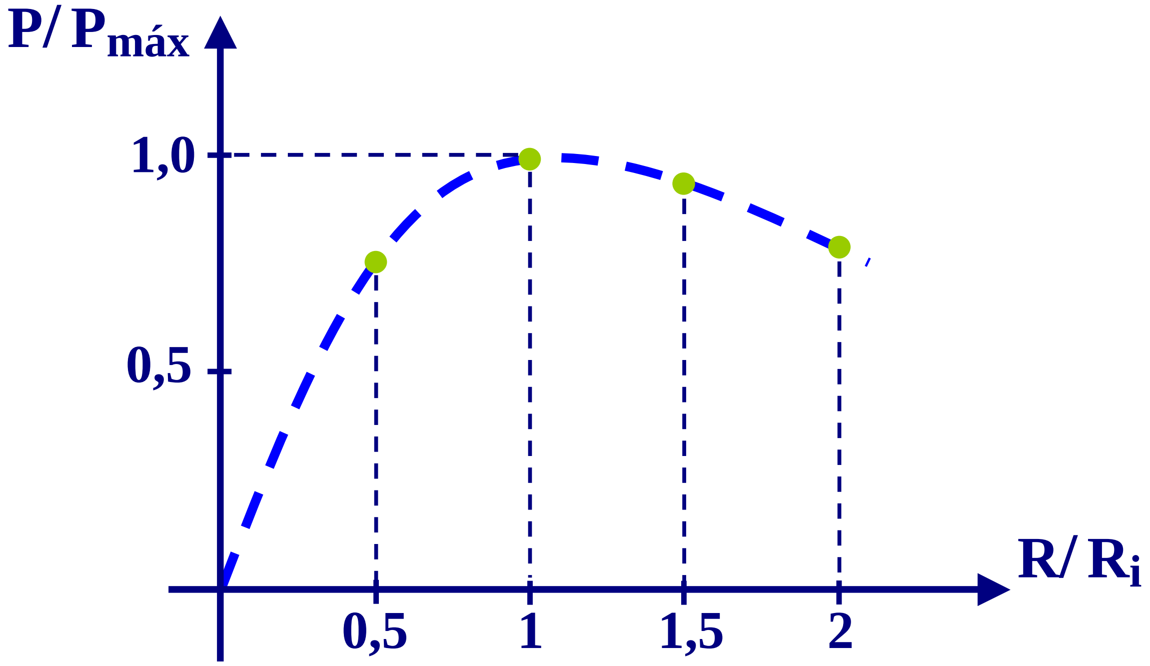
Exemplo : com Transformador Ideal

tal que
$$\begin{cases} R = R_i \\ X = -X_i \end{cases}$$

Transferência de Potência em RPS

$$\frac{P}{P_{máx}} = \frac{4R / R_i}{(1 + R/R_i)^2 + (X + X_i)^2 / R_i^2}$$

$$\frac{P}{P_{máx}} = 4 \frac{R}{R_i} \frac{1}{(1 + R/R_i)^2}$$



Condições:

$$\begin{cases} (X + X_i)^2 / R_i^2 << 1 \\ R \approx R_i \end{cases}$$