Eletromagnetismo II

Segunda Prova - Noturno

Gabarito

1.

a) $\vec{R} = \vec{r} - \vec{r}_q = d\hat{e}_x - z_0\cos(\omega t_r)\hat{e}_z \;\therefore\; R = \left[d^2 + z_0^2\cos^2(\omega t_r)\right]^{1/2}$

$t_r = t - \frac{R}{c} \;\therefore\; c(t-t_r) = \left[d^2 + z_0^2\cos^2(\omega t_r)\right]^{1/2} \;\therefore\; \boxed{(t-t_r)^2 - \frac{z_0^2}{c^2}\cos^2(\omega t_r) = \frac{d^2}{c^2}}$

b) $\vec{v} = \frac{d\vec{r}_q}{dt_r} = -z_0\omega\sin(\omega t_r)$; $\beta = \frac{|v|}{c} \therefore \beta \ll 1 \Rightarrow \boxed{\frac{z_0\omega}{c} \ll 1}$

c) $\beta \ll 1 \Rightarrow \frac{dP}{d\Omega} = \frac{1}{c^3}\frac{q^2}{16\pi^2\epsilon_0}\left[\hat{n}\times(\hat{n}\times\vec{a})\right]^2$

$\frac{z_0}{r} \ll 1 \Rightarrow \cancel{\frac{dP}{d\Omega}}\;\; \hat{n} = \frac{\vec{r}-\vec{r}_q}{|\vec{r}-\vec{r}_q|} \approx \hat{e}_r$

$\therefore \left[\hat{n}\times(\hat{n}\times\vec{a})\right] = \hat{n}\times\left[\hat{e}_r \times a\hat{e}_z\right] = -a\sin\theta\,(\hat{n}\times\hat{e}_\varphi)$;

$\therefore \left[\hat{n}\times(\hat{n}\times\vec{a})\right]^2 = a^2\sin^2\theta$; $a = -z_0\omega^2\cos(\omega t_r)$

$\therefore \frac{dP}{d\Omega} = \frac{1}{c^3}\frac{q^2}{16\pi^2\epsilon_0} z_0^2\omega^4\cos^2(\omega t_r)\sin^2\theta$

$\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{q^2z_0^2\omega^4}{16\pi^2\epsilon_0c^3}\sin^2\theta\underbrace{\left\langle\cos^2(\omega t_r)\right\rangle}_{=1/2} \;\therefore\; \boxed{\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{q^2z_0^2\omega^4\sin^2\theta}{32\pi^2\epsilon_0c^3}}$

d) $\langle P\rangle = \int_0^{2\pi}d\varphi\int_0^{\pi}\sin\theta\left\langle\frac{dP}{d\Omega}\right\rangle d\theta = \frac{q^2z_0^2\omega^4}{32\pi^2\epsilon_0c^3}\cdot 2\pi\cdot\underbrace{\int_0^{\pi}\sin^3\theta\,d\theta}_{=4/3}$

$\therefore \boxed{\langle P\rangle = \frac{q^2z_0^2\omega^4}{12\pi\epsilon_0c^3}}$

2. $\phi(\vec r,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi_\omega(\vec r)e^{-i\omega t}\,d\Omega$

∴

$$\phi(\vec r,t) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{|\vec r-\vec r'|}\left[\frac{1}{\sqrt{2\pi}}\int \rho_\omega(\vec r')\,e^{\pm ik|\vec r-\vec r'|-i\omega t}\,d\omega\right]d\tau'$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{d\tau'}{|\vec r-\vec r'|}\left\{\frac{1}{\sqrt{2\pi}}\int \rho_\omega(\vec r')\,e^{-i\omega\left[t\mp\frac{|\vec r-\vec r'|}{c}\right]}\,d\omega\right\}$$

Mas $\frac{1}{\sqrt{2\pi}}\int \rho_\omega(\vec r)\,e^{-i\omega\left[t\mp\frac{|\vec r-\vec r'|}{c}\right]}\,d\omega = \rho\left(\vec r, t\mp\frac{|\vec r-\vec r'|}{c}\right)$

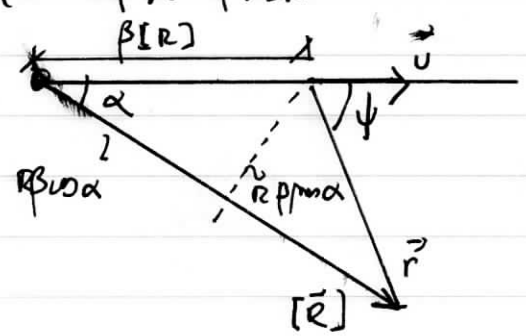
Escolhemos o sinal (-) porque o argumento resultante representa o tempo retardado, t_r, ou seja, o tempo t menos o tempo que levou para a informação sobre o valor instantaneo da fonte se propagar de $\vec r'$ a $\vec r$, com a velocidade da luz. Portanto

$$\boxed{\phi(\vec r,t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec r', t_r)}{|\vec r-\vec r'|}\,d\tau' \; ; \quad t_r = t - \frac{|\vec r-\vec r'|}{c}}$$

3.

a) $\vec a = 0 \Rightarrow \vec E(\vec r,t) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{s^3}(\vec R - R\vec\beta)(1-\beta^2)\right]_{ret}$

$s = R - \vec R\cdot\vec\beta$



$s = R - \vec R\cdot\vec\beta = R - R\beta\cos\alpha$

Mas $R\beta\cos\alpha = R - \sqrt{r^2 - R^2\beta^2\sin^2\alpha}$

∴ $s = R - (R - \sqrt{r^2-\beta^2R^2\sin^2\alpha}) = \sqrt{r^2 - R^2\beta^2\sin^2\alpha}$

Mas $\cancel{R\beta}$ $R\sin\alpha = r\sin\psi$ ∴ $s = \sqrt{r^2 - r^2\beta^2\sin^2\psi}$

∴ $s = r\sqrt{1-\beta^2\sin^2\psi}$

Por outro lado, da figura temos que

$$\vec{R} = R\vec{\beta} + \vec{r} \therefore \vec{R} - \vec{\beta}R = \vec{r};$$

assim

$$\boxed{\vec{E}(\vec{r},t) = \frac{q\vec{r}}{4\pi\epsilon_0 r^3} \frac{1-\beta^2}{[1-\beta^2 \sin^2\psi]^{3/2}}}$$

b) $\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}$; $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{1}{\mu_0 c^2} (\vec{E} \times (\vec{v} \times \vec{E}))$

$$\therefore \vec{S} = \frac{1}{\mu_0 c^2} [E^2 \vec{v} - (\vec{E}\cdot\vec{v})\vec{E}] = \frac{1}{\mu_0 c^2} [E^2\vec{v} - Ev\cos\psi\, \vec{E}]$$

$$\therefore \vec{S} = \epsilon_0 [E^2\vec{v} - E^2 v\cos\psi (\cos\psi\, \hat{e}_v + \sin\psi\, \hat{e}_\perp)]$$

$$\therefore \boxed{\vec{S} = \epsilon_0 E^2 [\sin^2\psi\, \vec{v} - v\sin\psi\cos\psi\, \hat{e}_\perp]}$$

onde $\hat{e}_\perp$ é o versor normal ao plano formado por $\vec{v}$ e $\vec{r}$.

4.

a) $[\vec{R}] \times \vec{E}_v = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{s^3} (-R(\vec{R}\times\vec{\beta}))(1-\beta^2) \right] = \frac{qR}{4\pi\epsilon_0} \left[\frac{\vec{\beta}\times\vec{R}}{s^3}(1-\beta^2) \right]$

$$\therefore [\vec{R}] \times \vec{E}_v = [R]c\,\vec{B}_v \therefore \vec{B}_v = \frac{[\vec{R}]\times\vec{E}}{[R]c}$$

$$[\vec{R}] \times \vec{E}_a = \frac{q}{4\pi\epsilon_0} \left[\frac{R}{c^2 s^3} (\vec{R}\cdot\vec{a})(\vec{\beta}\times\vec{R}) - \frac{R\, \vec{R}\times\vec{a}}{c^2 s^2} \right] = [R]c\,\vec{B}_a \checkmark$$

b) $\frac{\vec{R}}{R} \times \{ \vec{R} \times ((\vec{R} - R\vec{\beta}) \times \vec{a}) \} = \frac{\vec{R}}{R} \times [(\vec{R}\cdot\vec{a})(\vec{R} - R\vec{\beta}) - (R^2 - R\vec{R}\cdot\vec{\beta})\vec{a}]$

$$= \frac{\vec{R}}{R} \times [(\vec{R}\cdot\vec{a})(\vec{R} - R\vec{\beta}) - Rs\vec{a}] = \frac{1}{R}[(\vec{R}\cdot\vec{a})(\vec{\beta}\times\vec{R}) - Rs\,\vec{R}\times\vec{a}]$$

$$\therefore \boxed{\vec{B}_a(\vec{r},t) = \frac{q}{4\pi\epsilon_0 c} \left[\frac{\vec{R}\cdot\vec{a}}{c^2 s^3}(\vec{\beta}\times\vec{R}) - \frac{\vec{R}\times\vec{a}}{c^2 s^2} \right]}$$

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4.

$$[\overset{\circ}{R}].\vec{E}_a = \frac{q}{4\pi\epsilon_0}\frac{1}{c^2 s^2}\left[\frac{1}{s}Rs(\vec{R}.\vec{a}) - R(\vec{R}.\vec{a})\right] = 0$$