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Eletromagnetismo II

Segunda Prova - Diurno

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1. $R = c(t - t_r)$; $R = [(x-x_q)^2 + (y-y_q)^2 + (z-z_q)^2]^{1/2}$

$\therefore$

$$\left(\frac{\partial R}{\partial t}\right)_{\vec{r}} = \left(\frac{\partial R}{\partial t_r}\right)_{\vec{r}}\left(\frac{\partial t_r}{\partial t}\right)_{\vec{r}} = c\left(1 - \frac{\partial t_r}{\partial t}\right) \therefore \frac{\partial t_r}{\partial t} = \frac{1}{1 + \frac{1}{c}\left(\frac{\partial R}{\partial t}\right)_{\vec{r}}}$$

$$\left(\frac{\partial R}{\partial t}\right)_{\vec{r}} = -\frac{1}{R}\left[(x-x_q)\frac{dx_q}{dt_r} + (y-y_q)\frac{\partial y_q}{\partial t_r} + (z-z_q)\frac{\partial z_q}{\partial t_r}\right] = -\frac{\vec{R}.\vec{v}}{R}$$

$$\therefore \left(\frac{\partial t_r}{\partial t}\right)_{\vec{r}} = \frac{1}{1 - \frac{1}{c}\frac{\vec{R}.\vec{v}}{R}} = \frac{R}{s}\ ;\ s = R - \vec{R}.\vec{\beta}$$

$$\therefore \left(\frac{\partial}{\partial t}\right)_{\vec{r}} = \left(\frac{\partial t_r}{\partial t}\right)_{\vec{r}}\left(\frac{\partial}{\partial t_r}\right)_{\vec{r}} = \frac{R}{s}\left(\frac{\partial}{\partial t_r}\right)_{\vec{r}} \quad \therefore \boxed{\left(\frac{\partial}{\partial t}\right)_{\vec{r}} = \frac{R}{s}\left(\frac{\partial}{\partial t_r}\right)_{\vec{r}}}$$

2.

a) $\vec{E}(\vec{r},t) = \vec{E}_v(\vec{r},t) + \vec{E}_a(\vec{r},t)$; $\vec{E}_v(\vec{r},t) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{s^3}(\vec{R} - R\vec{\beta})(1-\beta^2)\right]$

$$\vec{E}_a(\vec{r},t) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{c^2 s^3}(\vec{R}.\vec{a})(\vec{R} - R\vec{\beta}) - \frac{R\vec{a}}{c^2 s^2}\right.$$

$[\vec{R}] = [R]\hat{e}_x$; $[\vec{\beta}] = [\beta]\hat{e}_x$; $[\vec{a}] = [a]\hat{e}_x$ $\therefore$ $s = R(1-\beta)$

$$\therefore \vec{E}_v = \frac{q}{4\pi\epsilon_0}\left[\frac{R}{s^3}(1-\beta)(1-\beta^2)\right]\hat{e}_x = \frac{q}{4\pi\epsilon_0[R]^2}\frac{1+\beta}{1-\beta}\hat{e}_x$$

$$\vec{E}_a = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{c^2 s^3}\{R^2 a(1-\beta) - Ra(R - R\beta)\}\right] = 0$$

$$\therefore \boxed{\vec{F} = Q\vec{E}_v = -\frac{Qe}{4\pi\epsilon_0[R]^2}\frac{1+\beta}{1-\beta}\hat{e}_x}$$

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b) A força que a carga positiva exerce no elétron é dada por

$$\vec{F}_e = \frac{Q|e|}{4\pi\epsilon_0 [R]^2}\hat{e}_x$$

Portanto a terceira lei de Newton não se verifica porque, enquanto o campo produzido pela carga Q é estático, ou seja, a força produzida no elétron só depende de sua posição, o campo produzido pelo elétron não atua "instantaneamente" sobre a carga Q.

c) $$\frac{dP}{d\Omega} = \frac{1}{c^3}\frac{e^2}{16\pi^2\epsilon_0}\frac{[\hat{n}\times\{(\hat{n}-\vec{\beta})\times\vec{a}\}]^2}{[1-\hat{n}\cdot\vec{\beta}]^5}$$

$$x=0 \Rightarrow \beta = 0 \therefore \hat{n}\times\{(\hat{n}-\vec{\beta})\times\vec{a}\} = \hat{n}\times(\hat{n}\times\vec{a}) = (\hat{n}\cdot\hat{e}_x)a\hat{n} - a\hat{e}_x$$

$$\therefore [\hat{n}\times(\hat{n}\times\vec{a})]^2 = a^2[\cos^2\theta - \cos^2\theta - \cos^2\theta + 1] = a^2\sin^2\theta$$

$$\therefore \frac{dP}{d\Omega} = \frac{1}{c^3}\frac{e^2}{16\pi^2\epsilon_0}a^2\sin^2\theta \Rightarrow P = \frac{1}{c^3}\frac{e^2}{16\pi^2\epsilon_0}a^2 \cdot 2\pi \cdot \underbrace{\int_0^{\pi}\sin^3\theta\, d\theta}_{=4/3}$$

$$\therefore P = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\frac{a^2}{c^3}; \quad m_e a = \frac{Q|e|}{4\pi\epsilon_0 D^2}$$

$$\therefore \boxed{P = \frac{1}{96\pi^3\epsilon_0^3 c^3}\frac{e^4Q^2}{m_e^2 D^4}}$$

3.

a) $\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}',t_r)}{R}d\tau'$; $R = |\vec{r}-\vec{r}'|$; $t_r = t - \frac{R}{c}$

$\vec{J}(\vec{r},t) = -\omega q_0 \sin(\omega t)\delta(x)\delta(y)\hat{e}_z$; $\vec{r} = \vec{r}$; $\vec{r}' = z'\hat{e}_z$

$$\therefore \ \vec{A}(\vec r,t) = -\frac{\mu_0 q_0 \omega}{4\pi}\int_{-\infty}^{\infty}\delta(x')dx'\int_{-\infty}^{\infty}\delta(y')dy'\int_{-d/2}^{d/2}\frac{\text{sen}\left(\omega\left(t-\frac{|\vec r - z'\hat e_z|}{c}\right)\right)}{|\vec r - z'\hat e_z|}dz'\,\hat e_z$$

$$\therefore \ \boxed{\vec{A}(\vec r,t) = -\frac{\mu_0 q_0 \omega}{4\pi}\hat e_z\int_{-d/2}^{d/2}\frac{\text{sen}\left(\omega\left(t-\frac{|\vec r - z'\hat e_z|}{c}\right)\right)}{|\vec r - z'\hat e_z|}dz'}$$

b) máximo valor de z' é $d/2$. Portanto, para $d \ll r$, $|\vec r - z'\hat e_z| \approx r$, em mais baixa ordem e

$$\vec{A}(\vec r,t) = -\frac{\mu_0 q_0 \omega}{4\pi}\hat e_z\frac{\text{sen}\left(\omega\left(t-\frac{r}{c}\right)\right)}{r}\int_{-d/2}^{d/2}dz'$$

$$\therefore \ \boxed{\vec{A}(\vec r,t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\,\text{sen}\left(\omega\left(t-\frac{r}{c}\right)\right)\hat e_z}$$

c) $\vec B = \nabla\times\vec A$; $\vec{A}(\vec r,t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\,\text{sen}\left(\omega\left(t-\frac{r}{c}\right)\right)(\cos\theta\,\hat e_r - \text{sen}\,\theta\,\hat e_\theta)$

$$\therefore \ \nabla\times\vec A = -\frac{\mu_0 p_0 \omega}{4\pi}\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{-\text{sen}\,\theta\,\text{sen}\left(\omega\left(t-\frac{r}{c}\right)\right)\right\} - \frac{1}{r^2}\text{sen}\left(\omega\left(t-\frac{r}{c}\right)\right)\frac{\partial\cos\theta}{\partial\theta}\right]\hat e_\varphi$$

A segunda parcela cai com $1/r^2$ e, portanto, não constitui campo de radiação

$$\therefore \ \boxed{\vec B(\vec r,t)\Big|_{rad} = -\frac{\mu_0 p_0 \omega^2}{4\pi c}\left(\frac{\text{sen}\,\theta}{r}\right)\cos\left(\omega\left(t-\frac{r}{c}\right)\right)\hat e_\varphi}$$

$$\vec S = \frac{1}{\mu_0}\vec E\times\vec B =$$

$$\vec E = -\frac{\partial\vec A}{\partial t} = -\frac{\mu_0 p_0 \omega^2}{4\pi r}\cos\left(\omega\left(t-\frac{r}{c}\right)\right)\hat e_z$$

$$\therefore \ \vec E\times\vec B = \left[\frac{\mu_0 p_0 \omega^3}{4\pi}\right]^2\frac{1}{c}\cos^2\left(\omega\left(t-\frac{r}{c}\right)\right)\left(\frac{\text{sen}\,\theta}{r}\right)\frac{1}{r}(\cos\theta\,\hat e_r - \text{sen}\,\theta\,\hat e_\theta)\times\hat e_\varphi$$

$$\therefore \vec{S} = \frac{-1}{\mu_0 c}\left[\frac{\mu_0 p_0 \omega^2}{4\pi}\right]^2 \omega^2(\omega(t-\frac{r}{c}))\left[\left(\frac{\sin\theta}{r}\right)^2 \hat{r} + \frac{\sin\theta\cos\theta}{r^2}\hat{\theta}\right]$$

$$\therefore \boxed{\langle \vec{S} \rangle = \frac{-1}{2\mu_0 c}\left[\frac{\mu_0 p_0 \omega^2}{4\pi}\right]^2 \left[\left(\frac{\sin\theta}{r}\right)^2 \hat{r} + \frac{\sin 2\theta}{2r^2}\hat{\theta}\right]}$$

4. $$\nabla^2\phi_\omega + k^2\phi_\omega = -\frac{\rho_\omega}{\epsilon_0}$$

$$\nabla^2 G(\vec{r},\vec{r}\,') + k^2 G(\vec{r},\vec{r}\,') = -\delta(\vec{r}-\vec{r}\,')$$

$\therefore$

$$G(\vec{r},\vec{r}\,')\nabla^2\phi_\omega - \phi_\omega\nabla^2 G(\vec{r},\vec{r}\,') = \phi_\omega\delta(\vec{r}-\vec{r}\,') - G(\vec{r},\vec{r}\,')\frac{\rho_\omega}{\epsilon_0}$$

$$\therefore \int_V [G\nabla^2\phi_\omega - \phi_\omega\nabla^2 G]\,d\tau = \int \phi_\omega \delta(\vec{r}-\vec{r}\,')\,d\tau - \int G\frac{\rho_\omega}{\epsilon_0}\,d\tau$$

Pela Segunda Identidade de Green, integral do lado esquerdo se transforma em uma integral de superfície que vai para zero quando tomamos a superfície indo para infinito. Por outro lado, utilizando a propriedade da função delta, temos finalmente

$$\boxed{\phi_\omega(\vec{r}) = \int G(\vec{r},\vec{r}\,')\frac{\rho_\omega(\vec{r}\,')}{\epsilon_0}\,d\tau'}$$