14.3 DERIVADAS PARCIAIS

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1-14 Determine as derivadas parciais indicadas.

1. $f(x, y) = x^3y^5$; $f_x(3, -1)$

2. $f(x, y) = \sqrt{2x + 3y}$; $f_y(2, 4)$

3. $f(x, y) = xe^{-y} + 3y$; $\dfrac{\partial f}{\partial y}(1, 0)$

4. $f(x, y) = \operatorname{sen}(y - x)$; $\dfrac{\partial f}{\partial y}(3, 3)$

5. $z = \dfrac{x^3 + y^3}{x^2 + y^2}$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

6. $z = x\sqrt{y} - \dfrac{y}{\sqrt{x}}$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

7. $z = \dfrac{x}{y} + \dfrac{y}{x}$; $\dfrac{\partial z}{\partial x}$

8. $z = (3xy^2 - x^4 + 1)^4$; $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$

9. $u = xy\sec(xy)$; $\dfrac{\partial u}{\partial x}$

10. $u = \dfrac{x}{x + t}$; $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial t}$

11. $f(x, y, z) = xyz$; $f_y(0, 1, 2)$

12. $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$; $f_z(0, 3, 4)$

13. $u = xy + yz + zx$; u_x, u_y, u_z

14. $u = x^2y^3t^4$; u_x, u_y, u_t

15-41 Determine as derivadas parciais de primeira ordem da função.

15. $f(x, y) = x^3y^5 - 2x^2y + x$

16. $f(x, y) = x^2y^2(x^4 + y^4)$

17. $f(x, y) = x^4 + x^2y^2 + y^4$

18. $f(x, y) = \ln(x^2 + y^2)$

19. $f(x, y) = e^x \operatorname{tg}(x - y)$

20. $f(s, t) = s/\sqrt{s^2 + t^2}$

21. $g(x, y) = y\operatorname{tg}(x^2y^3)$

22. $g(x, y) = \ln(x + \ln y)$

23. $f(x, y) = e^{xy}\cos x \operatorname{sen} y$

24. $f(s, t) = \sqrt{2 - 3s^2 - 5t^2}$

25. $z = \operatorname{senh}\sqrt{3x + 4y}$

26. $z = \log_x y$

27. $f(u, v) = \operatorname{tg}^{-1}(u/v)$

28. $f(x, t) = e^{\operatorname{sen}(t/x)}$

29. $z = \ln\left(x + \sqrt{x^2 + y^2}\right)$

30. $z = x^{x^y}$

31. $f(x, y) = \displaystyle\int_x^y e^{t^2}\,dt$

32. $f(x, y) = \displaystyle\int_y^x \frac{e^t}{t}\,dt$

33. $f(x, y, z) = x^2yz^3 + xy - z$

34. $f(x, y, z) = x\sqrt{yz}$

35. $f(x, y, z) = x^{yz}$

36. $f(x, y, z) = xe^y + ye^z + ze^x$

37. $u = z\operatorname{sen}\dfrac{y}{x + z}$

38. $u = xy^2z^3\ln(x + 2y + 3z)$

39. $u = x^{y^z}$

40. $f(x, y, z, t) = \dfrac{x - y}{z - t}$

41. $f(x, y, z, t) = xy^2z^3t^4$

42-45 Use a derivação implícita para encontrar $\partial z/\partial x$ e $\partial z/\partial y$.

42. $xy + yz = xz$

43. $xyz = \cos(x + y + z)$

44. $x^2 + y^2 - z^2 = 2x(y + z)$

45. $xy^2z^3 + x^3y^2z = x + y + z$

46. Determine $\partial z/\partial x$ e $\partial z/\partial y$ se $z = f(ax + by)$.

47-52 Determine todas as derivadas parciais de segunda ordem.

47. $f(x, y) = x^2y + x\sqrt{y}$

48. $f(x, y) = \operatorname{sen}(x + y) + \cos(x - y)$

49. $z = (x^2 + y^2)^{3/2}$

50. $z = \cos^2(5x + 2y)$

51. $z = t\operatorname{sen}^{-1}\sqrt{x}$

52. $z = x^{\ln t}$

53-56 Verifique que a conclusão do Teorema de Clairaut é válida, isto é, $u_{xy} = u_{yx}$.

53. $u = x^5y^4 - 3x^2y^3 + 2x^2$

54. $u = \operatorname{sen}^2 x\cos y$

55. $u = \operatorname{sen}^{-1}(xy^2)$

56. $u = x^2y^3z^4$

57-63 Determine as derivadas parciais indicadas.

57. $f(x, y) = x^2y^3 - 2x^4y$; f_{xxx}

58. $f(x, y) = e^{xy^2}$; f_{xxy}

59. $f(x, y, z) = x^5 + x^4y^4z^3 + yz^2$; f_{xyz}

60. $f(x, y, z) = e^{xyz}$; f_{yzy}

61. $z = x\operatorname{sen} y$; $\dfrac{\partial^3 z}{\partial y^2\,\partial x}$

62. $z = \ln\operatorname{sen}(x - y)$; $\dfrac{\partial^3 z}{\partial y\,\partial x^2}$

63. $u = \ln(x + 2y^2 + 3z^3)$; $\dfrac{\partial^3 u}{\partial x\,\partial y\,\partial z}$

64. Se f e g são funções duas vezes deriváveis de uma única variável, mostre que a função

$$u(x, y) = xf(x + y) + yg(x + y)$$

satisfaz a equação $u_{xx} - 2u_{xy} + u_{yy} = 0$.

65. Mostre que a função

$$f(x_1, \ldots, x_n) = (x_1^2 + \cdots + x_n^2)^{(2-n)/2}$$

satisfaz a equação

$$\frac{\partial^2 f}{\partial x_1^2} + \cdots + \frac{\partial^2 f}{\partial x_n^2} = 0$$