Matemática Aplicada I
Integrais

Prof. Dr. José Eduardo Holler Branco

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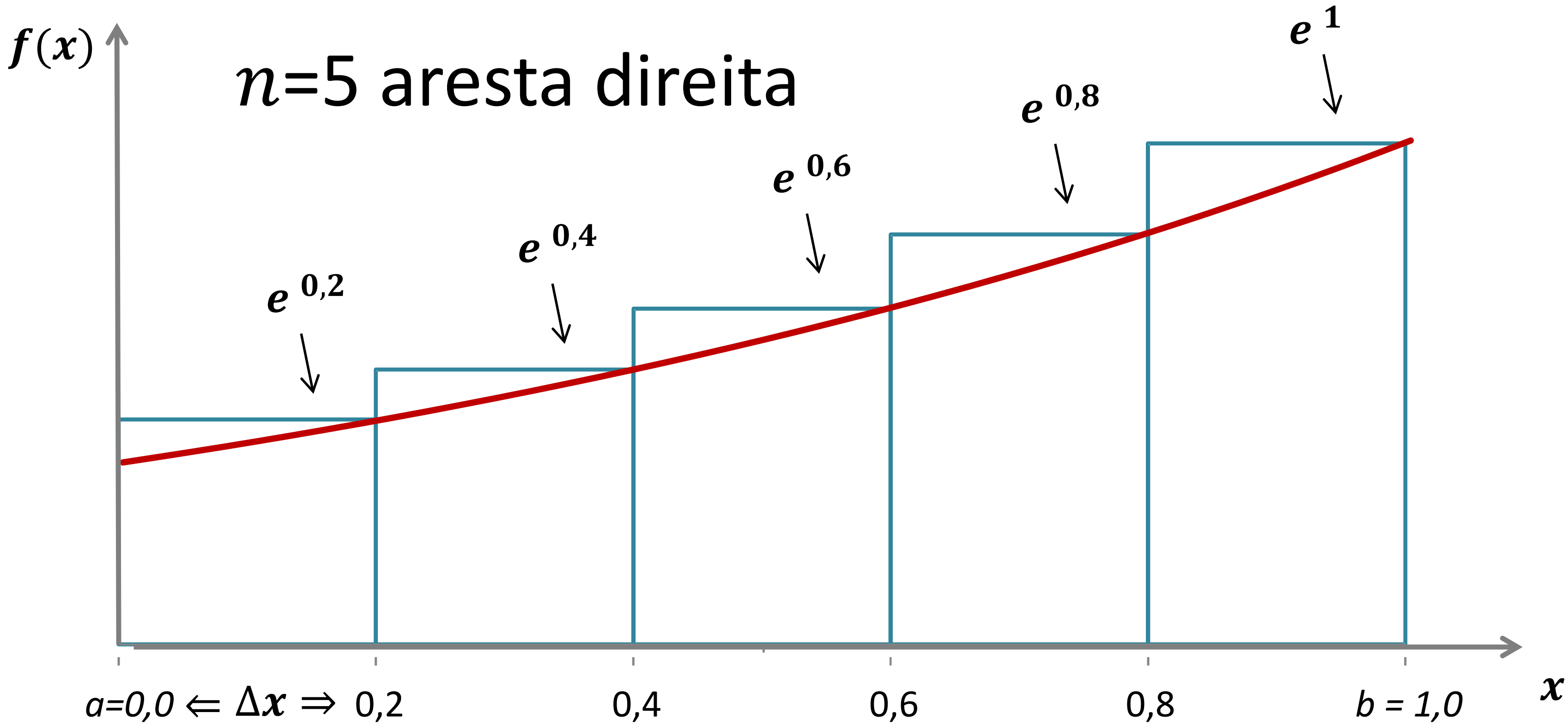
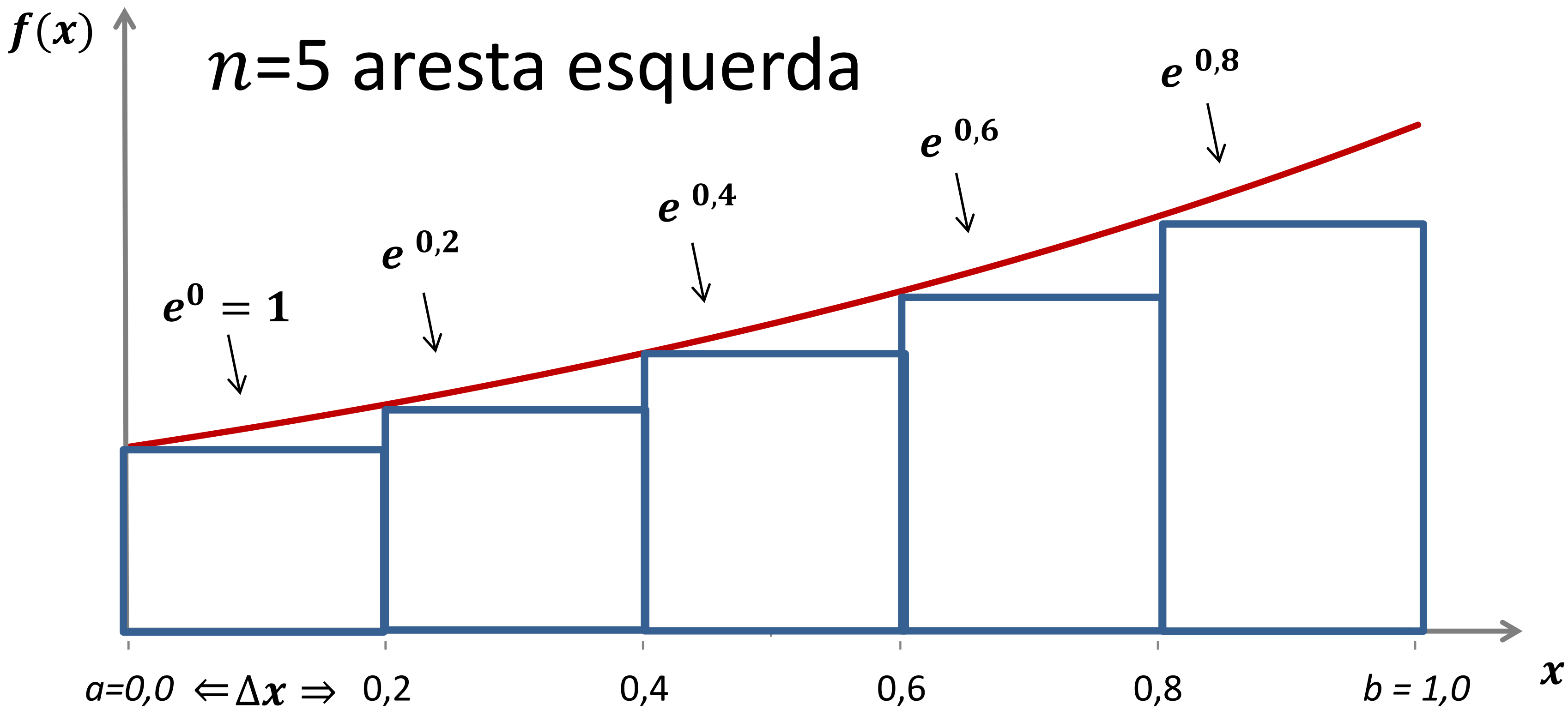
Área abaixo da curva $f(x) = e^x$



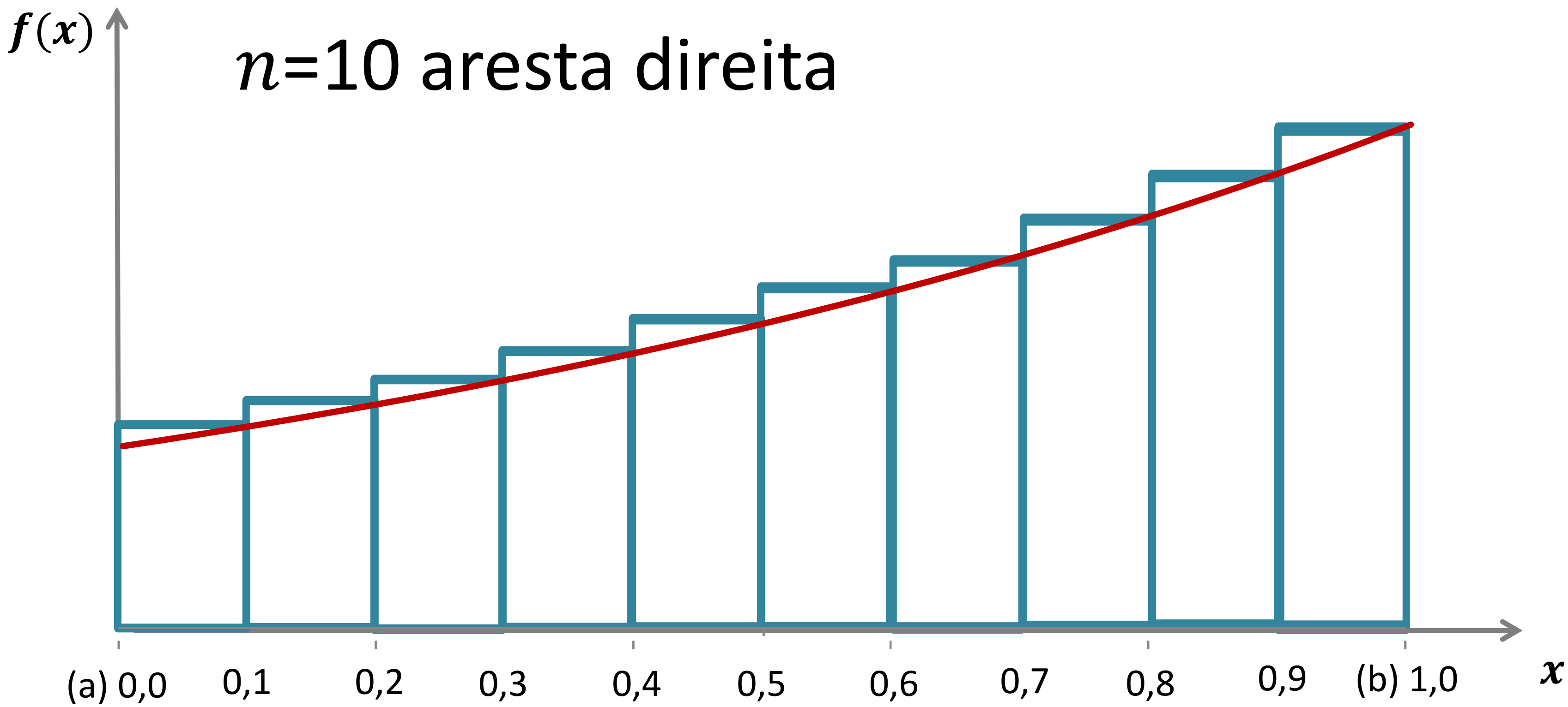
$$A_s = \sum_{i=1}^{n} f(x_i).\Delta x \qquad \Delta x = \left(\frac{b-a}{n}\right) \qquad A_s = 1{,}8958$$

Área abaixo da curva $f(x) = e^x$



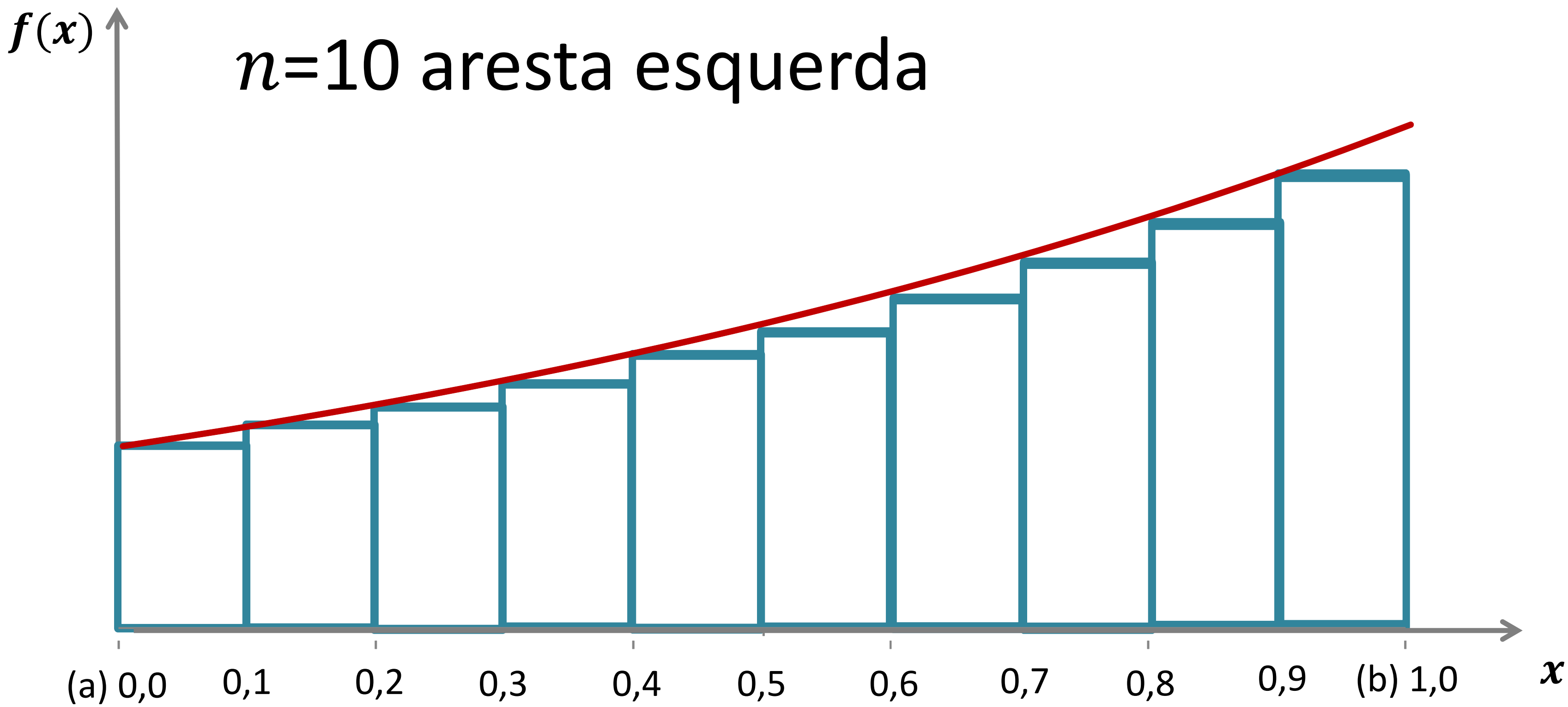
$$A_i = \sum_{i=1}^{n} f(x_i).\Delta x \qquad \Delta x = \left(\frac{b-a}{n}\right) \qquad A_i = 1{,}5522$$

Área abaixo da curva $f(x) = e^x$



$$A_s = \sum_{i=1}^{n} f(x_i).\Delta\boldsymbol{x} \qquad \Delta\boldsymbol{x} = \left(\frac{\boldsymbol{b}-\boldsymbol{a}}{\boldsymbol{n}}\right) \qquad A_s = 1{,}8056$$

Área abaixo da curva $f(x) = e^x$



$$A_i = \sum_{i=1}^{n} f(x_i).\Delta \boldsymbol{x} \qquad \Delta \boldsymbol{x} = \left(\frac{\boldsymbol{b}-\boldsymbol{a}}{\boldsymbol{n}}\right) \qquad A_i = 1{,}6338$$

Integral Definida

Proxy da área S abaixo da curva $f(x) = e^x$ no intervalo $[0,1]$

n	(Aresta Direita)				(Aresta Esquerda)
5	1,8958	>	S	>	1,5522
10	1,8056	>	S	>	1,6338
20	1,7616	>	S	>	1,6757
40	1,7398	>	S	>	1,6969
100	1,7269	>	S	>	1,7097
500	1,7200	>	S	>	1,7166
1000	1,7191	>	S	>	1,7174

$$\int_a^b f(x) = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i).\boldsymbol{\Delta x}$$

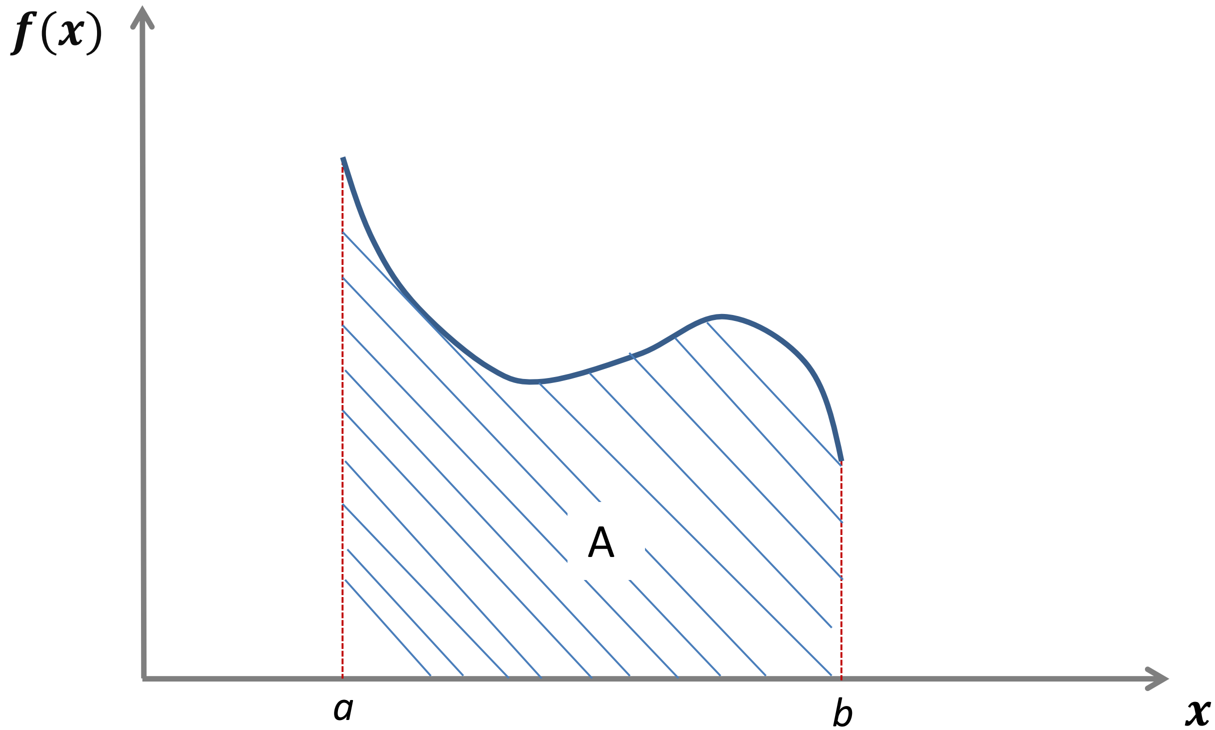
Se $f(x)$ *for integrável em* $[a,b]$

$$\boldsymbol{\Delta x} = {}^{(\boldsymbol{b}-\boldsymbol{a})}/_{\boldsymbol{n}} \quad \text{e} \quad \boldsymbol{x_i} = \boldsymbol{a} + i.\boldsymbol{\Delta x}$$

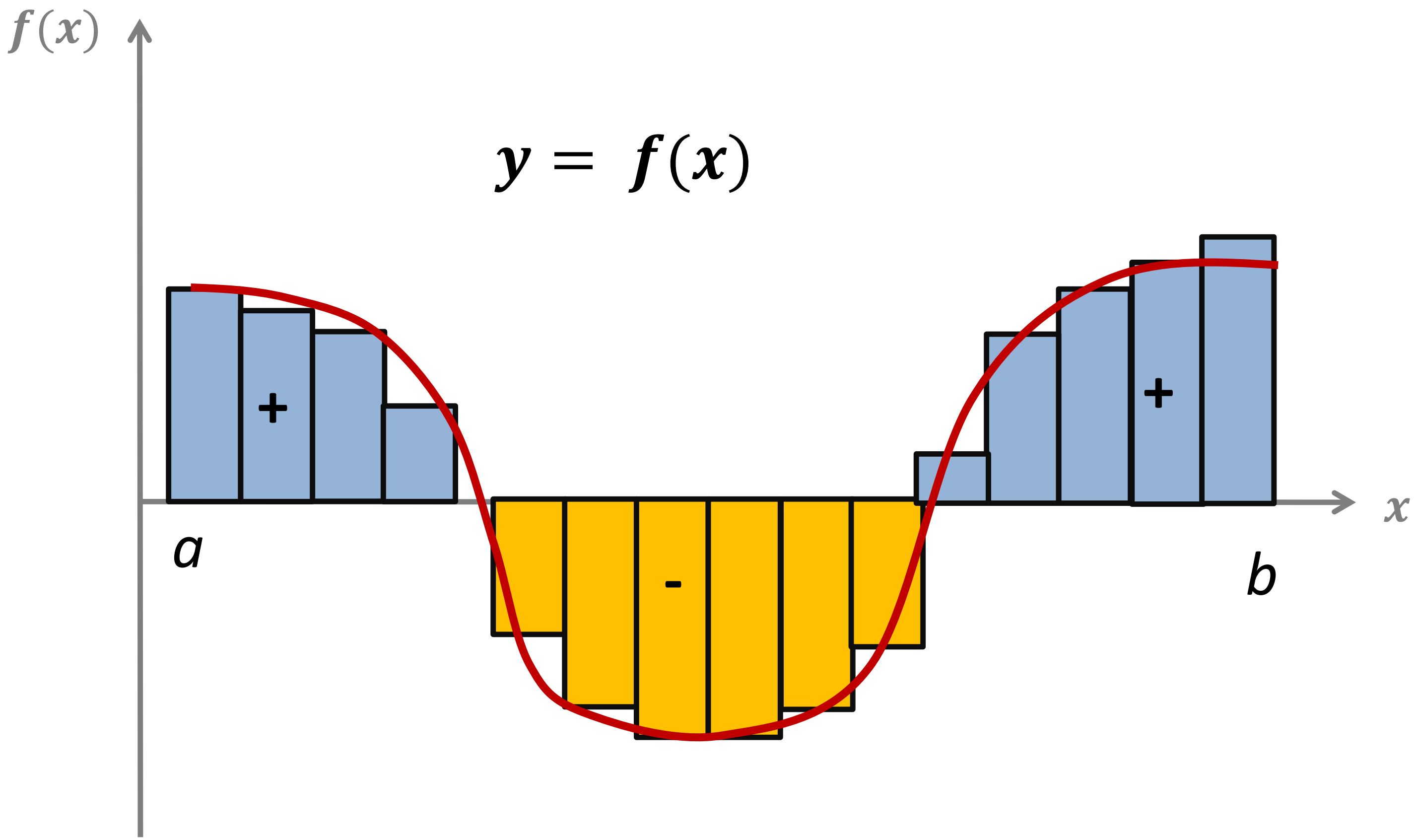
Integral Definida

$$\int_a^b f(x)\ dx \ corresponde\ à\ área\ sob\ a\ curva\ y = f(x), no\ intervalo\ [a, b]$$

Obs.: $f(x) > 0, no\ intervalo\ [a, b]$



Integral Definida



$$\int_a^b f(x)\ dx = A_1 - A_2$$

A_1: Área da parte superior $f(x) > 0$

A_2: Área da parte inferior $f(x) < 0$

Teorema Fundamental do Cálculo

Se f for contínua em $[a, b]$, então a função g é definida por:

$$g(x) = \int_a^x f(t)\, dt \quad a \leq x \leq b \Rightarrow g'(x) = f(x)$$

Se f for contínua em $[a, b]$, então:

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

Onde F é qualquer primitiva de f, isto é, uma função tal que $F' = f$

Propriedades da Integral

$$\int_b^a f(x)\ dx = -\int_a^b f(x)\,dx$$

$$\int_a^a f(x)\ dx = 0$$

$$\int_a^b c\,dx = c.(b-a)$$

$$\int_a^b [f(x) \pm g(x)]\,dx\ = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b cf(x)\,dx\ = \mathrm{c}\int_a^b f(x)\,dx$$

(Stewart, 2013)

Propriedades da Integral

$$\int_a^c f(x)\ dx + \int_c^b f(x)\ dx = \int_a^b f(x)\,dx$$

$$se\ f(x) \geq 0\ para\ a \leq x \leq b, então\ \int_a^b f(x)\,dx\ \geq 0$$

$$se\ f(x) \geq g(x)\ para\ a \leq x \leq b, então\ \int_a^b f(x)\,dx\ \geq \int_a^b g(x)\,dx$$

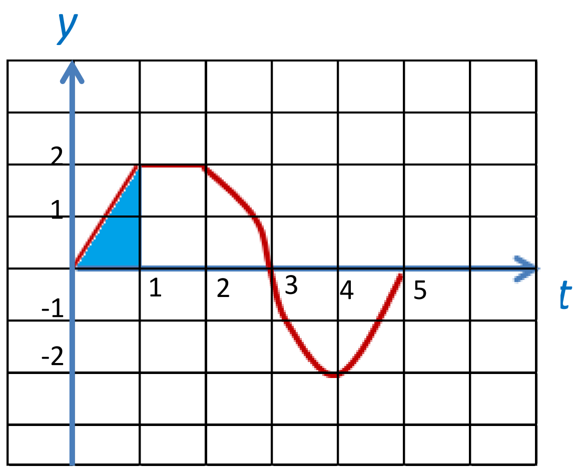
$$se\ m \leq f(x) \leq M\ para\ a \leq x \leq b, então:$$

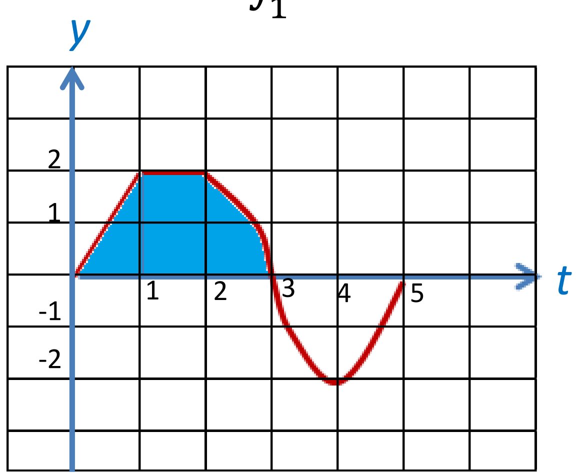
$$m.(b-a) \leq \int_a^b f(x)\,dx\ \leq M.(b-a)$$

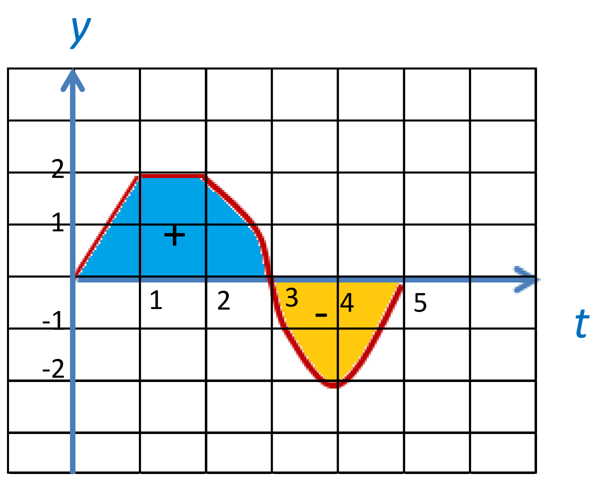
(Stewart, 2013)

Interpretação da Função Integral

$$g(3) = g(1) + \int_1^3 f(t)\,dt = 4{,}3$$

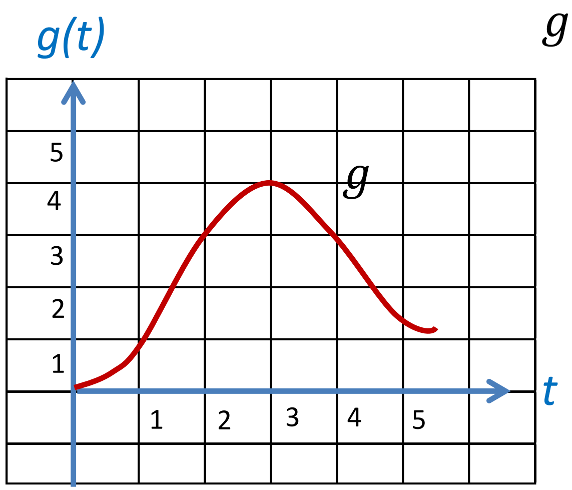






$$g(1) = \int_0^1 f(t)\,dt = 1$$

$$g(5) = g(3) + \int_3^5 f(t)\,dt = 1{,}7$$



(Stewart, 2013)

Regra da Substituição

$$Ache\ a\ Integral \int 2x\sqrt{1+x^2}\,dx$$

$$Se\ fizermos\ u = (1+x^2) \Rightarrow \frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$$

$$\int 2x\sqrt{1+x^2}\,dx = \int \sqrt{1+x^2}\,2x\,dx = \int \sqrt{u}\,du$$

$$\frac{2}{3}u^{3/2} + C = \frac{2}{3}(1+x^2)^{3/2} + \mathrm{C}$$

$$\frac{d}{dx}\left[\frac{2}{3}(1+x^2)^{3/2} + \mathrm{C}\right] = \frac{2}{3}.\frac{3}{2}(1+x^2)^{1/2}.2x = 2x\sqrt{1+x^2}$$

(Stewart, 2013)

Regra da Substituição

$$Calcule: \int \sqrt{2x+1}\, dx$$

$$Seja\ u = 2x+1 \Rightarrow \frac{du}{dx} = 2 \Rightarrow du = 2\,dx \Rightarrow dx = \frac{1}{2}\,du$$

$$\int \sqrt{2x+1}\, dx = \int \sqrt{u}\,.\frac{1}{2}\,du$$

$$= \frac{1}{2}\int (u)^{1/2}.\,du = \frac{1}{2}.\frac{2}{3}.(u)^{3/2} + C = \boxed{\frac{1}{3}\,(2x+1)^{3/2} + C}$$

$$\frac{d}{dx}\left[\frac{1}{3}\,(2x+1)^{3/2} + C\right] = \frac{1}{3}.\frac{3}{2}(2x+1)^{1/2}.2 = \sqrt{2x+1}$$

(Stewart, 2013)

Regra da Substituição - Integrais Definidas

$$\int_0^4 \sqrt{2x+1}\, dx$$

$$Seja\ u = 2x + 1 \Rightarrow \frac{du}{dx} = 2 \Rightarrow du = 2\,.\,dx \Rightarrow dx = \frac{1}{2}\, du$$

$$intervalo\ de\ f(u){:}\ se\ x = 0 \Rightarrow u = 1\ e\ se\ x = 4 \Rightarrow u = 9$$

$$\int_0^4 \sqrt{2x+1}\, dx = \int_1^9 \frac{1}{2}\sqrt{u}\, du$$

$$= \frac{1}{2}.\frac{2}{3}.u^{3/2}\Big|_1^9 = \frac{1}{3}\left(9^{3/2} - 1^{3/2}\right) = \frac{26}{3}$$

(Stewart, 2013)

Regra da Substituição

Se $u = g(x)$ for uma função derivável cuja imagem é um intervalo I e f for contínua em I, *então:*

$$\int f\big(g(x)\big)\, g'(x)\, dx = \int f(u)\, du$$

$$\int F'\big(g(x)\big)\, g'(x)\, dx = F\big(g(x)\big) + C, pois:$$

$$\frac{d}{dx} F\big(g(x)\big) + C = F'\big(g(x)\big).\, g'(x) = f\big(g(x)\big).\, g'(x)$$

(Stewart, 2013)

Regra da Substituição - Integrais Definidas

Se g' for contínua em $[a, b]$ e f for contínua na imagem de u $= g(x)$, então:

$$\int_a^b f\big(g(x)\big).\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$$

(Stewart, 2013)