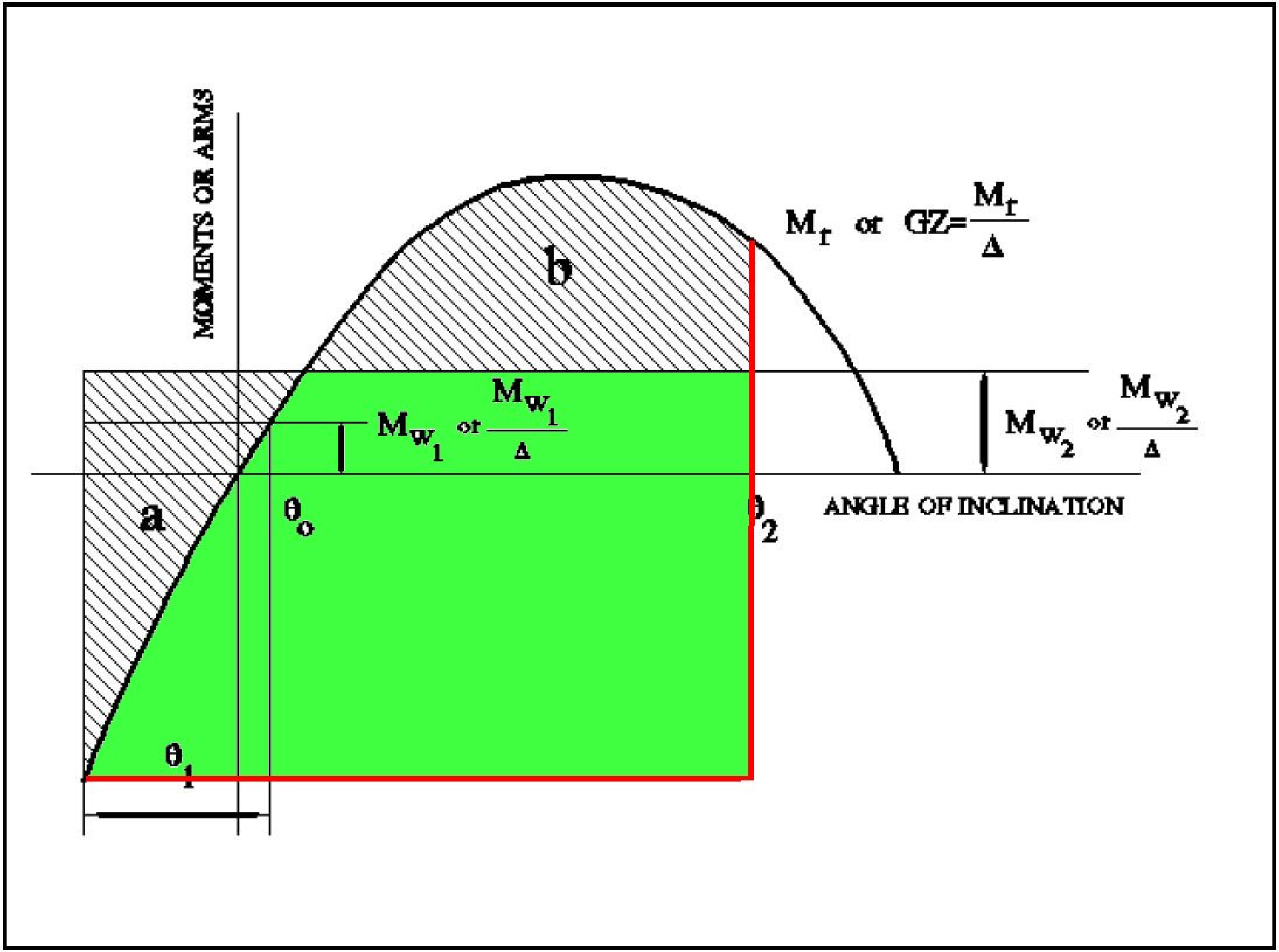


Figure 2.2 - The IMO Weather Criteria

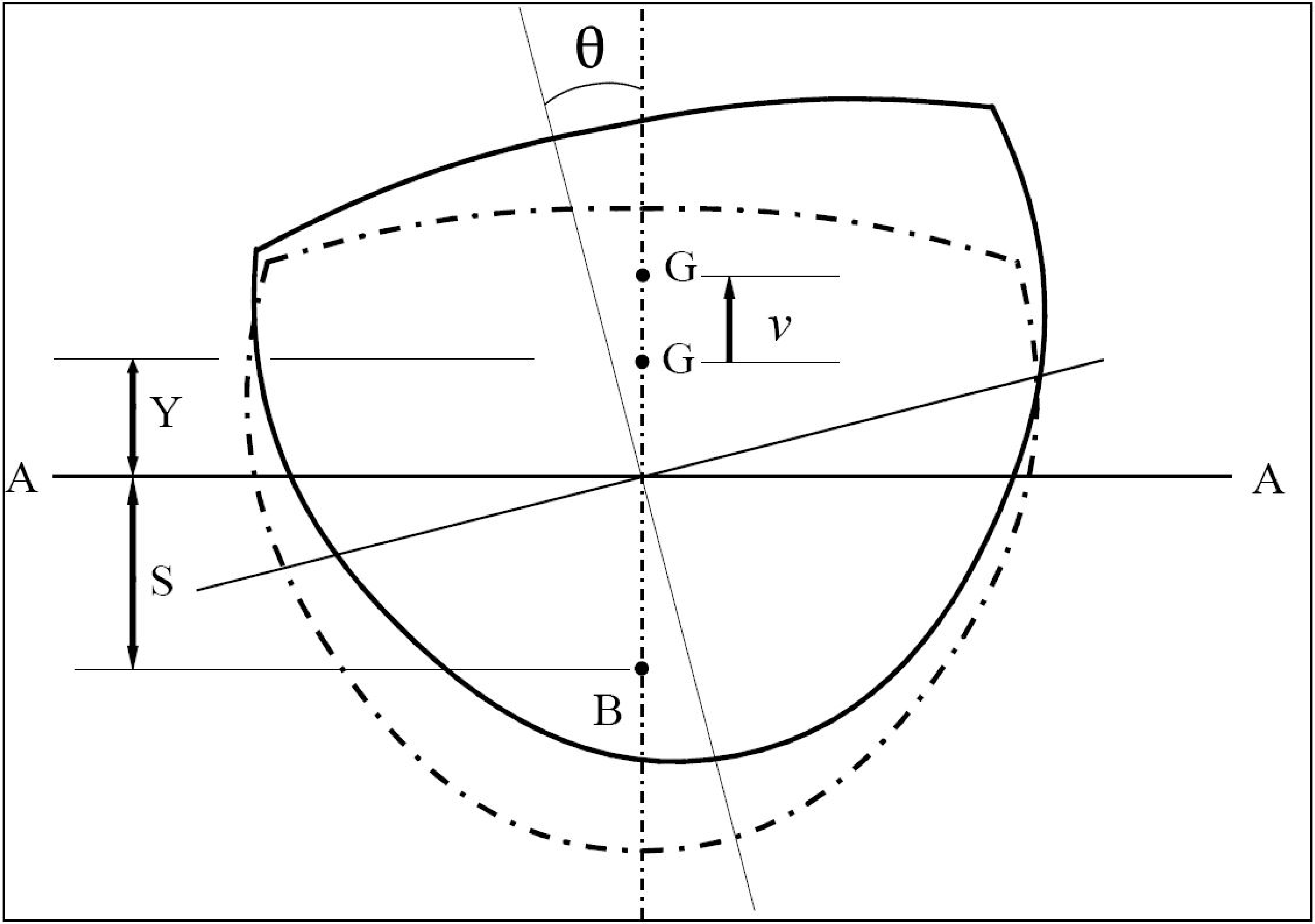


Figure 3.3 - Geometrical parameters for coupled heave-roll motion

As equações SIR (Symmetric Internal Resonance)

$$\frac{2}{R^2}\left(\ddot{y} + 2\zeta R\dot{y}\right) + 2y = x^2$$

$$\ddot{x} + 2\zeta\dot{x} + x - 2xy - b = F\,sin\,\omega t$$

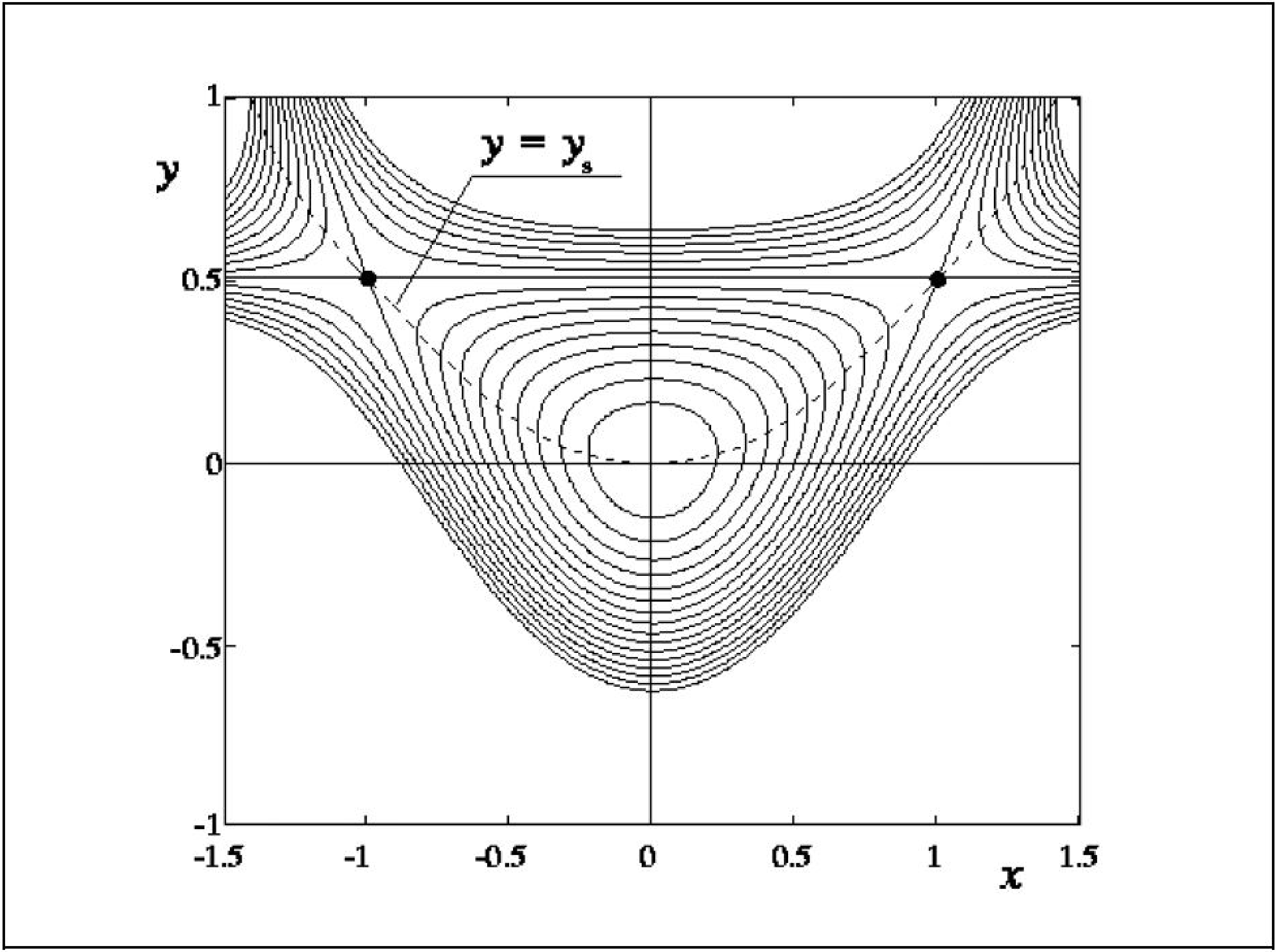


Figure 3.4 - Potential well for heave-roll model

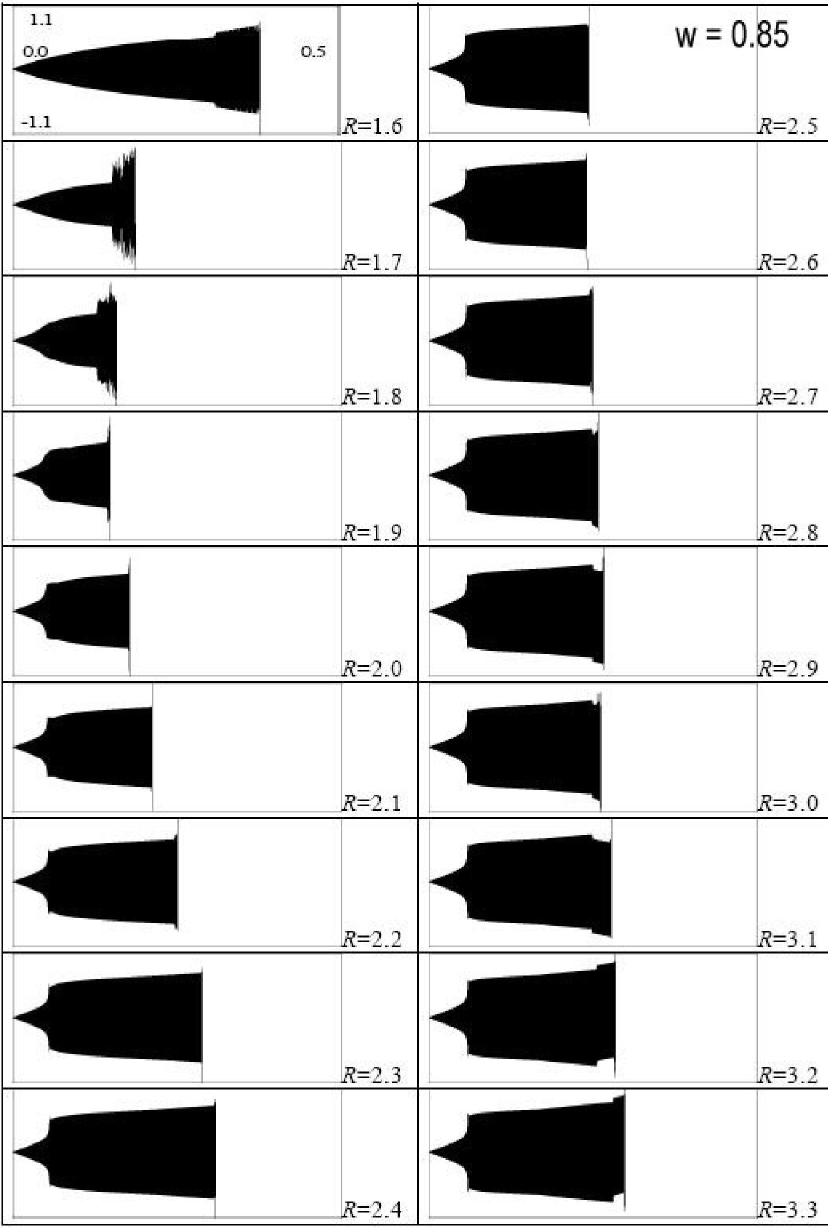


Figure 5.1 - SIR equations: escape under slowly increasing amplitude of forcing. The constant bias parameter b is set to 0, and damping level is given by $\varsigma = 0.05$.

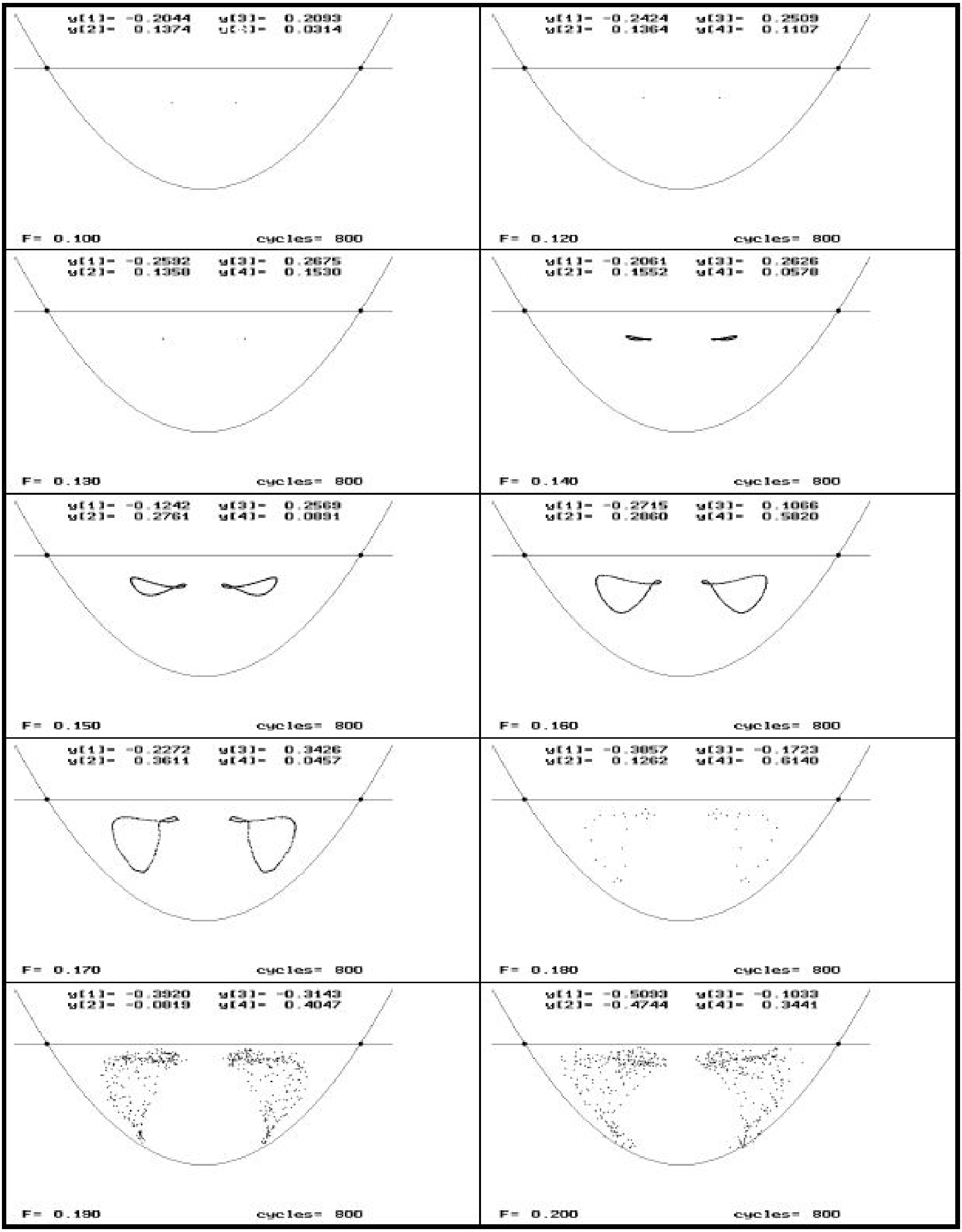


Figure 5.2 - SIR equations selection of attractors in the main sequence: Poincaré points

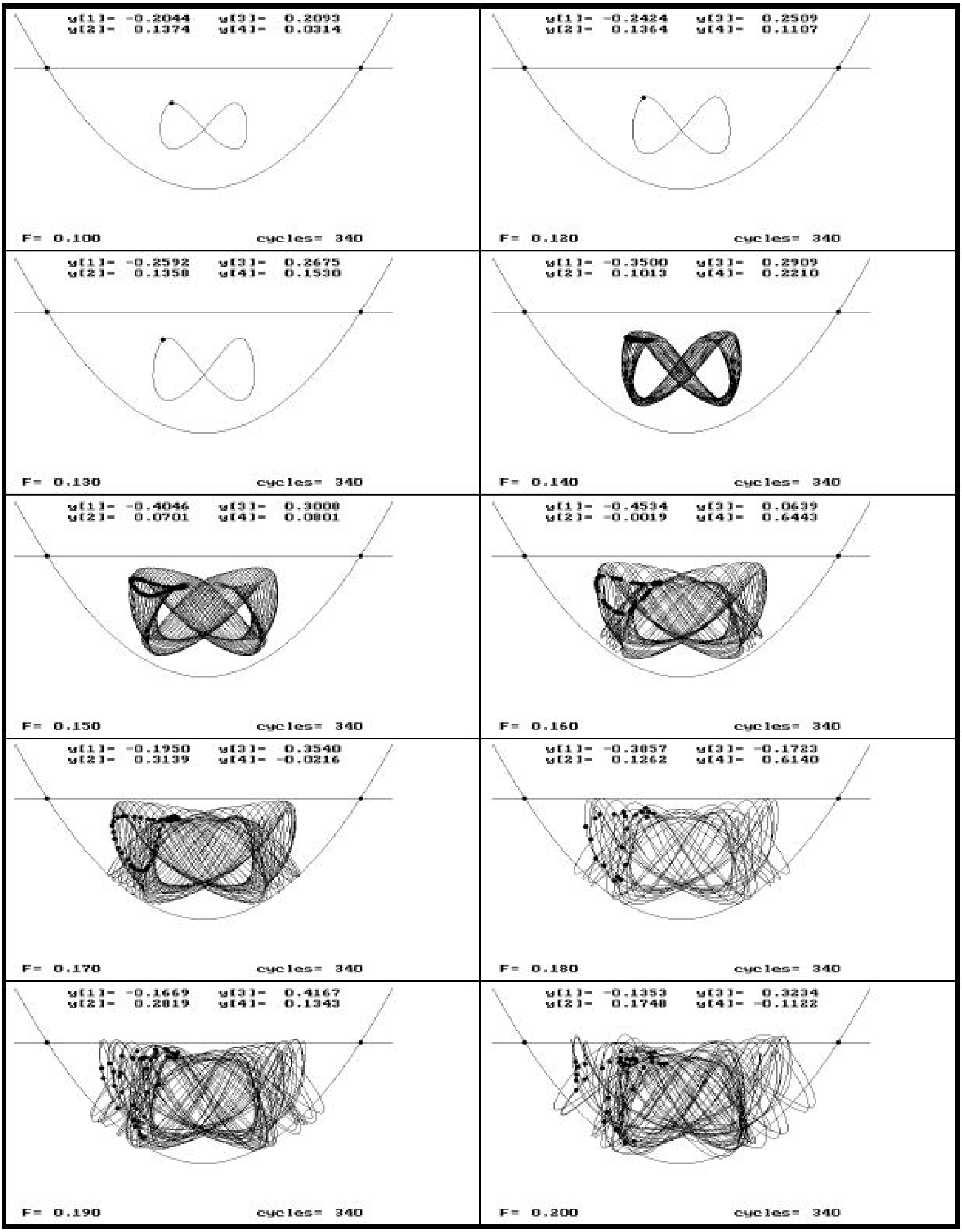


Figure 5.3 - SIR equations selection of attractors in the main sequence: trajectories

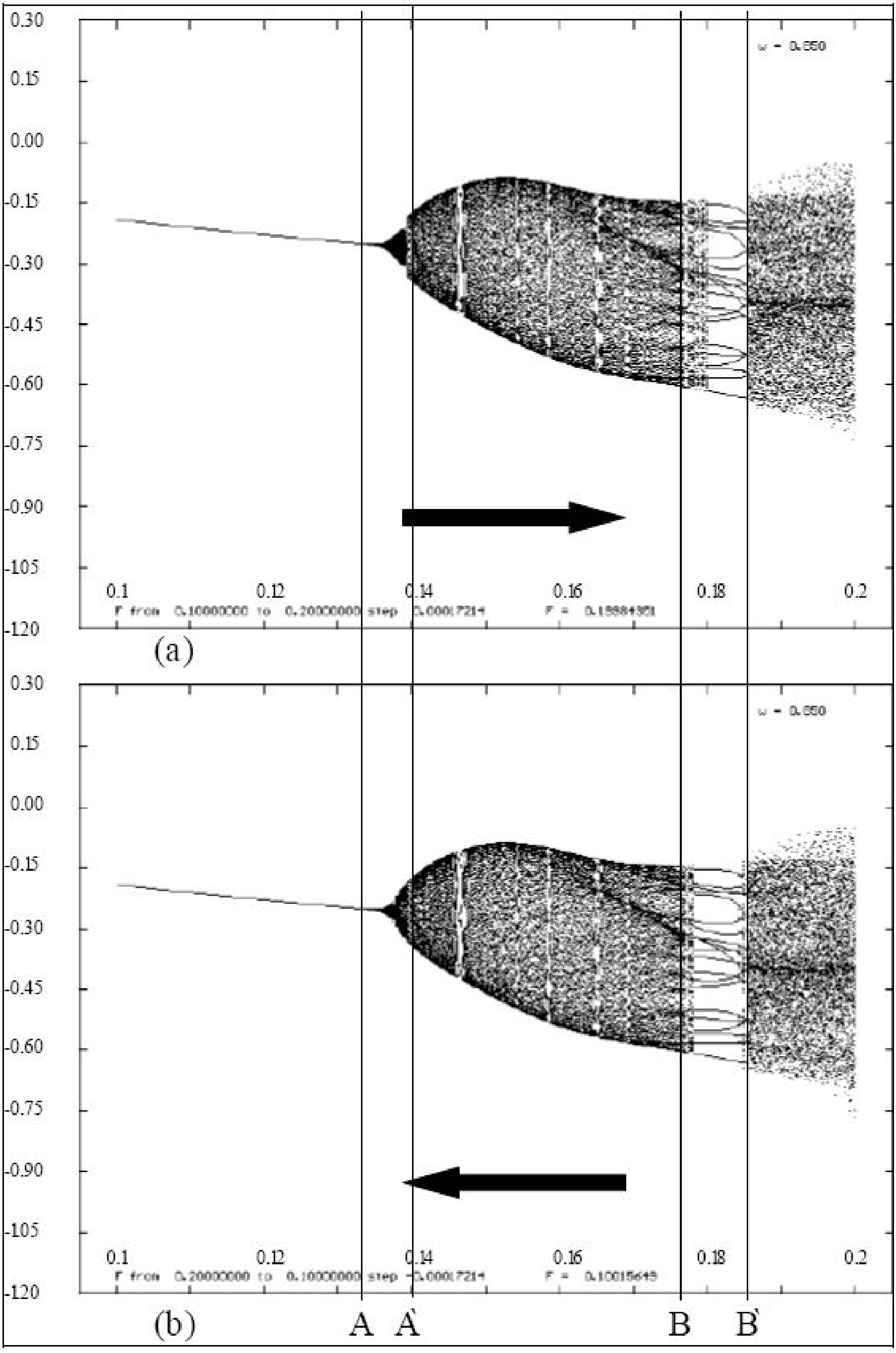


Figure 5.4 - Bifurcation diagrams for SIR equations

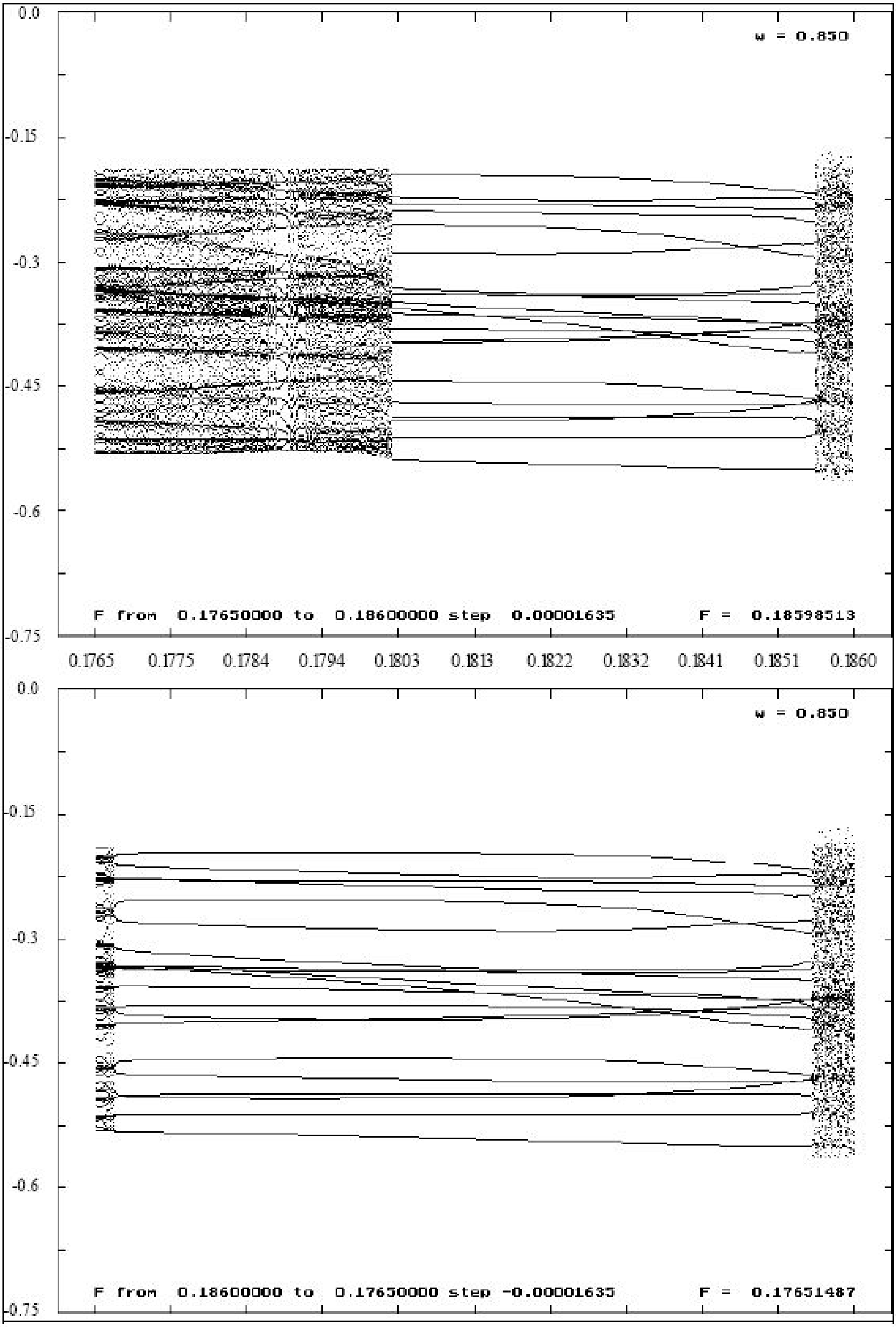


Figure 5.6 - Bifurcation diagrams for SIR equations - detail of region BB'

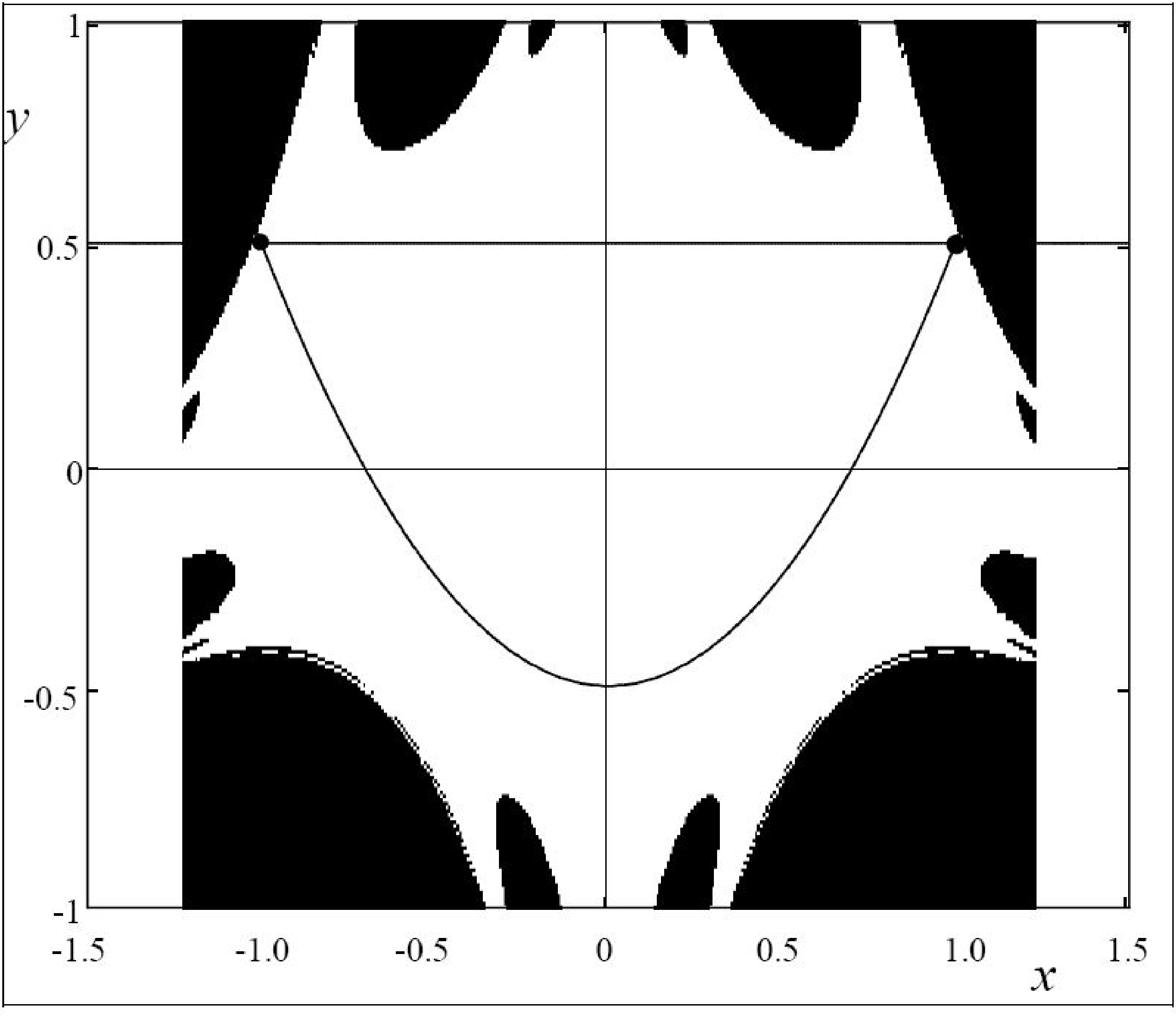


Figure 5.9 - Basin of attraction of the origin for the unforced SIR equations

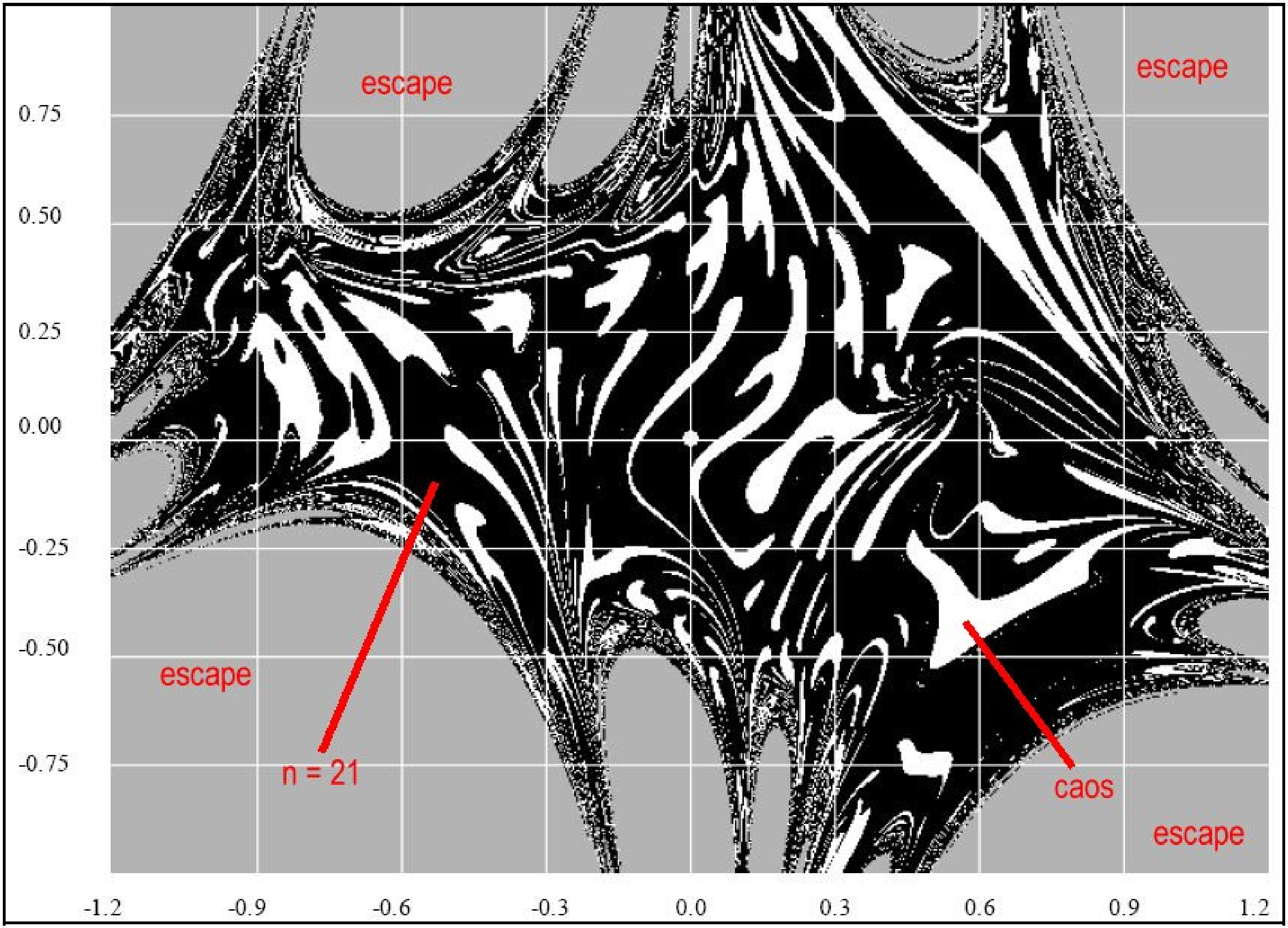


Figure 5.10 - Basins of attraction for the forced SIR equations, $F = 0.18,\ \omega = 0.85$

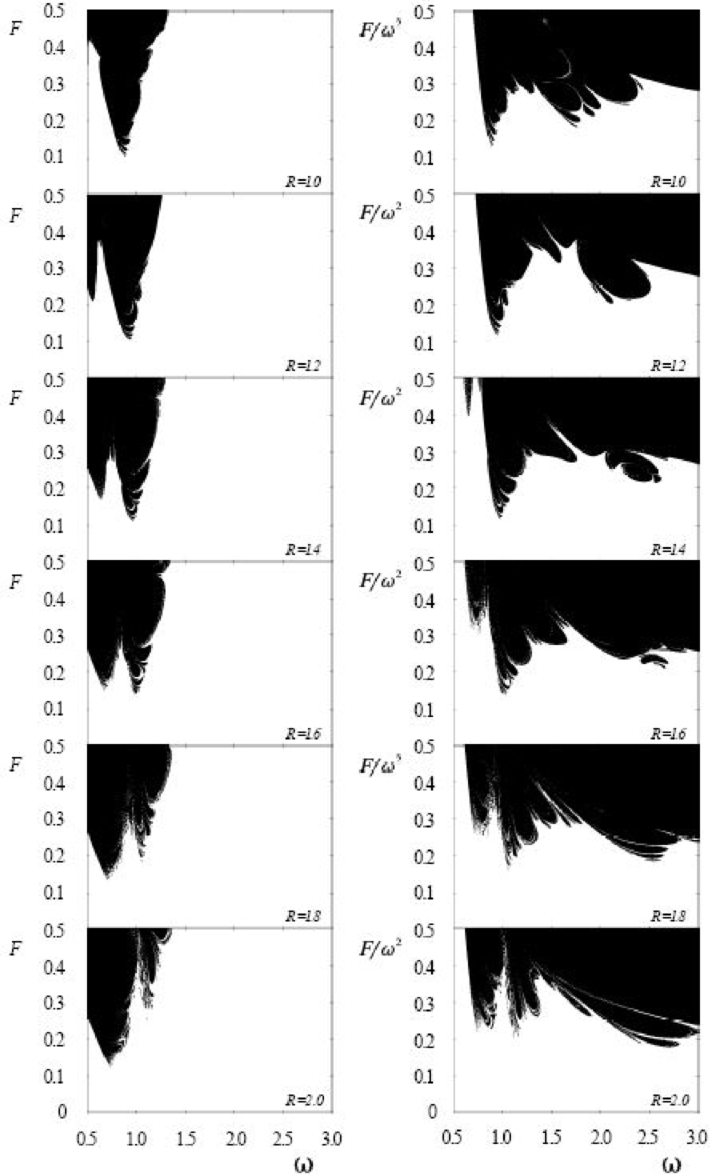
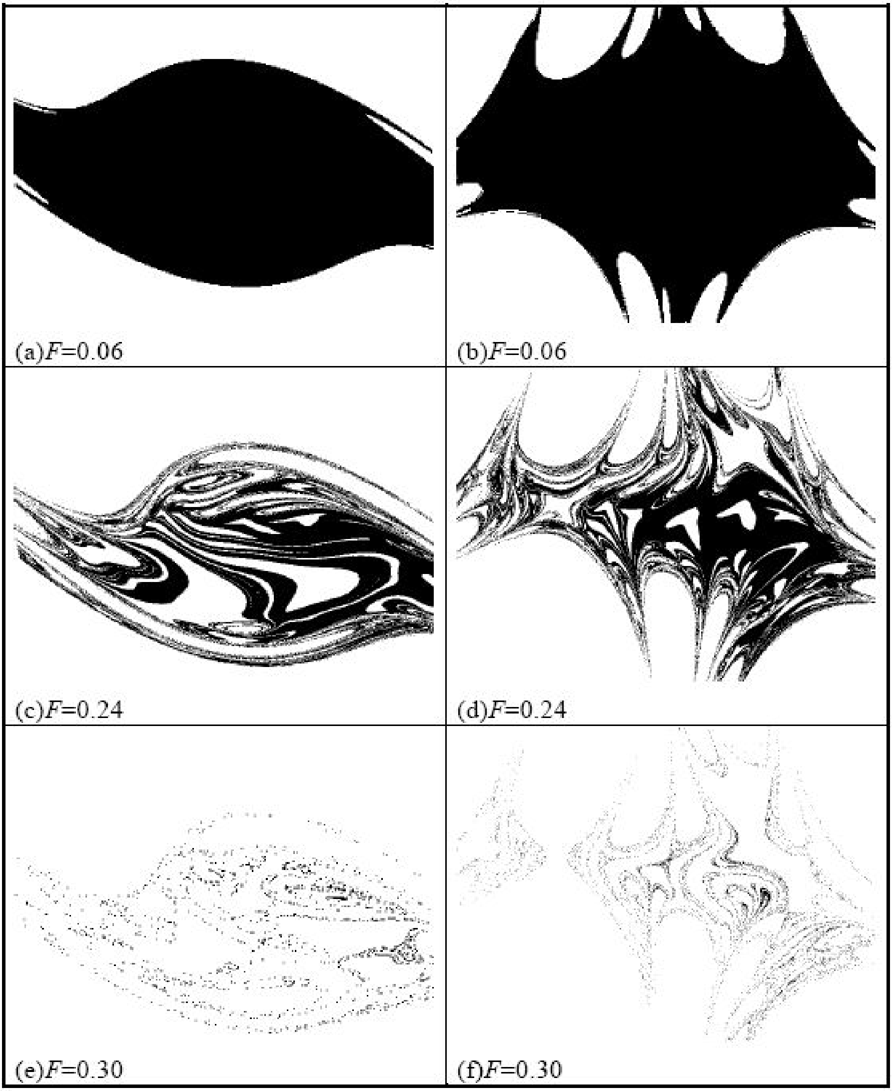


Figure 5-14 - SIR model: safe basins in the control-parameter spaces of (F, ω) (left column), and $(F/\omega^2, \omega)$ (right column) for a range of R values

$$\ddot{x} + \beta\dot{x} + x - x^3 = F\,sin(\omega t) \qquad 4.11$$

PARAMETERS OF THE MODEL	$\omega = 0.85,\ \varsigma = 0.05,\ R = 1.7$
PARAMETERS OF THE GRID	$(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}, y_{inf}, y_{sup}, \dot{y}_{inf}, \dot{y}_{sup}) = (-1.1, 1.1, -1.0, 1.0, -0.6, 0.6, -0.5, 0.5)$ $(N_1, N_2, N_3, N_4) = (320, 1, 240, 1)$
NUMERICAL PARAMETERS	$p = T/80,\ m = 8,\ d_{esc} = 1.2$



(a) F=0.06 (b) F=0.06

(c) F=0.24 (d) F=0.24

(e) F=0.30 (f) F=0.30

Figure 5.17 - Evolution of safe basins for the SIR equations with $\omega = 0.85$

Table 5.2 - Parameters for figure 5.18

PARAMETERS OF THE MODEL	$\omega = 0.85, \varsigma = 0.05$
PARAMETERS OF THE GRID	$(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}) = (-1.2, 1.2, -1.0, 1.0)$ $(N_1, N_2) = (640, 480)$
NUMERICAL PARAMETERS	$p = T/80,\ m = 8,\ d_{esc} = 1.2$

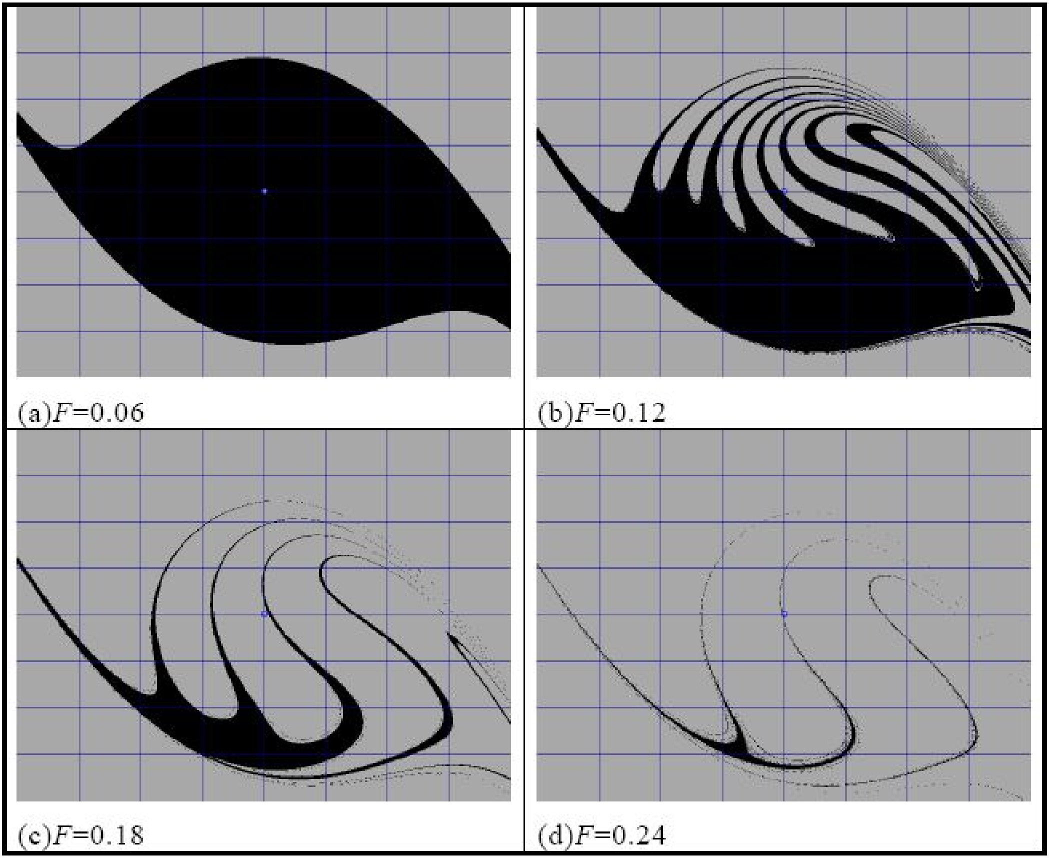
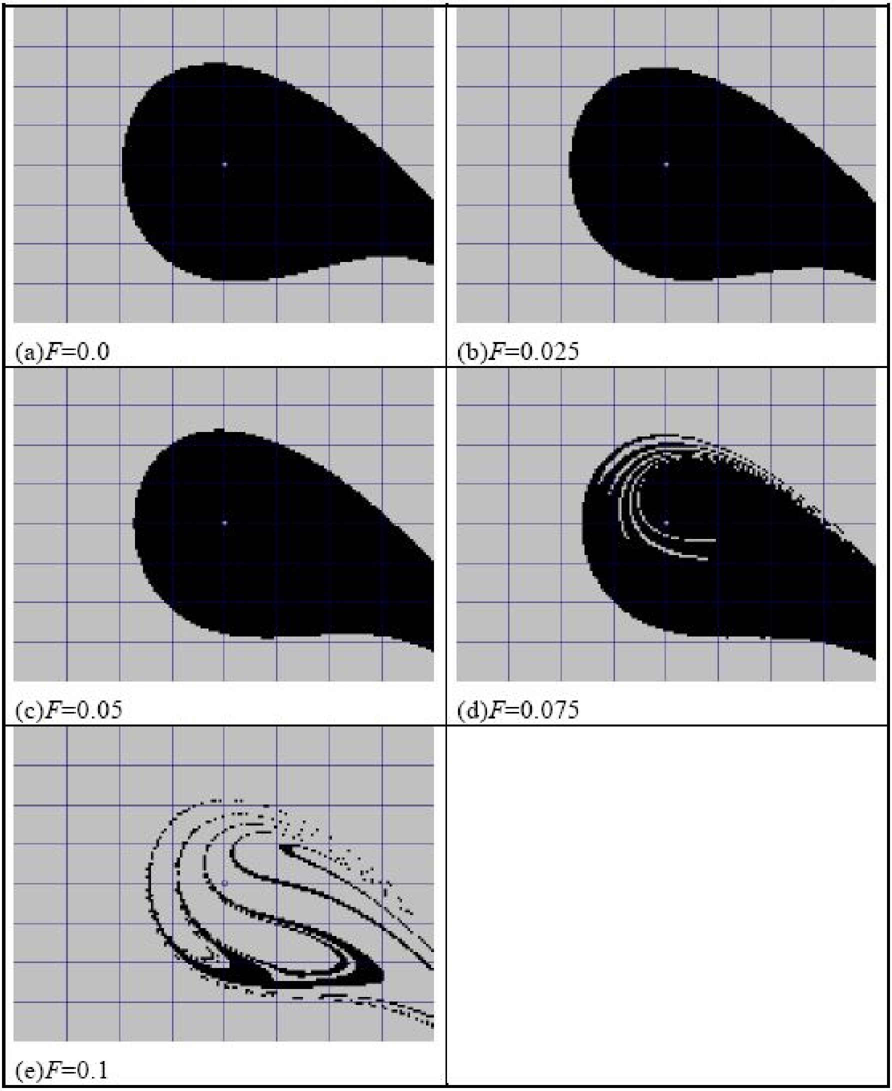


Figure 5.18 - Evolution of safe basins for the roll model (4.11) with $\omega = 0.85$

Parameters of the model	$\zeta = 0.05,\ \omega = 0.85$
Parameters of the grid	$(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}) = (-1.2, 1.2, -1.0, 1.0)$ $(N_1, N_2) = (200, 200)$
Numerical parameters	$p = T/80,\ m = 6,\ d_{esc} = 1.2$



(a) F=0.0 (b) F=0.025 (c) F=0.05 (d) F=0.075 (e) F=0.1

Figure 5.19 - Evolution of safe basins for the escape equation with $\omega = 0.85$

Table 5.4 - Parameters for figure 5.20	
Parameters of the model	$\varsigma = 0.05,\ \omega = 0.55$
Parameters of the grid	$(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}) = (-1.2, 1.2, -1.0, 1.0)$ $(N_1, N_2) = (200, 200)$
Numerical parameters	$p = T/80,\ m = 6,\ d_{esc} = 1.2$

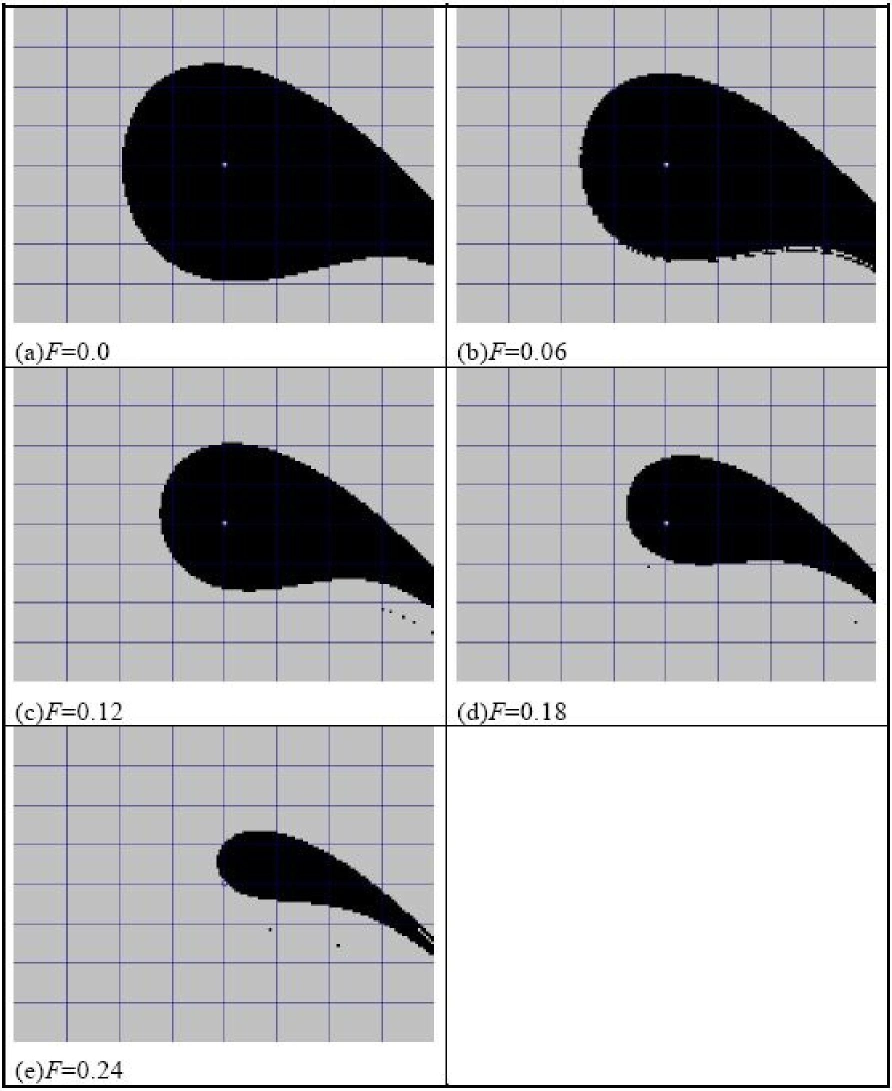
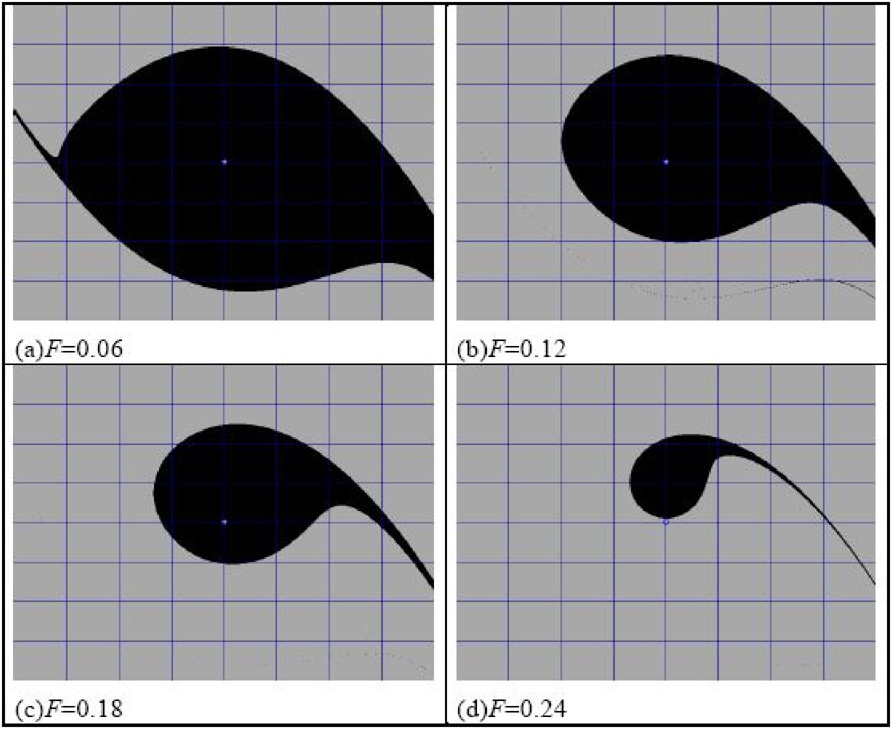


Figure 5.20 - Evolution of safe basins for the escape equation with $\omega = 0.55$

Table 5.5 - Parameters for figure 5.21

Parameters of the model	$\omega = 0.85, \varsigma = 0.05$
Parameters of the grid	$(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}) = (-1.2, 1.2, -1.0, 1.0)$ $(N_1, N_2) = (640, 480)$
Numerical parameters	$p = T/_{80}$, $m = 8$, $d_{esc} = 1.2$



(a) F=0.06

(b) F=0.12

(c) F=0.18

(d) F=0.24

Figure 5.21 - Evolution of safe basins for the roll model (4.11) with $\omega = 0.55$

Table 5.6 - Parameters for figure 5.22	
PARAMETERS OF THE MODEL	$\omega = 0.55,\ \varsigma = 0.05,\ R = 1.7$
PARAMETERS OF THE GRID	$(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}, y_{inf}, y_{sup}, \dot{y}_{inf}, \dot{y}_{sup}) = (-1.1, 1.1, -1.0, 1.0, -0.6, 0.6, -0.5, 0.5)$ $(N_1, N_2, N_3, N_4) = (640, 1, 480, 1)$
NUMERICAL PARAMETERS	$p = T/80,\ m = 8,\ d_{esc} = 1.2$

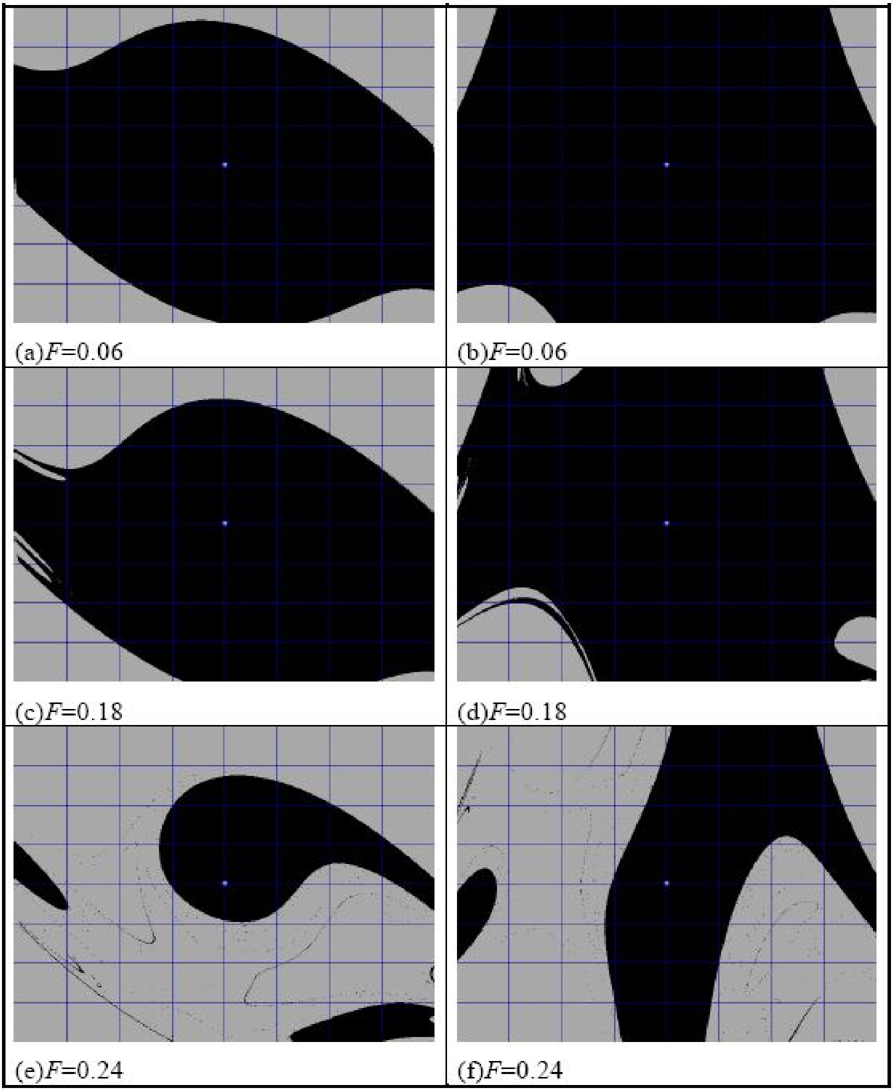


Figure 5.22- Evolution of safe basins for the SIR equations with $\omega = 0.55$

Parameters of the model	$\varsigma = 0.05$, ω as shown
Parameters of the grid	$(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}) = (-1.2, 1.2, -1.0, 1.0)$ $(N_1, N_2) = (200, 200)$
Numerical parameters	$p = T/80$, $m = 6$, $d_{esc} = 1.2$

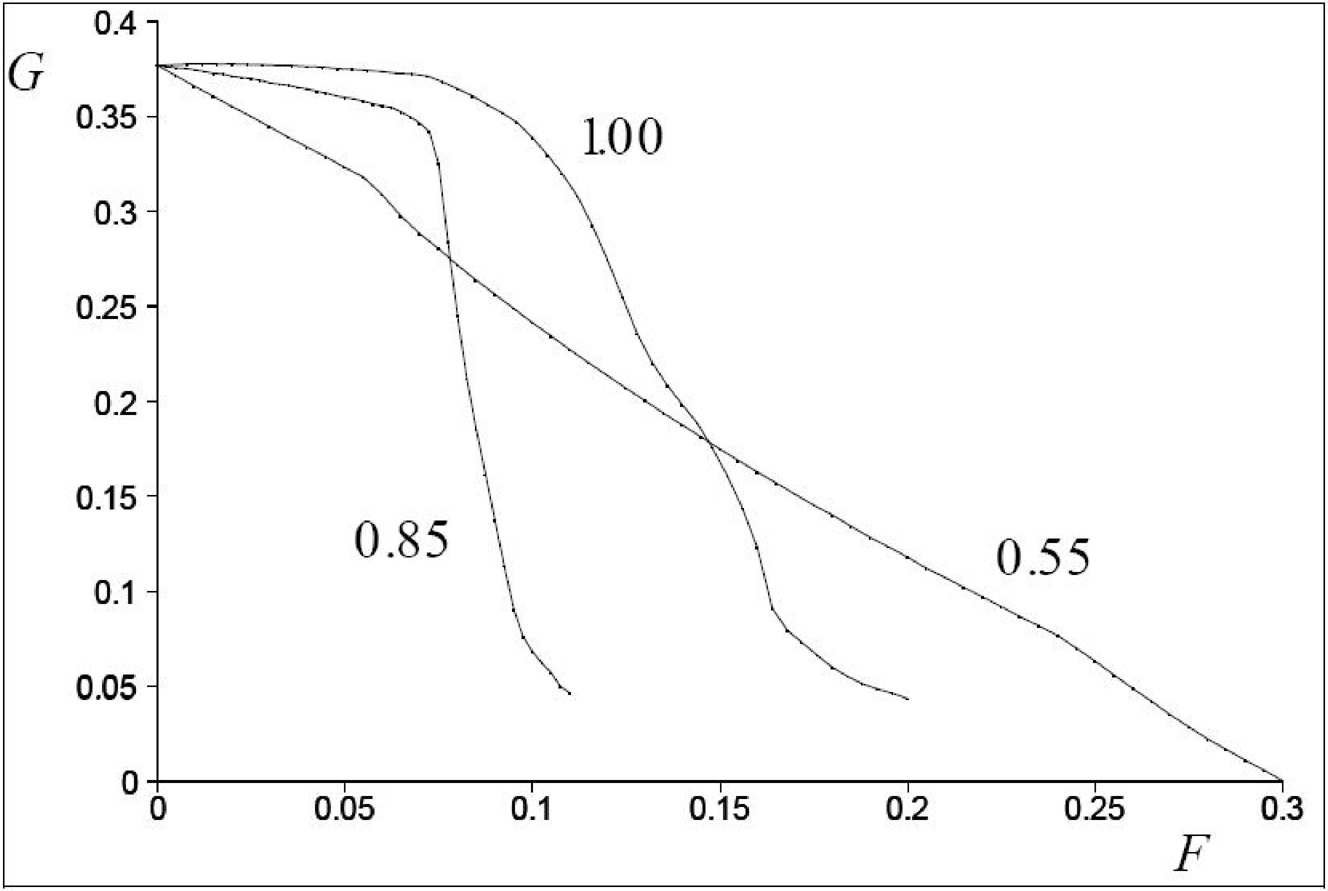


Figure 5.23 - Integrity diagrams for the escape equation (3.8)

Table 5.8 - Parameters for figure 5.24

Parameters of the model	$\omega = 0.85,\ \varsigma = 0.05,\ R$ as shown
Parameters of the grid	$(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}, y_{inf}, y_{sup}, \dot{y}_{inf}, \dot{y}_{sup}) = (-1.1, 1.1, -1.0, 1.0, -0.6, 0.6, -0.5, 0.5)$ $(N_1, N_2, N_3, N_4) = (10, 10, 10, 10)$
Numerical parameters	$p = T/80,\ m = 8,\ d_{esc} = 1.2$

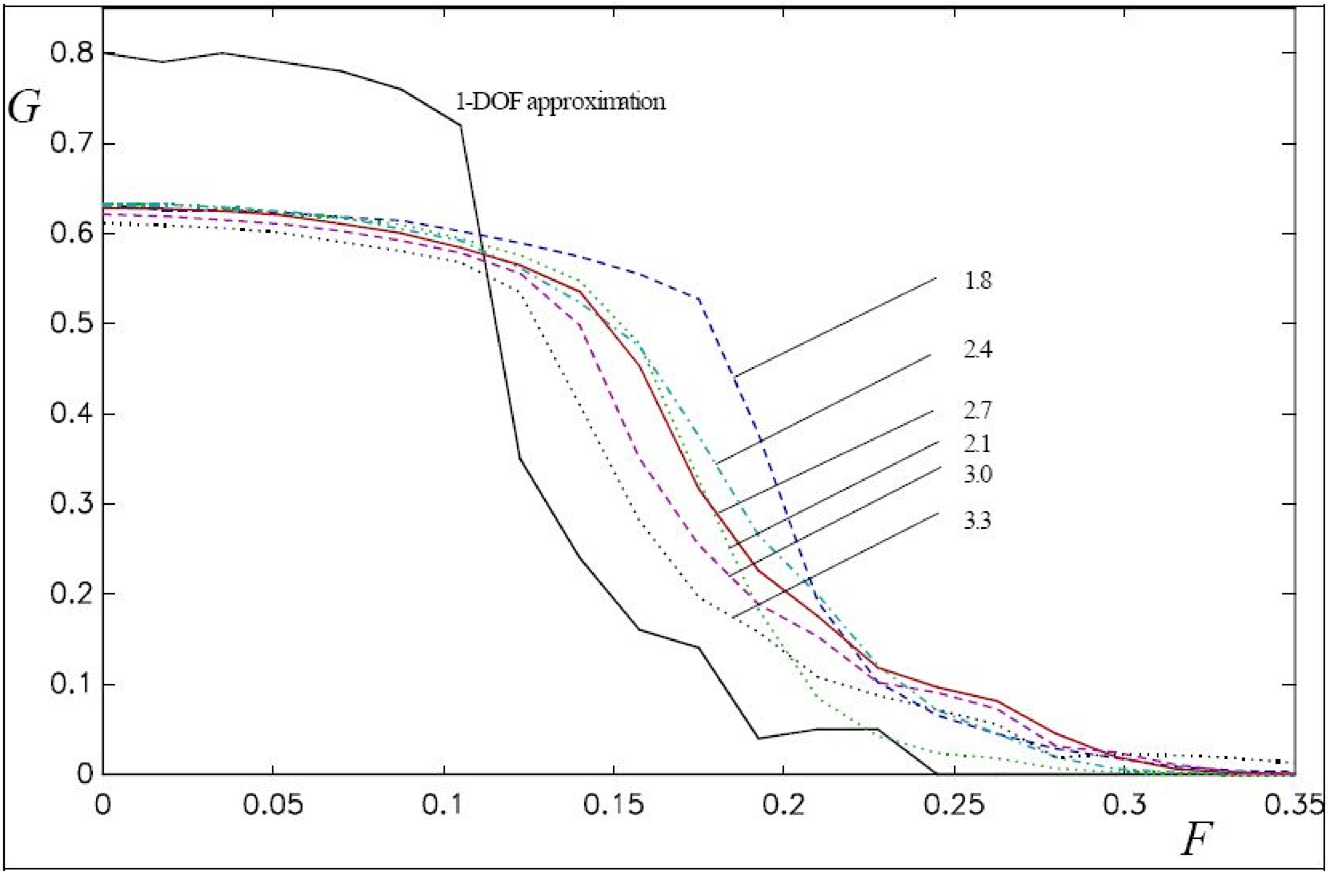


Figure 5.24 - Integrity diagrams for the SIR equations with varying R

Table 5.9 - Parameters for figure 5.25	
PARAMETERS OF THE MODEL	$\varsigma = 0.05$, $R = 1.7$, ω as shown
PARAMETERS OF THE GRID	$(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}, y_{inf}, y_{sup}, \dot{y}_{inf}, \dot{y}_{sup}) = (-1.1, 1.1, -1.0, 1.0, -0.6, 0.6, -0.5, 0.5)$ $(N_1, N_2, N_3, N_4) = (10,10,10,10)$
NUMERICAL PARAMETERS	$p = T/80$, $m = 8$, $d_{esc} = 1.2$

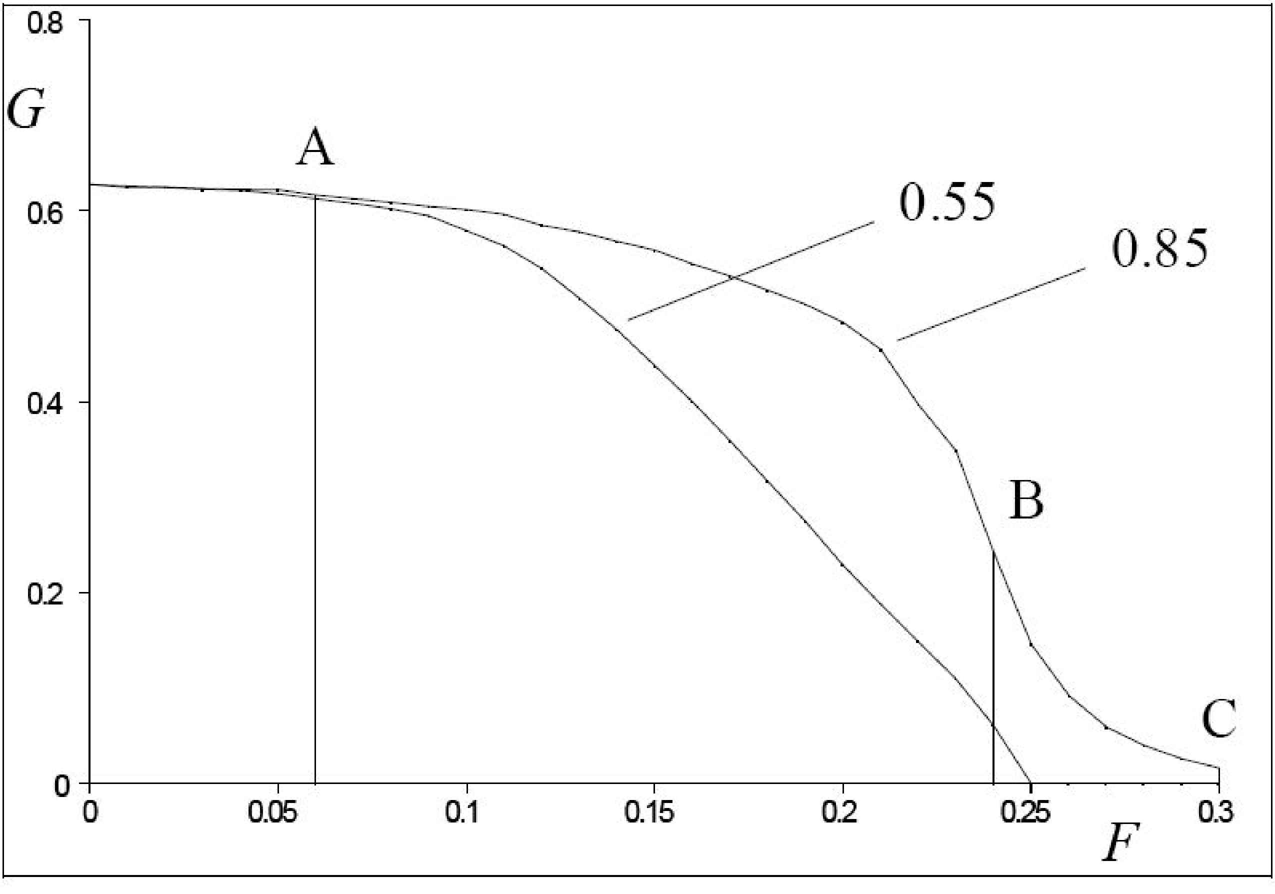


Figure 5.25 - Integrity diagram for the SIR equations with $R = 1.7$

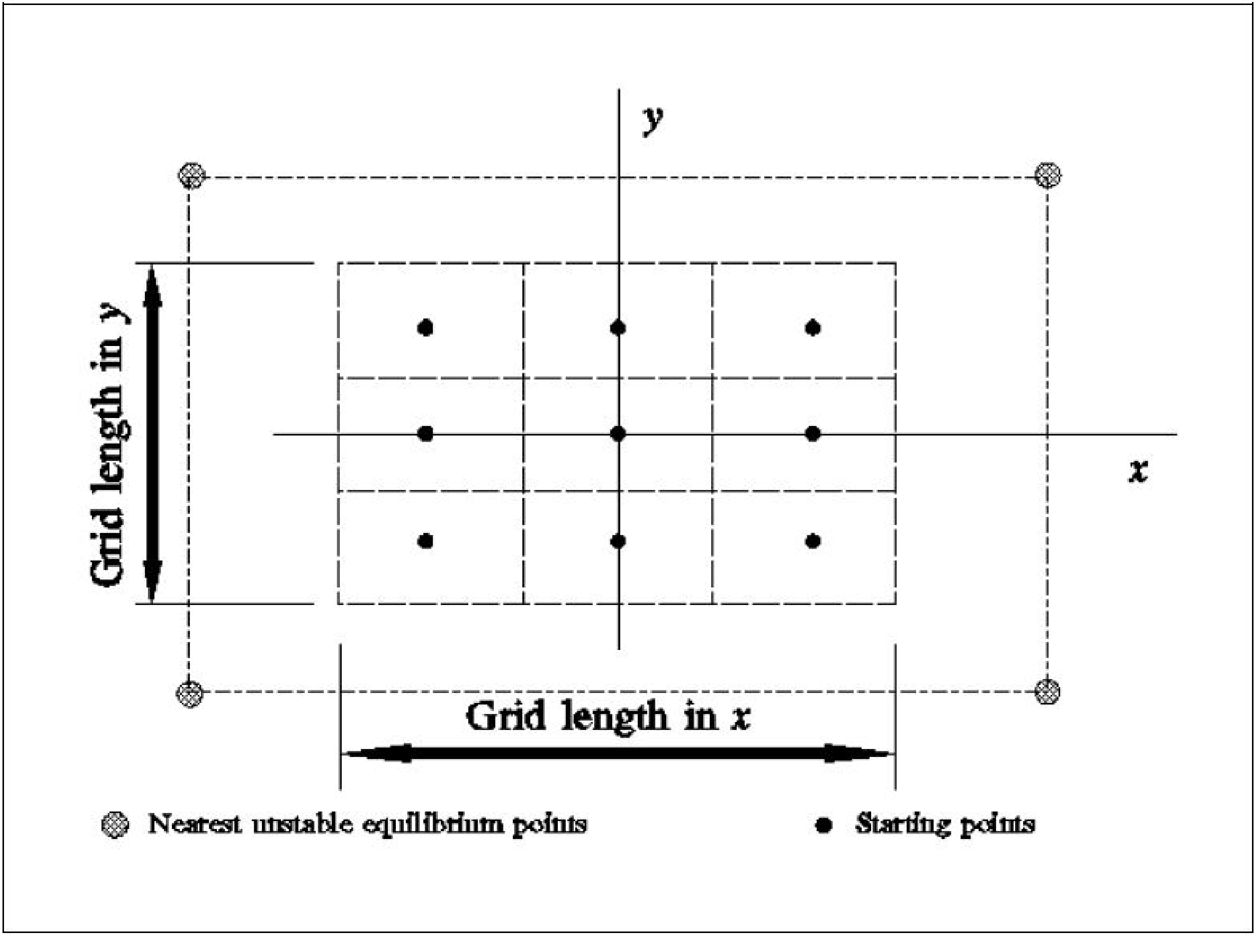


Figure 6.4 - Example of a two-dimensional grid-of-starts

$$\ddot{x} + \beta\dot{x} + x - x^2 = F\sin(\omega t + \delta) \quad 6.13$$

The symmetric single well roll model (4.11) (a Duffing equation):

$$\ddot{x} + \beta\dot{x} + x - x^3 = F\sin(\omega t + \delta) \quad 6.14$$

An asymmetric double-well model with quadratic and cubic nonlinearities from Virgin *et al* (1992):

$$\ddot{x} + \beta\dot{x} + \omega_0{}^2 x + \alpha_2 x^2 + \alpha_3 x^3 = F\sin(\omega t + \delta) \quad 6.15$$

A symmetric single-well roll model with cubic and fifth order nonlinearities from Soliman and Thompson (1991):

$$\ddot{x} + \beta_1\dot{x} + \beta_2|\dot{x}|\dot{x} + c_1 x + c_3 x^3 + c_5 x^5 = F\sin(\omega t) \quad 6.16$$

A symmetric two-dimensional potential well (2-DOF system) from Virgin *et al* (1992):

$$\begin{aligned} \ddot{x}_1 + \beta\dot{x}_1 + x_1 - 4x_1 x_2{}^2 &= F\sin(\omega t) \\ \ddot{x}_2 + \beta\dot{x}_2 + 2.25x_2 - 4x_1{}^2 x_2 &= F\sin(\omega t) \end{aligned} \quad 6.17$$

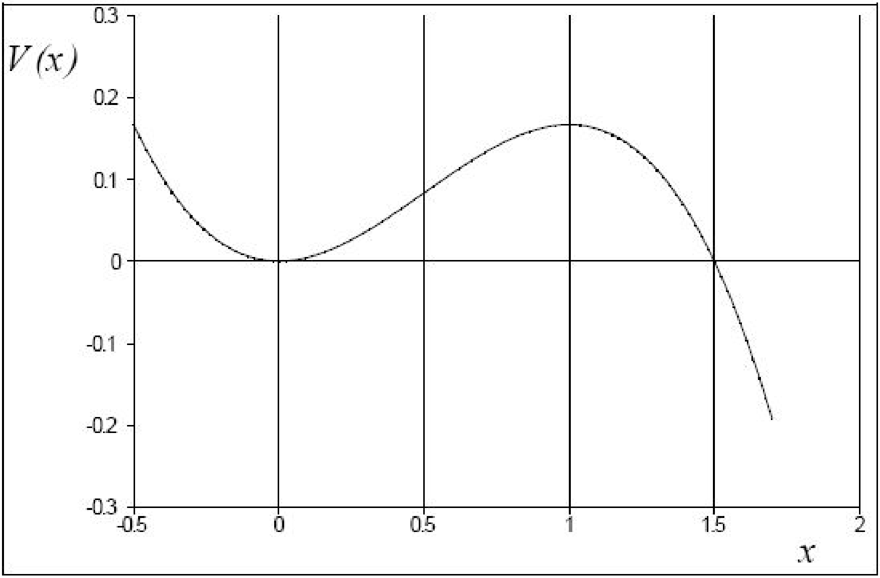


Figure 6.5 - Potential well for equation (6.13)

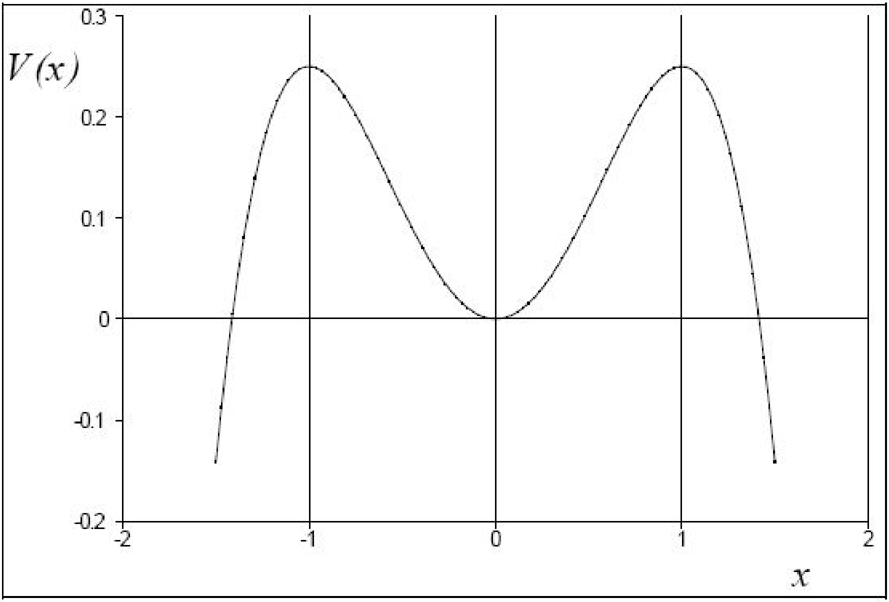


Figure 6.6 - Potential well for equation (6.14)

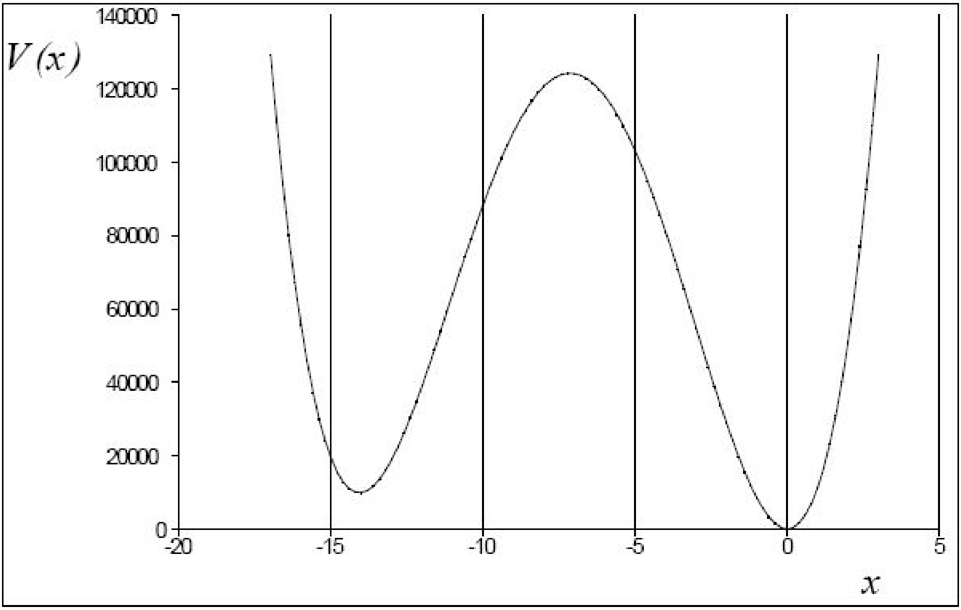


Figure 6.7 - Potential well for equation (6.15)

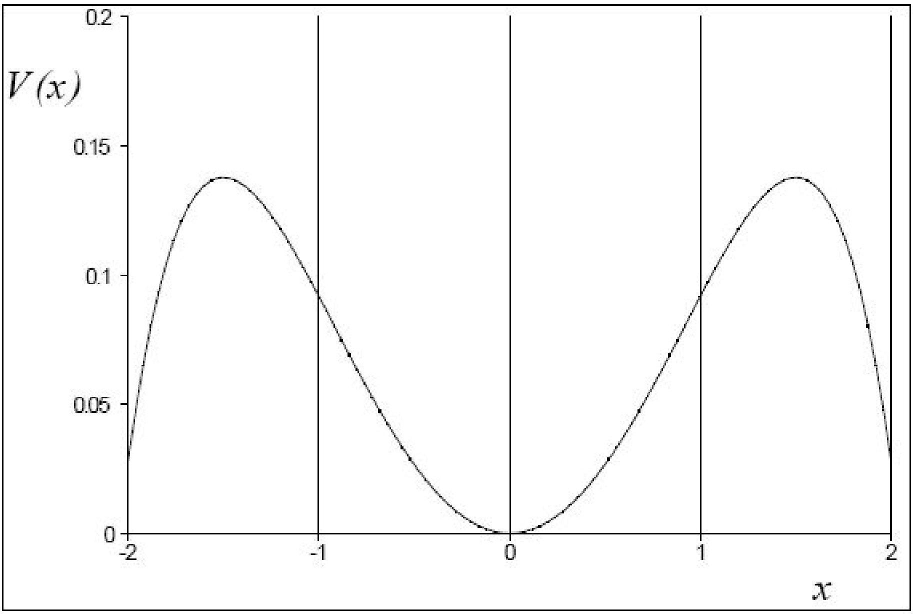


Figure 6.8 - Potential well for equation (6.16)

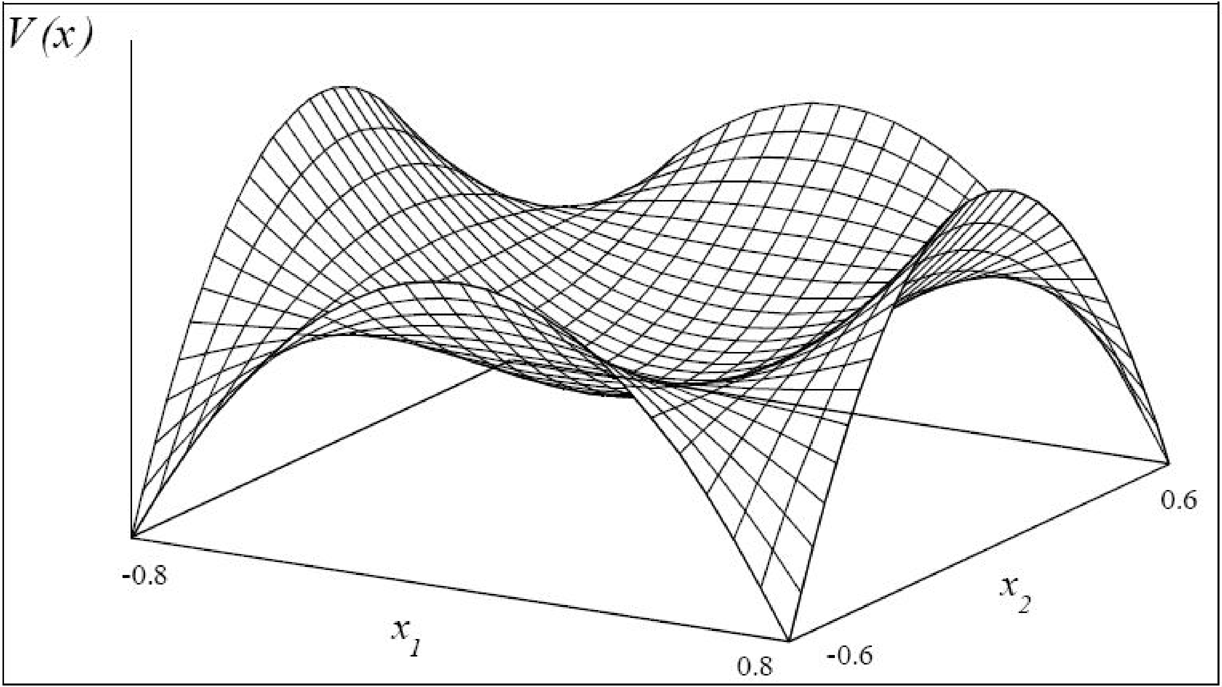


Figure 6.9 - Potential well for equation (6.17)

Table 6.1 - Main features of selected systems

FEATURE → EQUATION ↓	REFERENCE	PARAMETERS	STARTING POINTS	MAXIMUM TRANSIENT DURATION	ESCAPE CRITERION
(6.13)	Thompson (1989a)	$\beta = 0.1$	$x = \dot{x} = 0$	32 T	$x \geq 20$
(6.13)	Virgin *et al* (1992)	$\beta = 0.1$	$x = \dot{x} = 0$	30 T	$x > 1$
(6.14)	Virgin *et al* (1992)	$\beta = 0.1$	$x = \dot{x} = 0$	30 T	$\lvert x \rvert > 1$
(6.14)	Kan (1992)	$\beta = 0.04455$	$x = \dot{x} = 0$	20 T	$\lvert x \rvert > 2$
(6.15)	Virgin *et al* (1992)	$\beta = 0.1$ $\omega_0 = 139.93$ $\alpha_2 = 4132.7$ $\alpha_3 = 194.82$	$x = \dot{x} = 0$	30 T	$x < -7.143$
(6.16)	Soliman and Thompson (1991)	$\beta_1 = 0.0555$ $\beta_2 = 0.1659$ $c_1 = 0.2227$ $c_3 = -0.0694$ $c_5 = -0.0131$	$x = \dot{x} = 0$	16 T	$\lvert x \rvert > 1.57$
(6.17)	Virgin *et al* (1992)	$\beta = 0.1$	$x_1 = x_2 = \dot{x}_1 = \dot{x}_2 = 0$	30 T	$\sqrt{x_1^{\;2} + x_2^{\;2}} > 0.901$

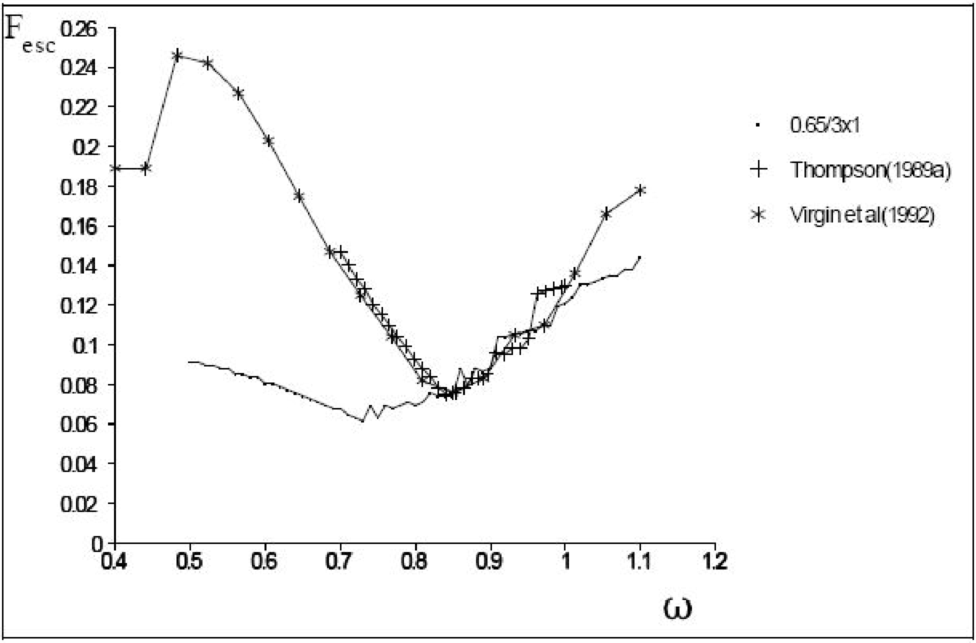


Figure 6.16 - Boundaries of safe motion: equation (6.13)

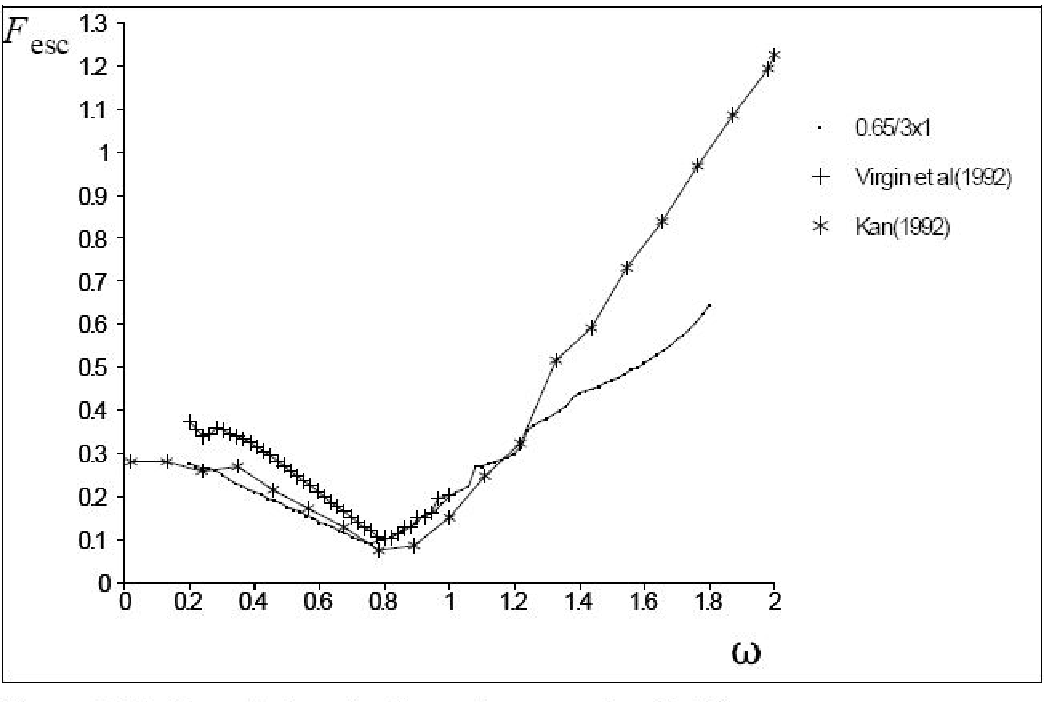


Figure 6.17 - Boundaries of safe motion: equation (6.14)

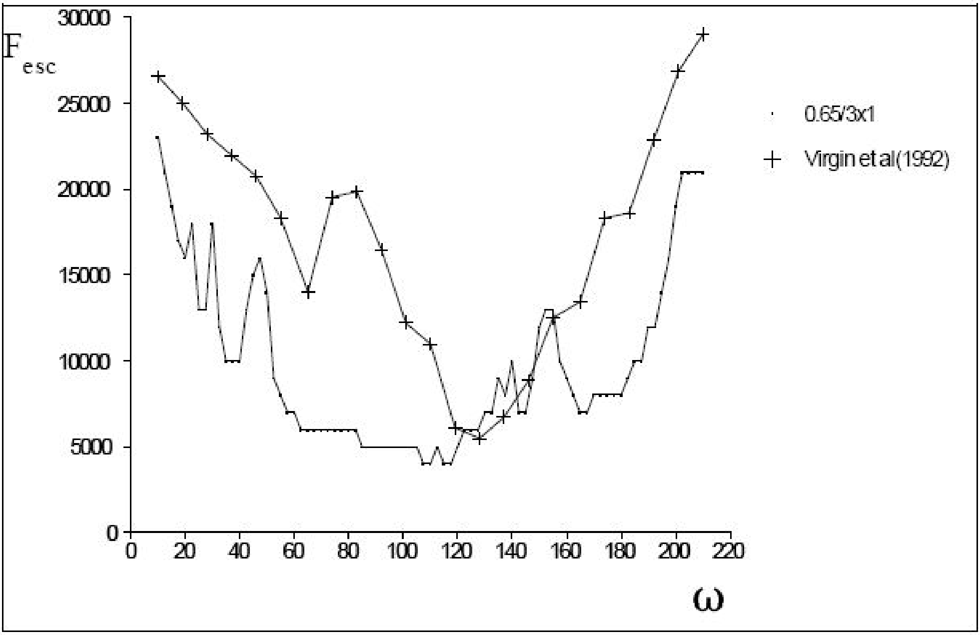


Figure 6.18 - Boundaries of safe motion: equation (6.15)

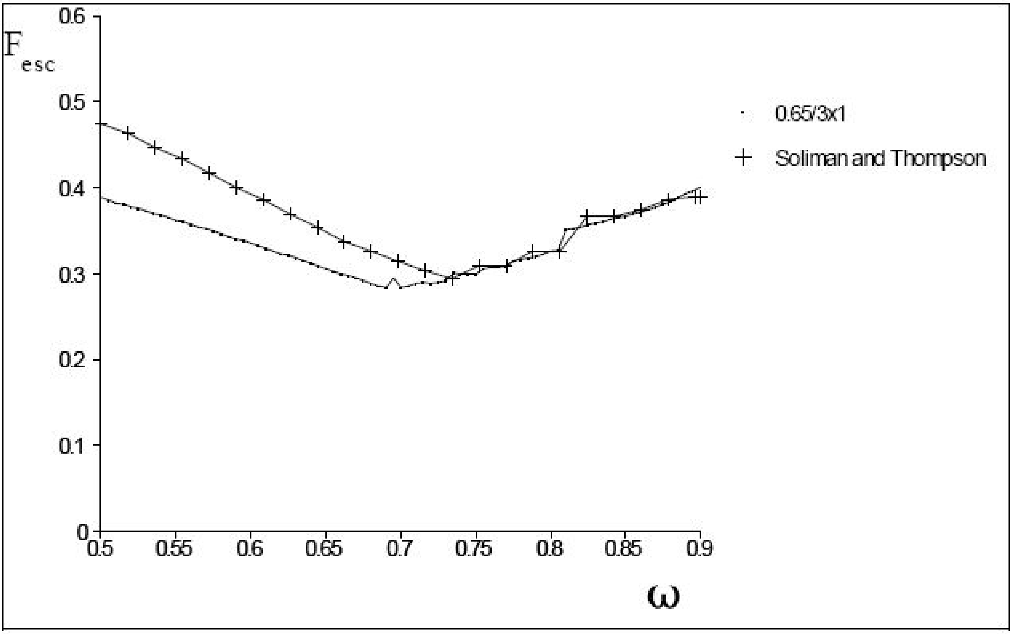


Figure 6.19 - Boundaries of safe motion: equation (6.16)

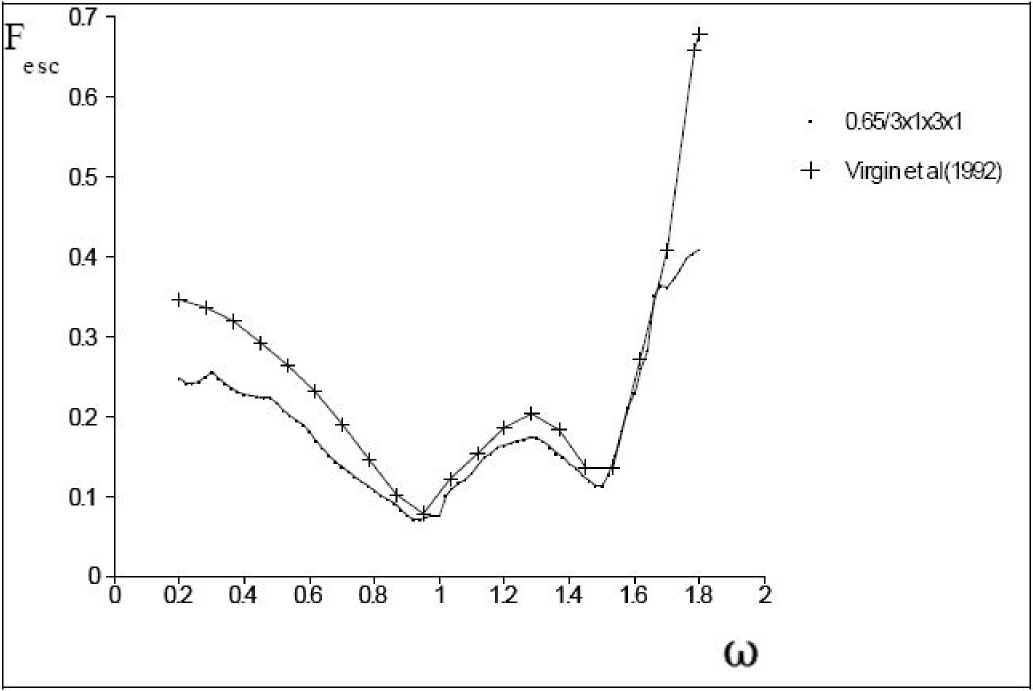


Figure 6.20 - Boundaries of safe motion: equation (6.17)

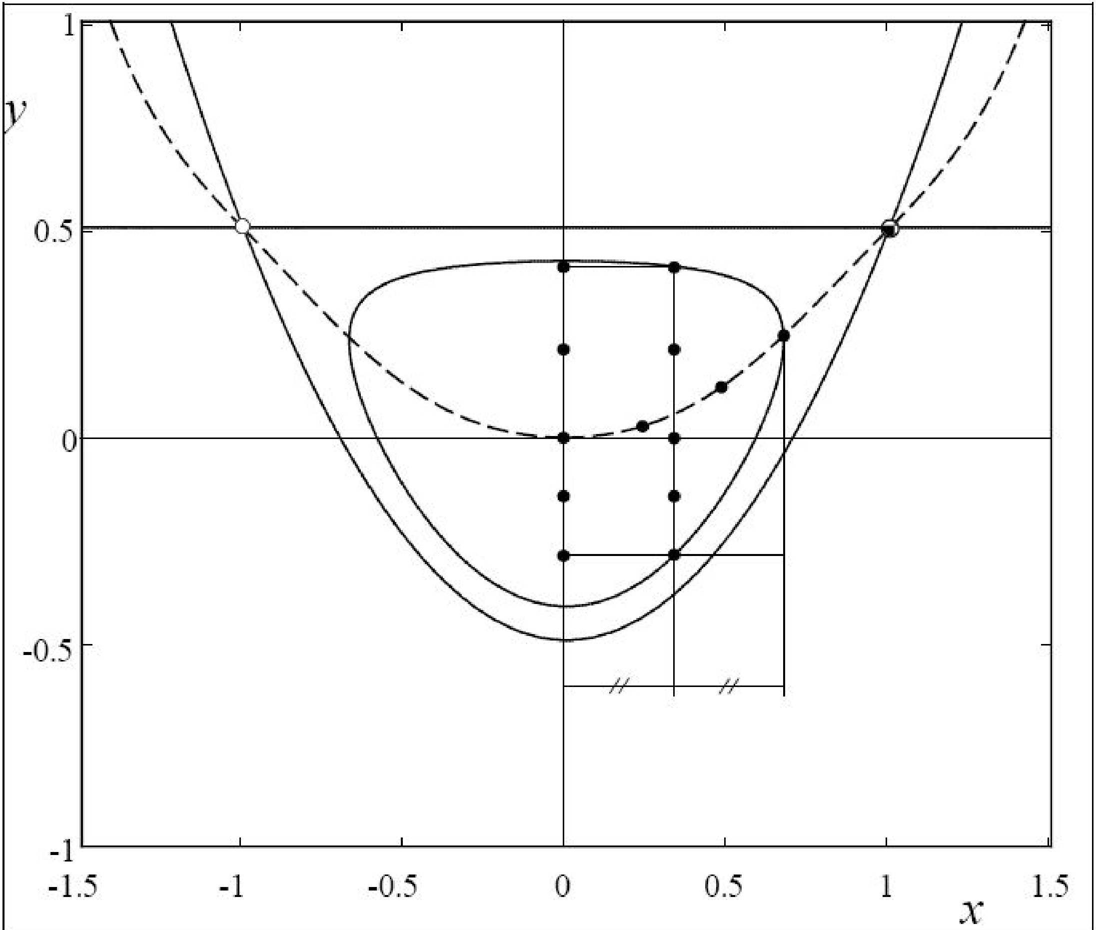


Figure 6.21 - One- and two-dimensional grids for the SIR model

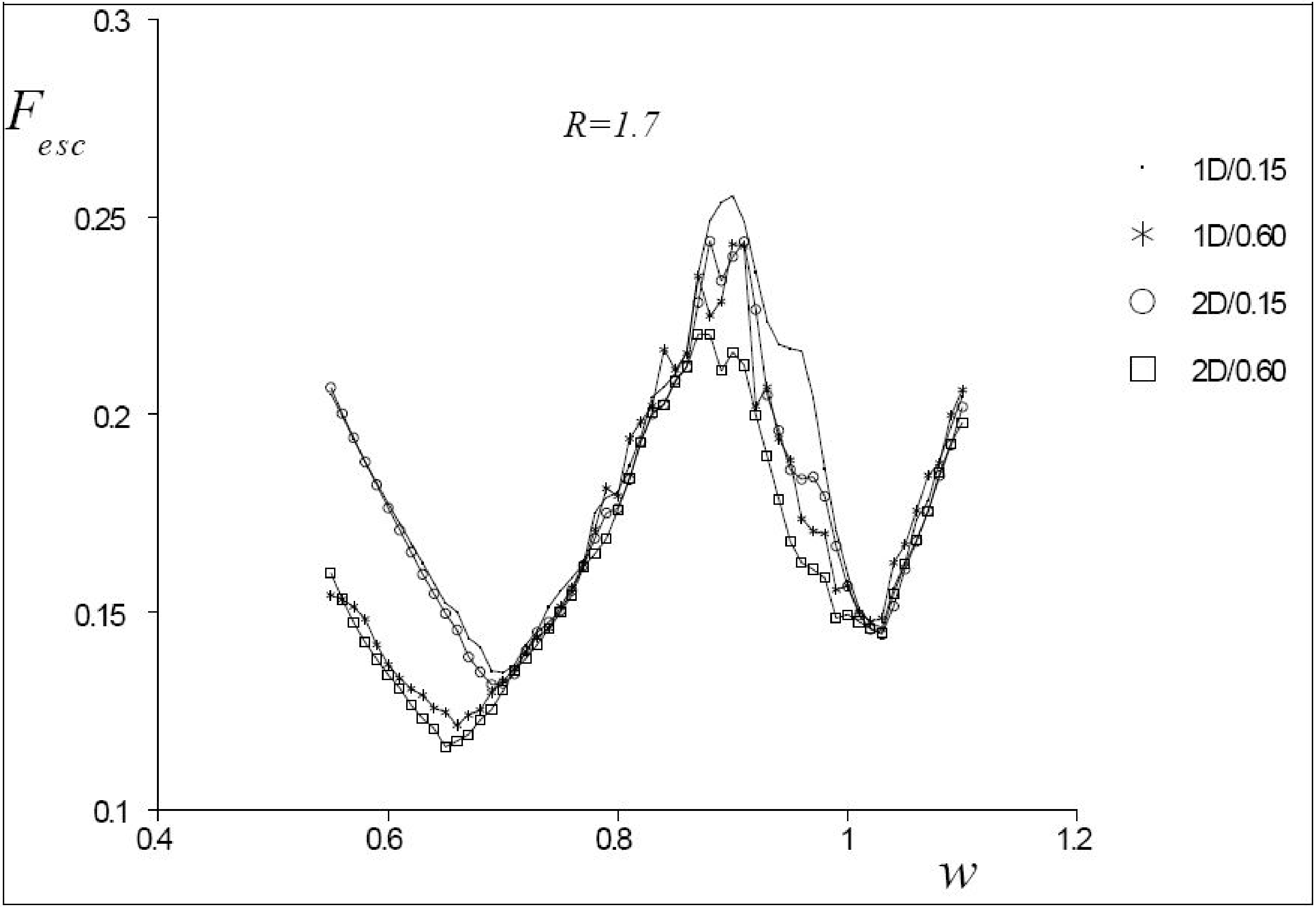


Figure 6.22 - Boundaries of safe motion for the SIR model: 1D and 2D grids

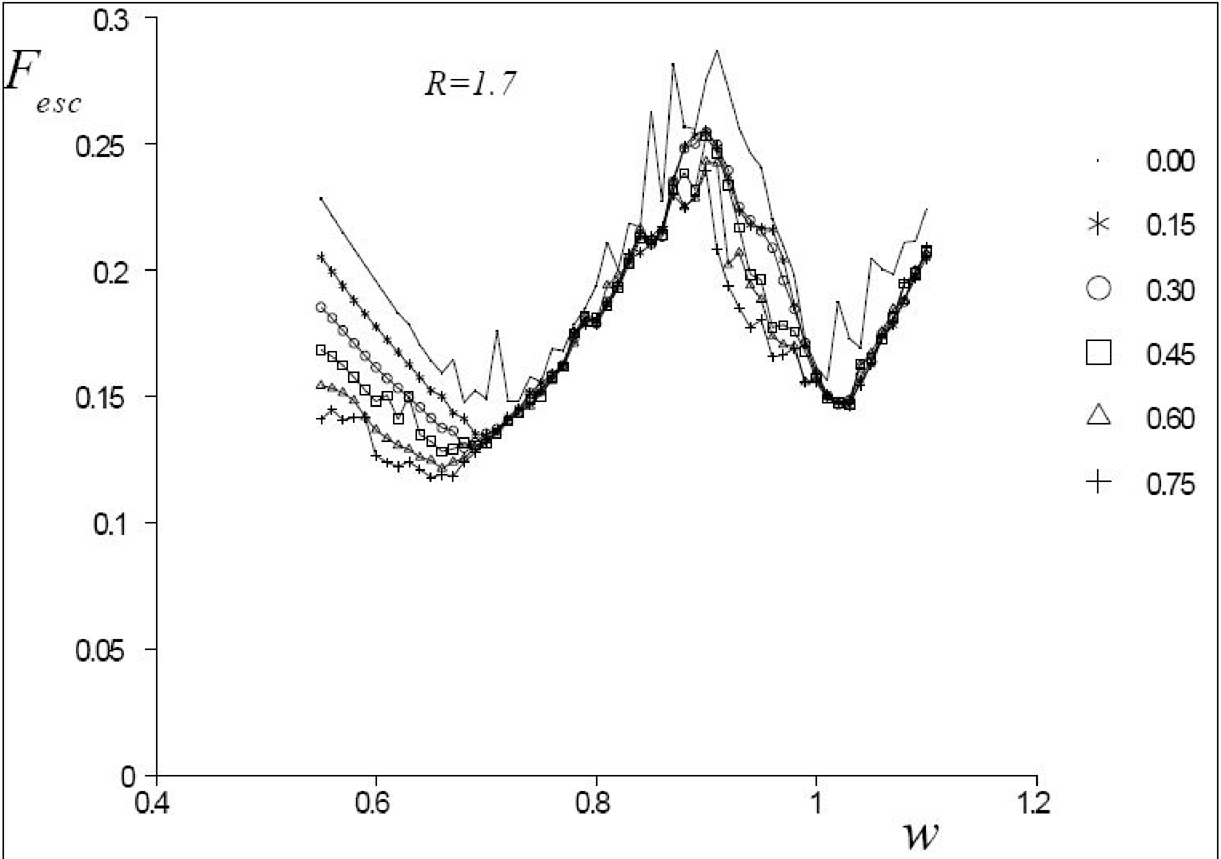


Figure 6.23 - Boundaries of safe motion for the SIR model: influence of grid size

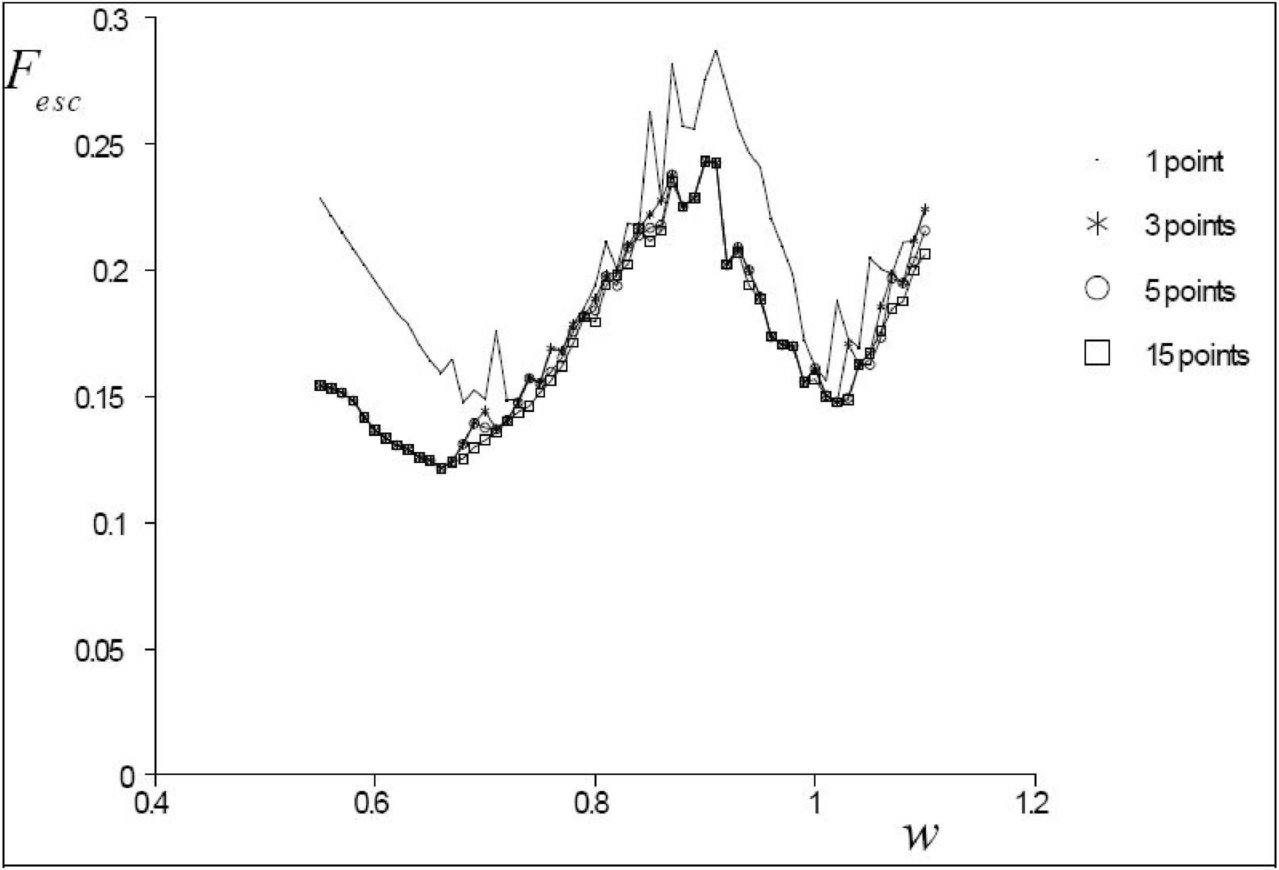


Figure 6.24 - Boundaries of safe motion for the SIR model: effect of grid density