

Figure 2.2 - The IMO Weather Criteria

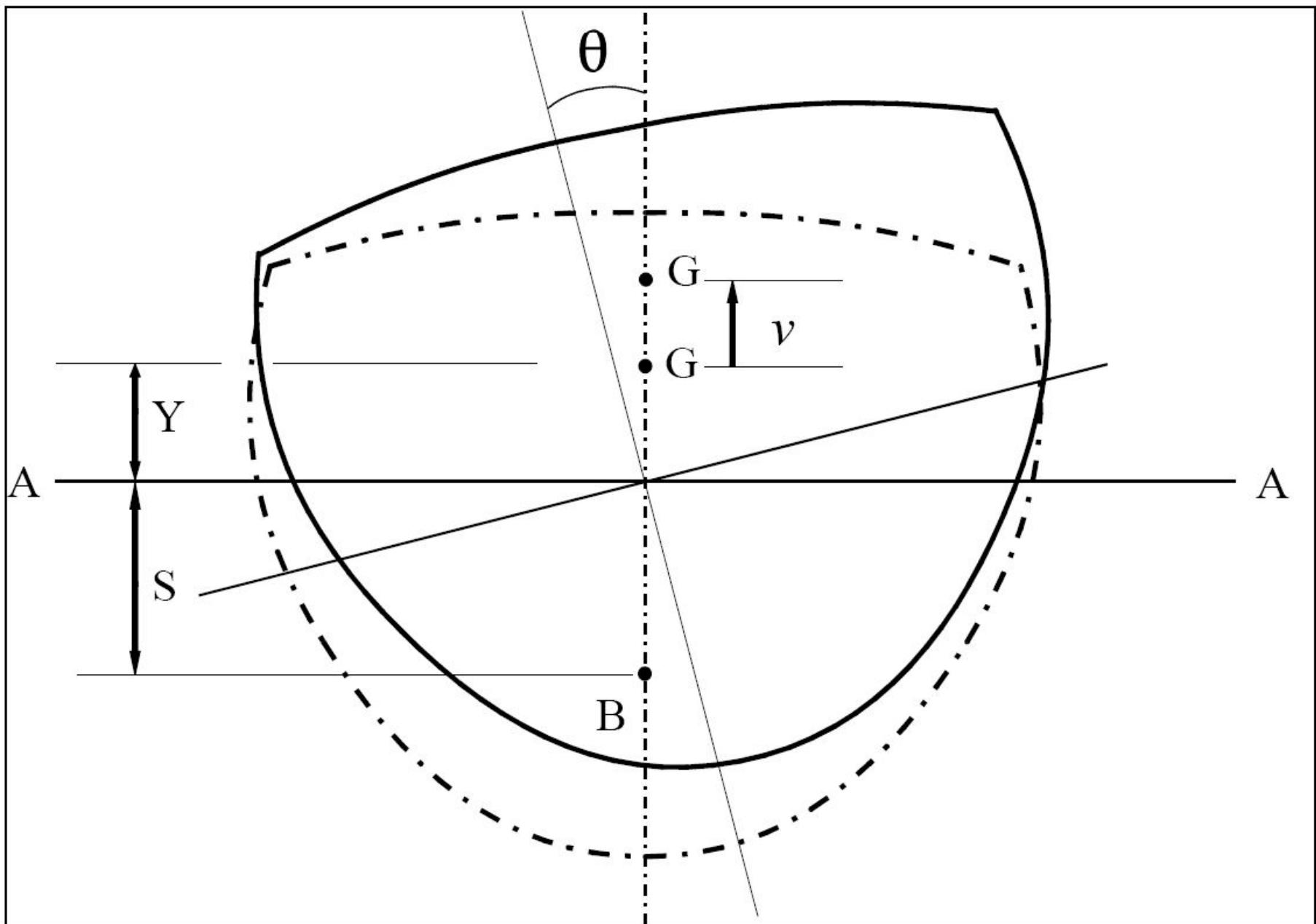


Figure 3.3 - Geometrical parameters for coupled heave-roll motion

As equações SIR (Symmetric Internal Resonance)

$$\frac{2}{R^2}(\ddot{y} + 2\zeta R\dot{y}) + 2y = x^2$$

$$\ddot{x} + 2\zeta\dot{x} + x - 2xy - b = F \sin \omega t$$

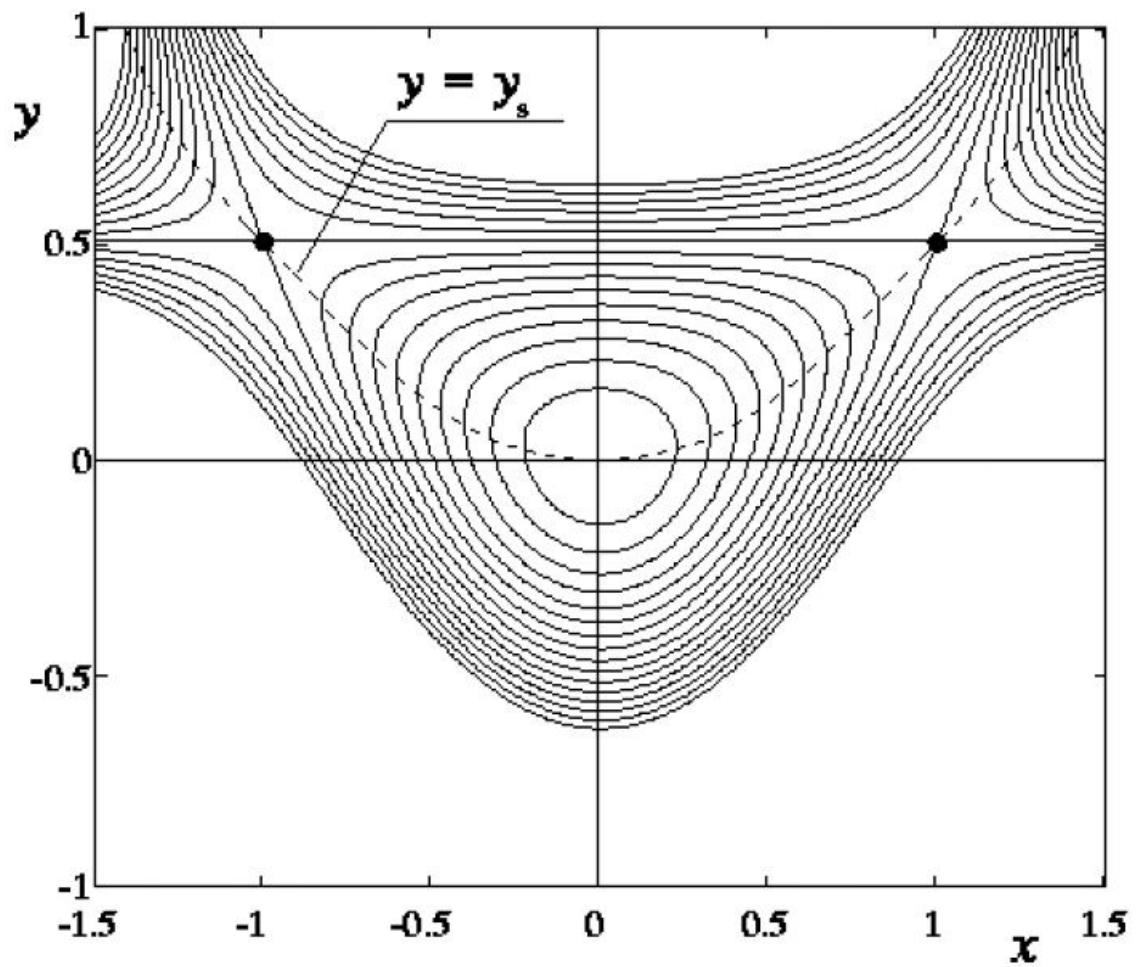


Figure 3.4 - Potential well for heave-roll model

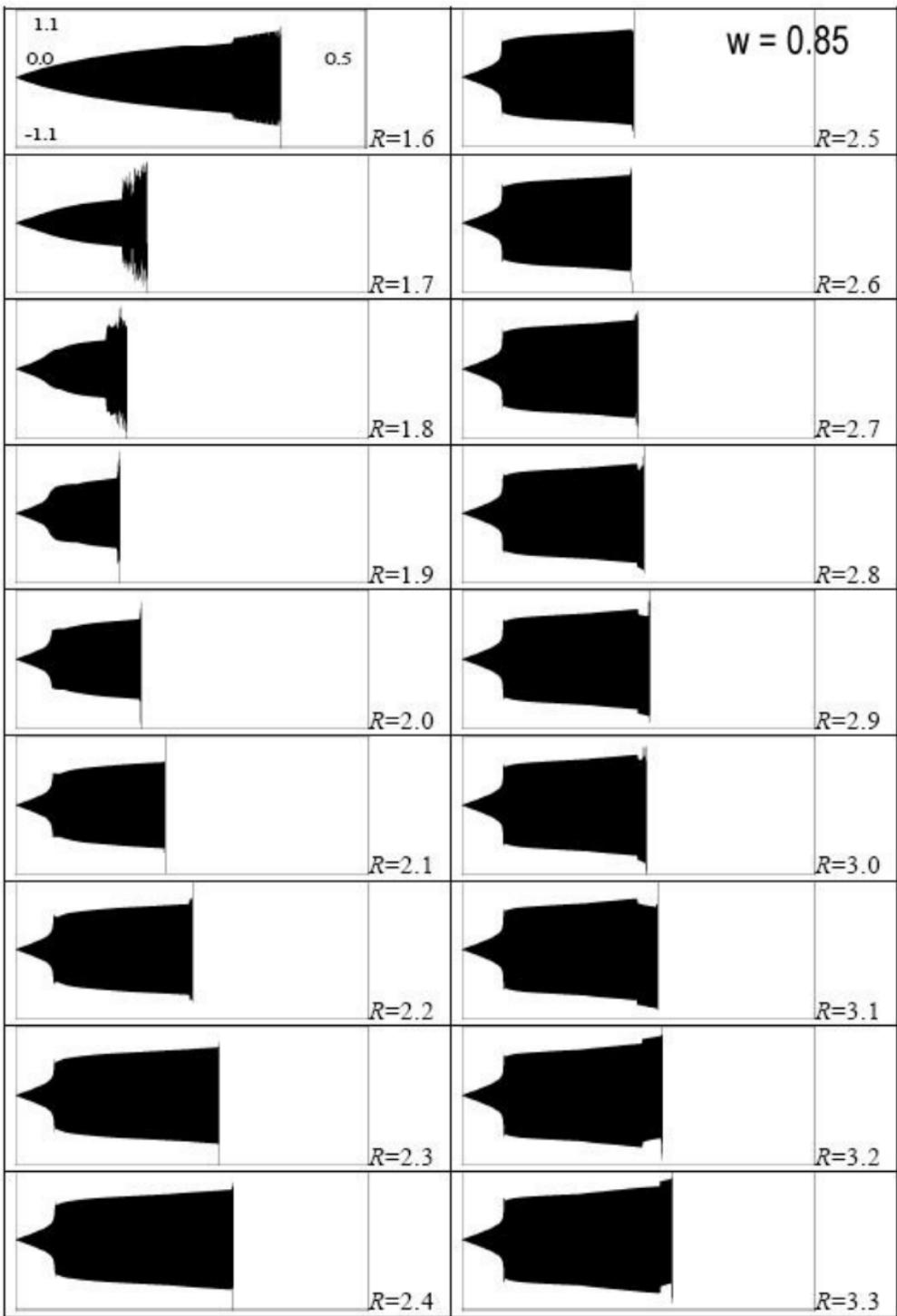


Figure 5.1 - SIR equations: escape under slowly increasing amplitude of forcing. The constant bias parameter b is set to 0, and damping level is given by $\zeta = 0.05$.

u(1)= -0.2044 u(3)= 0.2093
u(2)= 0.1374 u(4)= 0.0314

F= 0.100

cycles= 800

u(1)= -0.2592 u(3)= 0.2675
u(2)= 0.1358 u(4)= 0.1530

u(1)= -0.2424 u(3)= 0.2509
u(2)= 0.1364 u(4)= 0.1107

F= 0.120

cycles= 800

u(1)= -0.2061 u(3)= 0.2626
u(2)= 0.1552 u(4)= 0.0578

F= 0.130

cycles= 800

u(1)= -0.1242 u(3)= 0.2569
u(2)= 0.2761 u(4)= 0.0891

F= 0.140

cycles= 800

u(1)= -0.2715 u(3)= 0.1066
u(2)= 0.2860 u(4)= 0.5820

F= 0.150

cycles= 800

u(1)= -0.2272 u(3)= 0.3426
u(2)= 0.3611 u(4)= 0.0457

F= 0.160

cycles= 800

u(1)= -0.3057 u(3)= -0.1723
u(2)= 0.1262 u(4)= 0.6140

F= 0.170

cycles= 800

u(1)= -0.3920 u(3)= -0.3143
u(2)= -0.0819 u(4)= 0.4047

F= 0.180

cycles= 800

u(1)= -0.5093 u(3)= -0.1033
u(2)= -0.4744 u(4)= 0.3441

F= 0.190

cycles= 800

F= 0.200

cycles= 800

Figure 5.2 - SIR equations selection of attractors in the main sequence: Poincaré points

u(1)= -0.2044 u(3)= 0.2093
u(2)= 0.1374 u(4)= 0.0314

F= 0.100

cycles= 340

u(1)= -0.2592 u(3)= 0.2675
u(2)= 0.1358 u(4)= 0.1530

u(1)= -0.2424 u(3)= 0.2509
u(2)= 0.1364 u(4)= 0.1107

F= 0.120

cycles= 340

u(1)= -0.3500 u(3)= 0.2909
u(2)= 0.1013 u(4)= 0.2210

F= 0.130

cycles= 340

u(1)= -0.4046 u(3)= 0.3008
u(2)= 0.0701 u(4)= 0.0801

F= 0.140

cycles= 340

u(1)= -0.4534 u(3)= 0.0639
u(2)= -0.0019 u(4)= 0.6443

F= 0.150

cycles= 340

u(1)= -0.1950 u(3)= 0.3540
u(2)= 0.3139 u(4)= -0.0216

F= 0.160

cycles= 340

u(1)= -0.3857 u(3)= -0.1723
u(2)= 0.1262 u(4)= 0.6140

F= 0.170

cycles= 340

u(1)= -0.1669 u(3)= 0.4167
u(2)= 0.2819 u(4)= 0.1343

F= 0.180

cycles= 340

u(1)= -0.1253 u(3)= 0.3234
u(2)= 0.1748 u(4)= -0.1122

F= 0.190

cycles= 340

F= 0.200

cycles= 340

Figure 5.3 - SIR equations selection of attractors in the main sequence:
trajectories

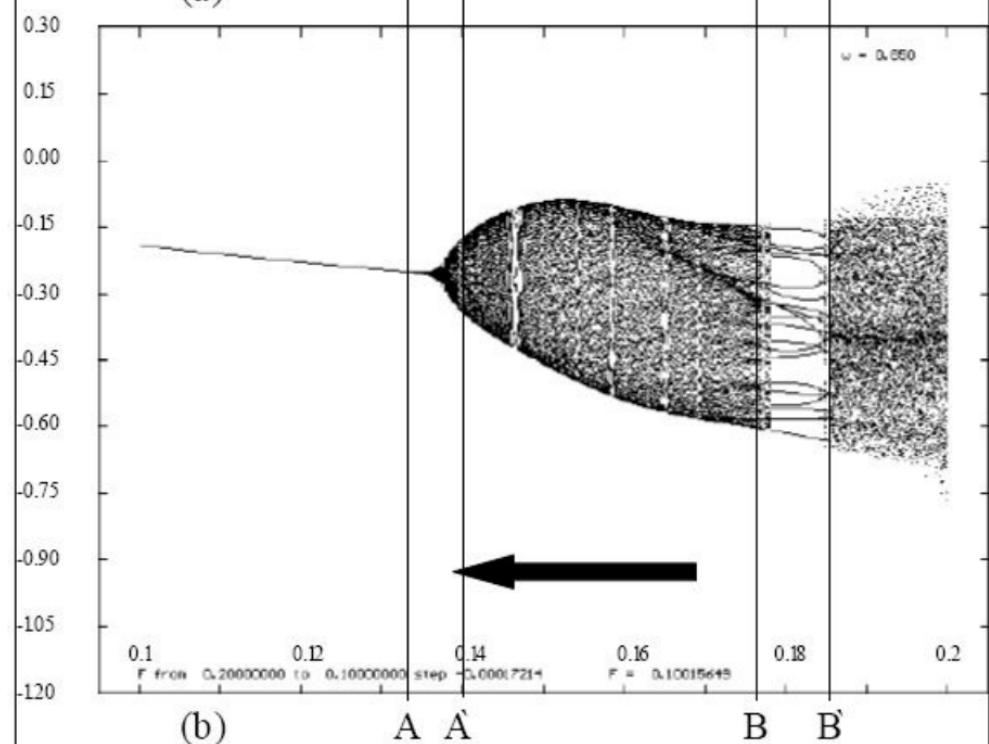
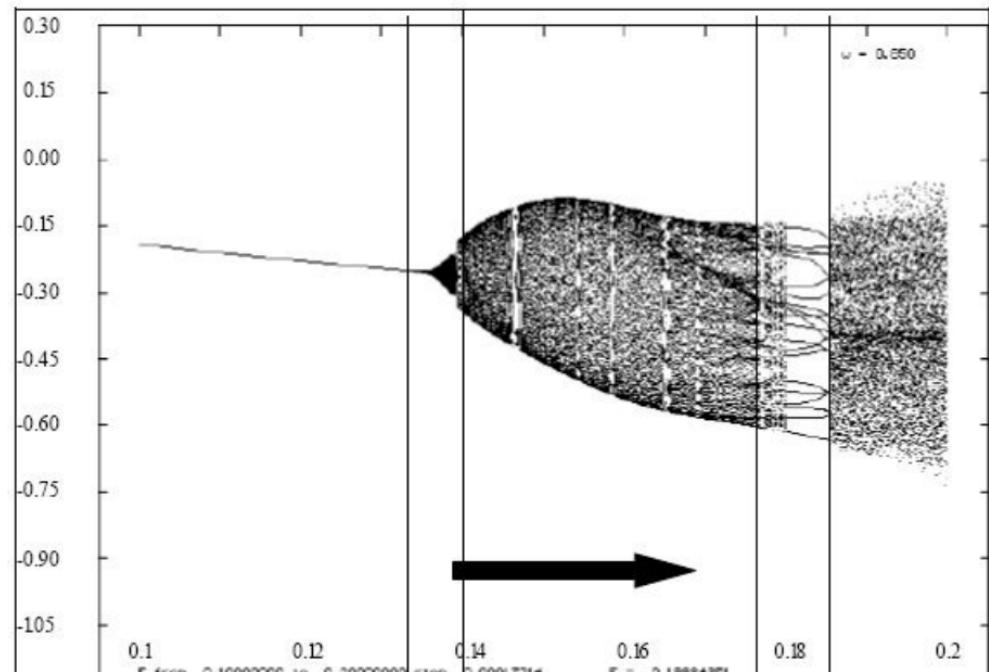


Figure 5.4 - Bifurcation diagrams for SIR equations

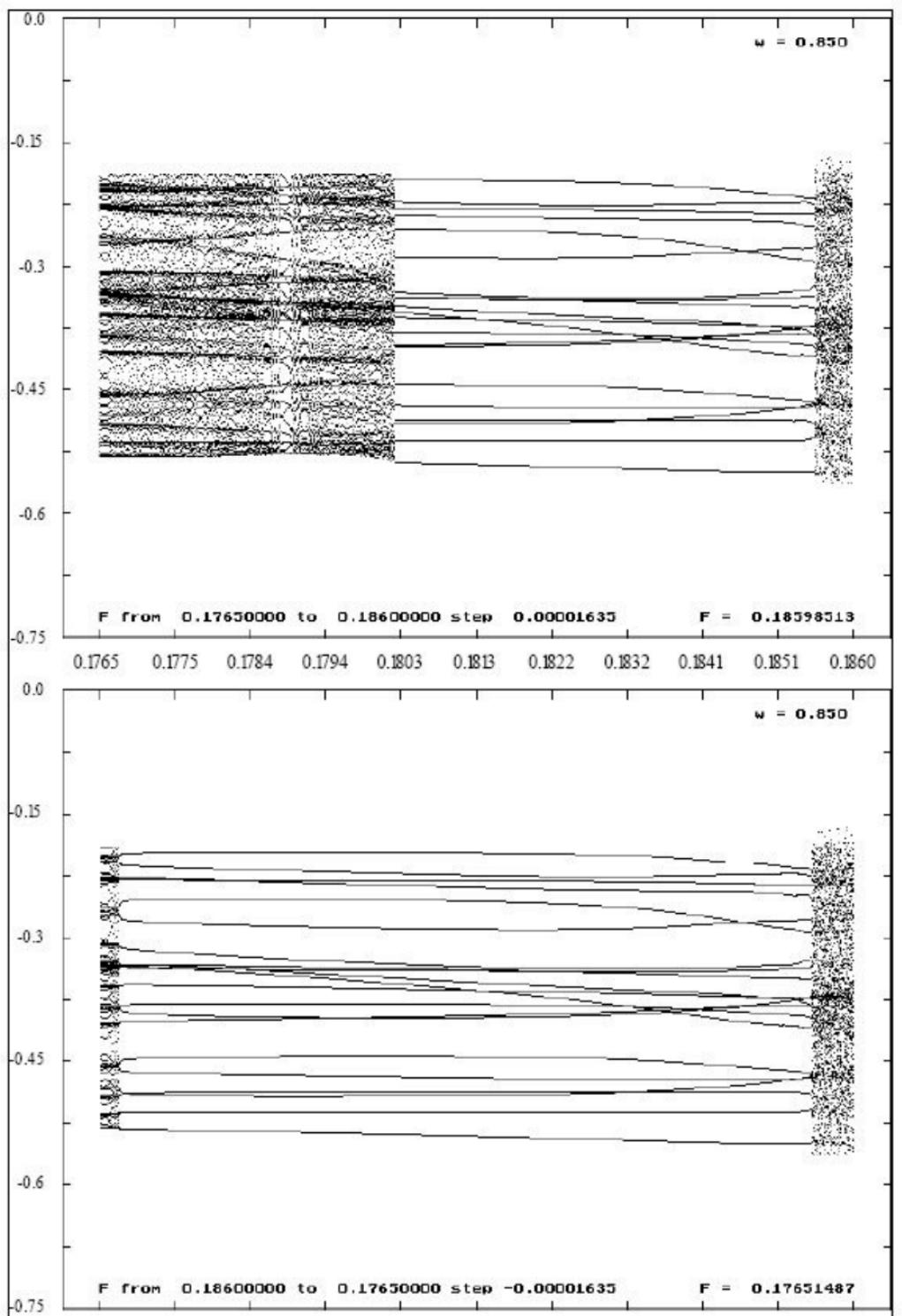


Figure 5.6 - Bifurcation diagrams for SIR equations - detail of region BB'

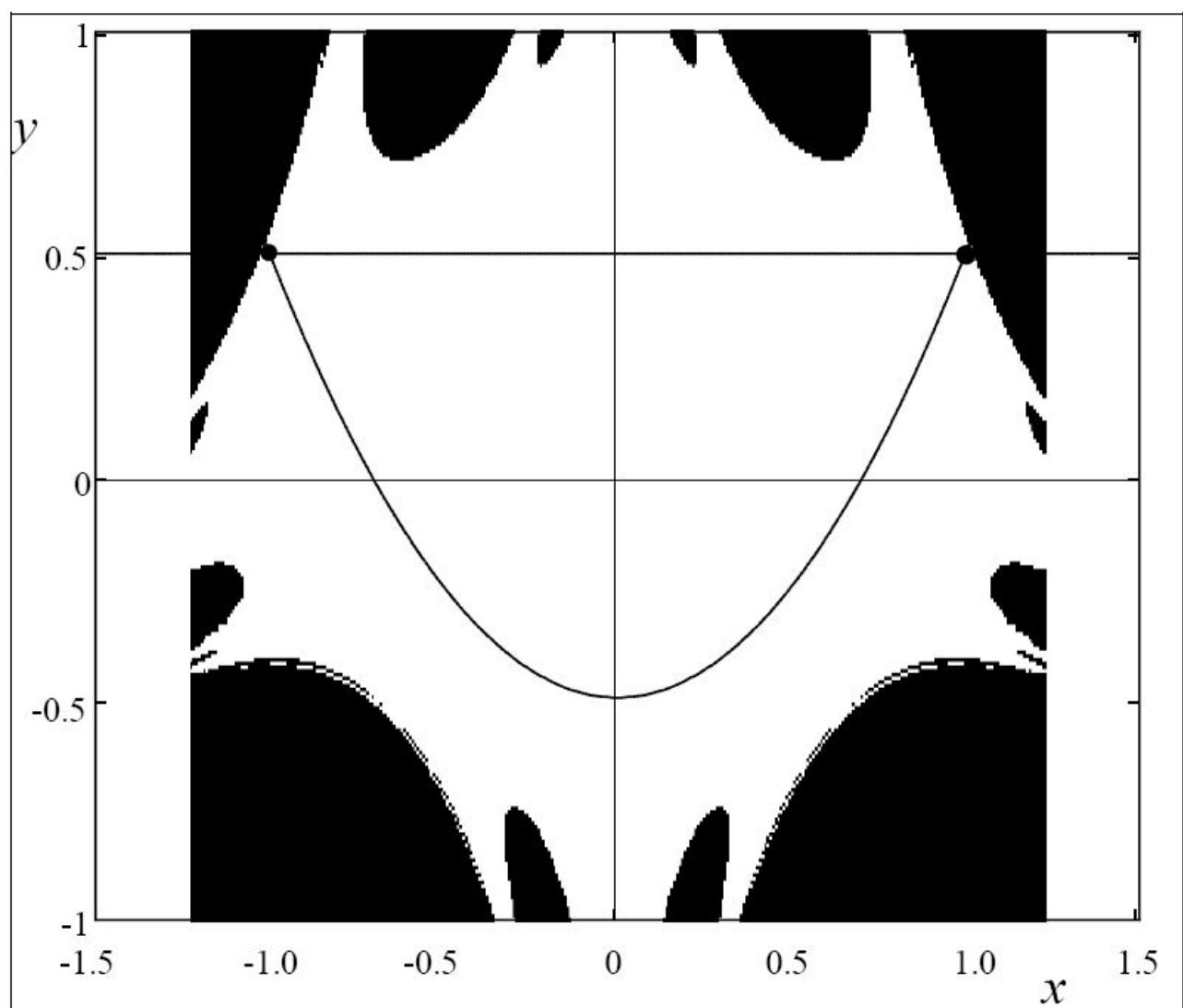


Figure 5.9 - Basin of attraction of the origin for the unforced SIR equations

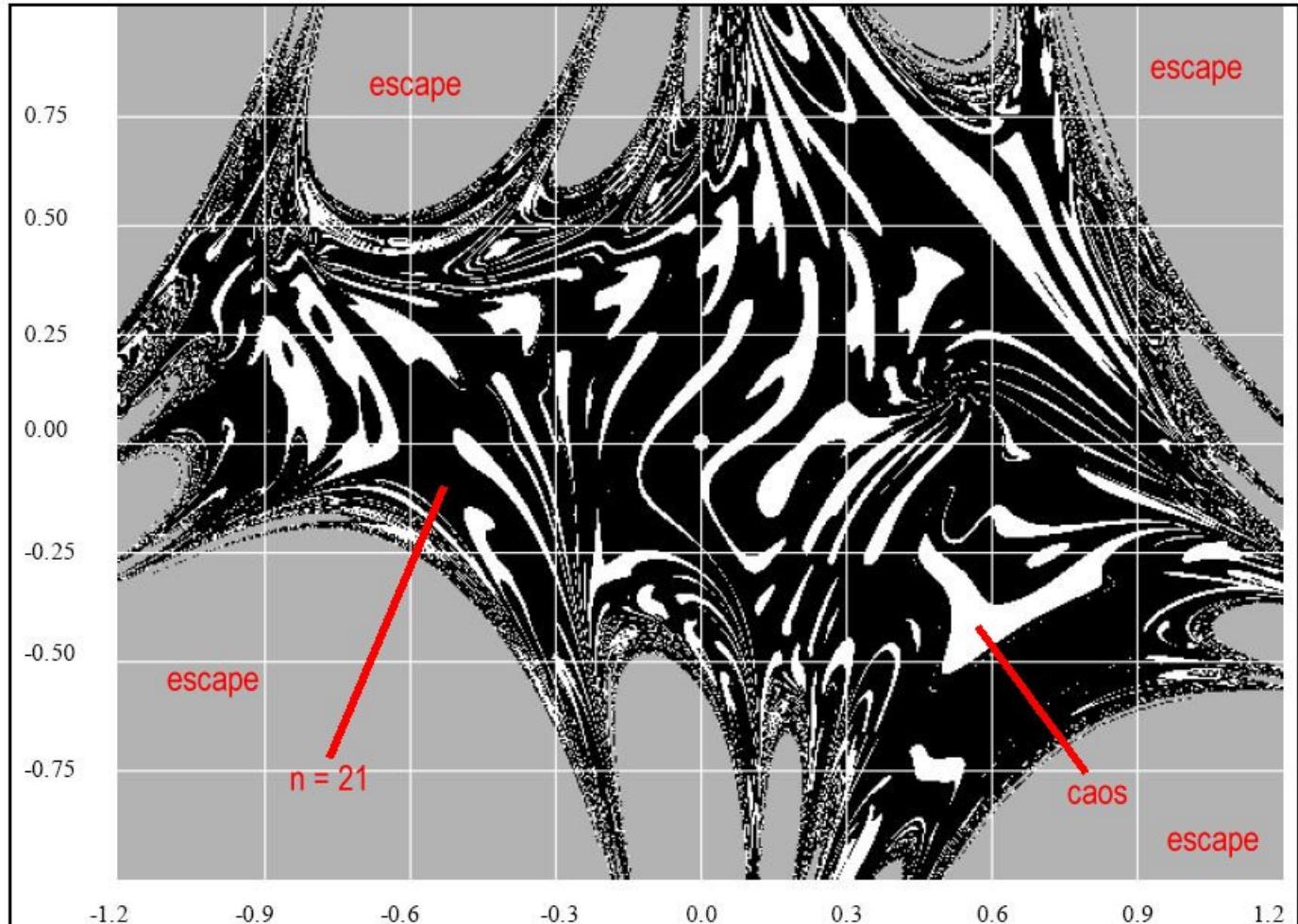


Figure 5.10 - Basins of attraction for the forced SIR equations,
 $F = 0.18$, $\omega = 0.85$

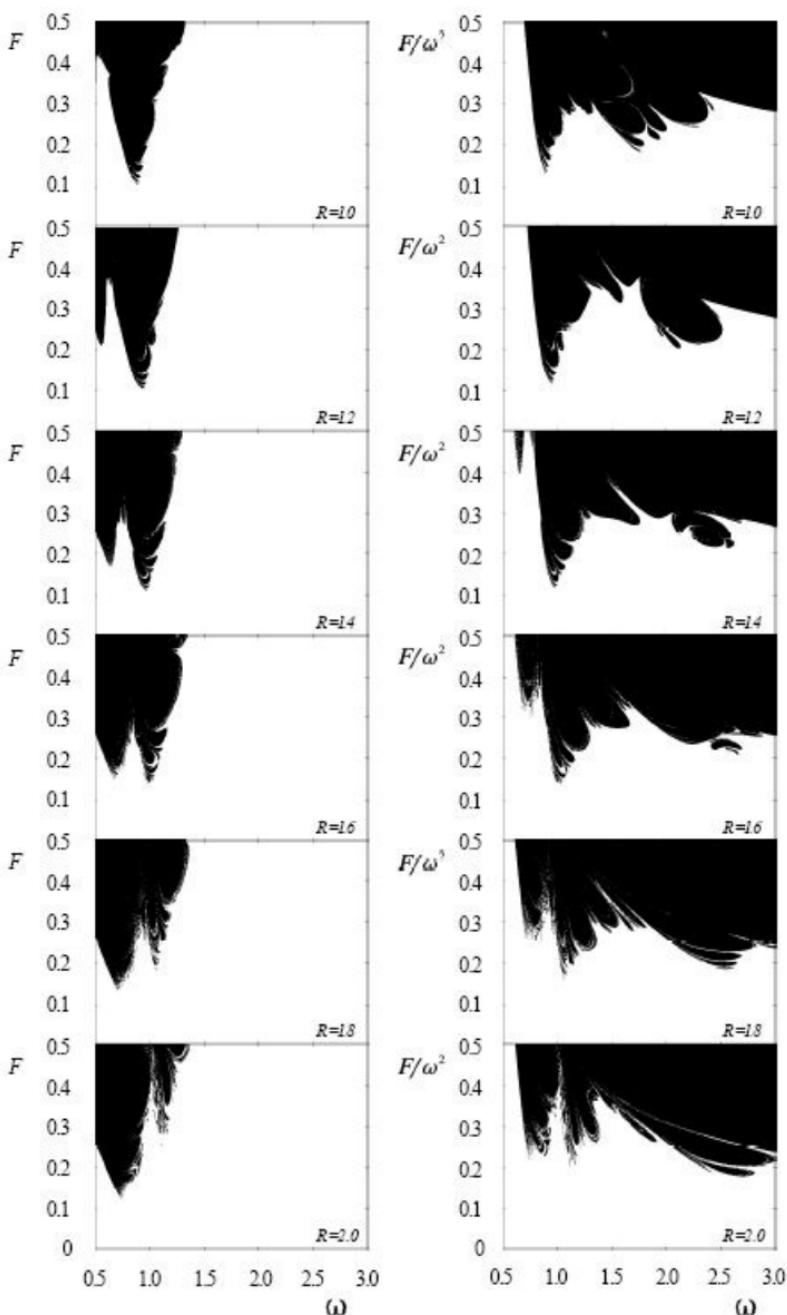


Figure 5-14 - SIR model: safe basins in the control-parameter spaces of (F, ω) (left column), and $(F/\omega^2, \omega)$ (right column) for a range of R values

$$\ddot{x} + \beta \dot{x} + x - x^3 = F \sin(\omega t)$$

4.11

| | |
|-------------------------|--|
| PARAMETERS OF THE MODEL | $\omega = 0.85, \zeta = 0.05, R = 1.7$ |
| PARAMETERS OF THE GRID | $(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}, y_{inf}, y_{sup}, \dot{y}_{inf}, \dot{y}_{sup}) = (-1, 1, -1, 1, 0, 1, -0.6, 0.6, -0.5, 0.5)$ $(N_1, N_2, N_3, N_4) = (320, 1, 240, 1)$ |
| NUMERICAL PARAMETERS | $p = T/80, m = 8, d_{exc} = 1.2$ |

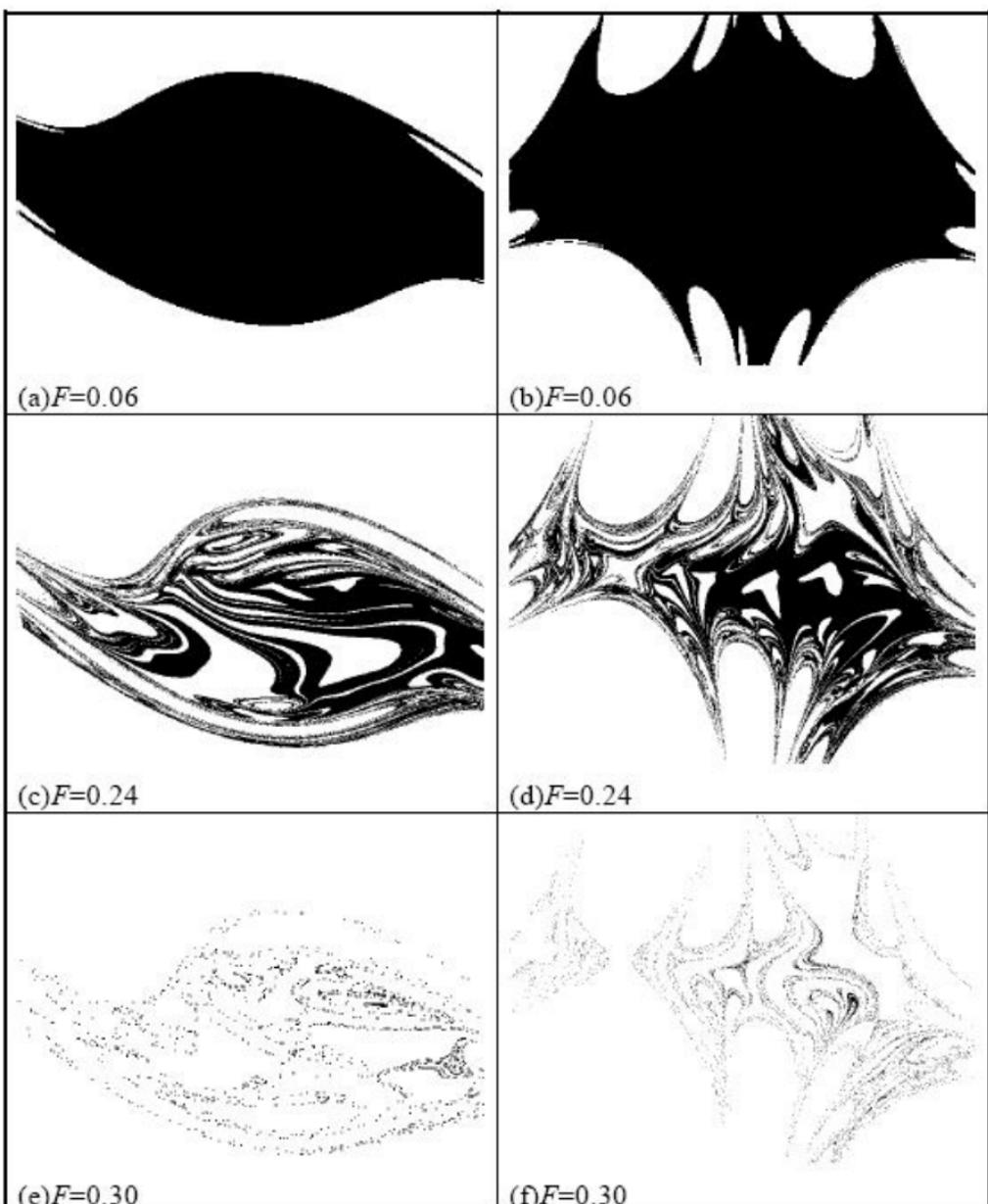
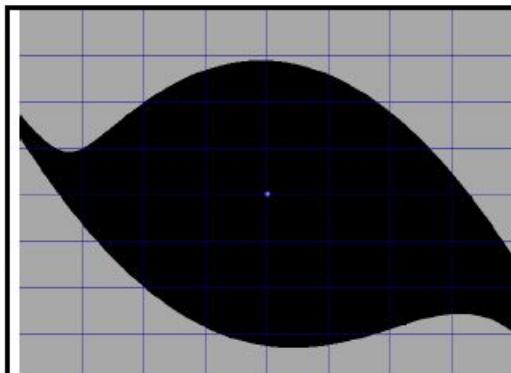


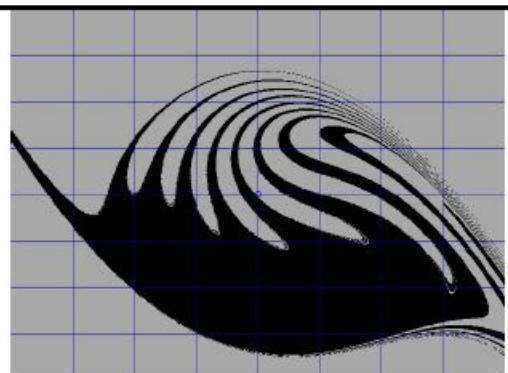
Figure 5.17 - Evolution of safe basins for the SIR equations with $\omega = 0.85$

Table 5.2 - Parameters for figure 5.18

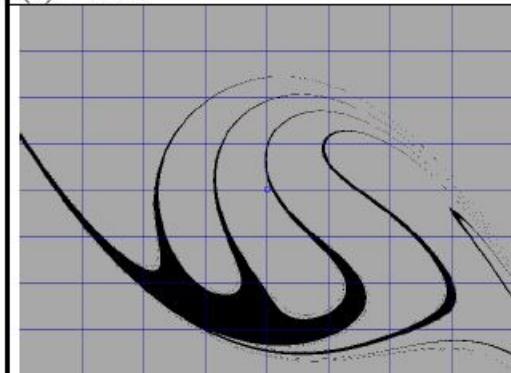
| | |
|-------------------------|--|
| PARAMETERS OF THE MODEL | $\omega = 0.85, \zeta = 0.05$ |
| PARAMETERS OF THE GRID | $(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}) = (-1.2, 1.2, -1.0, 1.0)$ $(N_1, N_2) = (640, 480)$ |
| NUMERICAL PARAMETERS | $p = T/80, m = 8, d_{sc} = 1.2$ |



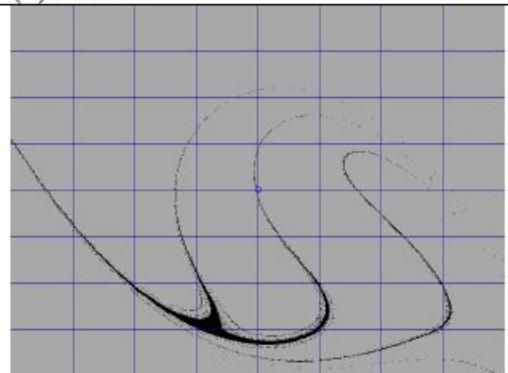
(a) $F=0.06$



(b) $F=0.12$



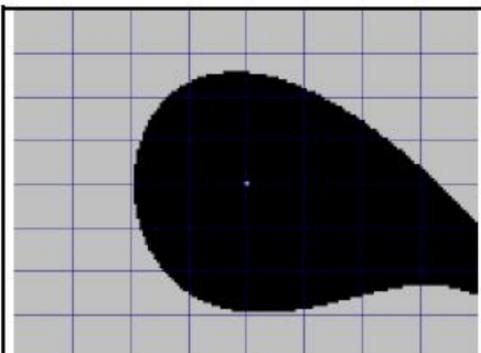
(c) $F=0.18$



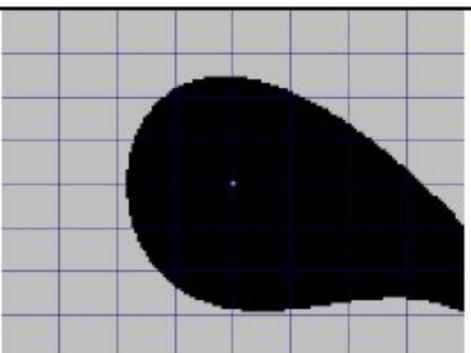
(d) $F=0.24$

Figure 5.18 - Evolution of safe basins for the roll model (4.11) with $\omega = 0.85$

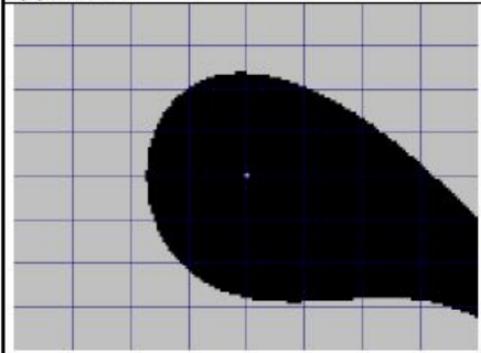
| | |
|-------------------------|--|
| PARAMETERS OF THE MODEL | $\zeta = 0.05, \omega = 0.85$ |
| PARAMETERS OF THE GRID | $(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}) = (-1.2, 1.2, -1.0, 1.0)$ $(N_1, N_2) = (200, 200)$ |
| NUMERICAL PARAMETERS | $p = T/80, m = 6, d_{esc} = 1.2$ |



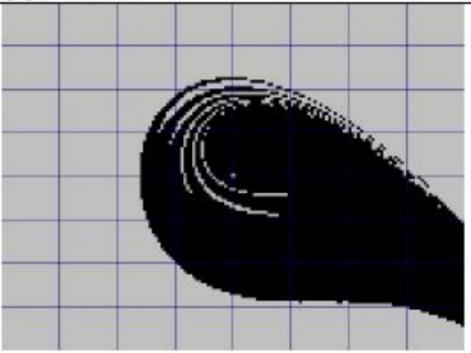
(a) $F=0.0$



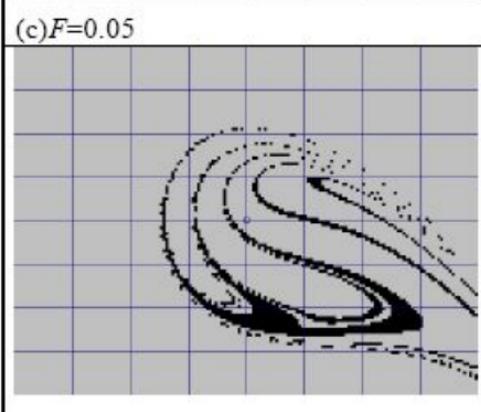
(b) $F=0.025$



(c) $F=0.05$



(d) $F=0.075$



(e) $F=0.1$

Figure 5.19 - Evolution of safe basins for the escape equation with $\omega = 0.85$

Table 5.4 - Parameters for figure 5.20

| | |
|-------------------------|--|
| PARAMETERS OF THE MODEL | $\zeta = 0.05, \omega = 0.55$ |
| PARAMETERS OF THE GRID | $(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}) = (-1.2, 1.2, -1.0, 1.0)$ $(N_1, N_2) = (200, 200)$ |
| NUMERICAL PARAMETERS | $p = T/80, m = 6, d_{esc} = 1.2$ |

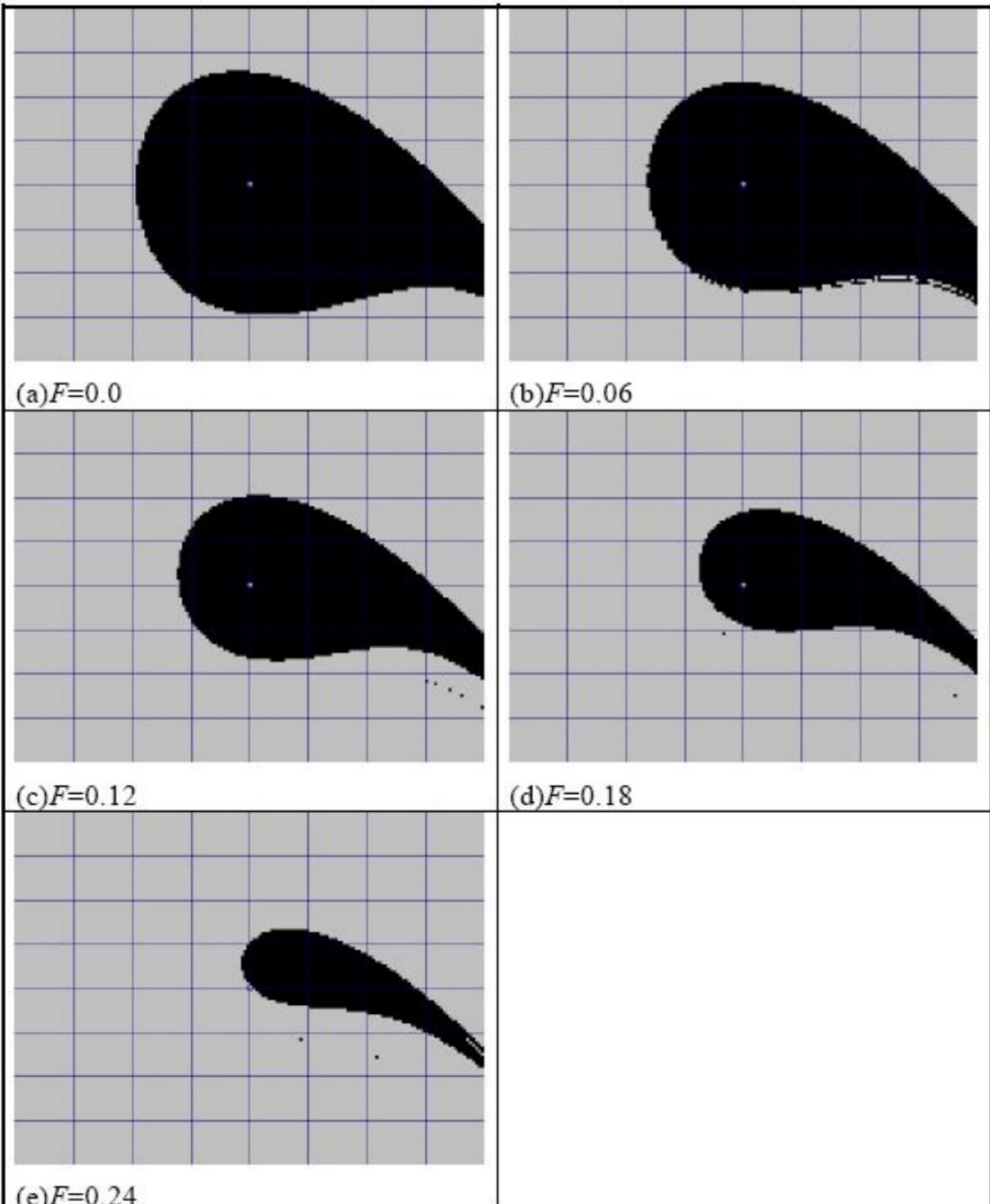
Figure 5.20 - Evolution of safe basins for the escape equation with $\omega = 0.55$

Table 5.5 - Parameters for figure 5.21

| | |
|-------------------------|--|
| PARAMETERS OF THE MODEL | $\omega = 0.85, \zeta = 0.05$ |
| PARAMETERS OF THE GRID | $(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}) = (-1.2, 1.2, -1.0, 1.0)$ $(N_1, N_2) = (640, 480)$ |
| NUMERICAL PARAMETERS | $p = T/80, m = 8, d_{exc} = 1.2$ |

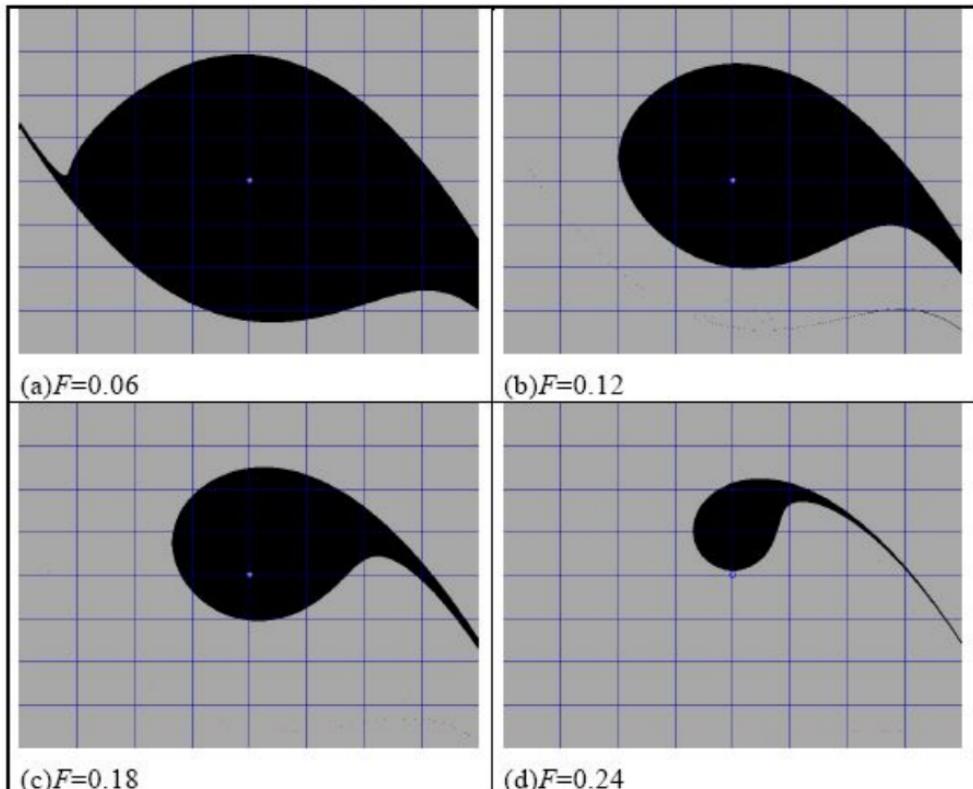
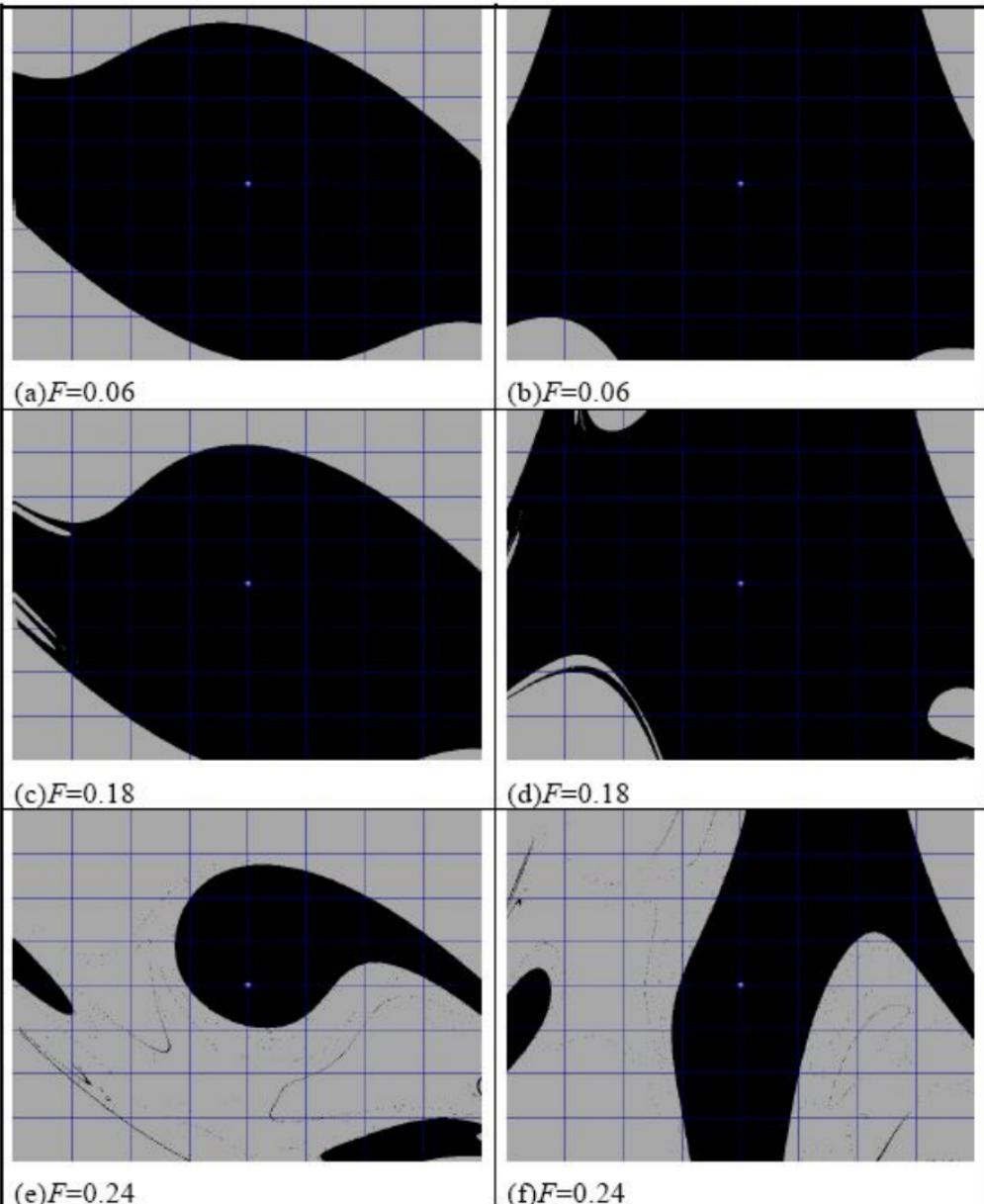


Figure 5.21 - Evolution of safe basins for the roll model (4.11) with $\omega = 0.55$

Table 5.6 - Parameters for figure 5.22

| | |
|-------------------------|--|
| PARAMETERS OF THE MODEL | $\omega = 0.55, \zeta = 0.05, R = 1.7$ |
| PARAMETERS OF THE GRID | $(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}, y_{inf}, y_{sup}, \dot{y}_{inf}, \dot{y}_{sup}) = (-1, 1, -1, 0, 1, 0, -0.6, 0.6, -0.5, 0.5)$ $(N_1, N_2, N_3, N_4) = (640, 1, 480, 1)$ |
| NUMERICAL PARAMETERS | $p = T/80, m = 8, d_{exc} = 1.2$ |

Figure 5.22- Evolution of safe basins for the SIR equations with $\omega = 0.55$

| | |
|-------------------------|--|
| PARAMETERS OF THE MODEL | $\zeta = 0.05, \omega$ as shown |
| PARAMETERS OF THE GRID | $(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}) = (-1.2, 1.2, -1.0, 1.0)$ $(N_1, N_2) = (200, 200)$ |
| NUMERICAL PARAMETERS | $p = T/80, m = 6, d_{esc} = 1.2$ |

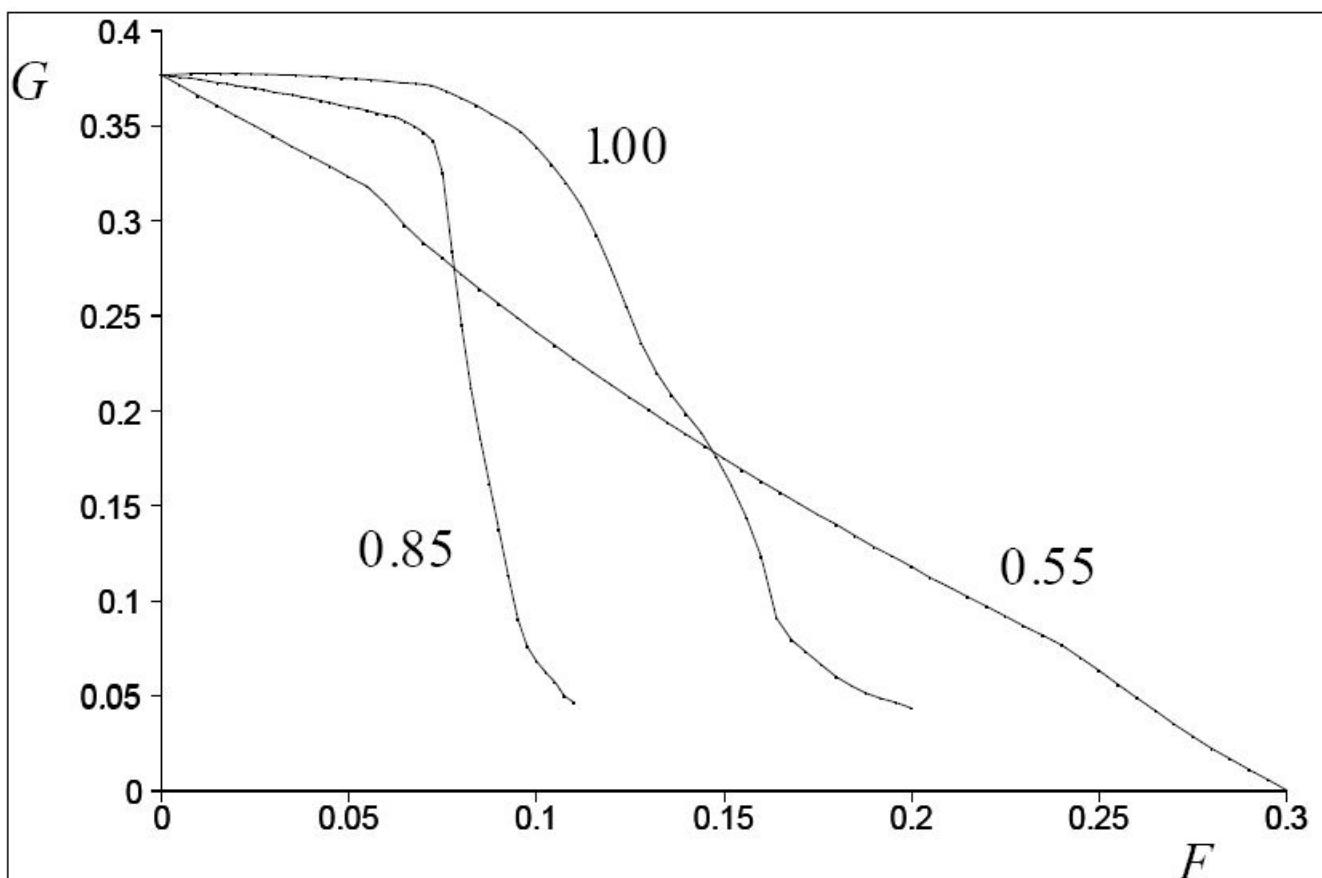


Figure 5.23 - Integrity diagrams for the escape equation (3.8)

Table 5.8 - Parameters for figure 5.24

| | |
|-------------------------|--|
| PARAMETERS OF THE MODEL | $\omega = 0.85, \zeta = 0.05, R$ as shown |
| PARAMETERS OF THE GRID | $(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}, y_{inf}, y_{sup}, \dot{y}_{inf}, \dot{y}_{sup}) = (-1.1, 1.1, -1.0, 1.0, -0.6, 0.6, -0.5, 0.5)$ $(N_1, N_2, N_3, N_4) = (10, 10, 10, 10)$ |
| NUMERICAL PARAMETERS | $p = T/80, m = 8, d_{esc} = 1.2$ |

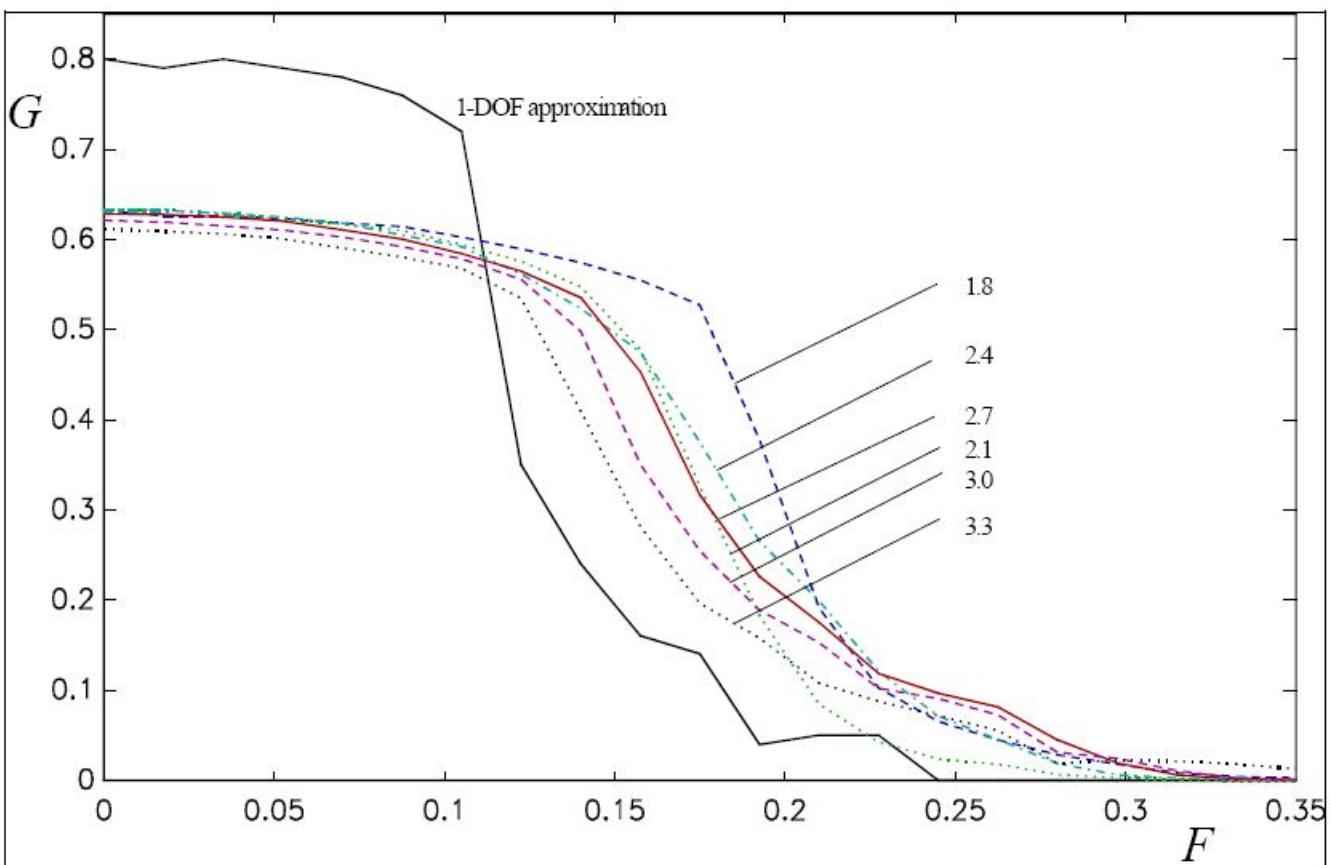

 Figure 5.24 - Integrity diagrams for the SIR equations with varying R

Table 5.9 - Parameters for figure 5.25

| | |
|-------------------------|--|
| PARAMETERS OF THE MODEL | $\zeta = 0.05, R = 1.7, \omega$ as shown |
| PARAMETERS OF THE GRID | $(x_{inf}, x_{sup}, \dot{x}_{inf}, \dot{x}_{sup}, y_{inf}, y_{sup}, \dot{y}_{inf}, \dot{y}_{sup}) = (-1.1, 1.1, -1.0, 1.0, -0.6, 0.6, -0.5, 0.5)$ $(N_1, N_2, N_3, N_4) = (10, 10, 10, 10)$ |
| NUMERICAL PARAMETERS | $p = T/80, m = 8, d_{esc} = 1.2$ |

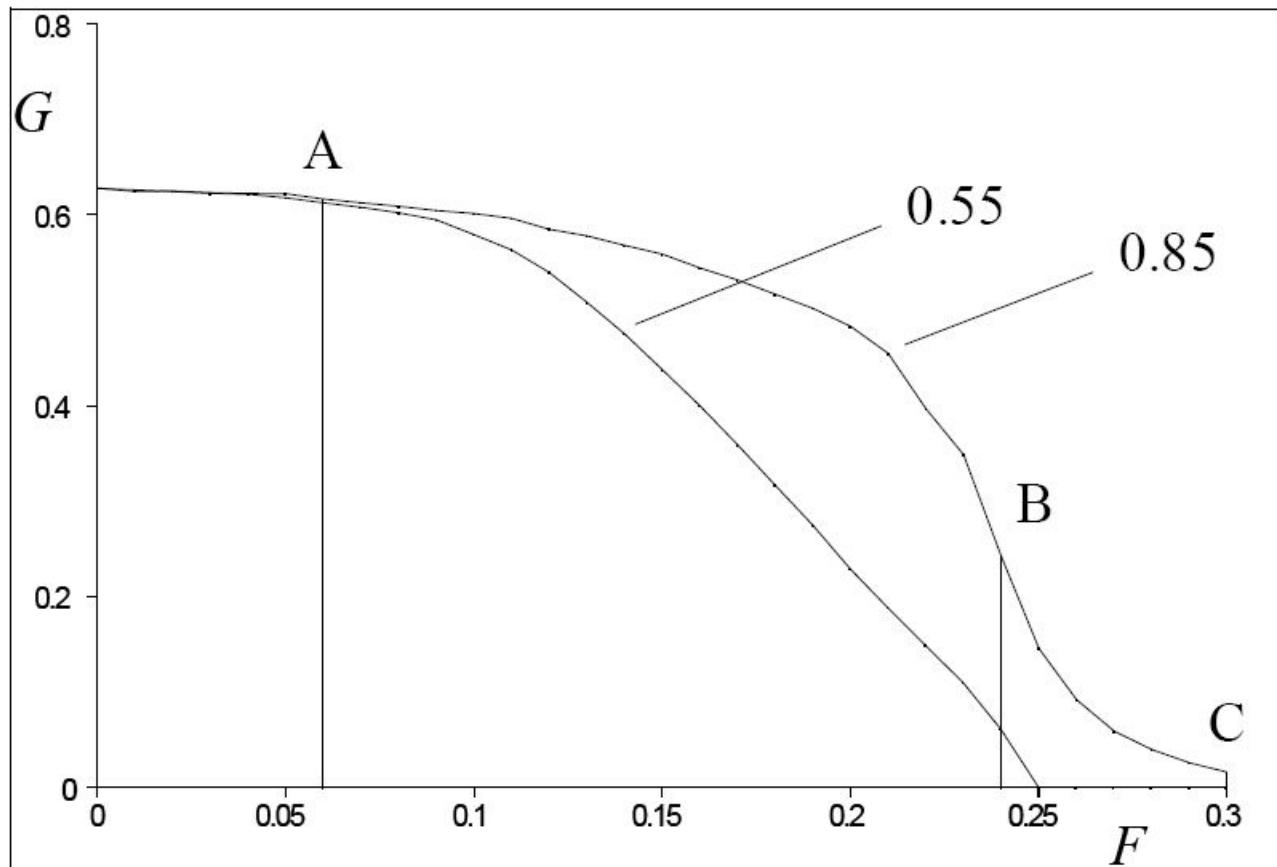


Figure 5.25 - Integrity diagram for the SIR equations with $R = 1.7$

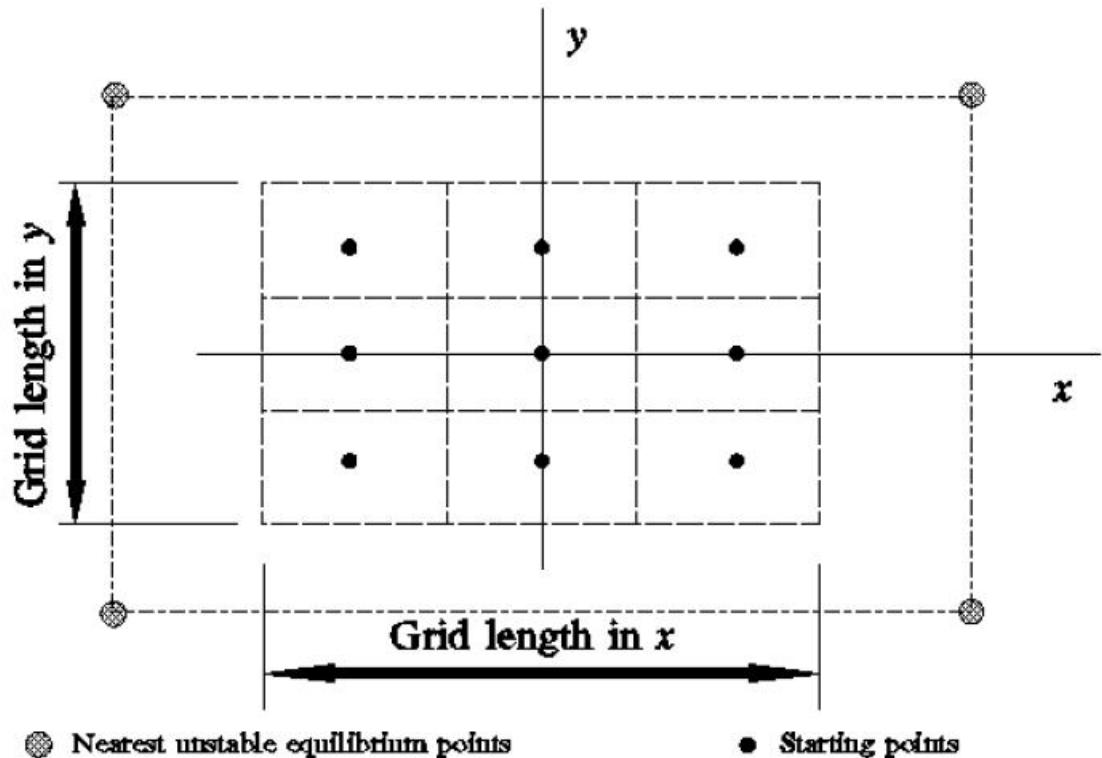


Figure 6.4 - Example of a two-dimensional grid-of-starts

$$\ddot{x} + \beta\dot{x} + x - x^2 = F\sin(\omega t + \delta) \quad 6.13$$

The symmetric single well roll model (4.11) (a Duffing equation):

$$\ddot{x} + \beta\dot{x} + x - x^3 = F\sin(\omega t + \delta) \quad 6.14$$

An asymmetric double-well model with quadratic and cubic nonlinearities from Virgin *et al* (1992):

$$\ddot{x} + \beta\dot{x} + \omega_0^2 x + \alpha_2 x^2 + \alpha_3 x^3 = F\sin(\omega t + \delta) \quad 6.15$$

A symmetric single-well roll model with cubic and fifth order nonlinearities from Soliman and Thompson (1991):

$$\ddot{x} + \beta_1\dot{x} + \beta_2|\dot{x}|\dot{x} + c_1x + c_3x^3 + c_5x^5 = F\sin(\omega t) \quad 6.16$$

A symmetric two-dimensional potential well (2-DOF system) from Virgin *et al* (1992):

$$\begin{aligned}\ddot{x}_1 + \beta\dot{x}_1 + x_1 - 4x_1x_2^2 &= F\sin(\omega t) \\ \ddot{x}_2 + \beta\dot{x}_2 + 2.25x_2 - 4x_1^2x_2 &= F\sin(\omega t)\end{aligned} \quad 6.17$$

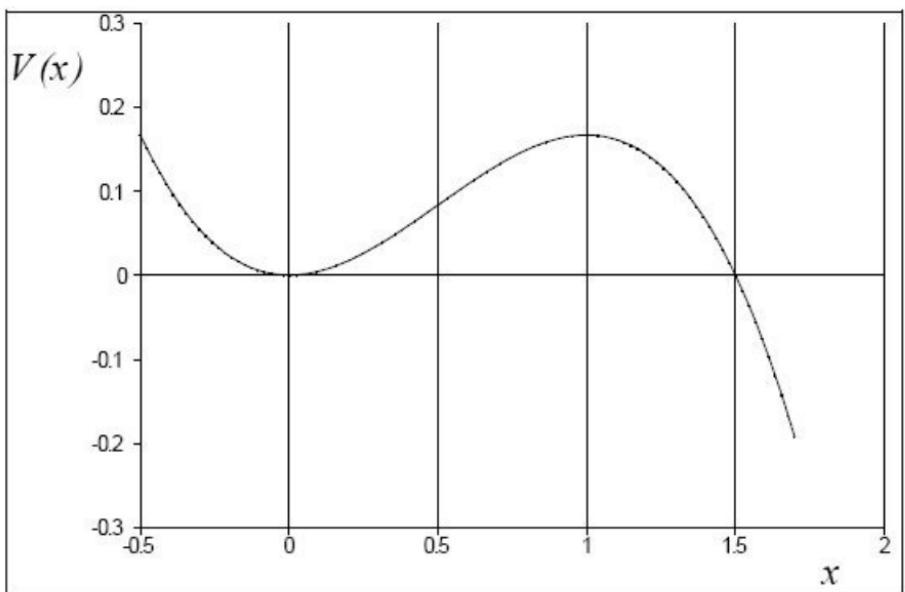


Figure 6.5 - Potential well for equation (6.13)

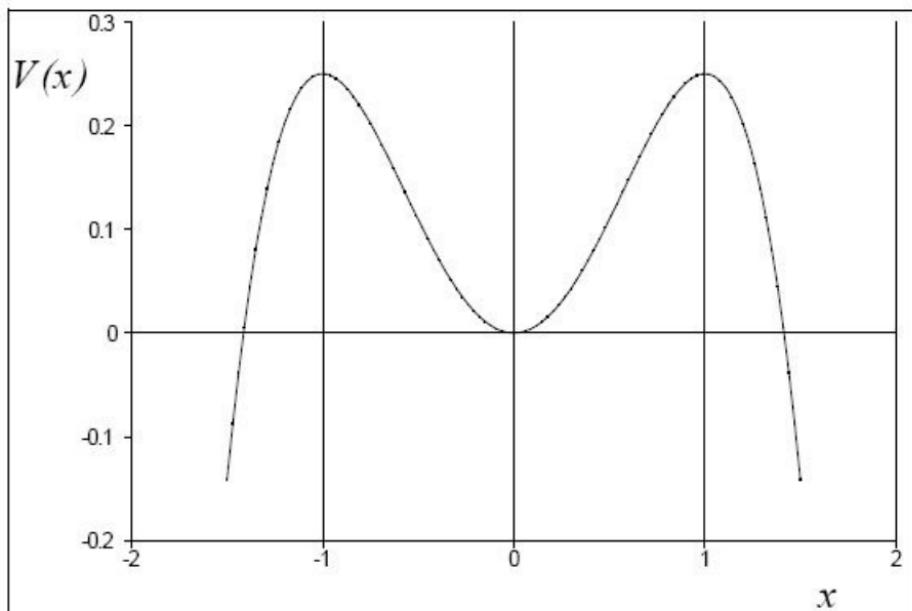


Figure 6.6 - Potential well for equation (6.14)

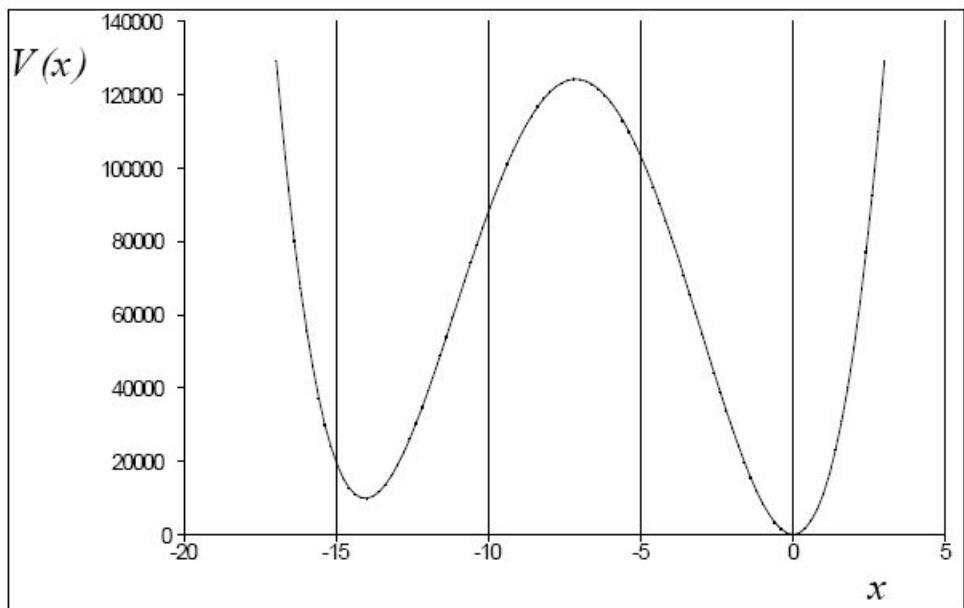


Figure 6.7 - Potential well for equation (6.15)

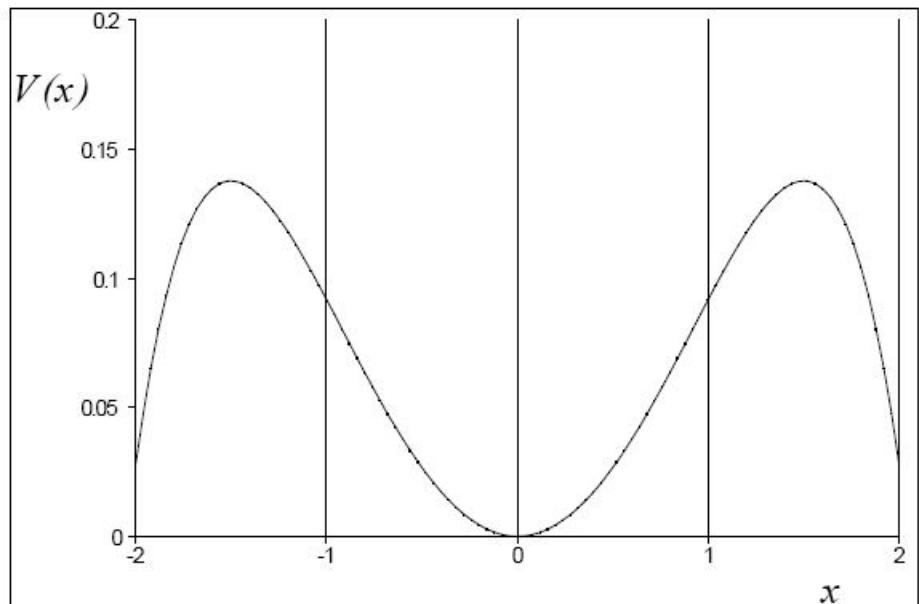


Figure 6.8 - Potential well for equation (6.16)

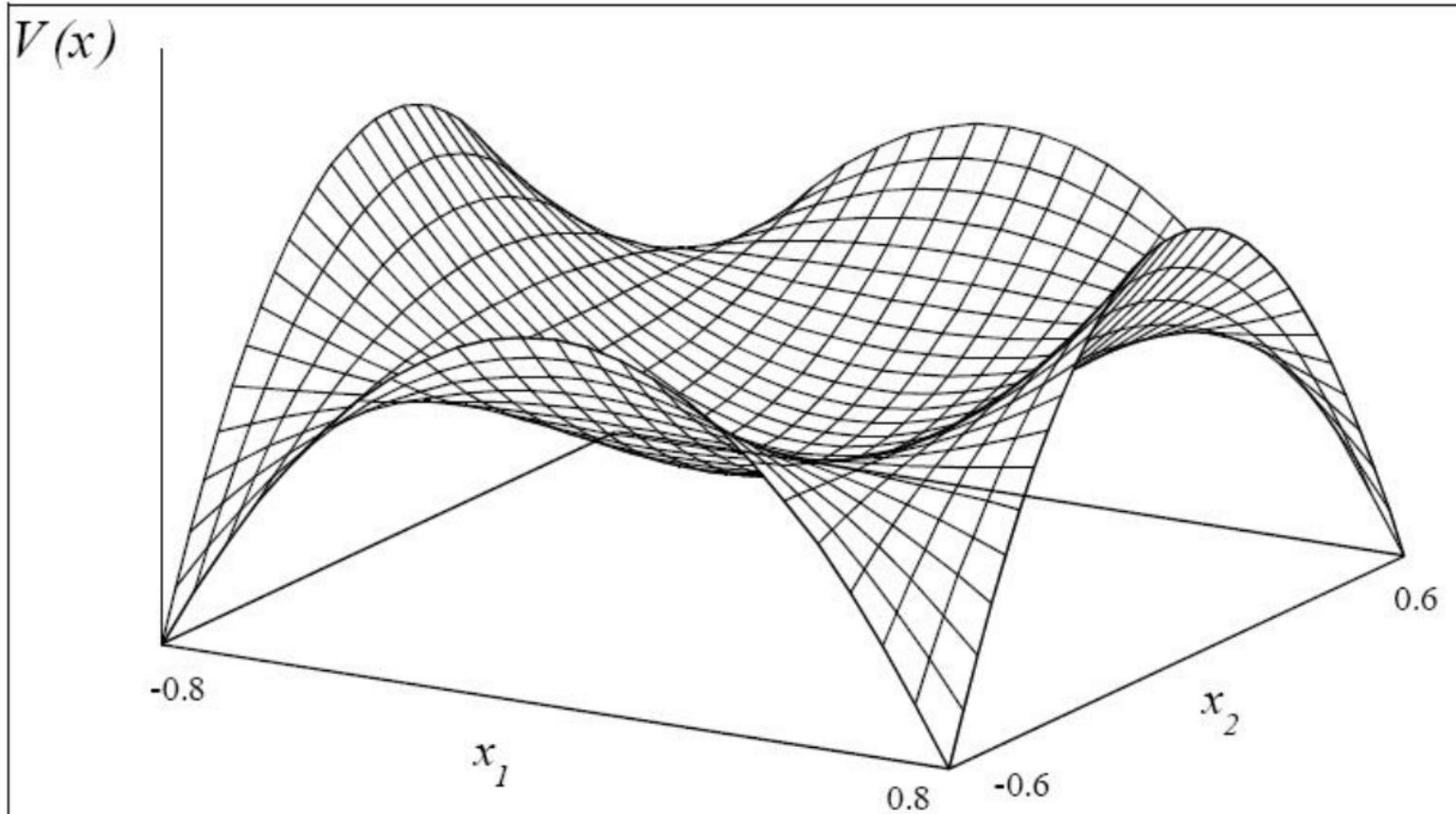
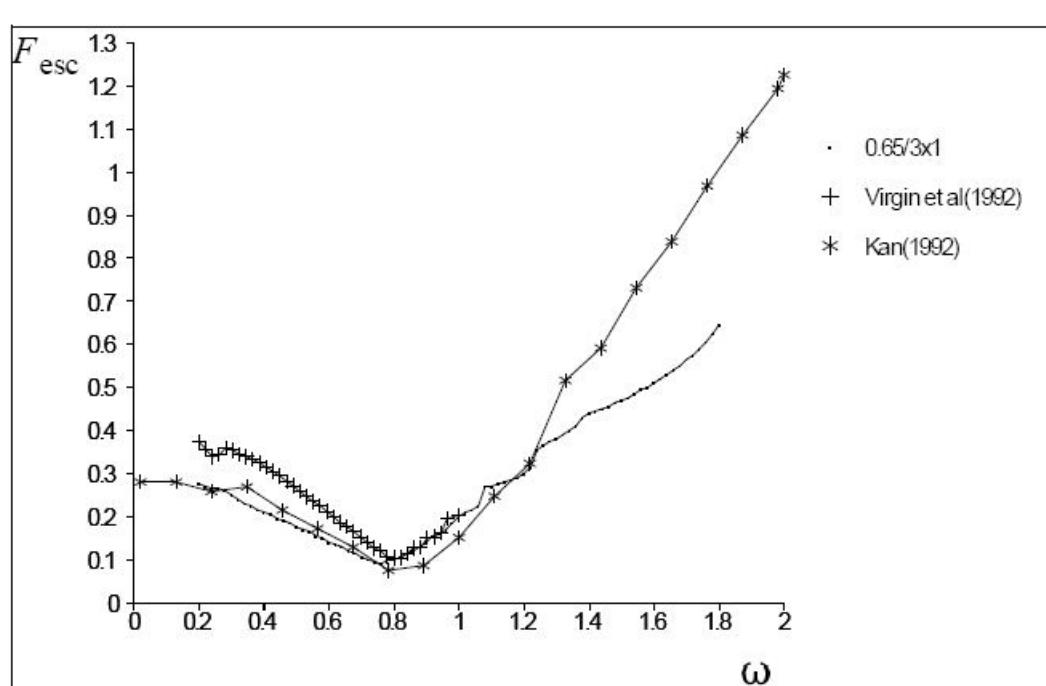
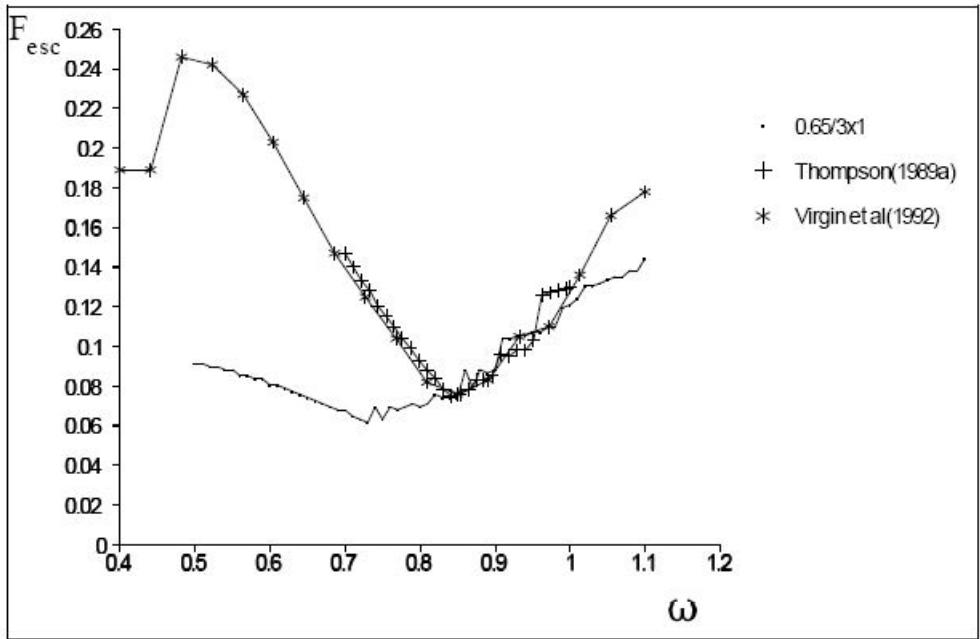


Figure 6.9 - Potential well for equation (6.17)

Table 6.1 - Main features of selected systems

| FEATURE → EQUATION ↓ | REFERENCE | PARAMETERS | STARTING POINTS | MAXIMUM TRANSIENT DURATION | ESCAPE CRITERION |
|-------------------------|-----------------------------|--|---|----------------------------------|--------------------------------|
| (6.13) | Thompson (1989a)) | $\beta = 0.1$ | $x = \dot{x} = 0$ | $32 T$ | $x \geq 20$ |
| (6.13) | Virgin <i>et al</i> (1992) | $\beta = 0.1$ | $x = \dot{x} = 0$ | $30 T$ | $x > 1$ |
| (6.14) | Virgin <i>et al</i> (1992) | $\beta = 0.1$ | $x = \dot{x} = 0$ | $30 T$ | $ x > 1$ |
| (6.14) | Kan (1992) | $\beta = 0.04455$ | $x = \dot{x} = 0$ | $20 T$ | $ x > 2$ |
| (6.15) | Virgin <i>et al</i> (1992) | $\beta = 0.1$ $\omega_0 = 139.93$ $\alpha_2 = 4132.7$ $\alpha_3 = 194.82$ | $x = \dot{x} = 0$ | $30 T$ | $x < -7.143$ |
| (6.16) | Soliman and Thompson (1991) | $\beta_1 = 0.0555$ $\beta_2 = 0.1659$ $c_1 = 0.2227$ $c_3 = -0.0694$ $c_5 = -0.0131$ | $x = \dot{x} = 0$ | $16 T$ | $ x > 1.57$ |
| (6.17) | Virgin <i>et al</i> (1992) | $\beta = 0.1$ | $x_1 = x_2 = \dot{x}_1 = \dot{x}_2 = 0$ | $30 T$ | $\sqrt{x_1^2 + x_2^2} > 0.901$ |



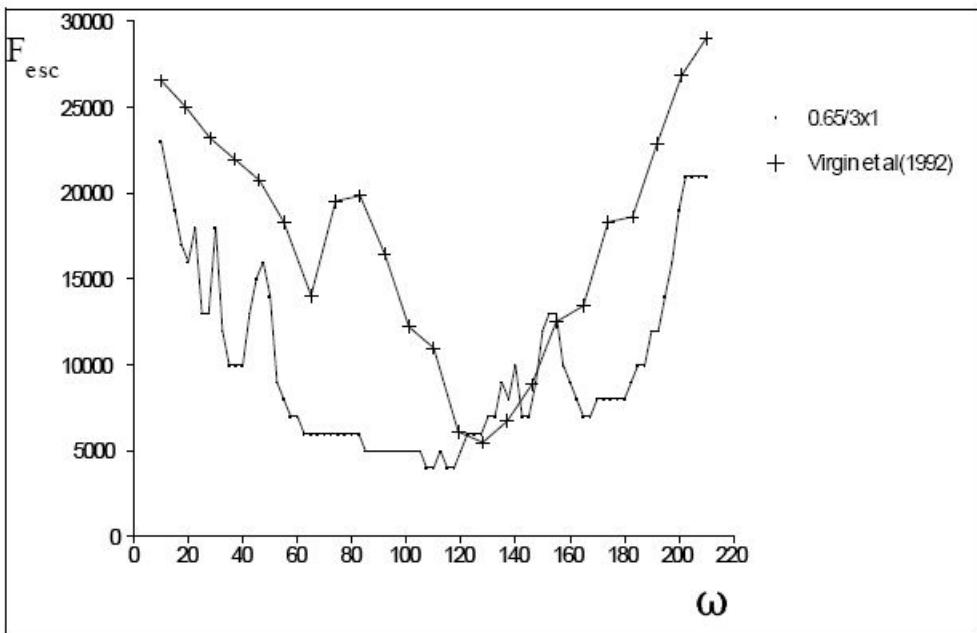


Figure 6.18 - Boundaries of safe motion: equation (6.15)

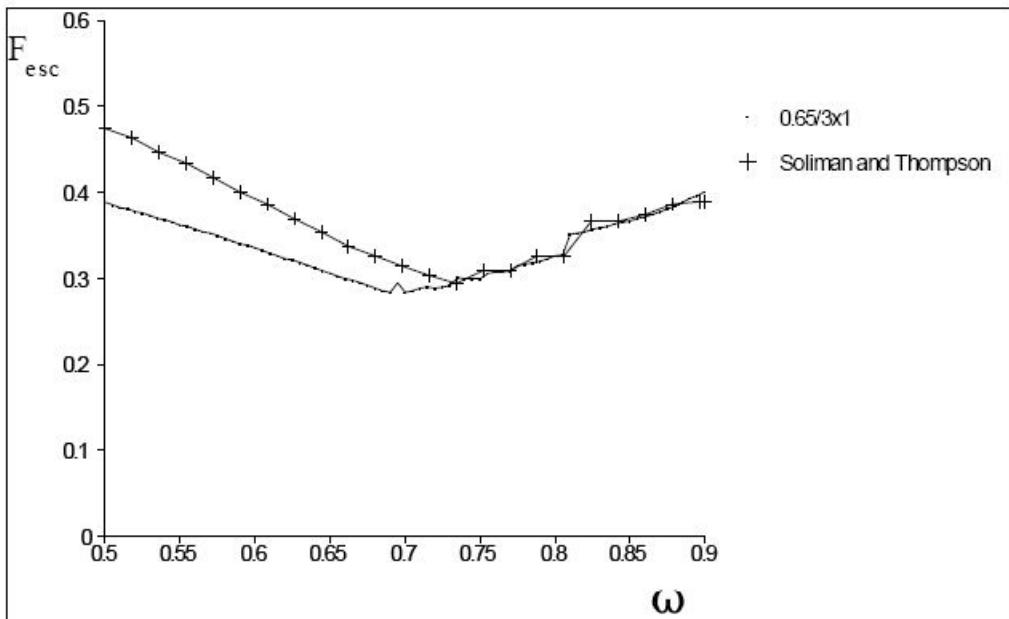


Figure 6.19 - Boundaries of safe motion: equation (6.16)

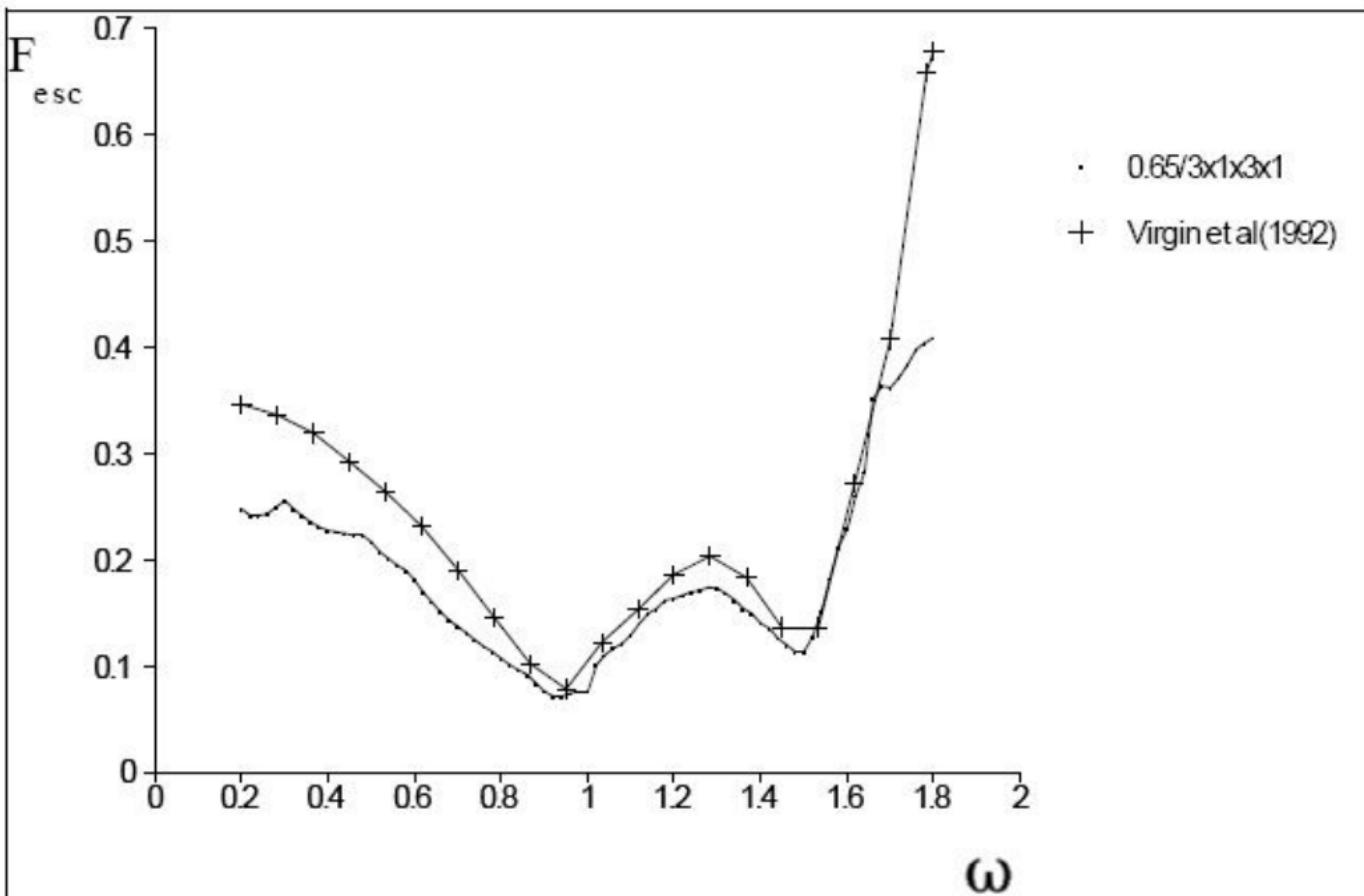


Figure 6.20 - Boundaries of safe motion: equation (6.17)

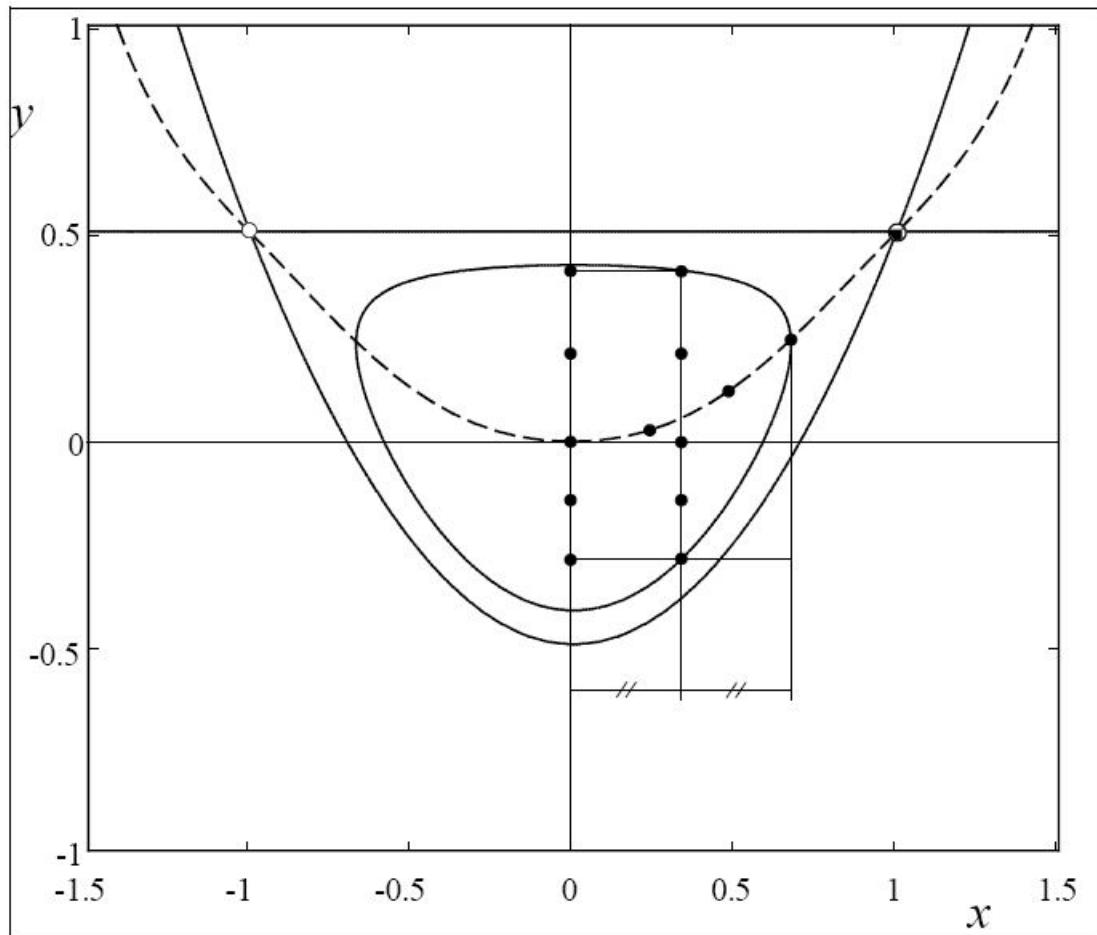


Figure 6.21 - One- and two-dimensional grids for the SIR model

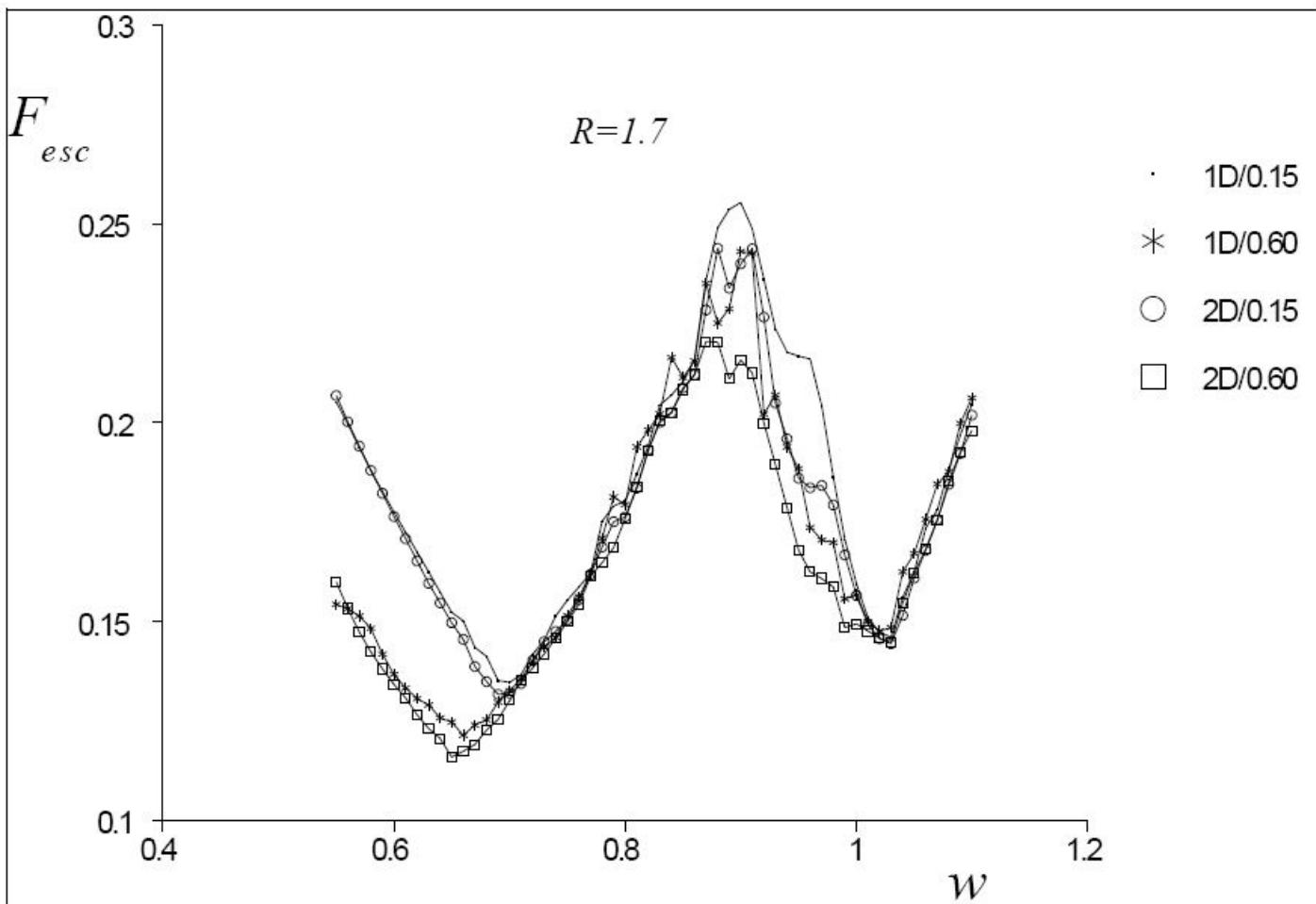


Figure 6.22 - Boundaries of safe motion for the SIR model: 1D and 2D grids

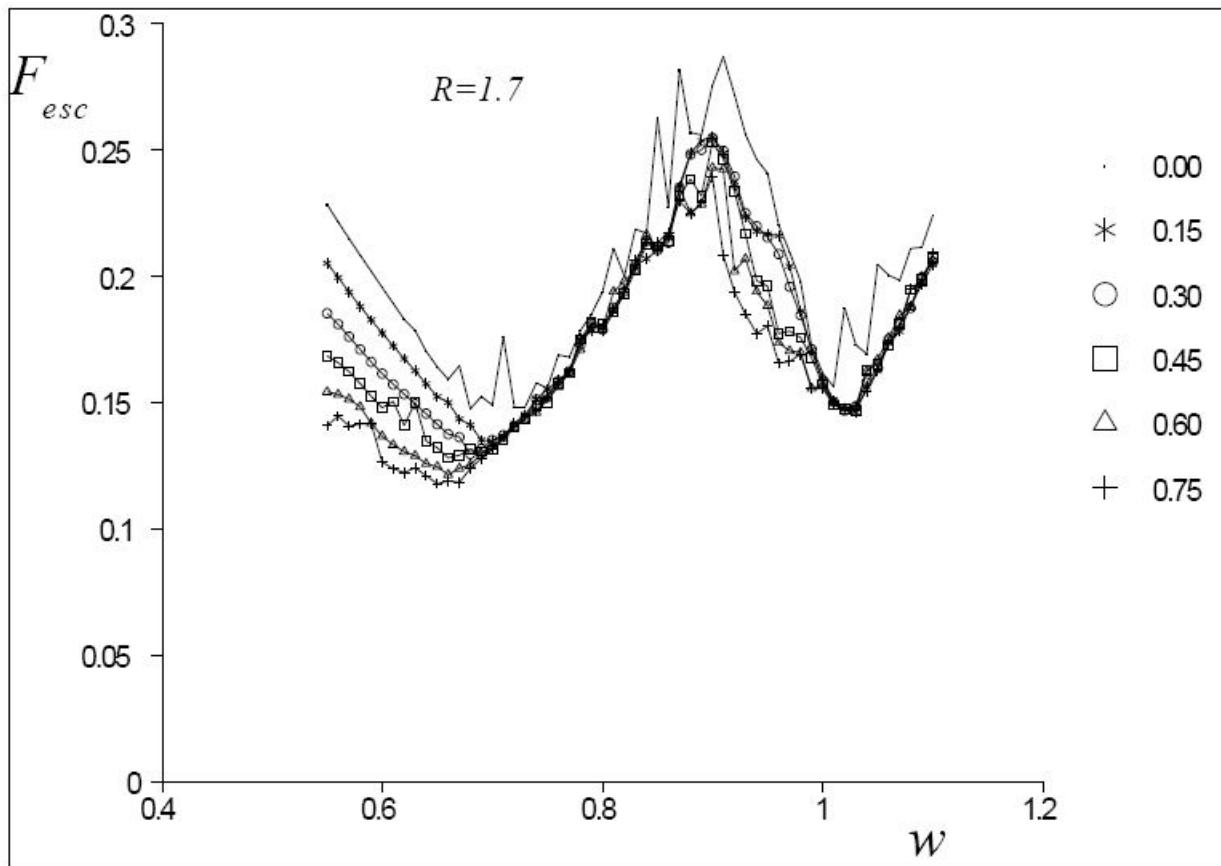


Figure 6.23 - Boundaries of safe motion for the SIR model: influence of grid size

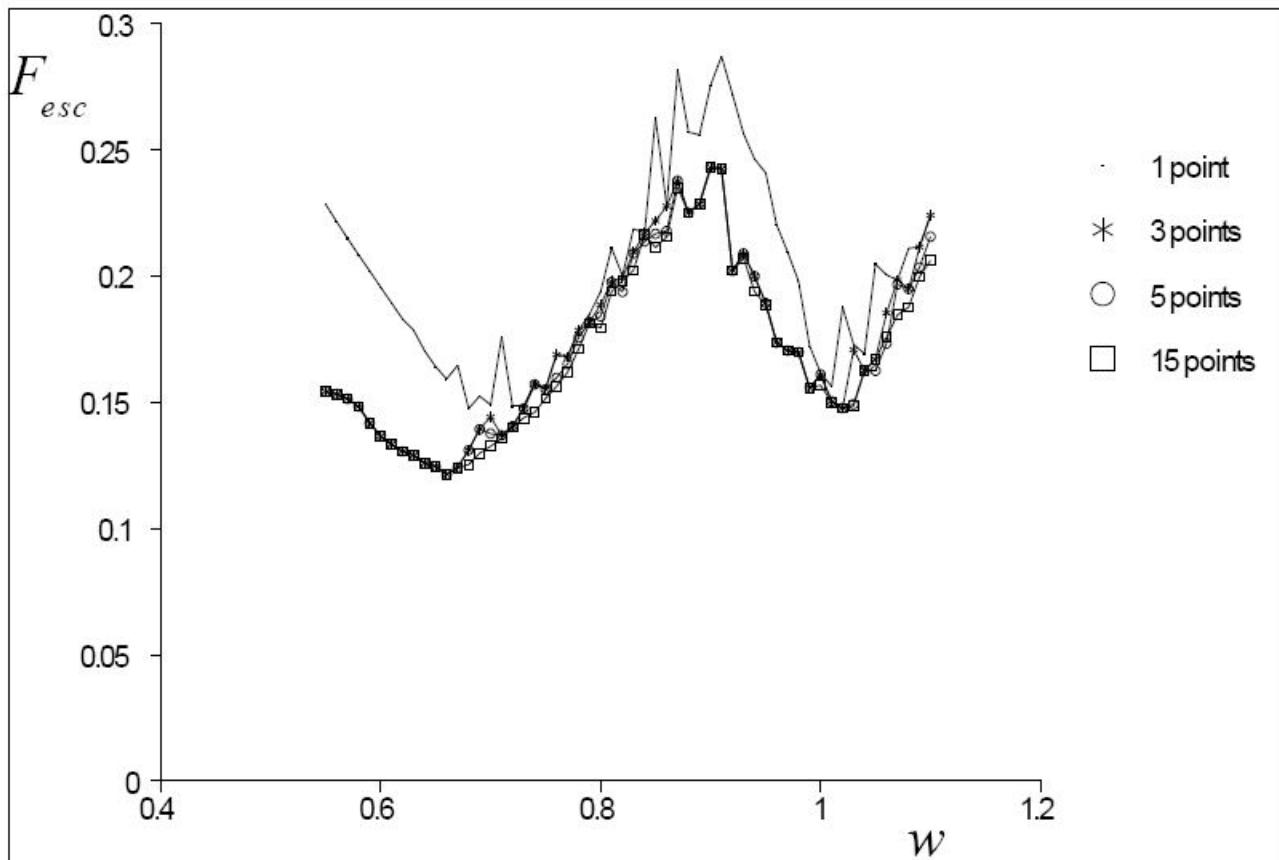


Figure 6.24 - Boundaries of safe motion for the SIR model: effect of grid density