Universidade de São Paulo
Escola Superior de Agricultura "Luiz de Queiroz"
Departamento de Ciências Exatas
LCE 0220 - Cálculo II
Professoras: Renata Alcarde Sermarini e Cristiane Mariana Rodrigues da Silva
Lista de Exercício - Funções de Várias Variáveis
Domínio, Gráficos, Curvas de Nível

1. Considere a função dada por $f(x,y) = \frac{2x+y}{y}$. Calcule:

   (a)) $f(1,1)$

   (b)) $f(0,3)$

   (c)) $f(-6,6)$

   (d)) $f(8,9)$

   (e)) $f(a,a)$, $a \neq 0$

   (f)) $f(0,3) + f(5,5)$

   (g)) $\frac{f(0,2)}{f(1,6)}$

   (h)) $f(3+\Delta x,\ 4) - f(3,4)$

   (i)) $f(3,\ 4+\Delta y) - f(3,4)$

2. Considere a função $f(x,y) = x+y$. Para que valores de x e y tem-se $f(x,y) = 2$? Represente graficamente a resposta.

3. Considere a função $f(x,y) = x \times y$. Represente graficamente os pontos (x,y) para os quais $f(x,y) = 1$.

4. Ache o domínio de cada uma das seguintes funções e represente-o graficamente:

   (a) $f(x,y) = \sqrt{x+y-2}$

   (b) $f(x,y) = \sqrt{y-x^2}$

   (c) $f(x,y) = \frac{1}{x+y-2}$

   (d) $f(x,y) = \sqrt{x^2+y^2-16}$

   (e) $f(x,y) = \frac{1}{\sqrt{x-y}}$

   (f) $f(x,y) = \sqrt{y-x} + \sqrt{y-2}$

   (g) $f(x,y) = \sqrt{xy}$

   (h) $f(x,y) = \log(x-y-2)$

   (i) $f(x,y) = \ln(x^2-y-1)$

   (j) $f(x,y) = \ln(y-x^3)$

5. Esboce o gráfico das seguintes funções:

   (a) $f(x,y) = xy$ com $D = \{(0,0),(1,0),(2,0),(0,1),(1,1),(2,1),(0,2),(1,2),(2,2)\}$

   (b) $f(x,y) = 3^x$ com $D = \{(0,0),(1,0),(2,0),(0,1),(1,1),(2,1),(0,2),(1,2),(2,2)\}$

   (c) $f(x,y) = 2$, $D = \mathbb{R}^2$

   (d) $f(x,y) = 5$, $D = \mathbb{R}^2$

   (e) $f(x,y) = 12 - 3x - 4y$, $D = \mathbb{R}^2$

   (f) $f(x,y) = x+y$, $D = \mathbb{R}^2$

(g) $f(x,y) = 3+x-y$, $D = \mathbb{R}^2$

(h) $f(x,y) = x^2+y^2$, $D = \mathbb{R}^2$

(i) $f(x,y) = 1-x^2$, $D = \mathbb{R}^2$

(j) $f(x,y) = 1-y^2$, $D = \mathbb{R}^2$

6. Esboce as curvas de nível das funções:

(a) $f(x,y) = 3x+4y$, nos níveis $c = 12$ e $c = 24$

(b) $f(x,y) = x-y$, nos níveis $c = 0$, $c = 1$ e $c = -1$

(c) $f(x,y) = 2x-3y$, nos níveis $c = 6$, $c = 10$ e $c = 12$

(d) $f(x,y) = \frac{1}{x^2+y^2}$, nos níveis $c = 1$ e $c = 4$

(e) $f(x,y) = y-x^2$, nos níveis $c = 0$ e $c = 1$

(f) $f(x,y) = y-x^2+4$, nos níveis $c = 0$ e $c = 5$

(g) $f(x,y) = y-x^3$, nos níveis $c = 0$ e $c = 1$

(h) $f(x,y) = \sqrt{x^2+y^2-2}$, nos níveis $c = 0$ e $c = 1$

(i) $f(x,y) = xy$, nos níveis $c = 1$, $c = -1$, $c = 2$ e $c = -2$

Respostas

1. (a)) 3

(b)) 1

(c)) -1

(d)) $\frac{25}{9}$

(e)) 3

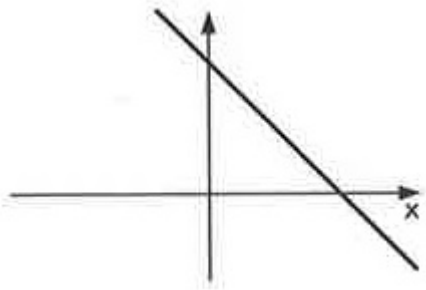
(f)) 4

(g)) $\frac{3}{4}$

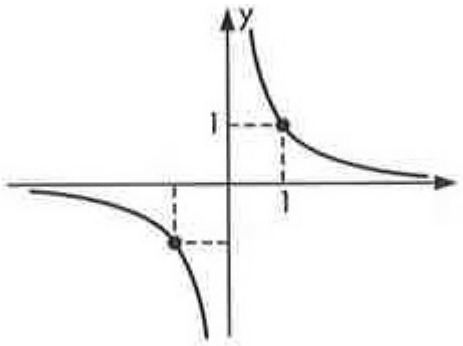
(h)) $\frac{f(\Delta x)}{2}$

(i)) $\frac{-3\Delta y}{2(4+\Delta y)}$

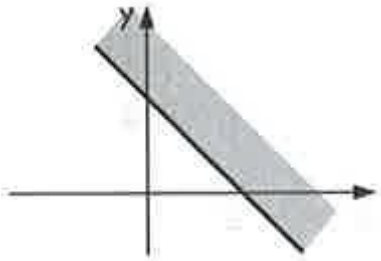
2.



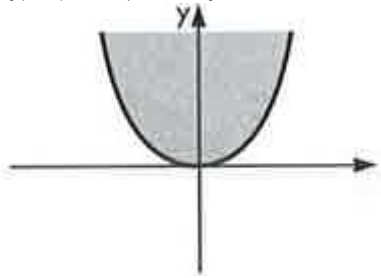
3.



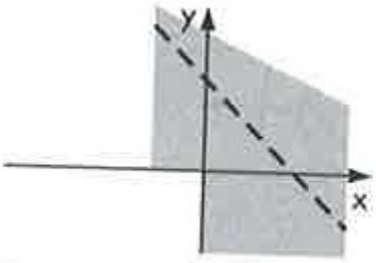
4. (a) $\{(x,y) \in R^2 / x+y-2 \geq 0\}$



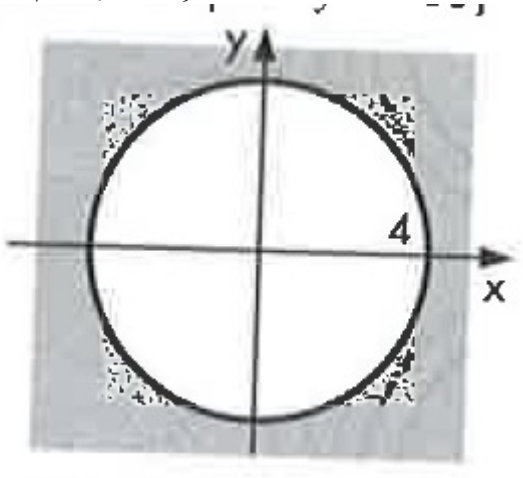
(b) $\{(x,y) \in R^2 / y \geq x^2\}$



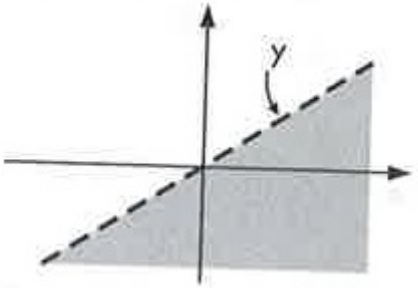
(c) $\{(x,y) \in R^2 / x+y-2 \neq 0\}$



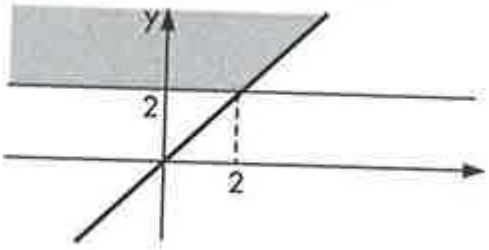
(d) $\{(x,y) \in R^2 / x^2+y^2 \geq 16\}$



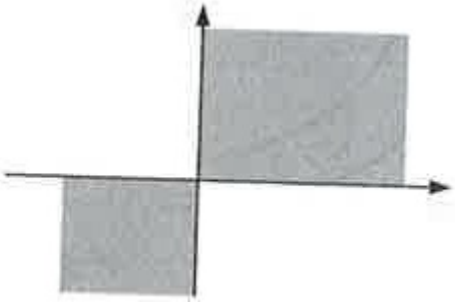
(e) $\{(x,y) \in R^2 / x - y > 0\}$



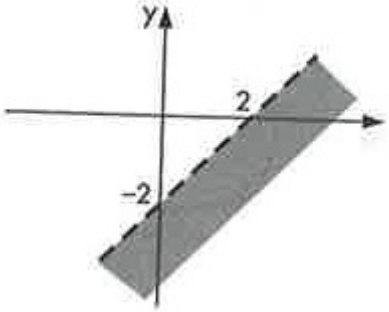
(f) $\{(x,y) \in R^2 / y \geq x \text{ e } y \geq 2\}$



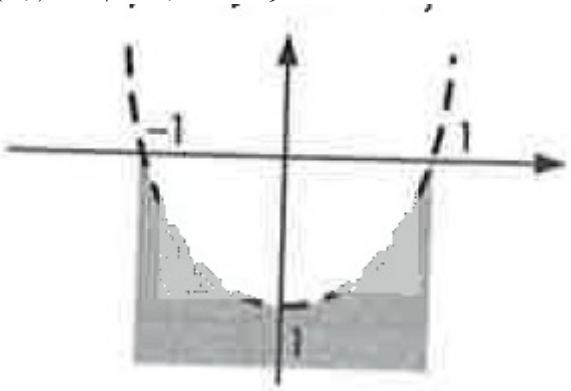
(g) $\{(x,y) \in R^2 / xy \geq 0\}$



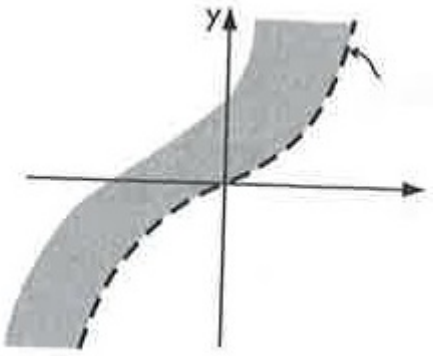
(h) $\{(x,y) \in R^2 / x - y - 2 > 0\}$



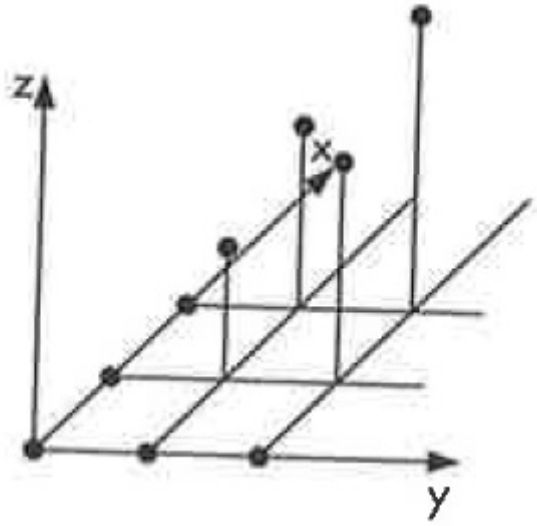
(i) $\{(x,y) \in R^2 / x^2 - y - 1 \geq 0\}$



(j) $\{(x,y) \in R^2 / y > x^3\}$



5. (a)



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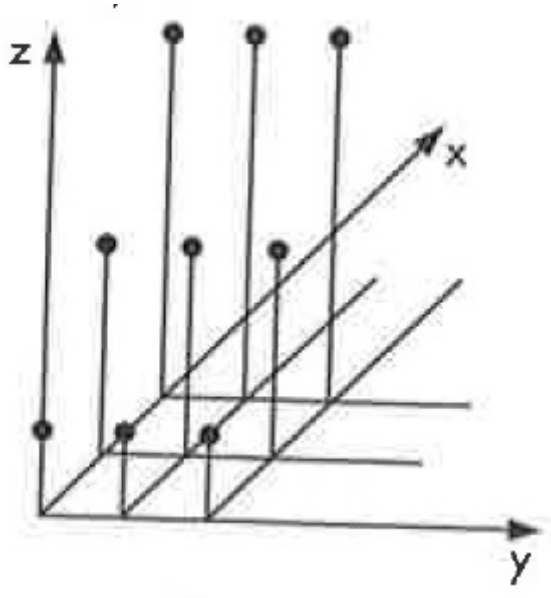

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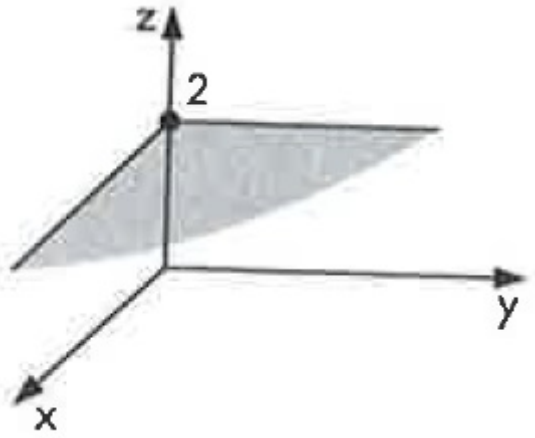

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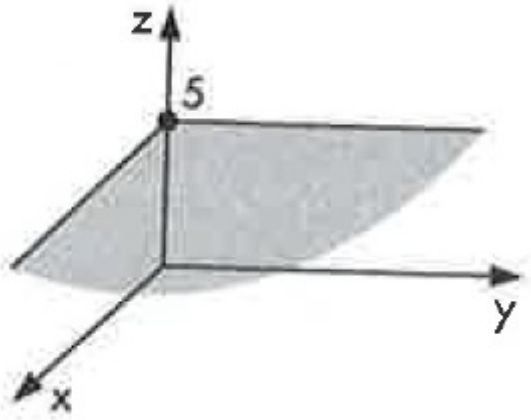

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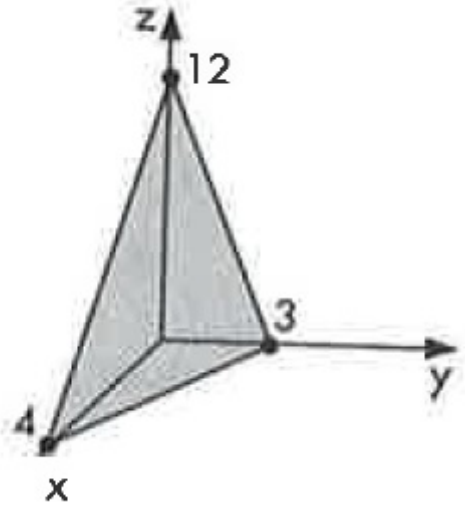

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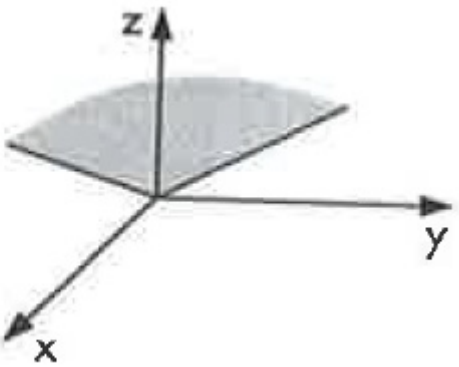

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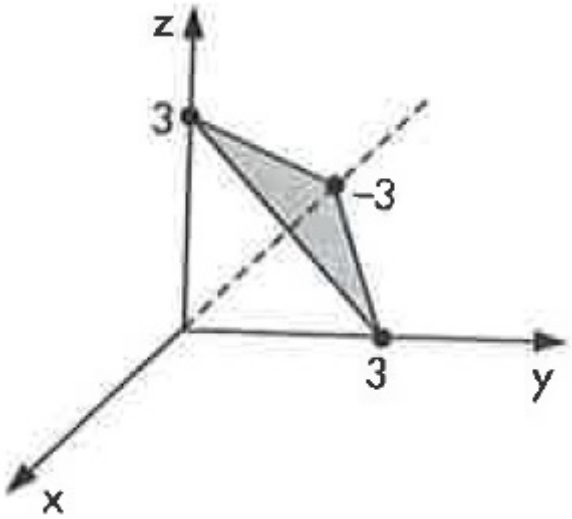

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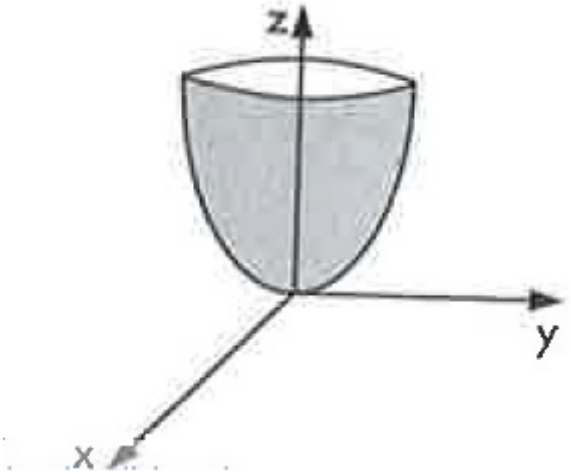

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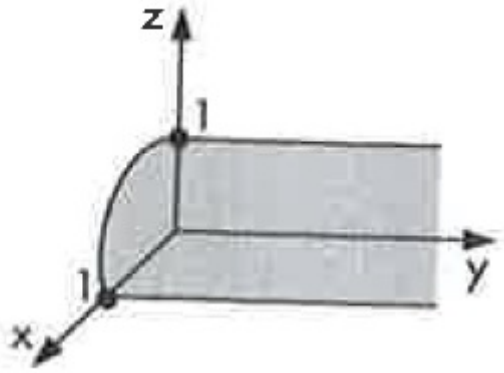

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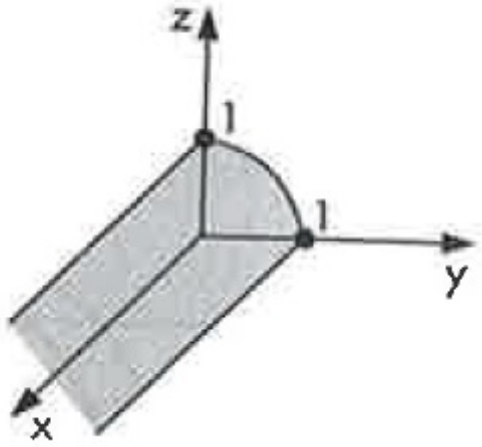
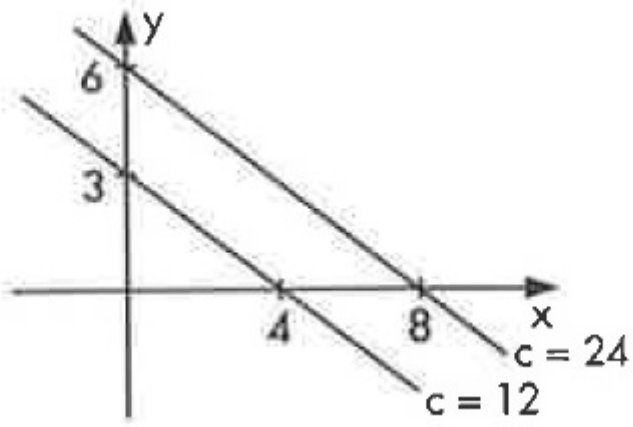


6. (a)



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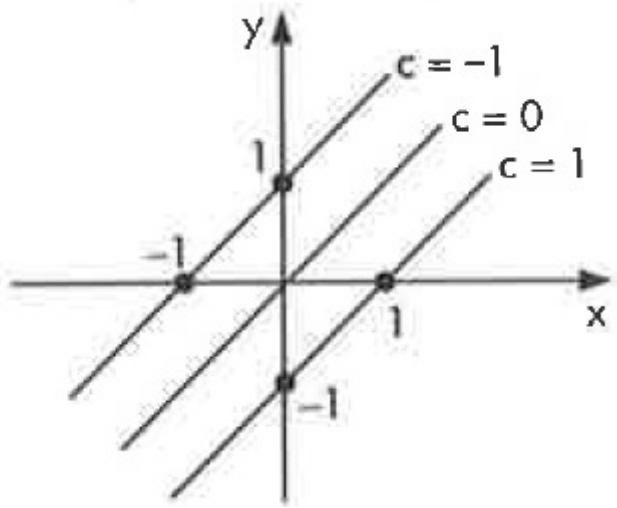

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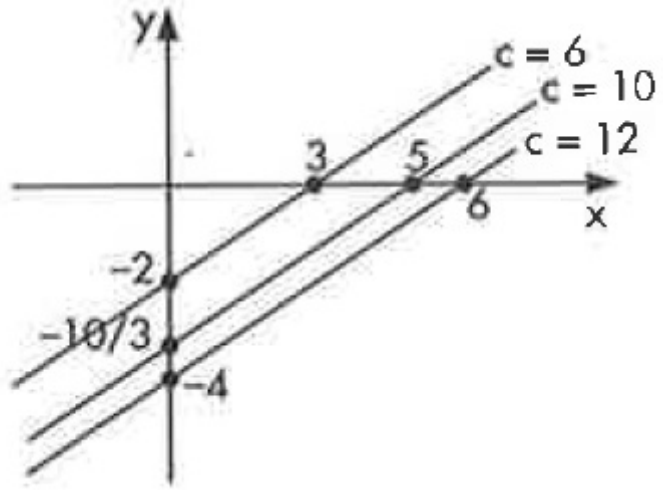
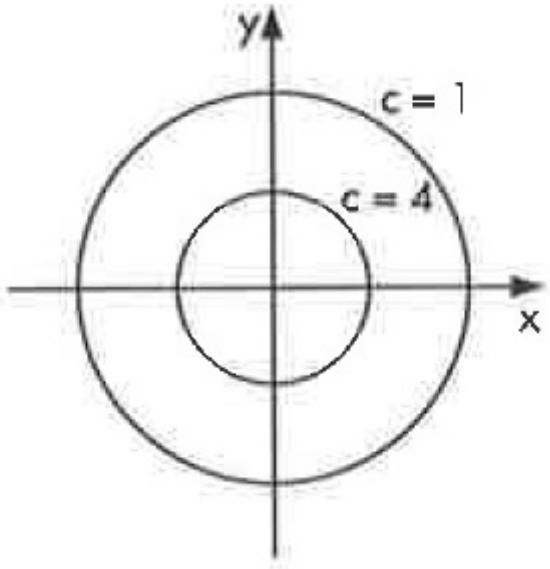


(d)



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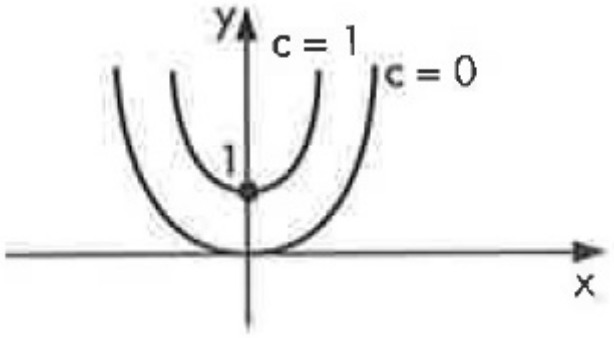

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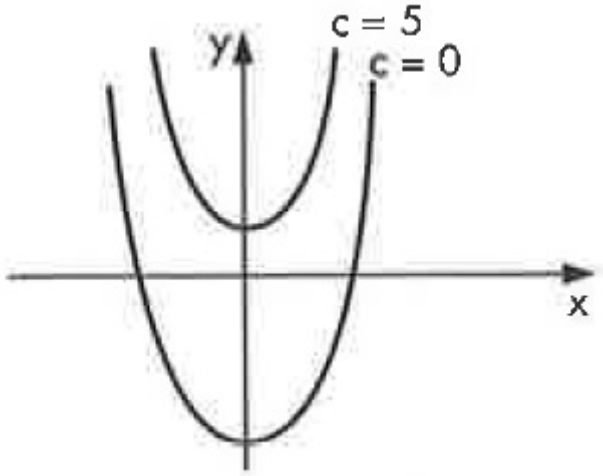

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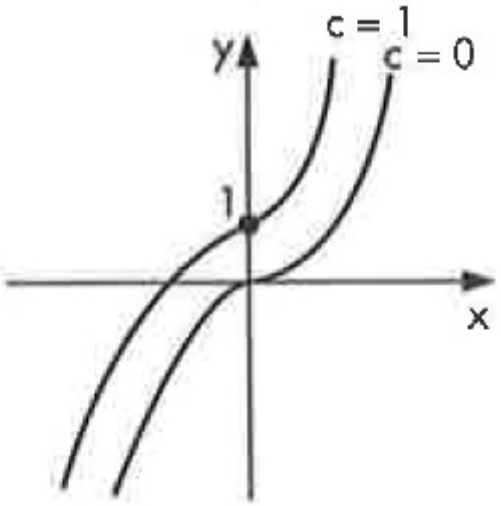

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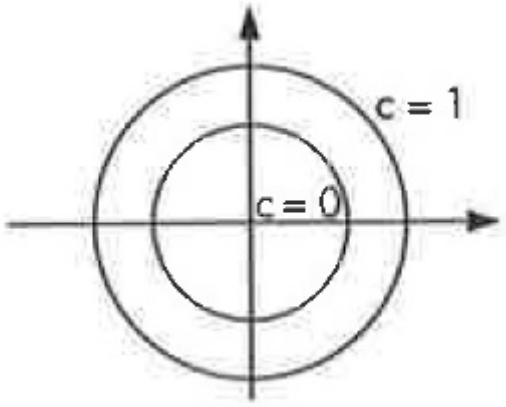

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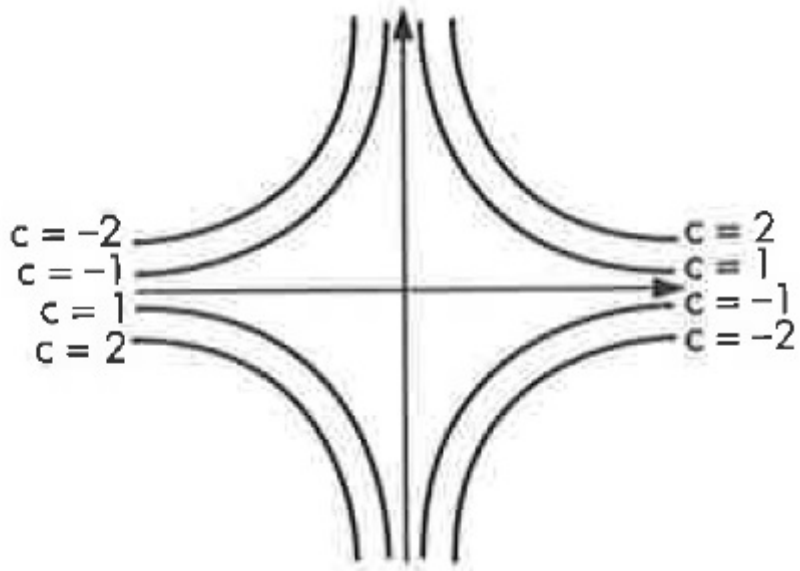

MORETTIN, P.A.; HAZZAN, S.; BUSSAB, W. O. Cálculo: funções de uma e várias variáveis. 2ª ed. São Paulo: Saraiva, 2012, 416p.