# Design model for electrical distribution systems considering renewable, conventional and energy storage units

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*Indexing terms: Distribution systems, Energy storage*

**Abstract:** A new model to solve the design problem of integrated electrical distribution systems is presented, considering special modelling for the load, different sources — conventional, renewable and energy storage — as well as connection lines. The resulting formulation is a linear programming problem, which is solved by a standard optimisation package. The advantages of this 'distributional approach' to the design problem of integrated distribution systems can be clearly seen from the results of the case study. The model determines the optimal size and site of all types of power supply units and connection lines. The same model can also be easily extended for the solution of the distribution expansion planning problem, when the planning period is divided into multiple subsequent stages.

## 1 Introduction

The structure of any power system is not solid. There are at least two reasons for this. First, in the demand side of the system, load has an intermittent nature and is continuously growing. The other reason, in the supply side, is concerned with the expansion of the existing plants to face the referred growing demand. Also, old power plants should be retired and new ones constructed.

With regard to these characteristics there is a constant design problem in the power system: to find out the optimal size and site for each new introduced power unit to face the load demand. The load demand is not definitely known so it is necessary to perform a load demand forecast before any attempt to plan the system expansion.

Determination of the optimal solution of the design problem means finding out an expansion alternative that minimises a cost function including capital cost and operating costs of the generation equipment as well as losses and capital costs in network components. Obviously the selected optimal alternative must satisfy a set of technical criteria, included in the model through a set of constraints.

When new types of generation units are introduced into the power system, complementing the existing conventional sources, all their particular advantageous features should be taken into account. When intermittent and energy storage sources are present, particular care must be taken to include into the planning model not only the fuel savings but also the decrease in installed capacities of conventional sources and connection lines as well as network losses. Studying new sources of energy has become a 'distributional problem' since there is the possibility to install them close to individual consumers.

New types of power units have been introduced so that they can replace a number of conventional units in the power system. The cost function must be changed accordingly so that the new units can be represented and modelled. It is clear that introduction of a new unit instead of a conventional one is cost effective only if the solution, obtained with the new cost function, is economically better than the previous one, without the new facility.

Two related problems are identified and briefly described as follows:

(*a*) Since the structure of the supply side of the power system is designed to face the maximum foreseen demand which in fact might never occur — everyday demand is not greater than the maximum — a control problem can be stated, that is to find out the optimal load for each generation unit to face the demand forecast for the next day.

(*b*) a current control problem, i.e. adjusting the solution of the control problem according to the real current demand.

The objective function for these problems corresponds to the operational part — fuel costs and losses costs — of the power system cost function. The present paper, however, is only concerned with the design problem and corresponding descriptions.

## 2 Power system description

In the proposed model the electrical power distribution system is represented by nodes interconnected through connection lines. Three types of nodes can be identified, namely pure generation, mixed load/generation and pure load. The supply side of the power system comprises conventional, storage and renewable sources. Each node is represented by a composition of models corresponding to the units — power supply sources or loads — connected to it. Fig. 1 illustrates the three types of nodes embedded in a possible power system configuration.

## 3 Design problem formulation

The design problem to be formulated is related to the determination of the optimal configuration of the power

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system and optimal location, type and sizing of generation units installed in certain nodes, so that the system meets demand requirements at minimum cost. The following Subsections are concerned with the modelling and characteristics of each system component, namely load, power sources — conventional, storage and renewable units — and connection lines. In sequence, the set of constraints and objective function are presented, leading to a linear programming problem formulation.

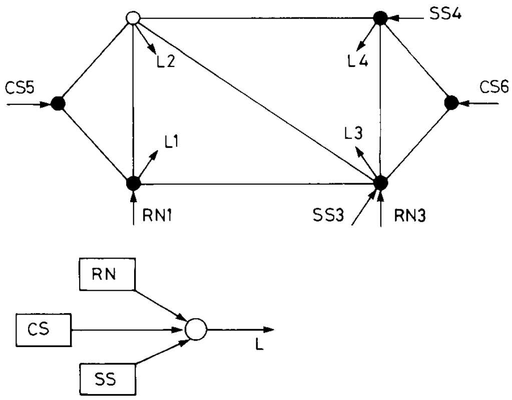


**Fig. 1** *Example of power system configuration*
RN = Renewable source
CS = Conventional source
SS = Storage source
L = Load

### 3.1 Load model

Load demand varies from day to day. For design purposes, however, the load characteristics must be given for the planning horizon and for the heaviest day in the year — usually a Tuesday or a Wednesday in December. Installed capacity should face this demand with a given reliability. During this day supply sources must generate the necessary amount of energy and be flexible enough to face the minimum demand. It is clear that if the supply side of the power system is able to face these requirements then it will successfully meet any variations of the load demand inside them. Therefore the model assumes that all the load curves in the system are approximated by two step curves, as presented in Fig. 2*a*. Energy demand could then be given by:

$$E_1 = \beta L \gamma \tau + L(1 - \gamma)\tau \tag{1}$$

where:

$E_1$ = energy demand during period $\tau$
$L, L_{min}$ = maximum and minimum load demands, respectively
$\beta = \frac{L_{min}}{L}$ = minimum load coefficient
$\gamma = \frac{t_{min}}{\tau}$ = minimum load duration coefficient
$t_{min}$ = duration of the minimum load demand
$\tau$ = load similarity period

and the minimum load duration coefficient is directly determined as: $\gamma = (L\tau - E_1)/[L\tau(1 - \beta)]$. Hence the load model for the design problem can be represented by four variables $L$, $\tau$, $\beta$ and $\gamma$ for each load node $i$ of the system. Fig. 2*a* illustrates the load model adopted.

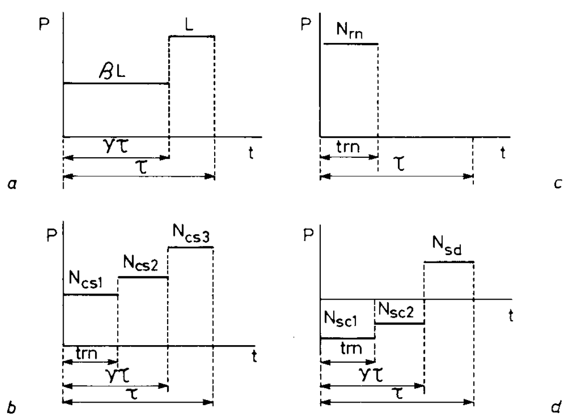


**Fig. 2** *Load curves*
*a* Load
*b* Conventional sources
*c* Renewable sources
*d* Storage sources

### 3.2 Source models

All the supply units are represented in the model by specific cost coefficients and a set of technical constraints. The three types of energy supply units — conventional, renewable and energy storage — are assumed to operate following the load curves illustrated in Fig. 2*b*, 2*c* and 2*d*. These assumptions are very acceptable for design purposes since the possible solutions will be related to the most severe conditions (e.g. the renewable sources contribute only during the minimum load period). It is also assumed that capital costs and installed capacities are linearly related. The cost function for each type of supply units is given as follows:

*3.2.1 Renewable sources:* since renewable sources generate free energy, their cost function ($f_{rn}$) includes only installation costs:

$$f_{rn} = \sum_{i \in \Omega_{rn}} (R_{rn,i} K_{rn,i} N_{rn,i}) \tag{2}$$

where:

$R_{rn,i}$ = discount rate for renewable source $i$
$K_{rn,i}$ = capital cost of renewable source $i$ per unit
$N_{rn,i}$ = installed capacity of renewable sources
$\Omega_{rn}$ = set of renewable sources

and the average energy generated ($E_{rn,i}$) is given by:

$$E_{rn,i} = N_{rn,i} F\tau \tag{3}$$

where

$F$ = capacity factor for renewable sources
$t_{rn} = F\tau$ = average duration in which installed renewable unit is available (assumption acceptable for design problem).

*3.2.2 Conventional sources:* the cost function for conventional sources can be divided into two parts, namely capital costs and operational costs. Capital costs are due to the installation of the supply units whereas operational costs are due to fuel consumption. Cost function ($f_c$) for these type of units can be written as:

$$f_c = \sum_{i \in \Omega_c} (R_{ci} K_{ci} N_{ci} + K_{fi} E_{ci}) \tag{4}$$

where:

$$E_{ci} = t_{rn} N_{cl,i} + (\gamma\tau - t_{rn}) N_{c2,i} + (1 - \gamma\tau) N_{c3,i} \tag{5}$$

$E_{ci}$ = Energy generated by conventional source $i$

$$N_{ci} = \max [N_{c1,i}\, N_{c2,i}\, N_{c3,i}] \tag{6}$$

$N_{ci}$ = Installed capacity of conventional source $i$
$R_{ci}$ = discount rates for conventional source $i$
$K_{ci}$ = capital cost of conventional source $i$ per unit
$N_{c1,i}\ N_{c2,i}\ N_{c3,i}$ = Power flow of conventional source $i$ in periods 1, 2 and 3, respectively. (See also Fig. 2*b*)
$K_{fi}$ = fuel cost for conventional source $i$
$\Omega_c$ = set of conventional sources

*3.2.3 Energy Storage:* these supply units can be charged during a certain period of time and discharged in any other period. The only requirement is that the charge-discharge cycle is completed during the period $\tau$. For modelling purposes it is assumed that the charge and discharge periods coincide with the minimum and maximum load periods, respectively. During the charge period, part of the absorbed energy is lost and the remaining energy is accumulated in the storage unit. The maximum amount of stored energy during the period $\tau$ is defined as the energy capacity. The total losses during the charge-discharge cycle are represented by the efficiency factors (see Fig. 3):

$$\eta = \frac{E_{sd}}{E_{sc}} = \frac{E_s}{E_{sc}} \frac{E_t}{E_s} \frac{E_{sd}}{E_t} = \eta_c \eta_s \eta_d \tag{7}$$

where:

$E_s$ = energy capacity for storage units
$E_{sc}, E_t, E_{sd}$ = energy amounts for storage units during charge period, during storage and during discharge periods, respectively
$\eta, \eta_c, \eta_s, \eta_d$ = total, charge, storage and discharge energy efficiencies

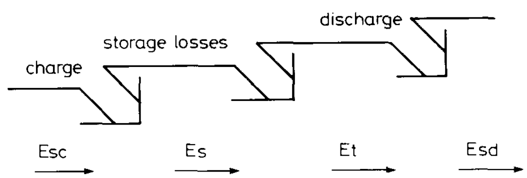


**Fig. 3** *Energy storage losses*

The cost function ($f_s$) for these units comprises the capital costs due to power and energy capacities and can be given as:

$$f_s = \sum_{i \in \Omega_{rn}} R_s(K_{e,i} E_{s,i} + K_{s,i} N_{s,i}) \tag{8}$$

where:

$$N_{s,i} = \max\,[N_{sc1,i} \quad N_{sc2,i} \quad N_{sd,i}] \tag{9}$$

$N_{s,i}$ = installed power capacity of storage unit $i$

$$E_{s,i} = \eta_c[N_{sc1,i}\,t_{rn} + N_{sc2,i}(\gamma t - t_{rn})] \tag{10}$$

$E_{s,i}$ = installed energy capacity of storage unit $i$
$R_s$ = discount rate for storage units
$K_{e,i}$ = capital cost per unit of energy capacity for storage unit $i$
$K_{s,i}$ = capital costs per unit of installed storage power capacity
$N_{sc1,i}, N_{sc2,i}, N_{sd,i}$ = Power flows to or from energy storage unit $i$ during periods 1, 2 and 3, respectively (see also Fig. 2*d*)

### 3.3 Connection lines model

Connection lines costs are also divided into capital and operational costs. Capital costs are due to the investment in installation and are assumed to be proportional to the maximum power flow (or to the installed capacity) of the line. Operational costs are due to Joule losses ($RI^2$) and are assumed to be proportional to the corresponding current power flows. Therefore, the resulting cost function for lines is given by:

$$f_1 = \sum_{ij \in \Omega_1} \left[ R_1 K_{lij} l_{ij} P_{cij} + \sum_{t=1}^{3} K_{ltij} l_{ij} P_{ijt} \right] \tag{11}$$

where

$\Omega_1$ = set of connection lines

$$P_{cij} = \max\,[P_{ijl}, P_{ij2}, P_{ij3}] \tag{12}$$

$R_1$ = discount rate for connection lines
$K_{lij}$ = capital cost per unit length and power flow of line $ij$
$l_{ij}$ = line length
$P_{cij}$ = installed capacity of connection line $ij$
$K_{ltij}$ = losses coefficient of line $ij$
$P_{ijt}$ = power flow at line $ij$, period $t$ ($t = 1, 2, 3$)

### 3.4 Set of constraints

The set of constraints assure that the possible set of solutions is operational and feasible in the sense that the obtained optimal solution satisfies a set of technical criteria. The following constraints are also included in the design model:

*3.4.1 Power balance equations:* The first Kirchhoff law must be satisfied for each node of the power system, so that power balance conditions are met. The period $\tau$ can be divided into three subperiods (see also Fig. 2):

(*a*) Period 1 $[0, t_{rn}]$: renewable and conventional sources are available, energy storage sources are being charged and demand is minimal

(*b*) Period 2 $[t_{rn}, \gamma\tau]$: conventional sources are generating, storage sources are being charged and demand is minimal

(*c*) Period 3 $[\gamma\tau, \tau]$: conventional sources are available, storage sources are discharging and demand is maximal

Load demand requirements are then assured by the following equations, written for each node $i$:

$$N_{rn,i} + N_{c,t,i} - N_{sc,t,i} + \sum_{j \in \Omega_i} P_{ijt} - \beta_i L_i = 0 \quad (t = 1)$$
$$N_{c,t,i} - N_{sc,t,i} + \sum_{j \in \Omega_i} P_{ijt} - \beta_i L_i = 0 \quad (t = 2)$$
$$N_{c,t,i} + N_{sd,t,i} + \sum_{j \in \Omega_i} P_{ijt} - L_i = 0 \quad (t = 3)$$
$$(13)$$

*3.4.2 Energy balance equations:* Energy balance must be represented in the model for each energy storage unit; that is, during the period $\tau$, the energy discharged from the storage equals the energy charged times the total efficiency. These equations can be given as:

$$\eta[N_{sc,i,1} F\tau + N_{sc,i,2}(\gamma\tau - F\tau)] - N_{sd,i}(1-\gamma)\tau = 0 \tag{14}$$

Energy demand and losses in the system, during the period $\tau$, must be supplied by conventional and renewable source units (energy storage does not generate energy). This requirement, total energy balance, is indirectly satisfied by the existing constraints — this assertion can be deduced by manipulating the power balance equations.

*3.4.3 Special limitations:* Possible installed capacity for renewable and conventional supply units, in any particular site, can be limited. This limitation is included in the model through the inequalities:

$$N_{rn,\,i} \leqslant N_{rnmax,\,i} \quad i \in \Omega_{rn} \tag{15}$$

$$N_{c,\,i} \leqslant N_{cmax,\,i} \quad i \in \Omega_{c} \tag{16}$$

where: $N_{rnmax,\,i}$ and $N_{cmax,\,i}$ are the maximum limits for renewable and conventional units at node $i$. Installed power and energy capacity for energy storage sources must also be bounded. In this case, however, limitation is due to load levelling. It is clear that their values should be no greater than what is needed for complete load levelling when the conventional generation curve is a straight line. This requirement is given by:

$$N_{s,\,i} \leqslant Li - \frac{E_{l,\,i}}{\tau} \quad i \in \Omega_s \tag{17}$$

$$E_{s,\,i} \leqslant \left[ Li - \frac{E_{l,\,i}}{\tau} \right] \Big/ [\tau(1 - \gamma)] \quad i \in \Omega_s \tag{18}$$

Renewable sources are intermittent. Capacity factor $F$ gives some average information but it is quite possible that renewable units are not available during the period $\tau$. In this case, the remaining types of supply units must be able to face the load demand. It means that conventional source should have enough capacity to generate all the energy required during the period $\tau$. This 'reserve' requirement, when applicable for an autonomous system, is given by:

$$\sum_{i \in \Omega_c} N_{ci} \geqslant \sum_{j=1}^{n} \frac{E_{lj}}{\tau} \tag{19}$$

The maximum capacity of connection lines is also included in the model by inserting an upper bound $P_{maxij}$ for each possible new path $ij$:

$$P_{cij} \leqslant P_{maxij} \quad ij \in \Omega_1 \tag{20}$$

where $P_{maxij}$ is the maximum capacity of connection line $ij$.

## 4 Optimisation problem formulation

Eqns. 2, 4, 8 and 11 define the cost function components and the set of constraints is defined by eqns. 5–6, 9–10 and 12–20. The optimisation problem formulation can be written as follows:

$$\begin{aligned} \min f(x) &= \quad cx \\ s.t. \quad & \quad Ax \leqslant b \end{aligned} \tag{21}$$

where:

$f = f_c + f_{rn} + f_{sc} + f_l$ – (linear) cost function
$x$ = vector of model variables
$A$ = matrix, resulted from the set of constraints
$b$ = right hand side of constraints

The given formulation in eqn. 21 is a linear programming (LP) problem and can be solved by any standard optimisation package.

## 5 Simplified design criterion

From the economical point of view, the main advantage of renewable sources is fuel saving. Therefore it is interesting to find out the minimal fuel cost when usage of renewable units can be justified. For a simplified system, when the conventional source is separated from the renewable and storage units and from the load by a single connection line (length $l$), a simple criterion can be formulated. It is given by:

$$K_f \geqslant \left[ \frac{R_{rn} K_{rn}}{\tau\gamma\eta} + R_s K_e - \frac{R_c K_c - R_s K_s + R_l K_l l}{[1 - \gamma]\tau} \right] \Big/ 365 \tag{22}$$

This criterion determines the minimum fuel cost coefficient that will result in the inclusion of renewable and energy storage units in the optimal solution for the power system expansion.

## 6 Computational model

Once the formulation of the model has been formally stated, the computational model can be specified. Moreover the computational model should allow an easy interaction man–machine with facilities for graphic visualisation of topology, data and obtained results.

The model stated in the previous sections can be solved by any LP standard package. The basic configuration of the framework developed is presented in Fig. 4, in which main programs, files and their interrelation are shown.

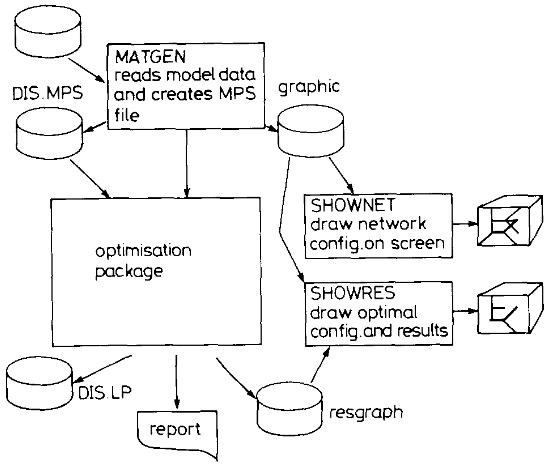


**Fig. 4** *Computational model*

The main programs for the model are MATGEN and the optimisation package modules. The other two auxiliary programs, SHOWNET and SHOWRES, allow the drawing of the proposed facilities as well as the presentation of results obtained by the model.

The program MATGEN reads the necessary data from the file DIS.DAT and creates two output files. The first one (DIS.MPS) is used by the optimisation package and contains the formulation data in a specific format (MPS format). The second output file (GRAPHIC) is used by the auxiliary programs. The optimisation package generates two output files: the file DIS.LP, a report of each simulation, and the file RESGRAPH, containing the necessary information for the auxiliary program SHOWRES.

## 7 Case study

A simplified case study is proposed to illustrate the model possibilities. It represents the design of a new independent electrical distribution system, supplying for example a small community. The proposed facilities — wind energy (renewable) units, hydropumped storage (energy storage) units, diesel (conventional) units, and lines — to be selected by the model-are presented in Fig. 1. Five different cases were simulated:

(i) only conventional sources are allowed to be installed in nodes 5 and 6

(ii) in addition to the possible conventional sources of case i, renewable sources are allowed to be installed in nodes 1 and 3

(iii) in addition to the possible conventional sources of case i, only energy storage sources are allowed to be installed in nodes 3 and 4

(iv) conventional, renewable and energy storage units are allowed to be installed in the nodes specified in cases i, ii and iii

(v) *network is not represented*, all the types of energy sources are allowed to be installed. This case is simulated by supplying the total load in one site (one node system). The result must provide only the total amounts of required capacity.

Load parameters are presented in Table 1. Table 2 contains the parameters [1–3] for energy sources and connection lines coefficients. Capacity limits are 15 MVA for lines 1–5 and 2–5 and 10 MVA for the remaining lines. Discount rates are 20% a.a. and the period $\tau$ is 24 h.

**Table 1: Load characteristics**

| Nodes | Load (MW) | $\beta$ | $\gamma$ |
|---|---|---|---|
| 1 | 10 | | |
| 2 | 10 | 0.5 | 0.5 |
| 3 | 12 | | |
| 4 | 12 | | |

**Table 2: Energy sources and lines characteristics**

| Type | Parameters | | |
|---|---|---|---|
| Conventional source | $K_c$ = £500/kW | $K_f$ = £0.0525/kWh | |
| Renewable source | $K_{rn}$ = £800/kW | $F$ = 0.30 | |
| Storage source | $K_s$ = £200/kW | $K_e$ = £10/kWh | $\eta$ = 0.8 |
| Lines (per km) | $K_l$ = £273/kW | $K_{lt}$ = £0.005/kWh | |

The final topology configurations and power flows per period for cases i–iv are presented in Fig. 5. Economical results, classified according to system facilities (network costs, conventional, energy storage and renewable units costs) and according to operational and capital costs are presented in Table 3.

The model determines optimal sizes and sites by selection among all possible given facilities. In case v — one node simulation — optimal sites are not identified since the network is not represented and only total capacity amounts of energy sources are determined.

Case i considers only conventional sources and new feeders as candidates for installation in the system. The optimal solution comprises installation of six feeders in the system, out of the nine proposed, that is feeders 1–5, 2–5, 3–6, 4–6, 2–4 and 1–3 with capacities of 12, 12, 10, 10, 2 and 2 MVA, respectively. The two conventional

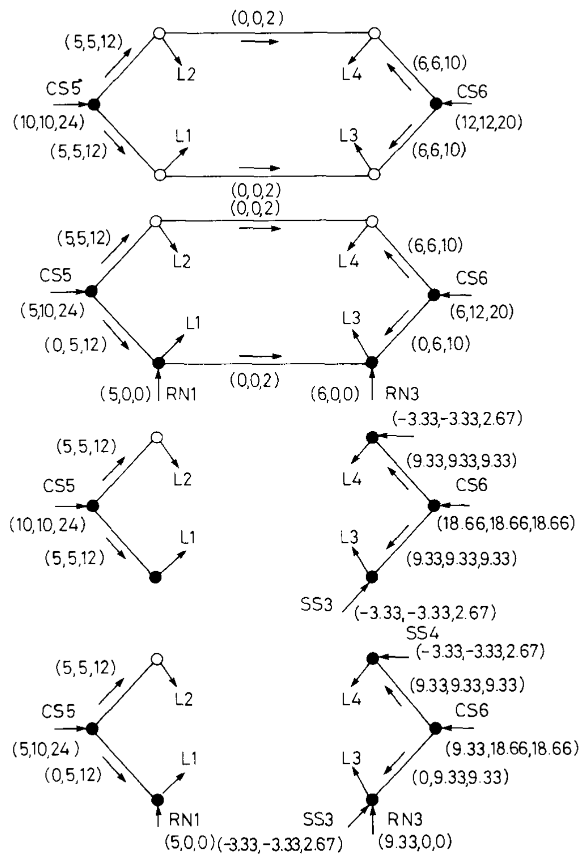


**Fig. 5** *Final configurations and power flows for cases (i)–(iv)*
[power flows (—, —, —) for periods 1, 2 and 3 in MW]
*a* case i
*b* case ii
*c* case iii
*d* case iv

units are installed at buses 5 and 6, with capacities of 24 and 20 MVA, respectively. It is noticed that feeders 2–4 and 1–3 are used only at peak demand periods. The optimal cost of case i sets an upper bound for the optimal cost of cases ii–iv, since in these cases a greater number of facilities are proposed for selection.

Case ii also considers renewable units in the set of possible alternatives. The optimal solution obtained includes all facilities chosen in case i and also the installation of renewable units at buses 1 and 3, with capacities of 5 and

**Table 3: Solution costs for cases i–v**

| Case | Costs $10^3$ £/day | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Network | | Conventional | | Energy storage | | Renewable | Total |
| | losses | installation | operation | installation | operation | installation | installation | |
| i | 4.20 | 7.20 | 41.58 | 12.10 | — | — | — | 65.08 |
| ii | 3.76 | 7.20 | 36.96 | 12.10 | — | — | 4.84 | 64.86 |
| iii | 4.04 | 5.80 | 42.42 | 10.64 | 0.44 | 0.73 | — | 64.07 |
| iv | 3.47 | 5.80 | 36.40 | 10.63 | 0.44 | 0.73 | 6.31 | 63.78 |
| v | — | — | 41.58 | 12.10 | 0.00 | 0.00 | 0.00 | 53.68 |

6 MVA, respectively. The installed capacity of conventional sources and network feeders is not changed with relation to case i, since renewable sources are not available during the maximum demand period. The installation of renewable units increases the total investment (installation costs) which is compensated by a cut in network losses and operational costs of conventional units, leading to savings in the total cost.

Case iii differs from case i by the fact that energy storage units are also considered in the set of possible alternatives. Four feeders are selected for installation by the model: 1–5, 2–5, 3–6 and 4–6 with capacities of 12, 12, 9.33 and 9.33 MVA, respectively. Conventional sources are installed at buses 5 and 6 with capacities of 24 and 18.66 MVA, respectively. Energy storage units are installed at buses 3 and 4, with capacities of 3.33 MVA each. The losses in the system are decreased with relation to case i. Investment costs in network and conventional units are also decreased since energy storage units are discharged during the maximum demand period. Operational costs are increased since the efficiency in the charge–discharge cycle of energy storage units is less than 100%. The resulting system leads to savings in total costs when compared with case i.

Case iv considers all possible energy source units and network feeders of the previous cases. The optimal solution, represented in Fig. 3*c*, comprises the same four lines, conventional units and energy storage units of case iii. However renewable units are installed at buses 1 and 3, with capacities of 5 and 9.33 MVA, respectively. It is noticed that the renewable unit at bus 3, available during the first period, has its capacity increased when compared to case ii, since it supplies not only the load but also the energy storage unit during the first part of the charging period. This solution presents the minimum expenses in operational and installation costs of conventional units and network feeders compensating the investment and operation costs of energy storage and renewable units, leading to savings in total costs.

Case v — one node simulation — does not take into account the network and corresponding installation and losses costs. The costs obtained and presented in Table 3 are due only to energy sources total amounts. The optimal solution comprises only installation of conventional sources, since the benefits in the network due to renewable and energy sources are not accounted.

Obviously the results obtained in this Section are not general for they are valid for this specific electrical system and data used. It is clear that for the given fuel and generation units' costs the optimal expansion plan comprises the three types of energy sources only when the network is taken into consideration. Network modelling therefore is of fundamental importance for justifying the use of renewable sources and designing integrated electric power systems.

## 8 Conclusions

We have described an approach to the modelling of different energy sources to power system design and planning studies. The model allows the accountancy of advantages due to new sources such as renewable and energy storage units, their impact on the installed capacities and fuel consumption of the conventional sources and corresponding distribution network. The case study results show that the model is very powerful to design integrated electrical distribution power systems.

The proposed model is formulated in a form suitable to be solved by a standard linear optimisation package. The model can be expanded for the representation of multistage expansion studies, when the planning period (present date to horizon), is accordingly divided into intervals. Other possible expansions in the formulation are related to the inclusion of different types of conventional and renewable sources, consideration of fixed and variable costs represented through binary decision variables (which leads to a mixed integer linear programming formulation) and consideration of social and environmental objectives [6]. Such model extensions are currently being addressed by the authors.

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