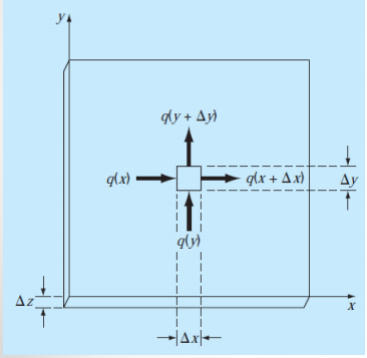
PHA 3002

Aula 11

Fluxo de Calor Bidimensional

$$q(x)\,\Delta y\,\Delta z\,\Delta t + q(y)\,\Delta x\,\Delta z\,\Delta t = q(x+\Delta x)\,\Delta y\,\Delta z\,\Delta t + q(y+\Delta y)\Delta x\,\Delta z\,\Delta t$$



$$q_i = -k\rho C\frac{\partial T}{\partial i}$$

$$T = \frac{H}{\rho C V}$$

Equação de Poisson

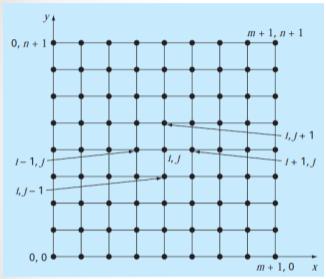
$$[q(x) - q(x+\Delta x)]\Delta y + [q(y) - q(y+\Delta y)]\Delta x = 0$$

$$\frac{q(x) - q(x+\Delta x)}{\Delta x}\Delta x\,\Delta y + \frac{q(y) - q(y+\Delta y)}{\Delta y}\Delta y\,\Delta x = 0 \quad \rightarrow \quad -\frac{\partial q}{\partial x} - \frac{\partial q}{\partial y} = 0$$

Equação de Poisson

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Solução Numérica



$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2}$$

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} = 0$$

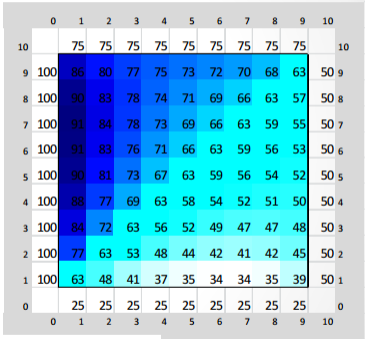
$$\Delta x = \Delta y$$

$$T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j} = 0$$

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}$$

Exemplo

- Calcular as temperaturas na placa plana ao lado



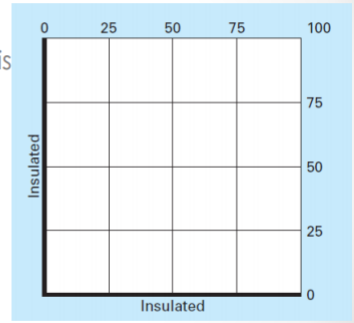
	0	1	2	3	4	5	6	7	8	9	10	
10		75	75	75	75	75	75	75	75	75		10
9	100	86	80	77	75	73	72	70	68	63	50	9
8	100	90	83	78	74	71	69	66	63	57	50	8
7	100	91	84	78	73	69	66	63	59	55	50	7
6	100	91	83	76	71	66	63	59	56	53	50	6
5	100	90	81	73	67	63	59	56	54	52	50	5
4	100	88	77	69	63	58	54	52	51	50	50	4
3	100	84	72	63	56	52	49	47	47	48	50	3
2	100	77	63	53	48	44	42	41	42	45	50	2
1	100	63	48	41	37	35	34	34	35	39	50	1
0		25	25	25	25	25	25	25	25	25		0
	0	1	2	3	4	5	6	7	8	9	10	

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}$$

```
Sub prepara()

    Columns("A:BM").Select
    Selection.ColumnWidth = 0.2
    Rows("1:65").Select
    Selection.RowHeight = 2
    Range(Cells(1, 1), Cells(65, 65)).Interior.Color = RGB(0, 255, 255)
    Range("a1").Select


End Sub
```

Exercício

- Resolver o problema da temperatura variável no tempo da placa com dois lados isolados utilizando 100 elementos..