CHAPTER 10
Cavity Theory

I. BRAGG-GRAY THEORY

The basis for cavity theory is contained in Eq. (8.27) of Chapter 8. If a fluence Φ of identical charged particles of kinetic energy T passes through an interface between two different media, g and w, as shown in Fig. 10.1a, then one can write for the absorbed dose on the g side of the boundary

$$D_g = \Phi\left[\left(\frac{dT}{\rho dx}\right)_{c,g}\right]_T \tag{10.1}$$

and on the w side,

$$D_w = \Phi\left[\left(\frac{dT}{\rho dx}\right)_{c,w}\right]_T \tag{10.2}$$

where $[(dT/\rho dx)_{c,g}]_T$ and $[(dT/\rho dx)_{c,w}]_T$ are the mass collision stopping powers of the two media, evaluated at energy T. Usually we may omit the brackets and subscript T, evaluation at an appropriate energy T being implied.

Assuming that the value of Φ is continuous across the interface (i.e., ignoring backscattering) one can write for the ratio of absorbed doses in the two media adjacent to their boundary

$$\frac{D_w}{D_g} = \frac{(dT/\rho dx)_{c,w}}{(dT/\rho dx)_{c,g}} \tag{10.3}$$

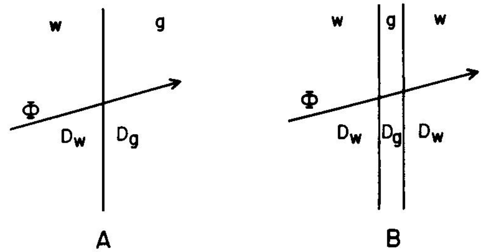

FIGURE 10.1. (*A*) **A fluence Φ of charged particles is shown crossing an interface between media w and g. Assuming Φ to be continuous across the boundary, the dose ratio D_w/D_g equals the corresponding ratio of mass collision stopping powers.** (*B*) **A fluence Φ of charged particles passes through a thin layer of medium g sandwiched between regions containing medium w. Assuming Φ to be continuous across layer g and both interfaces, the dose ratio D_w/D_g is again equal to the corresponding ratio of mass collision stopping powers.**

W. H. Bragg (1910) and L. H. Gray (1929, 1936) applied this equation to the problem of relating the absorbed dose in a probe inserted in a medium to that in the medium itself. Gray in particular identified the probe as a gas-filled cavity, whence the name "cavity theory". The simplest such theory is called the Bragg-Gray (B-G) theory, and its mathematical statement, referred to as the *Bragg-Gray relation*, will be developed next.

Suppose that a region of otherwise homogeneous medium w, undergoing irradiation, contains a thin layer or "cavity" filled with another medium g, as in Fig. 10.1*b*. *The thickness of the g-layer is assumed to be so small in comparison with the range of the charged particles striking it that its presence does not perturb the charged-particle field.* This assumption is often referred to as a "Bragg-Gray condition". It depends on the scattering properties of w and g being sufficiently similar that the mean path length (g/cm^2) followed by particles in traversing the thin g-layer is practically identical to its value if g were replaced by a layer of w having the same mass thickness. Similarity of backscattering at w-g, g-w, and w-w interfaces is also implied.

For heavy charged particles (either primary, or secondary to a neutron field), which undergo little scattering, this B-G condition is not seriously challenged so long as the cavity is very small in comparison with the range of the particles. However, for electrons even such a small cavity may be significantly perturbing unless the medium g is sufficiently close to w in atomic number.

Bragg-Gray cavity theory can be applied whether the field of charged particles enters from outside the vicinity of the cavity, as in the case of a beam of high-energy charged particles, or is generated in medium w through interactions by indirectly ionizing radiation. In the latter case it is also assumed that no such interactions occur in g. All charged particles in the B-G theory must originate elsewhere than in the cavity. Moreover charged particles entering the cavity are assumed not to stop in it.

A second B-G condition, incorporating these ideas, can be written as follows: *The absorbed dose in the cavity is assumed to be deposited entirely by the charged particles crossing it.*

This condition tends to be more difficult to satisfy for neutron fields than for photons, especially if the cavity gas is hydrogenous, thus having a large neutron-interaction cross section. The heavy secondary charged particles (protons, α-particles, and recoiling nuclei) also generally have shorter ranges than the secondary electrons that result from interactions by photons of quantum energies comparable to the neutron kinetic energies. Thus we see that the first B-G condition is the more difficult of the two to satisfy for photons and electrons, while the second B-G condition is the more difficult to satisfy for neutrons.

Under the terms of the two B-G conditions, the ratio of absorbed doses in the adjacent medium w to that in the cavity g is given by Eq. (10.3) for each monoenergetic component of the spectrum of charged particles crossing g. For a differential energy distribution Φ_T (particles per cm^2 MeV) the appropriate average mass collision stopping power in the cavity medium g is

$$
{}_m\bar{S}_g \equiv \frac{\int_0^{T_{\max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,g} dT}{\int_0^{T_{\max}} \Phi_T \, dT} = \frac{1}{\Phi}\int_0^{T_{\max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,g} dT = \frac{D_g}{\Phi} \tag{10.4}
$$

and likewise, for a thin layer of wall material w that may be inserted in place of g,

$$
{}_m\bar{S}_w \equiv \frac{\int_0^{T_{\max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,w} dT}{\int_0^{T_{\max}} \Phi_T \, dT} = \frac{1}{\Phi}\int_0^{T_{\max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,w} dT = \frac{D_w}{\Phi} \tag{10.5}
$$

Combining Eqs. (10.4) and (10.5) gives for the ratio of absorbed dose in w to that in g, which is the B-G relation in terms of absorbed dose in the cavity:

$$
\boxed{\frac{D_w}{D_g} = \frac{{}_m\bar{S}_w}{{}_m\bar{S}_g} \equiv {}_m\bar{S}_g^{\,w}} \tag{10.6}
$$

If the medium g occupying the cavity is a gas in which a charge Q (of either sign) is produced by the radiation, D_g can be expressed (in grays) in terms of that charge as

$$
D_g = \frac{Q}{m}\left(\frac{\overline{W}}{e}\right)_g \tag{10.7}
$$

where Q is expressed in coulombs, m is the mass (kg) of gas in which Q is produced, and $(\overline{W}/e)_g$ is the mean energy spent per unit charge produced (J/C; see Chapter 2, Section V.B, and Chapter 12, Section VI). By substituting Eq. (10.7) into Eq. (10.6), we obtain the B-G relation expressed in terms of cavity ionization:

$$\boxed{D_w = \frac{Q}{m}\left(\frac{\overline{W}}{e}\right)_g \cdot {}_m\overline{S}^w_g} \tag{10.8}$$

This equation allows one to calculate the absorbed dose in the medium immediately surrounding a B-G cavity, on the basis of the charge produced in the cavity gas, provided that the appropriate values of m, $(\overline{W}/e)_g$, and ${}_m\overline{S}^w_g$ are known.

Note that Q is generally greater than the charge Q' *collected* from the ion chamber, because of ionic recombination (as discussed in Chapter 12, Section V), requiring a correction.

m may be less than the total mass of gas contained in an ion chamber, if some of the volume is not active in providing measurable charge—for example, if some of the electrical lines of force terminate on a grounded guard ring. In most cases the value of m must be inferred from a chamber calibration in a known radiation field, a subject that is addressed in Chapter 13.

B-G theory also may be applied to solid- or liquid-filled "cavities" g, using Eq. (10.6) to calculate D_w from a value of D_g measured in some way. For example, medium g might be a thin plastic film that gradually darkens as a known function of absorbed dose. Thus D_g could be determined after an exposure by means of a densitometer measurement. However, it is relatively difficult to satisfy the B-G conditions with condensed cavity media, since the cavity thickness must be only ~0.001 times as great as for a gas-filled cavity at 1 atm to obtain a comparable mass thickness of g. Thus a 1-mm gas-filled cavity is comparable to a 1-μm layer of a condensed medium.

So long as ${}_m\overline{S}^w_g$ is evaluated for the charged-particle spectrum Φ_T that crosses the cavity, as in Eqs. (10.4)-(10.6), the B-G relation requires neither charged-particle equilibrium (CPE) nor a homogeneous field of radiation. However, the charged-particle fluence Φ_T must be the same in the cavity and in the medium w at the place where D_w is to be determined.

If CPE does exist in the neighborhood of a point of interest in the medium w, then the insertion of a B-G cavity at the point may be assumed not to perturb the "equilibrium spectrum" of charged particles existing there, since by definition a B-G cavity satisfies the B-G requirements. Thus a B-G cavity approximates an evacuated cavity in this respect. The presence of an equilibrium spectrum of charged particles allows some simplification in estimating Φ_T and hence ${}_m\overline{S}^w_g$, as will be seen later in Spencer's derivation of the B-G relation.

The medium w surrounding the cavity of an ionization chamber is ordinarily just the solid chamber wall itself, and one often refers to the B-G theory as providing a relation between the doses in the gas and in the wall.

II. COROLLARIES OF THE BRAGG-GRAY RELATION

Two useful corollaries of the B-G relation can be readily derived from it. The first relates the charge produced in different gases contained in the same chamber, while the second relates the charge in the same gas contained by different chamber walls.

A. First Bragg-Gray Corollary

A B-G cavity chamber of volume V with wall medium w is first filled with gas g_1 at density ρ_1, then with gas g_2 at density ρ_2. Identical irradiations are applied, producing charges Q_1 and Q_2, respectively. The absorbed dose in gas g_1 can be written as

$$D_1 = D_w \cdot {}_m\overline{S}^{g_1}_w = \frac{Q_1}{\rho_1 V}\left(\frac{\overline{W}}{e}\right)_1 \tag{10.9}$$

and the dose in gas g_2 as

$$D_2 = D_w \cdot {}_m\overline{S}^{g_2}_w = \frac{Q_2}{\rho_2 V} \cdot \left(\frac{\overline{W}}{e}\right)_2 \tag{10.10}$$

The ratio of charges therefore becomes

$$\frac{Q_2}{Q_1} = \frac{\rho_2 V}{\rho_1 V} \cdot \frac{(\overline{W}/e)_1}{(\overline{W}/e)_2} \cdot \frac{{}_m\overline{S}^{g_2}_w}{{}_m\overline{S}^{g_1}_w} \tag{10.11}$$

which reduces to the first B-G corollary:

$$\frac{Q_2}{Q_1} = \frac{\rho_2}{\rho_1} \cdot \frac{(\overline{W}/e)_1}{(\overline{W}/e)_2} \cdot {}_m\overline{S}^{g_2}_{g_1} \tag{10.12}$$

Note that Eq. (10.12) does not depend explicitly upon the wall material w, implying that the same value of Q_2/Q_1 would be observed if the experiment were repeated with different chamber walls. This is true as long as the spectrum Φ_T of charged particles crossing the cavity is not significantly dependent on the kind of wall material. For example, the starting spectrum of secondary electrons produced in different wall media by γ-rays is the same if the γ-energy is such that only Compton interactions can occur. Although different wall media modify the starting electron spectrum somewhat differently as the electrons slow down (to be discussed in Section III), the resulting *equilibrium spectrum* that crosses the cavity in different thick-walled ion chambers is sufficiently similar that Q_2/Q_1 is observed to be nearly independent of the wall material in this case.

B. Second Bragg-Gray Corollary

A single gas g of density ρ is contained in two B-G cavity chambers that have thick walls (exceeding the maximum charged-particle range), and that receive identical irradiations of penetrating x- or γ-rays, producing CPE at the cavity. The first chamber has a volume V_1 and wall material w_1, the second has a volume V_2 and wall w_2. The absorbed dose in the wall of the first chamber, adjacent to its cavity, can be written as

$$D_{w_1} \overset{\text{CPE}}{=} (K_c)_{w_1} = \Psi \left(\overline{\frac{\mu_{en}}{\rho}} \right)_{w_1}$$

$$= D_1 \cdot {}_m\overline{S}_g^{w_1} = \frac{Q_1}{\rho V_1} \left(\overline{\frac{W}{e}} \right)_g \cdot {}_m\overline{S}_g^{w_1} \tag{10.13}$$

where Ψ = photon energy fluence,
$(\overline{\mu_{en}/\rho})_{w_1}$ = mean mass energy-absorption coefficient of wall w_1 for those photons,
D_1 = absorbed dose in gas g in the first chamber,
Q_1 = charge produced in the first chamber.

The corresponding equation for the second chamber is

$$D_{w_2} \overset{\text{CPE}}{=} (K_c)_{w_2} = \Psi \left(\overline{\frac{\mu_{en}}{\rho}} \right)_{w_2}$$

$$= D_2 \cdot {}_m\overline{S}_g^{w_2} = \frac{Q_2}{\rho V_2} \left(\overline{\frac{W}{e}} \right)_g \cdot {}_m\overline{S}_g^{w_2} \tag{10.14}$$

The ratio of the ionizations in the two chambers is obtained from Eqs. (10.13) and (10.14) as

$$\frac{Q_2}{Q_1} = \frac{V_2}{V_1} \cdot \frac{(\overline{\mu_{en}/\rho})_{w_2}}{(\overline{\mu_{en}/\rho})_{w_1}} \cdot \frac{{}_m\overline{S}_g^{w_1}}{{}_m\overline{S}_g^{w_2}} \tag{10.15}$$

where the constancy of $(\overline{W}/e)_g$ for electron energies above a few keV allows its cancellation.

A further simplification of the final factor to ${}_m\overline{S}_{w_2}^{w_1}$ can be made only if the charged-particle spectrum Φ_T crossing the cavity is the same in the two chambers [see Eqs. (10.4)–(10.6)]. The Compton-interaction case cited in the preceding section allows such a simplification, for example. If such a cancellation of stopping powers thus eliminates g from Eq. (10.15), the same value of Q_2/Q_1 should result irrespective of the choice of gas.

An equation similar to (10.15) can be obtained for neutron irradiations in place of photons by substituting kerma factors F_n for the mass energy-absorption coefficients [see Eq. (2.9a)]:

$$\frac{Q_2}{Q_1} = \frac{V_2}{V_1} \cdot \frac{(\overline{F}_n)_{w_2}}{(\overline{F}_n)_{w_1}} \cdot \frac{{}_m\overline{S}_g^{w_1}}{{}_m\overline{S}_g^{w_2}} \cdot \frac{(\overline{W}/e)_1}{(\overline{W}/e)_2} \tag{10.16}$$

The ratio $\overline{W}/e$ may have to be retained here if w_1 and w_2 differ sufficiently to produce heavy charged-particle spectra that have somewhat different $\overline{W}/e$ values even in the same gas. Otherwise it can be canceled as in Eq. (10.15).