EXEMPLO

$$y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$

y	X_1	X_2
-4	0	3
5	1	1
4	2	2
11	3	0

$$X = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 0 \end{bmatrix} \qquad y = \begin{bmatrix} -4 \\ 5 \\ 4 \\ 11 \end{bmatrix}$$

$$X'X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 3 & 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 6 & 6 \\ 6 & 14 & 5 \\ 6 & 5 & 14 \end{bmatrix} = \begin{bmatrix} n & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{bmatrix}$$

$$X'y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 3 & 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \\ 4 \\ 11 \end{bmatrix} = \begin{bmatrix} 16 \\ 46 \\ 1 \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum X_1 y \\ \sum X_2 y \end{bmatrix}$$

$$\left(X'X\right)^{-1} = \begin{bmatrix} 171/36 & -54/36 & -54/36 \\ -54/36 & 20/36 & 16/36 \\ -54/36 & 16/36 & 20/36 \end{bmatrix}$$

$$b = (X'X)^{-1} X'y = \begin{bmatrix} 5{,}5 \\ 2 \\ -3 \end{bmatrix}$$

$$\hat{y} = 5{,}5 + 2X_1 - 3X_2$$

$$e = y - \hat{y}$$

$$\hat{y} = \begin{bmatrix} -3{,}5 \\ 4{,}5 \\ 3{,}5 \\ 11{,}5 \end{bmatrix} \qquad e = \begin{bmatrix} -0{,}5 \\ 0{,}5 \\ 0{,}5 \\ -0{,}5 \end{bmatrix} \qquad s^2 = \frac{e'e}{n-K} = \frac{1}{4-3} = 1$$

$$\left(X'X\right)^{-1} s^2 = \begin{bmatrix} 171/36 & -54/36 & -54/36 \\ -54/36 & 20/36 & 16/36 \\ -54/36 & 16/36 & 20/36 \end{bmatrix} \times 1 = \begin{bmatrix} \mathrm{var}(a) & \mathrm{cov}(ab_1) & \mathrm{cov}(ab_2) \\ \mathrm{cov}(ab_1) & \mathrm{var}(b_1) & \mathrm{cov}(b_1 b_2) \\ \mathrm{cov}(ab_2) & \mathrm{cov}(b_1 b_2) & \mathrm{var}(b_2) \end{bmatrix}$$

1) Testar H_0: β_1 = -2 a 5% de significância.

$$t = \frac{b_1 - \beta_1}{\sqrt{\text{var}(b_1)}} = \frac{2+2}{\sqrt{20/36}} = 5{,}37 \quad t \text{ da tabela a 5\% com 1 g.l.} = 12{,}71$$

Portanto, não se rejeita $H_{0.}$

Intervalo de confiança.

$$b_1 - t_{\alpha/2}\sqrt{\text{var}(b_1)} \le \beta_1 \le b_1 + t_{\alpha/2}\sqrt{\text{var}(b_1)}$$

$$2 - 12{,}71\sqrt{20/36} \le \beta_1 \le 2 + 12{,}71\sqrt{20/36}$$

$$-7{,}47 \le \beta_1 \le 11{,}47$$

Testar H_0: $\beta_2 = 4$ a 5% de significância.

$$t = \frac{b_2 - \beta_2}{\sqrt{\text{var}(b_2)}} = \frac{-3-4}{\sqrt{20/36}} = -9{,}39$$ t da tabela a 5% com 1 g.l. = 12,71

Portanto, não se rejeita H_0.

Intervalo de confiança.

$$-12{,}47 \leq \beta_2 \leq 6{,}47$$

2)Testar H_0: $\beta_1 = \beta_2 \Rightarrow \beta_1 - \beta_2 = 0$

H_A: $\beta_1 > \beta_2 \Rightarrow \beta_1 - \beta_2 > 0$ a 5% de significância.

$$t = \frac{r'b - r'\beta}{\sqrt{r'(X'X)^{-1} r\, s^2}} = \frac{5-0}{\sqrt{8/36}} = 10{,}61$$

$$r'b = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 5{,}5 \\ 2 \\ -3 \end{bmatrix} = 5$$

$$r'(X'X)^{-1} r\, s^2 = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 171/36 & -54/36 & -54/36 \\ -54/36 & 20/36 & 16/36 \\ -54/36 & 16/36 & 20/36 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \frac{8}{36}$$

$$\mathrm{var}(b_1 - b_2) = \mathrm{var}(b_1) + \mathrm{var}(b_2) - 2\,\mathrm{cov}(b_1 b_2) = \frac{20}{36} + \frac{20}{36} - 2\frac{16}{36} = \frac{8}{36}$$

t da tabela a 5% com 1 g.l. = 6,31

Portanto, rejeita-se $H_{0.}$

3) Testar H_0: $\beta_1 = \beta_2 = 0$ a 5% de significância.

$$F_{(J,n-K)} = \frac{(Rb-q)'\left[s^2 R(X'X)^{-1} R'\right]^{-1}(Rb-q)}{J}$$

$$Rb = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5{,}5 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$R(X'X)^{-1} R' s^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 171/36 & -54/36 & -54/36 \\ -54/36 & 20/36 & 16/36 \\ -54/36 & 16/36 & 20/36 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 20/36 & 16/36 \\ 16/36 & 20/36 \end{bmatrix}$$

$$\left[R(X'X)^{-1} R' s^2\right]^{-1} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \end{bmatrix} \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 113$$

$F = \dfrac{113}{2} = 56{,}5$ $\qquad$ F da tabela a 5% com 2 e 1 g.l. = 200

Portanto, não se rejeita $H_{0.}$

4) Testar H_0: $\beta_1 = -2$ e $\beta_2 = 4$ a 5% de significância.

$$Rb = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5{,}5 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \qquad q = \begin{bmatrix} -2 \\ 4 \end{bmatrix} \qquad Rb - q = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -7 \end{bmatrix} \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ -7 \end{bmatrix} = 549$$

$$F = \frac{549}{2} = 274{,}5$$ F da tabela a 5% com 2 e 1 g.l. = 200

Portanto, rejeita $H_{0.}$

Achar a região de confiança para β_1 e β_2

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad q = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \qquad Rb = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$\left[R(X'X)^{-1} R' s^2 \right]^{-1} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

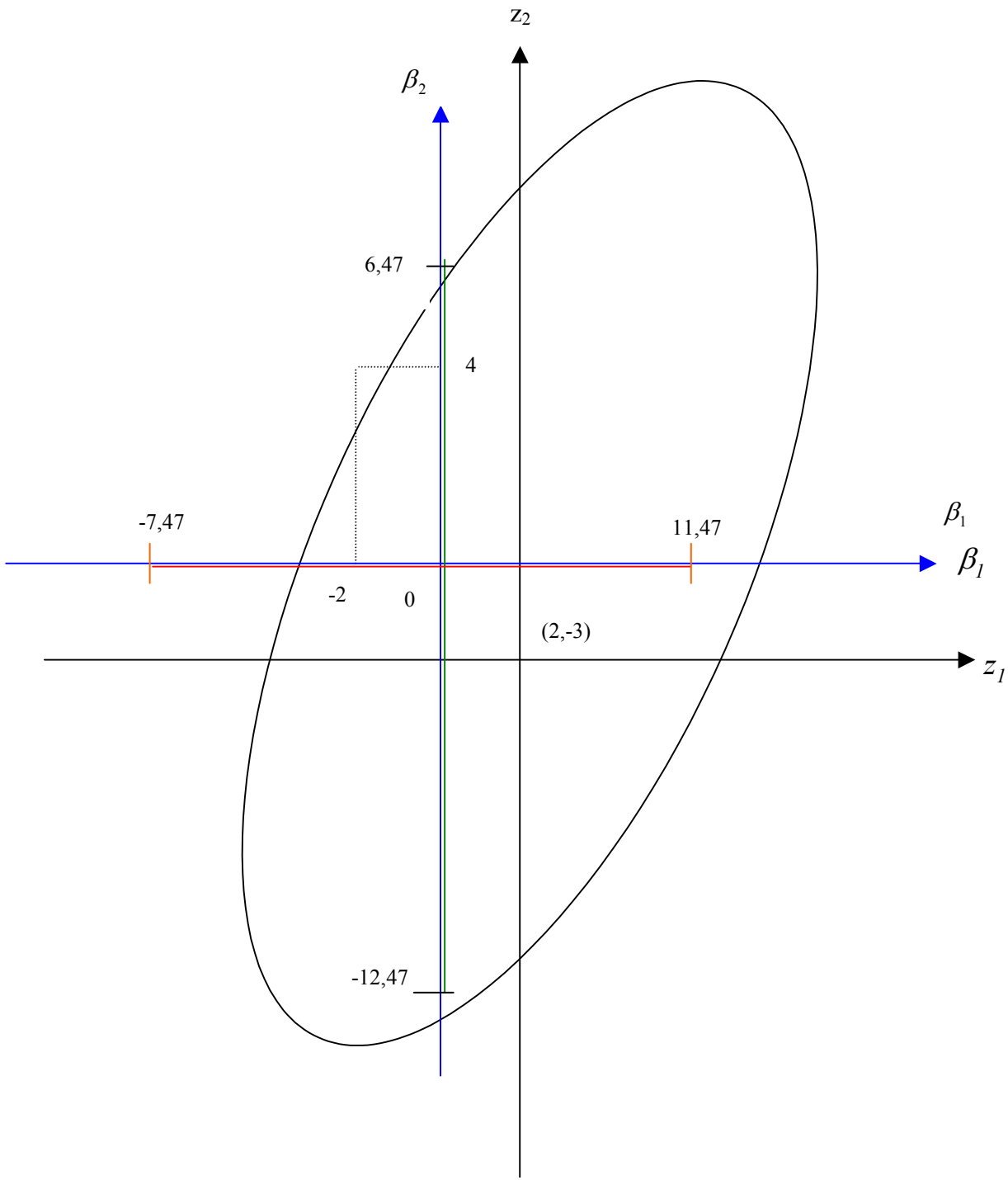
$$\frac{1}{J}(Rb-q)'\left[s^2R(X'X)^{-1}R'\right]^{-1}(Rb-q)\leq F(J,n-K)$$

$$\frac{1}{2}\begin{bmatrix}2-\beta_1 & -3-\beta_2\end{bmatrix}\begin{bmatrix}5 & -4\\ -4 & 5\end{bmatrix}\begin{bmatrix}2-\beta_1\\ -3-\beta_2\end{bmatrix}\leq F(2,1)$$

Seja $\beta_1-2=z_1$ e $\beta_2+3=z_2$

$$\begin{bmatrix}z_1 & z_2\end{bmatrix}\begin{bmatrix}5 & -4\\ -4 & 5\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}\leq 2\times 200$$

$$5z_1^2-8z_1z_2+5z_2^2\leq 400$$


z2
β2
6,47
4
-7,47
11,47
β1
β1
-2
0
(2,-3)
z1
-12,47