## Numerical Cognition Without Words: Evidence from Amazonia

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> Members of the Pirahã tribe use a "one-two-many" system of counting. I ask whether speakers of this innumerate language can appreciate larger numerosities without the benefit of words to encode them. This addresses the classic Whorfian question about whether language can determine thought. Results of numerical tasks with varying cognitive demands show that numerical cognition is clearly affected by the lack of a counting system in the language. Performance with quantities greater than 3 was remarkably poor, but showed a constant coefficient of variation, which is suggestive of an analog estimation process.

Is it possible that there are some concepts that we cannot entertain because of the language that we speak? At issue here is the strongest version of Benjamin Lee Whorf's hypothesis that language can determine the nature and content of thought. The strong version of Whorf's hypothesis goes beyond the weaker claim that linguistic structure simply influences the way that we think about things in our everyday encounters. For example, recent studies suggest that language might affect how people mentally encode spatial relations ( $1-3$ ), and how they conceive of the nature of individual objects and their material substances (4). However, none of these studies suggest that linguistic structure prevents us from entertaining the concepts that are available to speakers of alternative linguistic systems.

The question of whether linguistic determinism exists in the stronger sense has two parts. The first is whether languages can be incommensurate: Are there terms that exist in one language that cannot be translated into another? The second is whether the lack of such translation precludes the speakers of one language from entertaining concepts that are encoded by the words or grammar of the other language. For many years, the answer to both questions appeared to be negative. While languages might have different ways in which situations are habitually described, it has generally been accepted that there would always be some way in which one could capture the equivalent meaning in any other language (5). Of course, when speaking of translatable concepts, we do not mean terms like 'molecule' or 'quark', which would not exist in a
culture without advanced scientific institutions. Failure to know what molecules or quarks are does not signal an inability to understand the English language - surely people were still speaking English before such terms were introduced. On the other hand, one would question someone's command of English if they did not understand the basic vocabulary and grammar.

Words that indicate numerical quantities are clearly among the basic vocabulary of a language like English. But not all languages contain fully elaborated counting systems. Although no language has been recorded that completely lacks number words, there is a considerable range of counting systems that exists across cultures. Some cultures use a finite number of body parts to count 20 or 30 body tags (6). Many cultures use particular body parts like fingers as a recursive base for the count system as in our 10 -based system. Finally, there are cultures that base their counting systems on a small-number somewhere between 2 and 4. Sometimes, the use of a small-number base is recursive and potentially infinite. For example, it is claimed that the Gumulgal South Sea Islanders counted with a recursive binary system: $1,2,2^{\prime} 1,2^{\prime} 2,2^{\prime} 2^{\prime} 1$ and so on (6).

The counting system that differs perhaps most from our own is the "one-two-many" system, where quantities beyond two are not counted but are simply referred to as 'many'. If a culture is limited to such a counting system, is it possible for them to perceive or conceptualize quantities beyond the limited sets picked out by the counting sequence, or to make what we consider to be quite trivial distinctions such as that between 4 versus 5 objects? The Pirahã are such a culture. They live along the banks of the Maici River in the Lowland Amazonia region of Brazil. They maintain very much of a hunter-gatherer existence and reject assimilation into mainstream Brazilian culture. Almost completely monolingual in their own language, they have a population of less than 200 living in small villages of 10 to 20 people. They have only limited exchanges with outsiders, using primitive pidgin systems for communicating in trading goods without monetary exchange and without the use of Portuguese count words. The Pirahã counting system consists of the words: 'hói' (falling tone $=$ 'one') and 'hoí' (rising tone $=$ 'two'). Larger quantities are designated as 'baagi' or 'aibai' (= 'many').

I was able to take three field trips ranging from one week to two months living with the Pirahã along with Dr. Daniel Everett and Keren Everett, two linguists who have lived and worked with the tribe for over 20 years and are completely familiar with their language and cultural practices. Observations were informed by their background of continuous and extensive immersion in the Pirahã culture. During my visits, I became interested in the counting system of the Pirahã that I had heard about and wanted to examine whether they really did have only two numbers, and how this would affect their ability to perceive numerosities that extended beyond the limited count sequence.

Year 1: Initial observations. On my first week-long trip to the two most up-river Maici villages, I began with informal observations of the Pirahã use of the number words for one and two. I was also interested in the possibility that the one-two-many system might actually be a recursive base- 2 system, that their limited number words might be supplemented by more extensive finger counting, or that there might be taboos associated with counting certain kinds of objects as suggested by Zaslavsky in her studies of African counting systems $(7,8)$. Keren Everett developed some simple tasks to see if our two Pirahã informants could refer to numerosities of arrays of objects using Pirahã terms and any finger counting system they might have. Instructions and interactions with participants were in the Pirahã language. When it was necessary to refer to the numerosity of an array, Keren Everett used the Portuguese number words embedded in Pirahã dialogue. Such terms are understood by the Pirahã to be the language of Brazilians, but their meaning is not understood. In addition to this short session, during the first year trip, I continuously took opportunities to probe for counting abilities in everyday situations.

The outcome of these informal studies revealed the following: 1) There was no recursive use of the count system - the Pirahã never used the count words in combinations like 'hói-hoí' to designate larger quantities, 2) Fingers were used to supplement oral enumeration, but this was highly inaccurate even for small numbers less than five. In addition, 'hói', and 'hoí', the words for 'one' and 'two', were not always used to denote those quantities. While the word for 'two' always denoted a larger quantity than the word for 'one' (when used in the same context), the word for "one" was sometimes used to denote just a small quantity such as 2 or 3 or sometimes more. An example of the use of counting words and finger counting are given in Table 1 in one of the informal sessions with an informant who appeared to be in his 50s. Videotaped extracts from the session are included in the supporting online materials (movie S1).

The interpretation of these observations is limited by their informal nature and small sample size. However, the observations are supplemented with 20 years of observation by the Everetts as trained linguists in their analysis of the Pirahã language. One particularly interesting finding is that 'hói' appears to designate 'roughly one' -or a small quantity whose prototype is one. Most of the time, in enumeration task, 'hói' referred to one, but not always. An analogy might be when we ask for "a couple of $X \mathrm{~s}$ " in English, where the prototypical quantity is two, but we are not upset if we are given three or four objects. However, we surely would be upset if given only one object since the designation of a single object has a privileged status in our language. There is no concept of "roughly one" in a true integer system. Even the informal use of the indefinite article: "a $X$ " strictly requires a singular reference. In Pirahã, 'hói' can also mean "small", which contrasts with 'ogii' (= big), suggesting that the distinction between discrete and continuous quantification is quite fuzzy in the Pirahã language.

Year 2: Experiments in nonverbal numerical reasoning. On my second visit to the Pirahã villages for a two month period, I developed a mo re systematic set of procedures for evaluating the numerical competence of members of the tribe. The experiments were designed to employ some combination of cognitive skills such as the need for memory, speed of encoding, and mental-spatial transformations. This would reveal the extent to which such task demands interact with numerical ability, such as it is. Details of the methods are available on Science Online (9). There were seven participants, who included all six adult males from two villages and one female. Most of the data were collected on four of the men who were consistently available for participation. The tasks were devised to use objects that were available and familiar to the participants (sticks, nuts, batteries). The results of the tasks, along with schematic diagrams, are presented in Figure 1. These are roughly ordered in terms of increasing cognitive demand. Any estimation of a person's numerical competence will always be confounded with performance factors of the task. Since this is unavoidable, it makes sense to explore how performance is affected by a range of increasingly demanding tasks.

In the matching tasks (A, B, C, D, F), I sat across from the participant and with a stick dividing my side from theirs, I presented an array of objects on my side of the stick (below the line in the figures) and they responded by placing a linear array of AA batteries $(5 \mathrm{~cm} \times 1.4 \mathrm{~cm})$ on their side of the table (above the line). The matching task provides a kind of concrete substitute for counting. It shares the element of placing tokens in one-to-one correspondence with individuals in a to-be-counted group. The first
matching tasks began with simple linear arrays of batteries. This progressed to clusters of nuts matched to the battery line, orthogonal matching of battery lines, matching of battery lines that were unevenly spaced, and copying lines on a drawing. In all of these matching experiments, participants responded with relatively good accuracy with up to 2 or 3 items, but performance deteriorated considerably beyond that up to 8 to 10 items. In the first simple linear matching task A , performance hovered around $75 \%$ up to the largest quantities. Matching tasks with greater cognitive demands required mental transposition of the sample array to the match array without benefit of tagging for numerical quantity. Performance dropped precipitously to $0 \%$ for the larger target set sizes in these tasks. One exception was task $D$ with unevenly spaced objects. Although this was designed to be a difficult task, participants showed an anomalous superiority for large numerosities over small. Performance initially deteriorated with increased set size up to 6 items, then shot up to near perfect performance for set size 7 through 10. A likely interpretation of this result was that the uneven spacing for larger set sizes promoted recoding of arrays into smaller configurations of 2 or 3 items. This allowed participants to use a chunking strategy of treating each of the subgroups as a matching group.

When time constraints were introduced in task F (exposing the array for only one second), performance was drastically affected and there was a clear correlation between set size and accuracy beginning at set size 3 . A Line-drawing task (E) was highly affected by set size, being one of the worst performances of all. Not only do the Pirahã not count, but they also do not draw. Producing simple straight lines was accomplished only with great effort and concentration, accompanied by heavy sighs and groans. The final two tasks $(\mathrm{G}, \mathrm{H})$ required participants to keep track of a numerical quantity through visual displacement. In one case, they were first allowed to inspect an array of nuts for about 8 seconds. The nuts were placed in a can, and then withdrawn one at a time. Participants were required to say, after each withdrawal, if there were still any nuts left in the can, or if it was empty. Performance was predictably strongly affected by set size from the very smallest quantities. The final task involved hiding candy in a box, which had a picture of some number of fish on the lid. The box was then hidden behind the author's back, and two cases were revealed, the original with the candy, and another with one more or one less fish on the lid. For quite small comparisons such as 3 versus 4, performance rarely went over $50 \%$ chance responding.

There is a growing consensus in the field of numerical cognition that primitive numerical abilities are of two kinds: First, there is the ability to enumerate accurately
small quantities up to about 3 items, with only minimal processing requirements (10-16). I originally termed this ability "parallel individuation" $(17,18)$, referring to how many items one can encode as discrete unique individuals at the same time in memory. Without overt counting, humans and other animals possess an analog procedure whereby numerical quantities can be estimated with a limited degree of accuracy (11, 19-26). Many researchers believe that large-number estimation, although based on individuated elements, is coalesced into a continuous analog format for mental representation. For example, the discrete elements of a large number array might be represented as a continuous length of a line, where a longer line inexactly represents a larger numerosity.

When people employ this analog estimation procedure, the variability of their estimates tends to increase as the target set size increases. The ratio of average error to target set size is known as Weber's fraction and can be indexed by a measure known as the coefficient of variation --the standard deviation of the estimates divided by set size (23). Although performance by the Pirahã on the present tasks was quite poor for set sizes above 2 or 3 , it was not random. Figure 2 shows the mean response values mapped against the target values for all participants in the simple matching tasks $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and F . The top graph shows that mean responses and target values are almost identical. This means that the Pirahã participants were actually trying very hard to get the answers correct, and they clearly understood the tasks. The lower graph in Figure 2 shows that the standard deviation of the estimates increases in proportion to the set size, resulting in a constant coefficient of variation of about 1.5 after set size 3 , as predicted by the dual model of mental enumeration. This value for the coefficient of variation is about the same as one finds in college students engaged in numerical estimation tasks (23). Data for individual tasks and individual participants were consistent with the averaged trends Figure 2. Graphs are available in supporting online materials.

The results of these studies show that the Pirahã's impoverished counting system truly limits their ability to enumerate exact quantities when set sizes exceed two or three items. For tasks that required additional cognitive processing, performance deteriorated even on set sizes smaller than 3. Participants showed evidence of employing analog magnitude estimation and, in some cases, they took advantage of spatial chunking to decrease the cognitive demands of larger set sizes. This split between exact enumeration ability for set size smaller than 3 and analog estimation for larger set sizes parallels findings from laboratory experiments with adults who are prevented from explicit counting, studies of numerical abilities in prelinguistic infants, monkeys, birds, and rodents, and in
recent studies employing brain imaging techniques (11,2330).

The analog estimation abilities exhibited by the Pirahã are a kind of numerical competence that appears to be immune to numerical language deprivation. But since lower animals also exhibit such abilities, robustness in the absence of language is already established. The present experiments allow us to ask whether humans who are not exposed to a number system can represent exact quantities for medium-sized sets of 4 or 5 . The answer appears to be negative. The Pirahã inherit just the abilities to exactly enumerate small sets of less than 3 items if processing factors are not unduly taxing (31).

In evaluating the case for linguistic determinism, I suggest that the Pirahã language is incommensurate with languages that have counting systems that enable exact enumeration. Of particular interest is the fact that the Pirahã have no privileged name for the singular quantity. Instead, 'hói' meant "roughly one" or "small", which precludes any precise translation of exact numerical terms. The present study represents a rare and perhaps unique case for strong linguistic determinism. The study also provides a window into how the possibly innate distinction (26) between quantifying small versus large sets of objects is relatively unelaborated in a life without number words to capture those exact magnitudes (32).

## References and Notes

1. S. C. Levinson, J. Ling. Anth. 7, 98 (1997).
2. P. Li, L. Gleitman, Cogn. 3, 83 (2002).
3. S. C. Levinson, S. Kita, D. B. M. Haun, B. H. Rasch, Cogn. 4, 84 (2002).4. J. A. Lucy, Grammatical Categories and Cognition. (Cambridge University Press, Cambridge, 1992).
4. R. W. Brown, E. H. Lenneberg, J. Abnorm. Soc. Psychol. 49, 454 (1954).
5. K. Menninger, Number Words and Number Symbols: A Cultural History of Numbers. (MIT Press, Cambridge, 1969).
6. C. Zaslavsky, Africa Counts: Number and Pattern in African Culture (Prindle, Weber \& Schmidt, Boston, 1973).
7. R. Gelman, C. R. Gallistel, The Child's Understanding of Number (Harvard University Press, Cambridge 1978).
8. Materials and methods are available as supporting material on Science Online.
9. S. Carey, Mind Lang. 16, 37 (2001).
10. L. Feigenson, S. Carey, M. Hauser, Psychol. Sci. 13, 150 (2002).
11. B. J. Scholl, Cogn. 80, 1 (2001).
12. T. J. Simon, Cog. Dev., 12, 349 (1997).
13. L. Trick, Z. W. Pylyshyn, Psychol. Rev. 101, 80 (1994).

15 C. Uller, G. Huntley-Fenner, S. Carey, L. Klatt, Cog. Dev. 14, 1 (1999).
16. F. Xu, Cogn. 89, 15 (2003).
17. P. Gordon, Paper presented at the biennial meeting of the Society for Research in Child Development, New Orleans, LA, March, 1993.
18. P. Gordon, Paper presented at the European Society for Philosophy and Psychology, Paris, France, September 1994.
19. W. H. Meck, R. M. Church, J. Exp. Psych: Anim. Beh. Proc. 9, 320 (1983).
20. H.Barth, N. Kanwisher, E.Spelke, Cogn. 86, 201 (2003).
21. S. Cordes, R. Gelman, C. R. Gallistel, J. Whalen, Psychon. Bul. Rev. 8, 698 (2001).
22. C. R. Gallistel, The Organization of Learning (MIT Press Cambridge, 1990).
23. J. Whalen, C. R.Gallistel, R. Gelman, Psychol. Sci., 10, 130 (1999).
24 S. Cordes, R. Gelman, C. R. Gallistel, J. Whalen, Psychon. Bull. Rev., 8, 698 (2001).
25. S. Dehaene, The Number Sense (Oxford University Press, NY, 1997).
26. B. Butterworth, What Counts (Simon \& Schuster, NY, 1999).
27. J. S. Lipton, E. S. Spelke, Psychol. Sci. 14, 396 (2003).
28. M. D. Hauser, F. Tsao, P. Garcia, E. S. Spelke, Proc. Roy. Soc. London: Bio. Sci. 270, 1441 (2003).
29. S. Dehaene, E. Spelke, P. Pinel, R. Stanescu, S. Tsivkin, Sci. 284, 970 (1999).
30. J. R. Platt, D. M. Johnson, Learn. Motiv., 2, 386 (1971).
31. Cordes et al. (24) suggest that analog representations exist even for $\mathrm{n}=2$ since subjects made errors on a task in which counting was suppressed during rapid button pressing. However, errors in this range also occurred when subjects counted and might have been the result of perseveration errors rather than reflecting numerical representations.
32. One can safely rule out that the Pirahã are mentally retarded. Their hunting, spatial, categorization and linguistic skills are remarkable and they show no clinical signs of retardation.
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## Supporting Online Material

www.sciencemag.org/cgi/content/full/1094492/DC1
Methods
SOM Text
Figs. S1 to S3
Movie S1and S2
References
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Fig. 1. Results of number tasks with Pirahã villagers ( $\mathrm{n}=7$ ). Rectangles indicate ( $5 \mathrm{~cm} \times 1.4 \mathrm{~cm}$ ) AA batteries, and circles indicate ground nuts. Center line indicates a stick between the author's example array (below the line) and the participant's attempt to "make it the same" (above the line). Tasks A through D required the participant to match the lower array presented by the author using a line of batteries; E was similar, but using the unfamiliar task of copying lines drawn on paper; F was a matching task where the participant only saw the numerical display for about one second before it was hidden behind a screen; G involved putting nuts into a can and withdrawing them one by one; (participants responded after each withdrawal as to whether the can still contained nuts or was empty.) H involved placing candy inside a box with a number of fish drawn on the lid; (this was then hidden and brought out again with another box with one more or one less fish on the lid and participants had to choose which box contained the candy.)

Fig. 2. Mean Accuracy of Reponses in Matching Tasks and Coefficient of Variation. Figures for individual tasks and individual participants are available in the supporting online materials.

Table 1. Use of fingers and number words by Pirahã participant.

| No. of <br> Objects | Number word <br> used | No. of Fingers |
| :--- | :--- | :--- |
| 1 | hói (= 1) |  |
| 2 | hoí (= 2) <br> aibaagi (= many) | 2 |
| 3 | hoí (= 2) | 3 |
| 4 | hoí ( $=2$ ) <br> aibai (= many) | $5 \rightarrow 3$ |
| 5 | aibaagi (= many) | 5 |
| 6 | aibaagi (= many) | $6 \rightarrow 7$ |
| 7 | hói (= $)^{*}$ <br> aibaagi (= many) | 1 <br> $5 \rightarrow 8$ |
| 8 |  | $5 \rightarrow 8 \rightarrow 10$ |
| 9 | aibaagi (= many) | $5 \rightarrow 10$ |
| 10 |  | 5 |

$\rightarrow$ indicates a shift from one quantity to the next

* this use of "one" might have been a reference to adding one rather than to the whole set of objects.



