Escola Politécnica da Universidade de São Paulo

Departamento de Engenharia Mecatrônica

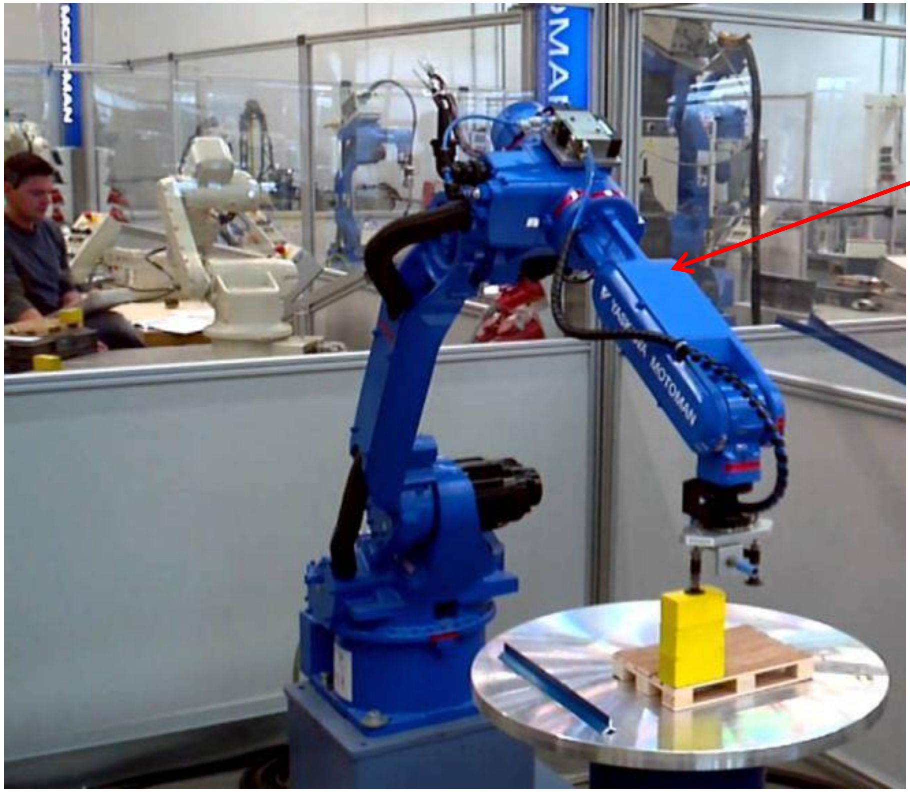
Análise cinemática 3D (parte 1)

Prof. Chi Nan **Pai**

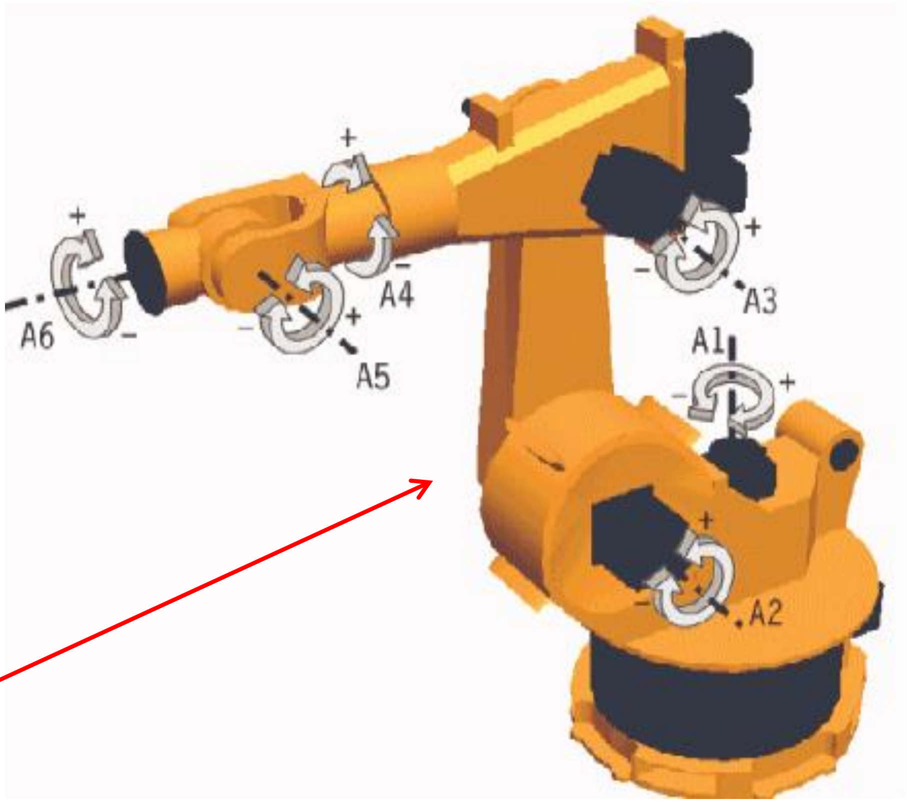
MS-14

chinan.pai@usp.br

Aplicação 3D



Yaskawa

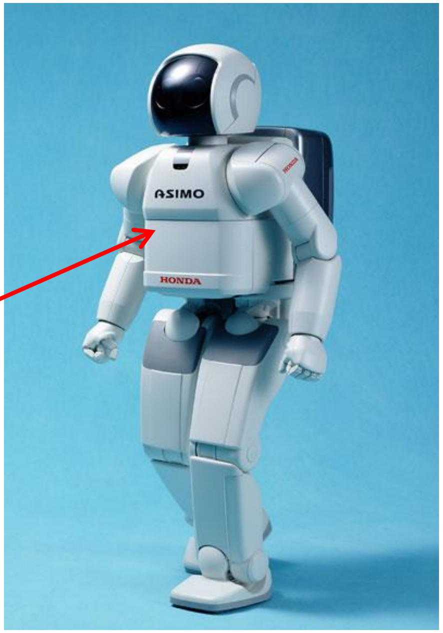


Kuka

- Disponível para alunos de PMR2560

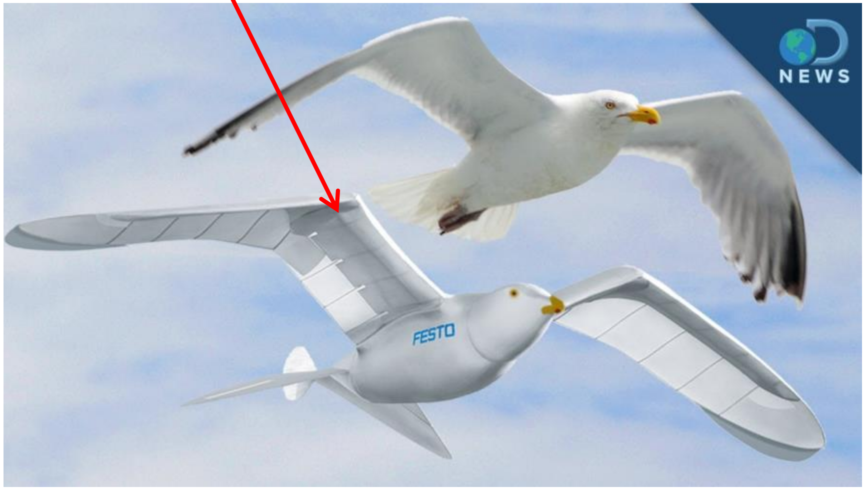
Aplicação 3D

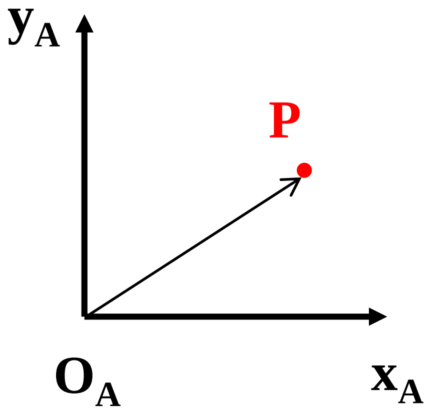
Simulador de voô



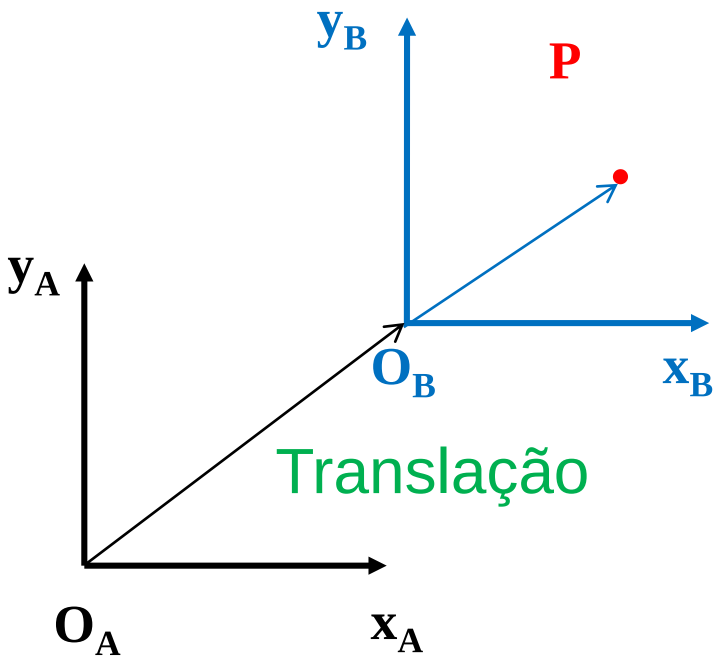
Robô humanóide

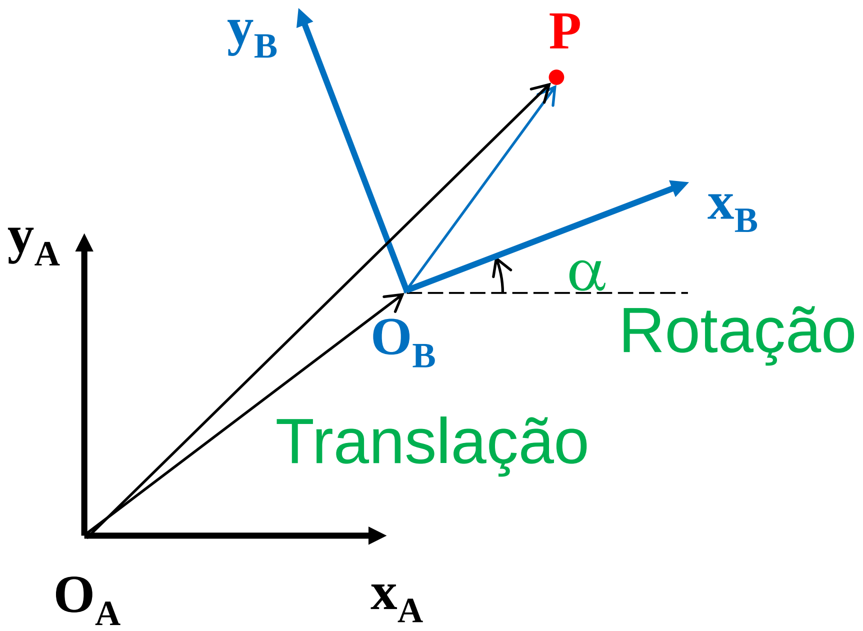
Smartbird

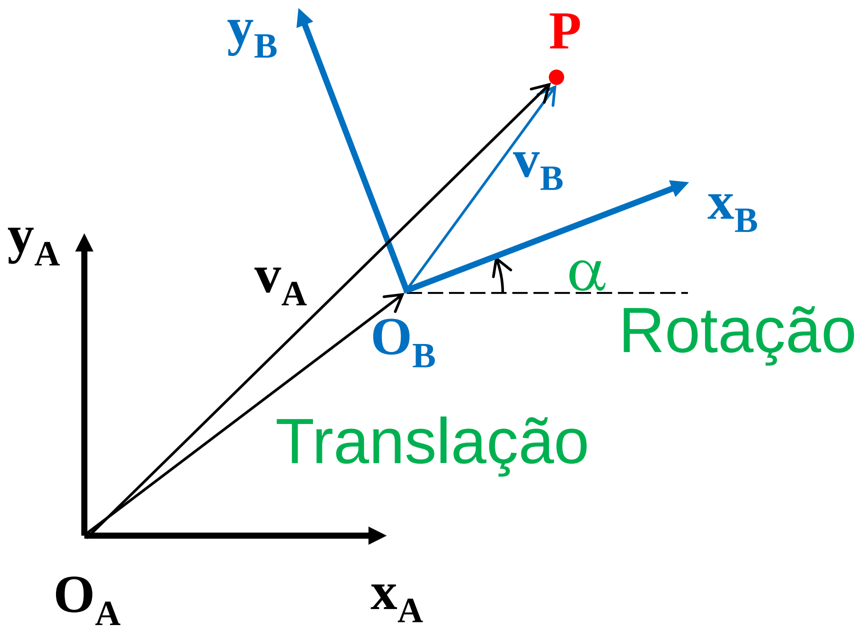



y_A
P
O_A
x_A

Análise cinemática 2D




y_B
P
x_B
y_A
α
O_B
Rotação
Translação
O_A
x_A



$$\begin{bmatrix} {}^{A}r_P \\ 1 \end{bmatrix} = \left[{}^{A}T_B\right] \begin{bmatrix} {}^{B}r_P \\ 1 \end{bmatrix}$$

${}^{A}r_P$ Ponto P na Base A

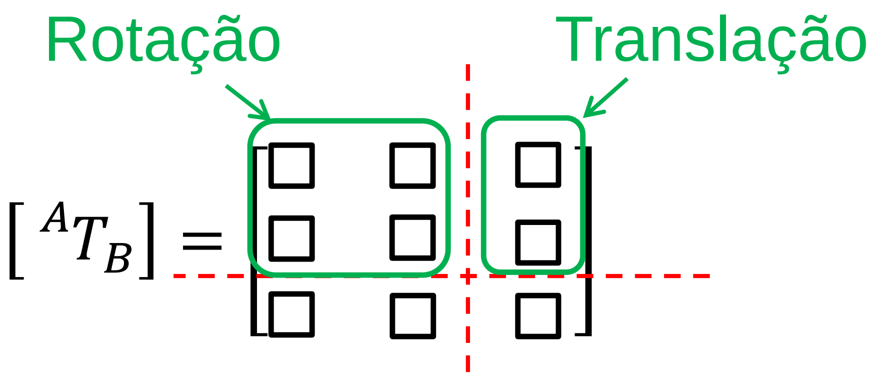
${}^{B}r_P$ Ponto P na Base B

$\left[{}^{A}T_B\right]$ Base B → Base A

$$\left[{}^{A}T_B\right] = \begin{bmatrix} \left[{}^{A}R_B\right] & {}^{A}r_{O_B} \\ 0^T & 1 \end{bmatrix}$$

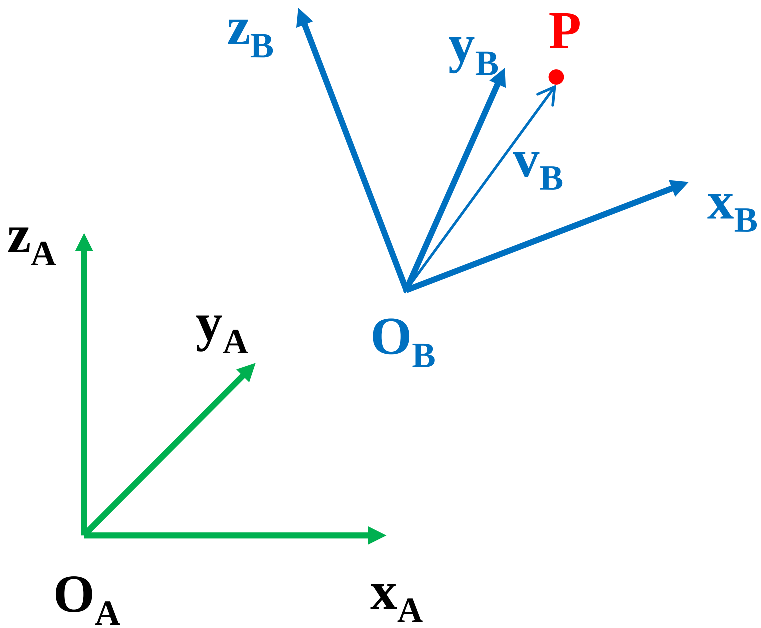
$$\left[{}^{A}R_B\right] = \begin{bmatrix} cos\alpha & -sen\alpha \\ sen\alpha & cos\alpha \end{bmatrix}$$

$$0^T = [0 \quad 0]$$



Matriz de transformação homogênea

Transformação homogênea em 3D



$$\begin{bmatrix} {}^{A}r_{P} \\ 1 \end{bmatrix} = \left[{}^{A}T_{B} \right] \begin{bmatrix} {}^{B}r_{P} \\ 1 \end{bmatrix}$$

${}^{A}r_{P}$ Ponto P na Base A

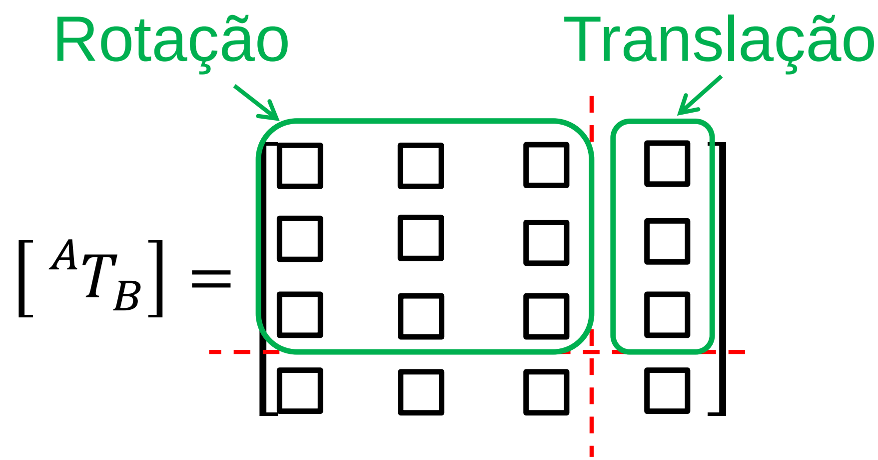
${}^{B}r_{P}$ Ponto P na Base B

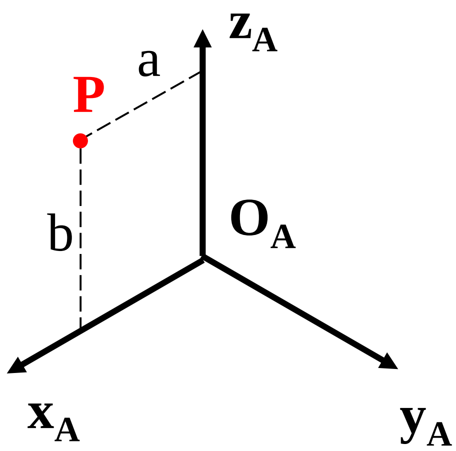
$\left[{}^{A}T_{B} \right]$ Base B $\rightarrow$ Base A

$$\left[{}^{A}T_{B} \right] = \begin{bmatrix} \left[{}^{A}R_{B} \right] & {}^{A}r_{O_B} \\ 0^T & 1 \end{bmatrix}$$

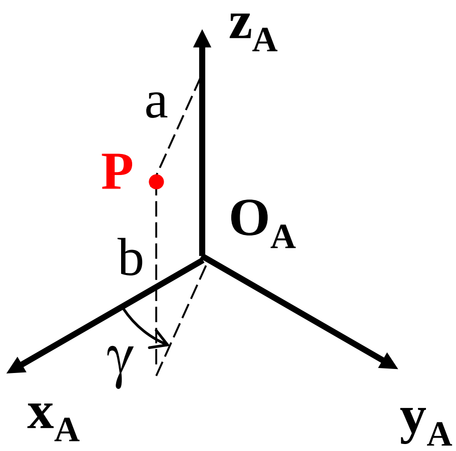
$$\left[{}^{A}R_{B} \right] = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$0^T = [0 \quad 0 \quad 0]$$



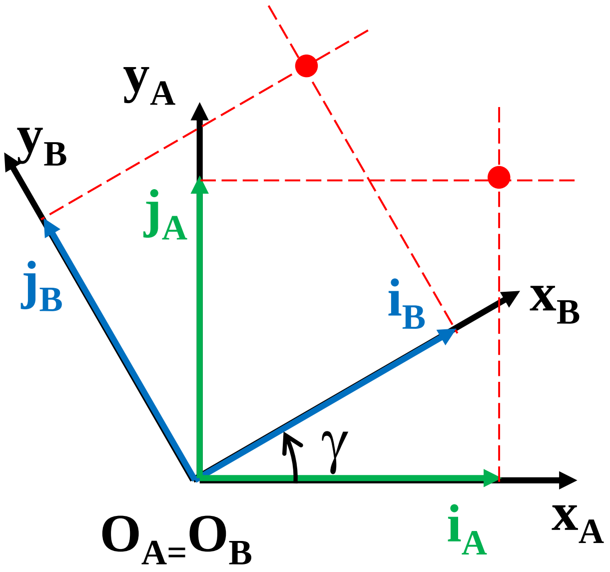


$$^{A}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$$



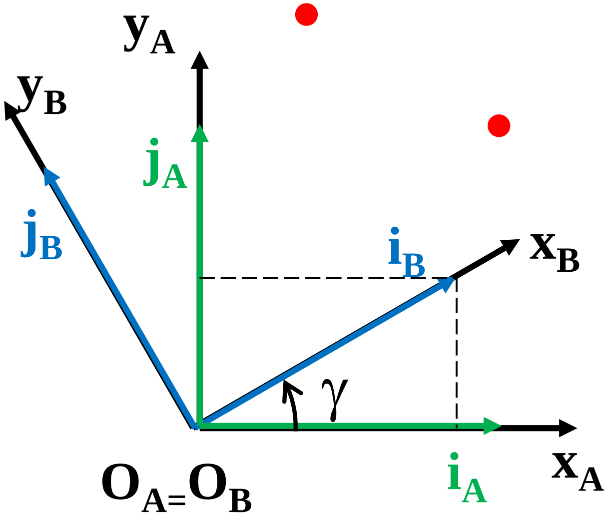
$$^{A}r_P = \begin{bmatrix} ? \\ ? \\ b \end{bmatrix}$$

Transformação homogênea em 3D

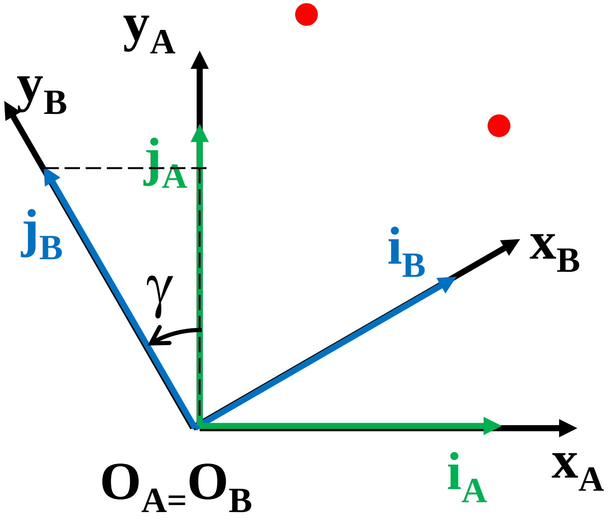


Transformação homogênea em 3D

$$\mathbf{i_B} = \cos\gamma\,\mathbf{i_A} + \mathrm{sen}\gamma\,\mathbf{j_A} + 0\,\mathbf{k_A}$$

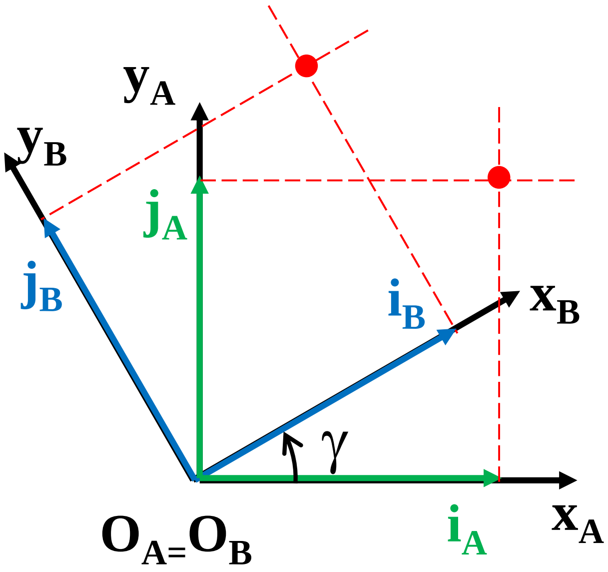


Transformação homogênea em 3D



$$\mathbf{i_B} = \cos\gamma\,\mathbf{i_A} + \mathrm{sen}\gamma\,\mathbf{j_A} + 0\mathbf{k_A}$$
$$\mathbf{j_B} = -\mathrm{sen}\gamma\,\mathbf{i_A} + \cos\gamma\,\mathbf{j_A} + 0\mathbf{k_A}$$

Transformação homogênea em 3D

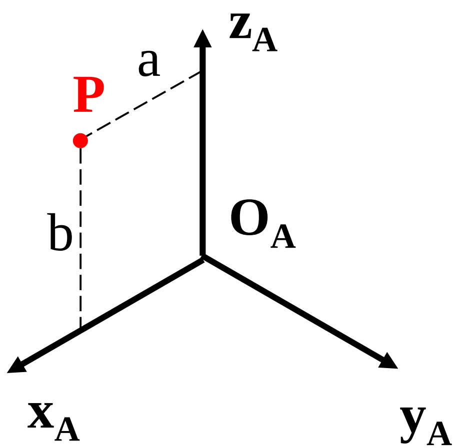


$$\mathbf{i_B} = \cos\gamma\,\mathbf{i_A} + \mathrm{sen}\gamma\,\mathbf{j_A} + 0\,\mathbf{k_A}$$
$$\mathbf{j_B} = -\mathrm{sen}\gamma\,\mathbf{i_A} + \cos\gamma\,\mathbf{j_A} + 0\,\mathbf{k_A}$$
$$\mathbf{k_B} = 0\,\mathbf{i_A} + 0\,\mathbf{j_A} + \mathbf{k_A}$$

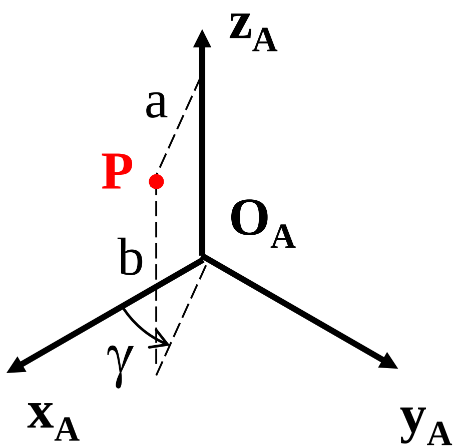
$$[\,{}^{A}R_B] = Rot(Z_B, \gamma) = [\,{}^{A}\mathrm{i}_B,\ {}^{A}\mathrm{j}_B,\ {}^{A}\mathrm{k}_B]$$

$$[\,{}^{A}R_B] = \left[\begin{array}{c|c|c} cos\gamma & -sen\gamma & 0 \\ sen\gamma & cos\gamma & 0 \\ 0 & 0 & 1 \end{array}\right]$$

Transformação homogênea em 3D



$$^{A}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$$

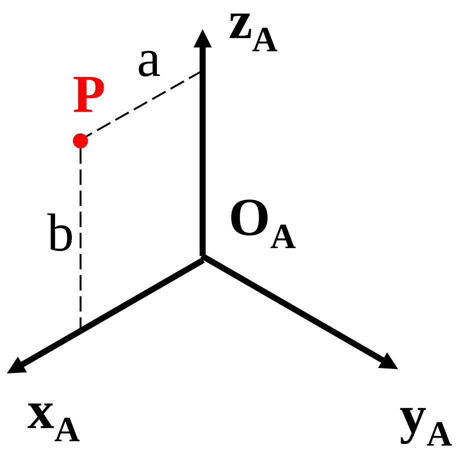


$$^{A}r_P = \begin{bmatrix} ? \\ ? \\ b \end{bmatrix}$$

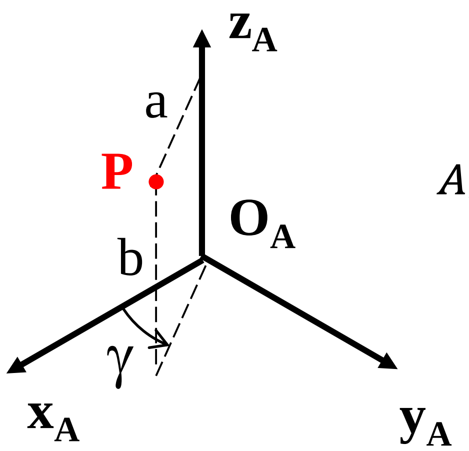
$$[\,^{A}R_B] = Rot(Z_B,\gamma) = \left[\begin{array}{c|c|c} cos\gamma & -sen\gamma & 0 \\ sen\gamma & cos\gamma & 0 \\ 0 & 0 & 1 \end{array}\right]$$

$$^{A}r_P = [\,^{A}R_B][\,^{B}r_P] = \left[\begin{array}{c|c|c} cos\gamma & -sen\gamma & 0 \\ sen\gamma & cos\gamma & 0 \\ 0 & 0 & 1 \end{array}\right]\begin{bmatrix} a \\ 0 \\ b \end{bmatrix} = \boxed{\begin{bmatrix} acos\gamma \\ asen\gamma \\ b \end{bmatrix}}$$

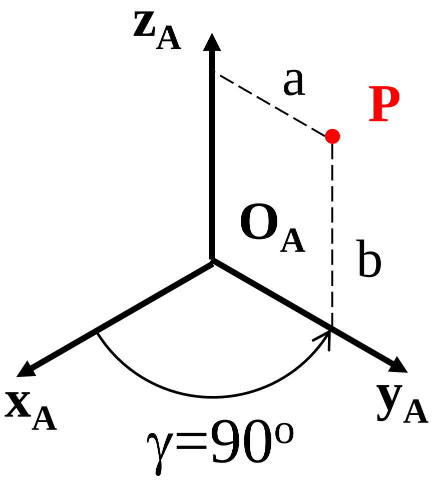
Transformação homogênea em 3D



$$ {}^{A}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} $$

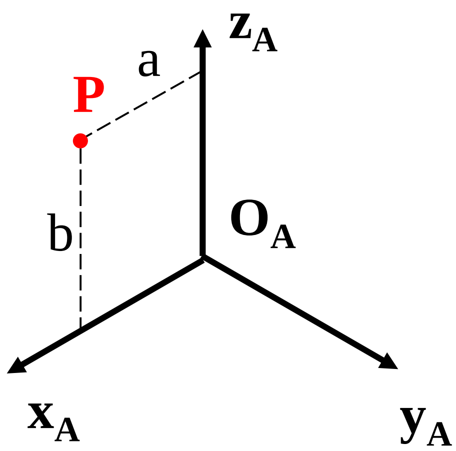


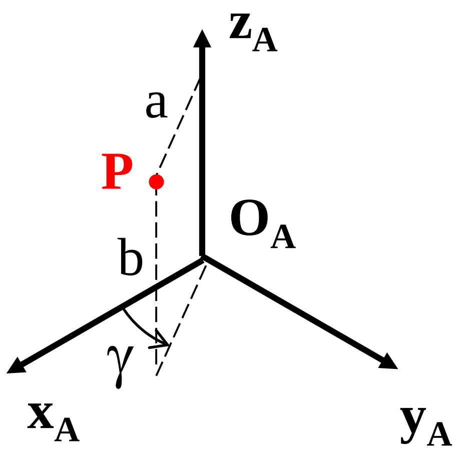
$$ {}^{A}r_P = \begin{bmatrix} acos\gamma \\ asen\gamma \\ b \end{bmatrix} $$



$$ {}^{A}r_P = \begin{bmatrix} 0 \\ a \\ b \end{bmatrix} $$

Transformação homogênea em 3D

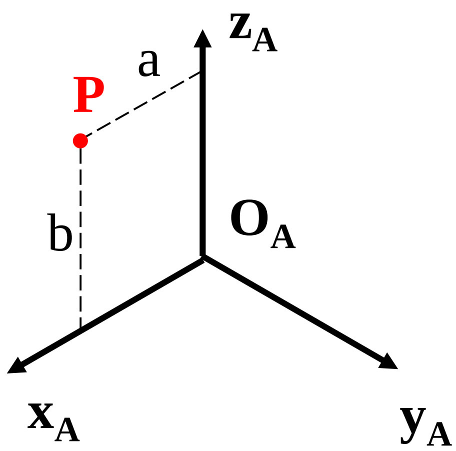


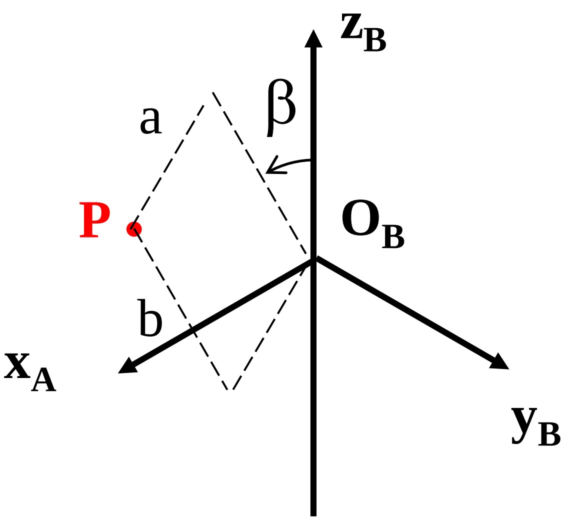


$$\left[{}^{A}R_{B}\right] = Rot(Z_B, \gamma) = \begin{bmatrix} cos\gamma & -sen\gamma & 0 \\ sen\gamma & cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}^{A}R_{B}\right] = Rot(X_B, \alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & -sen\alpha \\ 0 & sen\alpha & cos\alpha \end{bmatrix}$$

Transformação homogênea em 3D

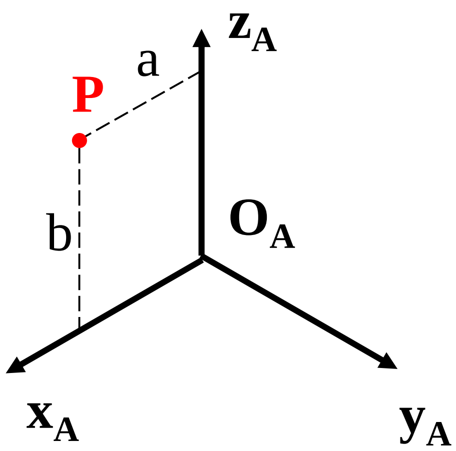




$$[\,{}^{A}R_B] = Rot(Y_B, \beta) = \left[\begin{array}{c|c|c} cos\beta & 0 & sen\beta \\ 0 & 1 & 0 \\ -sen\beta & 0 & cos\beta \end{array}\right]$$

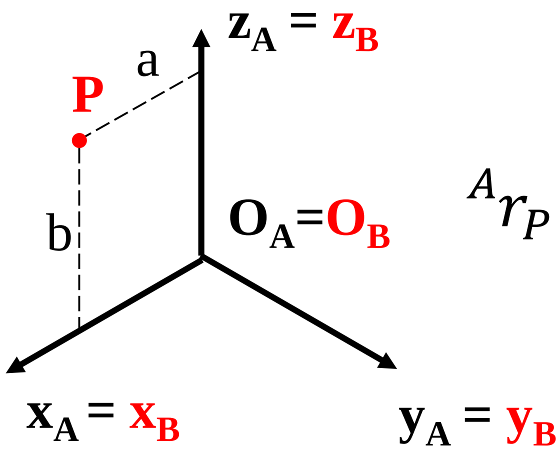
Transformação homogênea em 3D

1



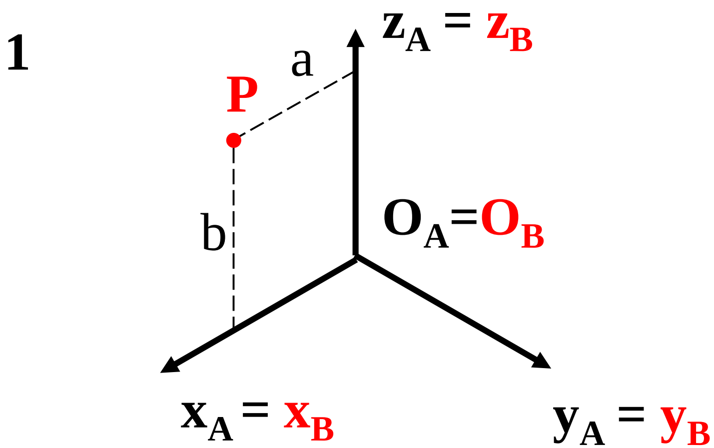
$$^{A}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$$

1

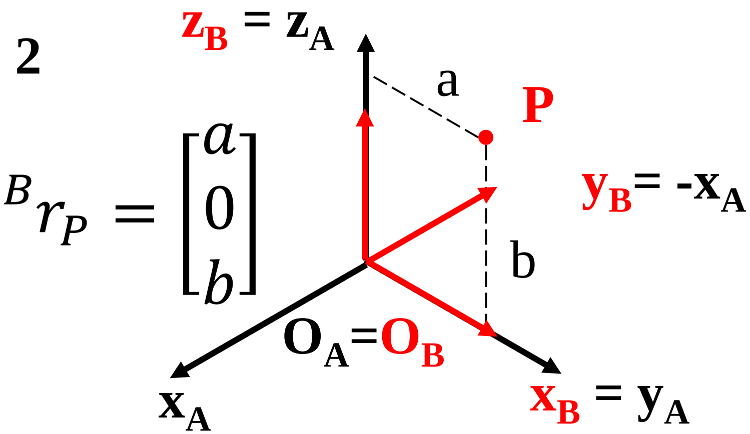


$$ {}^{A}r_P = {}^{B}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} $$

Transformação homogênea em 3D

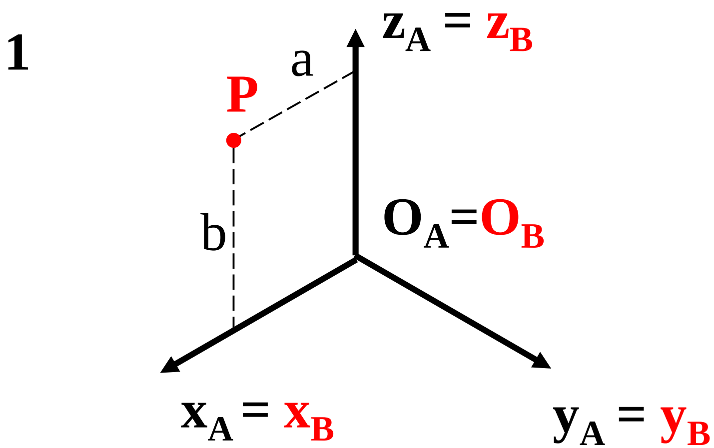


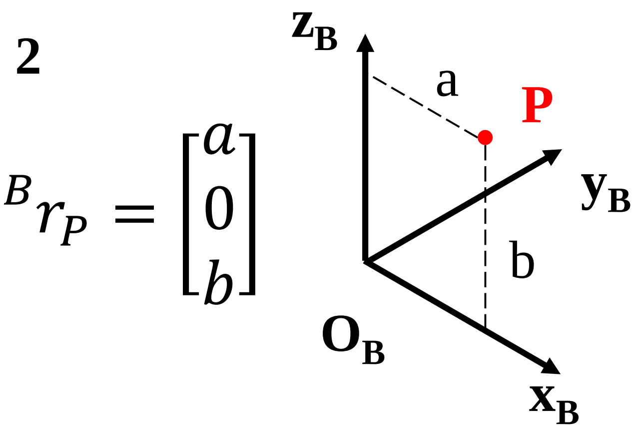
$$ {}^{B}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} $$



rotação (Z_B)

Transformação homogênea em 3D



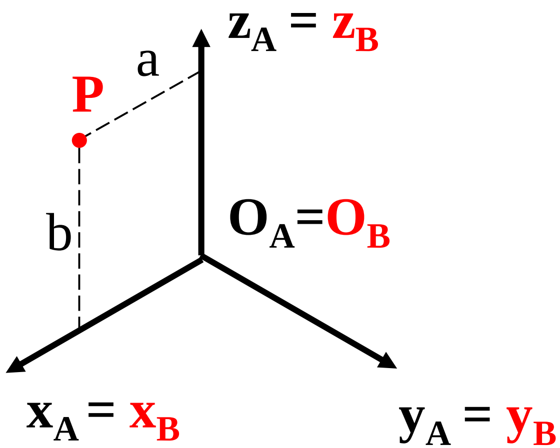


$$ {}^{B}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} $$

rotação (Z_B)

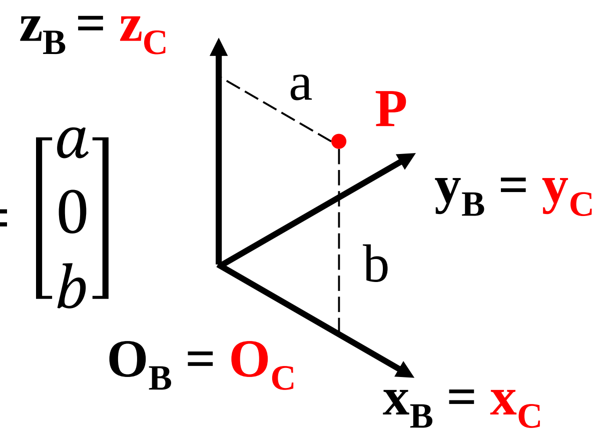
Transformação homogênea em 3D

1



2

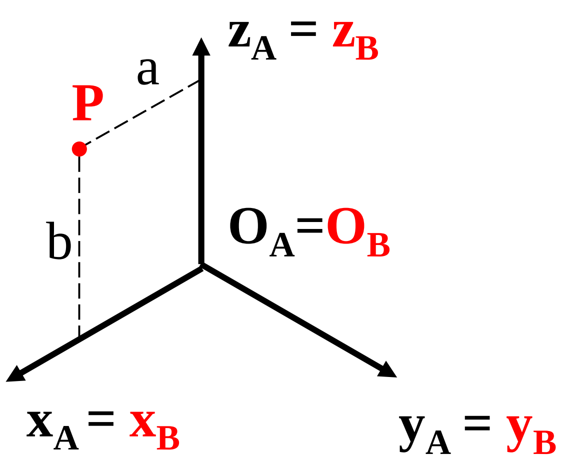
$$ {}^{B}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} $$



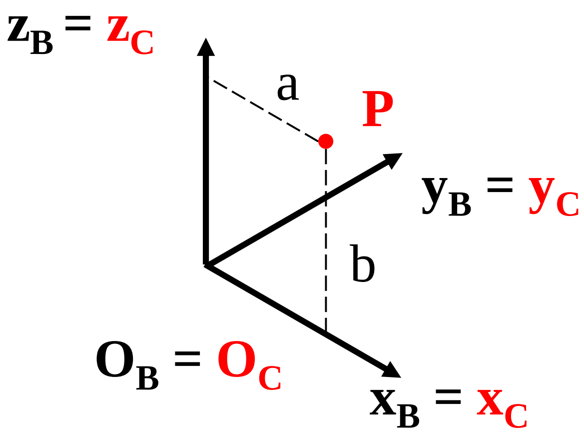
rotação (Z_B)

Transformação homogênea em 3D

1

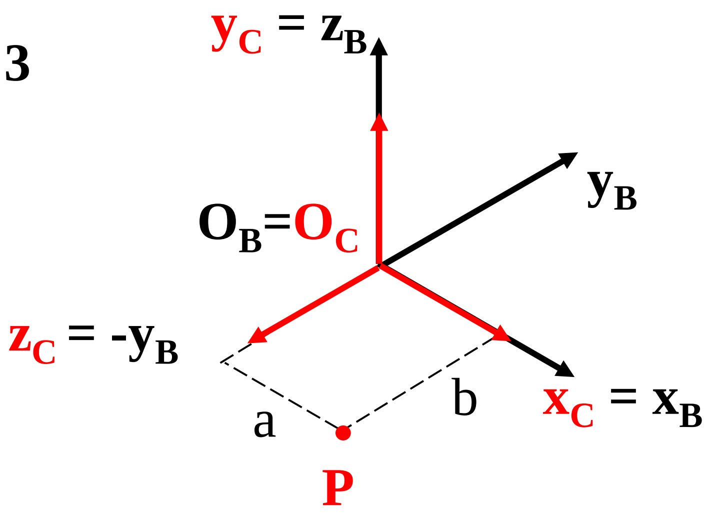


2



rotação (Z_B)

3

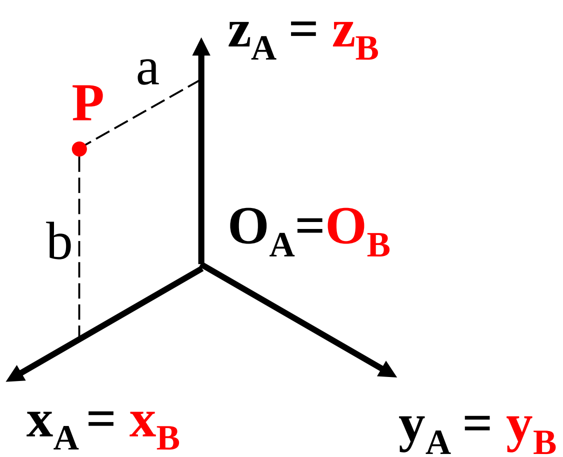


$$^{C}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$$

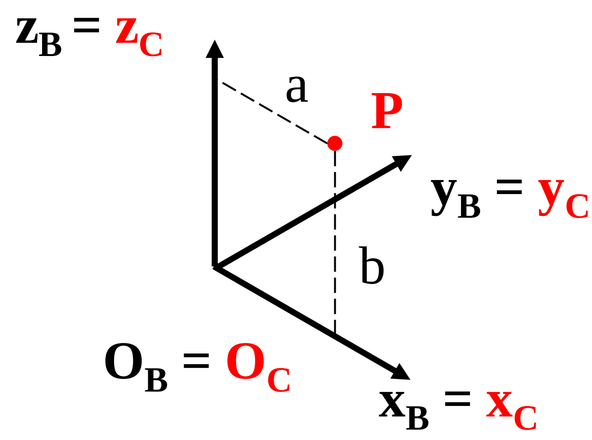
rotação (X_C)

Transformação homogênea em 3D

1

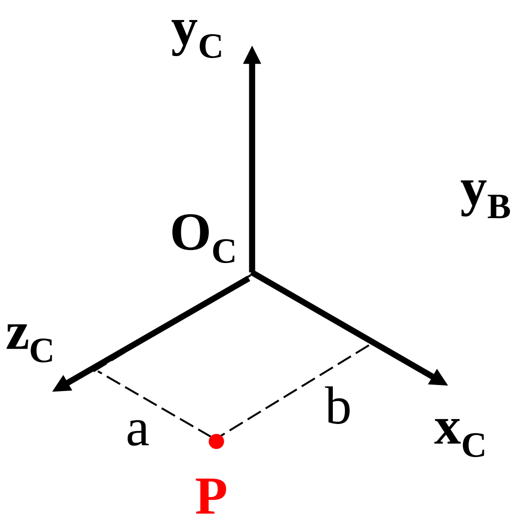


2



rotação (Z_B)

3

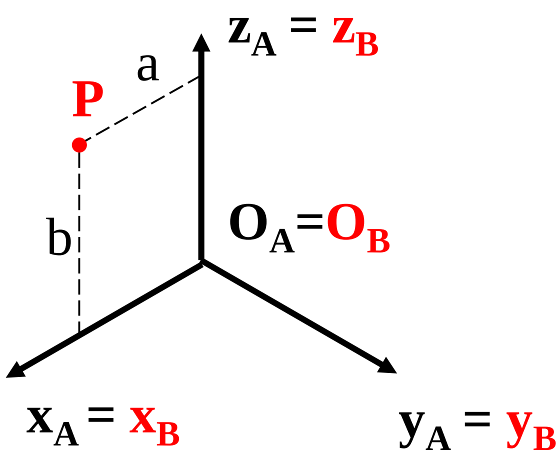


$$ {}^{c}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} $$

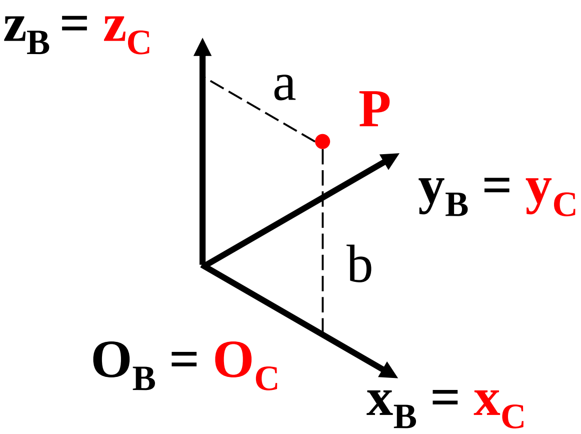
rotação (X_C)

Transformação homogênea em 3D

1

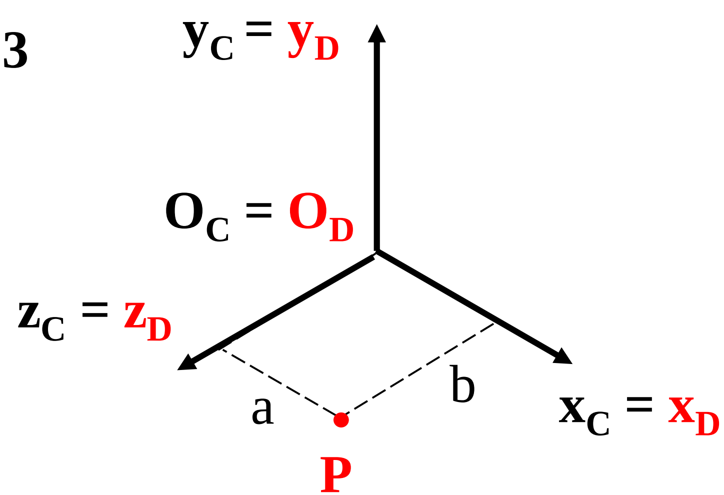


2



rotação (Z_B)

3

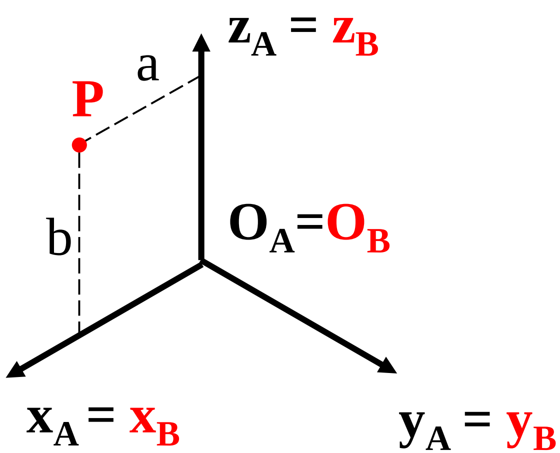


$$^{c}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$$

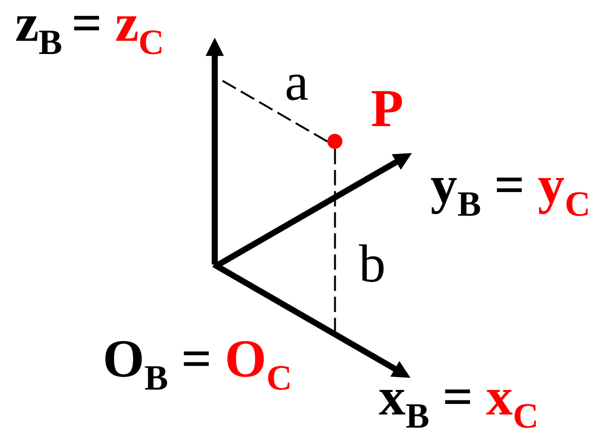
rotação (X_C)

Transformação homogênea em 3D

1

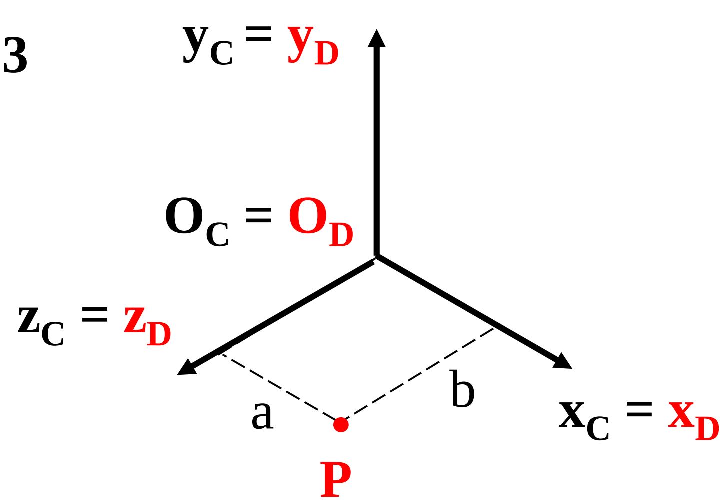


2



rotação (Z_B)

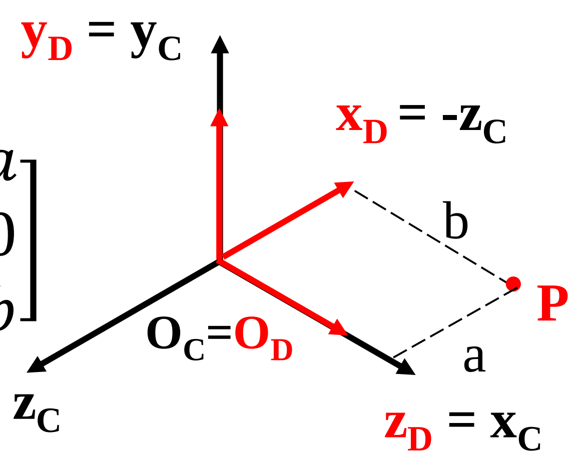
3



rotação (X_C)

4

$$ {}^{D}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} $$



$$ \boxed{ {}^{A}r_P = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} } $$

rotação (Y_D)

Transformação homogênea em 3D

$$\left[{}^{A}r_P\right] = \left[{}^{A}R_B\right]\left[{}^{B}r_P\right] = Rot(Z_B,\gamma)Rot(X_C,\alpha)Rot(Y_D,\beta)\begin{bmatrix} a \\ 0 \\ b \end{bmatrix} =$$

$$= \begin{bmatrix} cos\gamma & -sen\gamma & 0 \\ sen\gamma & cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & -sen\alpha \\ 0 & sen\alpha & cos\alpha \end{bmatrix}\begin{bmatrix} cos\beta & 0 & sen\beta \\ 0 & 1 & 0 \\ -sen\beta & 0 & cos\beta \end{bmatrix}\begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$$

$$\left[{}^{A}r_P\right] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ 0 \\ b \end{bmatrix} = \begin{bmatrix} -a \\ b \\ 0 \end{bmatrix}$$

Transformação homogênea em 3D

$$\left[{}^{A}r_P \right] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$$

$$\left[{}^{A}r_P \right] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$$

$$\left[{}^{A}r_P \right] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} = \begin{bmatrix} -a \\ b \\ 0 \end{bmatrix}$$

Custo computacional:

- 63 multiplicações
- 42 adições

Transformação homogênea em 3D

$$\left[{}^{A}r_P\right] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$$

$$\left[{}^{A}r_P\right] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} b \\ 0 \\ -a \end{bmatrix}$$

$$\left[{}^{A}r_P\right] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b \\ a \\ 0 \end{bmatrix} = \begin{bmatrix} -a \\ b \\ 0 \end{bmatrix}$$

Custo computacional:
- 27 multiplicações
- 18 adições

Transformação homogênea em 3D

$$\left[{}^{A}r_P\right] = \left[{}^{A}R_B\right]\left[{}^{B}r_P\right] = Rot(Z_B,\gamma)Rot(X_C,\alpha)Rot(Y_D,\beta)\begin{bmatrix} a \\ 0 \\ b \end{bmatrix} = \begin{bmatrix} -a \\ b \\ 0 \end{bmatrix}$$

$$\left[{}^{A}r_P\right] = \left[{}^{A}R_B\right]\left[{}^{B}r_P\right] = Rot(Y_D,\beta)Rot(X_C,\alpha)Rot(Z_B,\gamma)\begin{bmatrix} a \\ 0 \\ b \end{bmatrix} =?$$

Transformação homogênea em 3D

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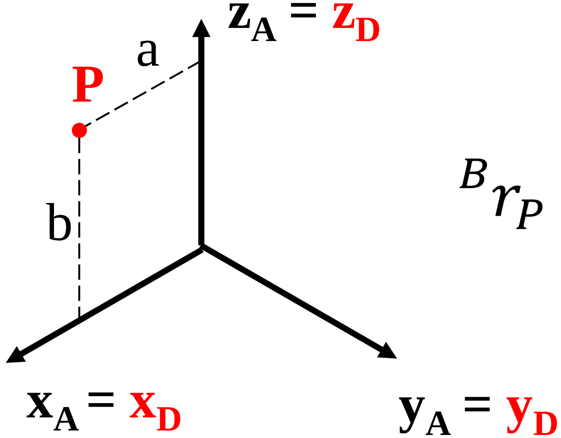
$$[{}^{A}r_P] = [{}^{A}R_B][{}^{B}r_P] = Rot(Y_D,\beta)Rot(X_C,\alpha)Rot(Z_B,\gamma)\begin{bmatrix} a \\ 0 \\ b \end{bmatrix} = ?$$

$$= \begin{bmatrix} cos\beta & 0 & sen\beta \\ 0 & 1 & 0 \\ -sen\beta & 0 & cos\beta \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\alpha & -sen\alpha \\ 0 & sen\alpha & cos\alpha \end{bmatrix}\begin{bmatrix} cos\gamma & -sen\gamma & 0 \\ sen\gamma & cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ 0 \\ b \end{bmatrix}$$

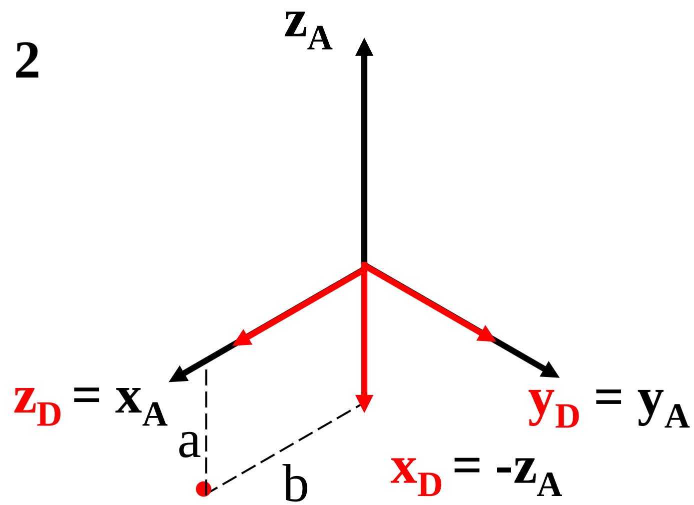
$$[{}^{A}r_P] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ 0 \\ b \end{bmatrix} = \begin{bmatrix} a \\ -b \\ 0 \end{bmatrix}$$

1



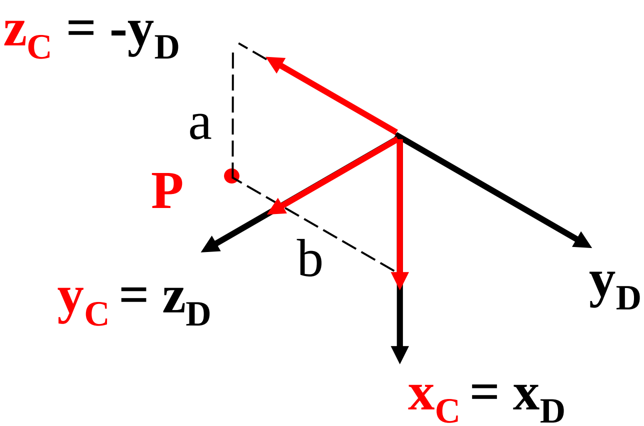
$$ {}^{B}r_P = \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} $$

2



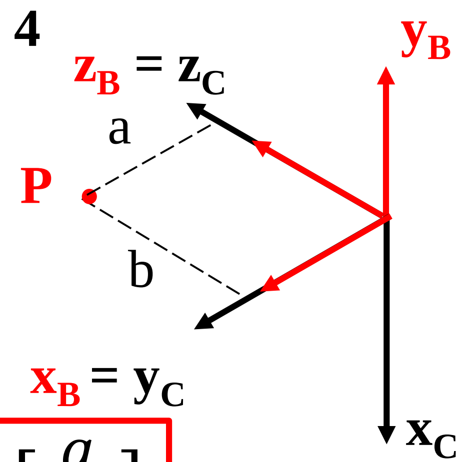
rotação (Y_D)

3



rotação (X_C)

4



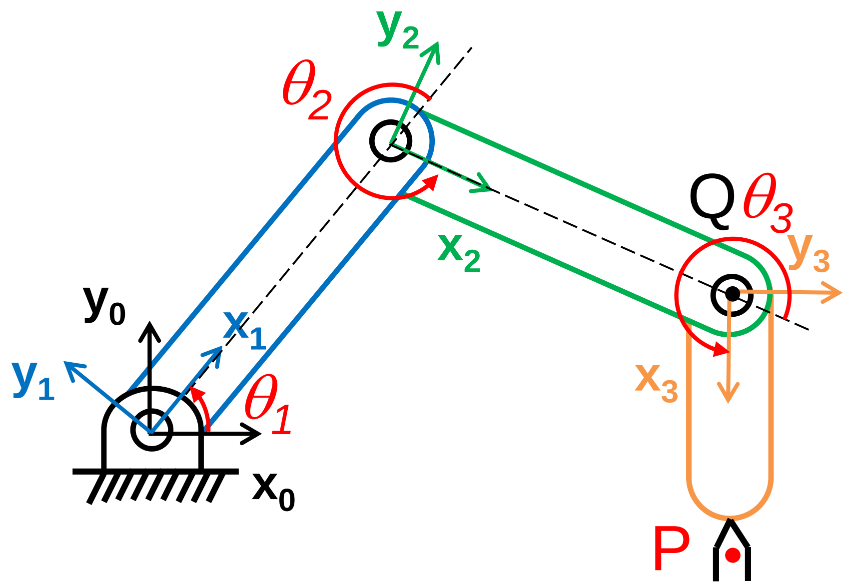
$$ \boxed{ {}^{A}r_P = \begin{bmatrix} a \\ -b \\ 0 \end{bmatrix} } $$

rotação (Z_B)

Para o robô da figura abaixo, considere que os comprimentos das peças 1, 2 e 3, respectivamente, sejam $L_1 = L_2 = 1$ m, $L_3 = 0,5$ m, que as coordenadas absolutas do ponto P sejam $[1 \;\; 0,5 \;\; 0]^T$ e que a orientação da garra seja $-\mathbf{j_0}$ (vertical, de cima para baixo). Determine as coordenadas do ponto Q (origem da base $O_3x_3y_3z_3$), em relação à base fixa $Ox_0y_0z_0$, bem como os ângulos θ_1, θ_2 e θ_3.

Dados:
- $L_1 = L_2 = 1$ m
- $L_3 = 0,5$ m
- $P = [1 \;\; 0,5 \;\; 0]^T$
- Orientação da garra: $-j_0$

Pede-se:
- Q = ?
- θ_1, θ_2 e θ_3?



Exercício 1 - resposta

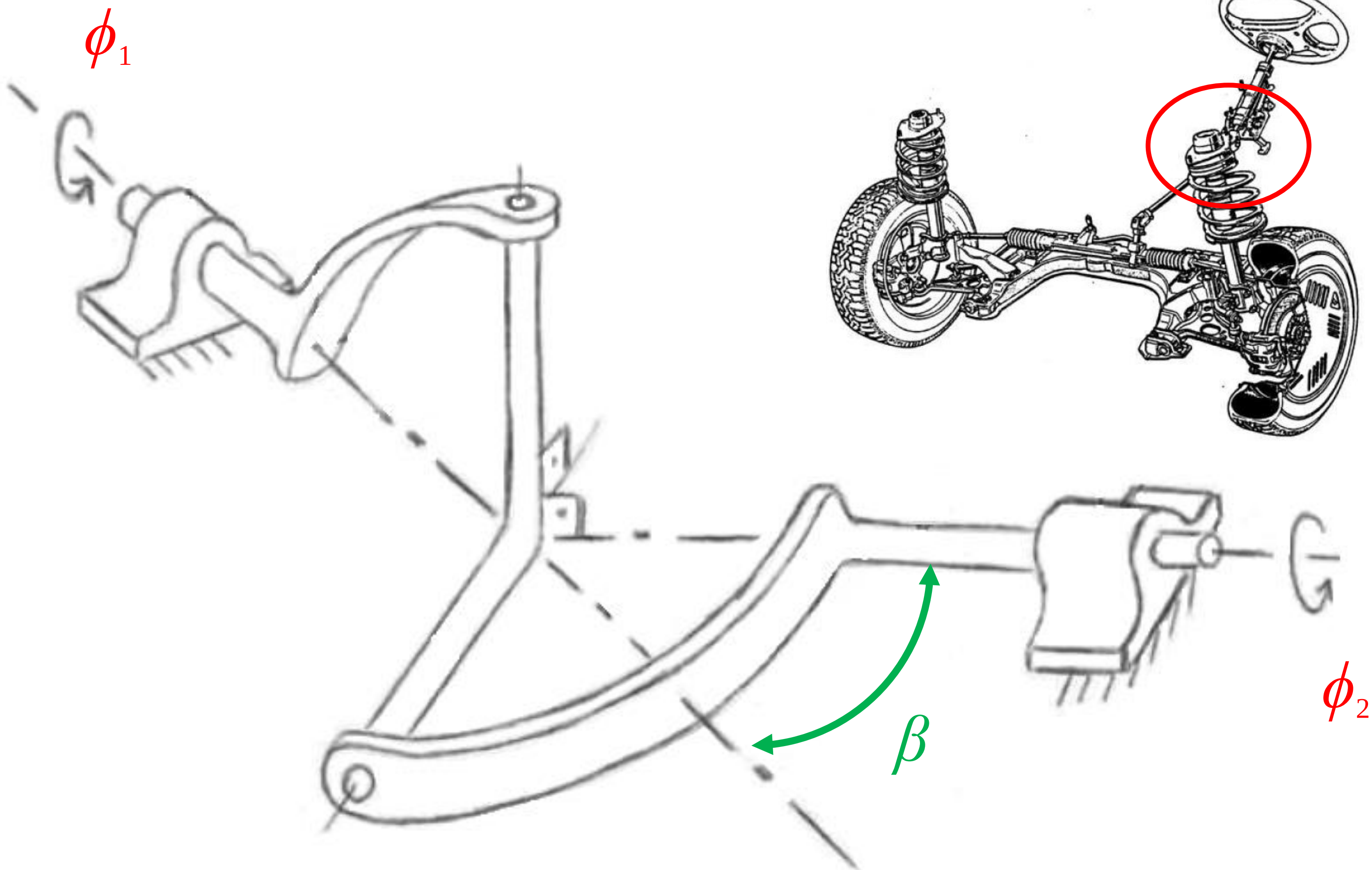
$$^{0}Q = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

1 solução: (θ_1 = 90º ; θ_2 = -90º e θ_3 = -90º)

2 solução: (θ_1 = 0º ; θ_2 = 90º e θ_3 = 180º)

Resposta correta

Exercício 2 - Mecanismo <u>R</u>UR



Variáveis: $\boldsymbol{\phi_1}$ e $\boldsymbol{\phi_2}$

Dados: $\boldsymbol{\beta}$

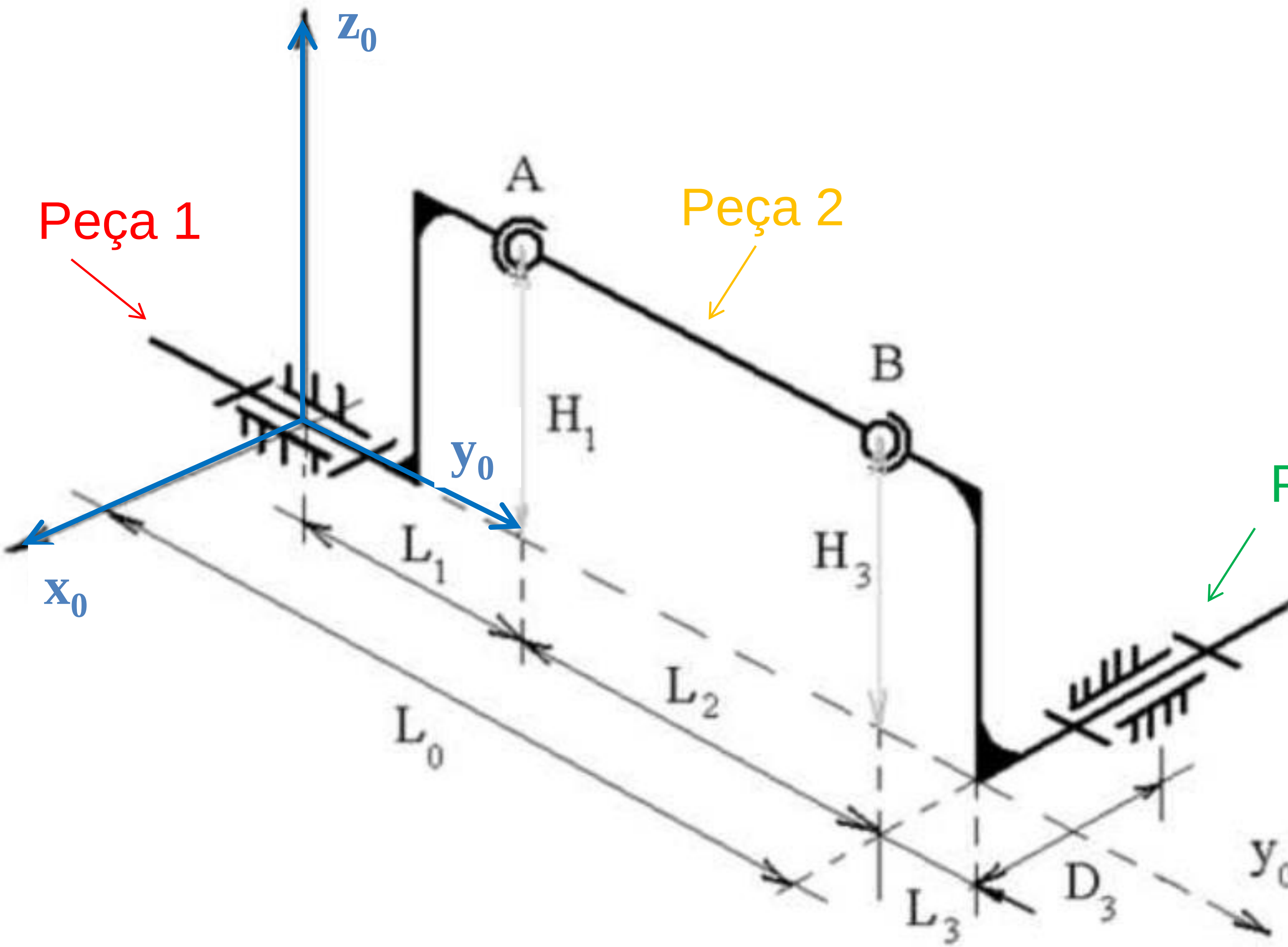
Pede-se: relação entre ϕ_1 e ϕ_2

Exercício 2 - resposta

$$c\beta \frac{s\phi_1}{c\phi_1} = \frac{s\phi_2}{c\phi_2}$$ ou $$c\beta tg\phi_1 = tg\phi_2$$

$$\dot{\phi}_2 = \frac{c\beta}{1 - s^2\beta s^2\phi_1}\dot{\phi}_1$$

Exercício 3 - Mecanismo RSSR



Pede-se: relação entre θ_1 e θ_3

Exercício 3 - resposta

$$Ec\theta_3 + Fs\theta_3 + G = 0$$

$$E = 2[(L_1 - L_0)L_3 - H_1 H_3 c\theta_1]$$

$$F = 2[(L_1 - L_0)H_3 + H_1 L_3 c\theta_1]$$

$$G = {H_1}^2 + {H_3}^2 + (L_1 - L_0)^2 + {L_3}^2 - {L_2}^2$$

Dados: $\theta_1 = 0$, $H_1 = H_3 = 1$ m, $L_1 = L_3 = 0$, $L_0 = L_2 = 1$ m

Determine: $\boldsymbol{\theta_3}$

$$E = -2\,; F = -2\,; G = 2$$

$$-2c\theta_3 - 2s\theta_3 + 2 = 0$$

$$c\theta_3 + s\theta_3 = 1$$

$$\theta_3 = 0°$$
$$\theta_3 = 90°$$

Resposta: $\boldsymbol{\theta_3 = 90^o}$