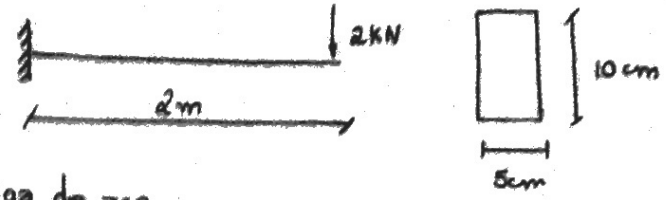
Exercícios 3 – Linha Elástica

1)



$\delta_{máx} = ?$

$\delta = ?$

$\phi = ?$

Considerando:

$E = 210\ GPa$

Viga de aço

$$I = \frac{bh^3}{12} = \frac{0{,}05 \times (0{,}1)^3}{12} = \frac{1}{240000} = 4{,}167 \times 10^{-6}\ m^4$$

$$E_{aço} = 200\ GPa = 200 \times 10^9 \frac{N}{m^2} = 200 \times 10^6 \frac{KN}{m^2}$$

$$E_{aço} = 2 \times 10^8 \frac{KN}{m^2}$$

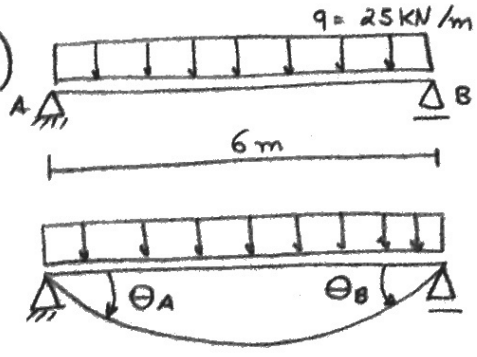
$$\sigma = \frac{M_{máx}\, y}{I} \qquad M_{máx} = 2 \times 2 = 4\ KN.m$$

$$\sigma_{máx} = \frac{4\ KN.m \times 0{,}1/2\ m}{\frac{1}{240000}\ m^4} = 48000 \frac{KN}{m^2} = 48\ MPa \Rightarrow \boxed{\sigma_{máx} = 48 MPa}$$

$$\delta_B = \frac{PL^3}{3EI} = 6{,}09 \times 10^{-3}\ m = 0{,}609\ cm \ //$$

$$\phi_B = \frac{PL^2}{2EI} = 4{,}57 \times 10^{-3}\ rad \ //$$

2)



$I_x = 74129{,}8\ cm^4 = 7{,}41298 \times 10^{-4}\ m^4$

$E = 210\ GPa = 210 \times 10^6\ KN/m^2$

$\delta_{máx} = ?$

θ_A e $\theta_B = ?$

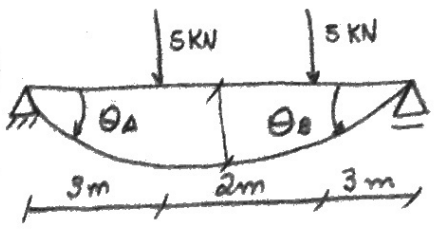
Pela tabela:

$$\delta_{máx} = \frac{5qL^4}{384\,EI} = 2{,}71 \times 10^{-3} = 0{,}0027\ m = 0{,}27\ cm$$

$\boxed{\delta_{máx} = 0{,}27 cm}$

$$\theta_A = -\frac{qL^3}{24EI} = -0{,}00145\ rad \ // \quad \text{e} \quad \boxed{\theta_B = +0{,}00145\ rad}$$

3)



$\delta_{máx} = ?$

Pela tabela

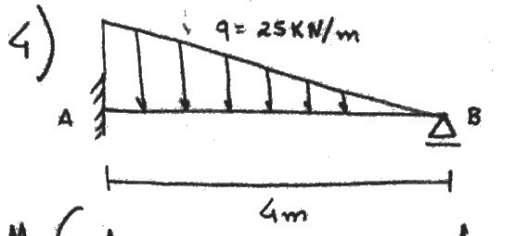
$$\delta_{máx} = \frac{Pa}{24EI}(3L^2 - 4a^2) \quad \therefore a = 3m \text{ e } L = 8m$$

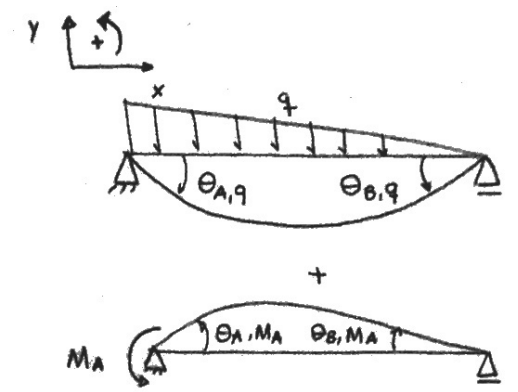
$$E = 25\ GPa = 25 \times 10^6\ KN/m^2$$

$$I = \frac{bh^3}{12} = \frac{0{,}2 \cdot 0{,}4^3}{12} = 1{,}067 \times 10^{-3}\ m^4$$

$$\delta_{máx} = 3{,}6 \times 10^{-3}\ m = 0{,}36\ cm \ //$$

4)





- 3 reações de apoio: V_A, V_B e M_A (RA)
- 2 eqs de equilíbrio: $\Sigma F_v = 0$ e $\Sigma M_B = 0$ (EE)

$\Sigma F_v = 0$

$V_A + V_B = \frac{25 \times 4}{2} \Rightarrow \boxed{V_A + V_B = 50}$ (1)

$\Sigma M_A = 0$

$M_A - 100 \times \frac{4}{3} + V_B \times 4 = 0$

$\boxed{M_A + 4V_B = \frac{400}{3}}$ (2)

- grau de hiperasticidade

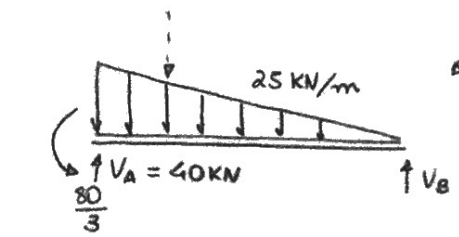
GH = RA - EE = 3 - 2 = 1 x hiperestática

1º) Escolhendo (M_A) como incógnita hiperestática:

2º) Equação de compatibilidade: $\boxed{\theta_{A,q} + \theta_{A,M_A} = 0}$

$\theta_{A,q} = -\frac{q\ell^3}{45EI}$ e $\theta_{A,M_A} = +\frac{M_A \ell}{3EI}$

$\frac{-q\ell^3}{45EI} + \frac{M_A \ell}{3EI} = 0 \Rightarrow M_A = +\frac{q\ell^2}{15} \Rightarrow M_A = \frac{25 \cdot 4^2}{15} = \frac{80}{3}$ KN.m $= 26{,}7$ KN m



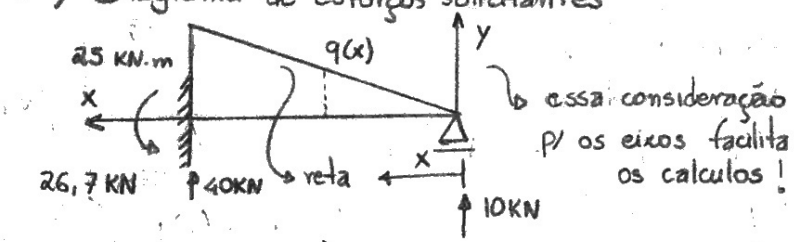
$+\circlearrowleft \Sigma M_A = 0 \quad \frac{80}{3} + 4V_B - \frac{25 \cdot 4}{2} \cdot \frac{1}{3} \cdot 4 = 0$

$4V_B = \frac{25 \cdot 4^2}{2 \cdot 3} - \frac{80}{3}$

$\boxed{V_B = 10 \text{ KN}}$

$+\uparrow \Sigma F_v = 0 \quad V_A + 10 - \frac{25 \cdot 4}{2} = 0 \quad \boxed{V_A = 40 \text{ KN}}$

3º) Diagrama de esforços solicitantes



essa consideração p/ os eixos facilita os cálculos!

$q(x) = ax + b$

$x = 0, q(0) = 0 \therefore 0 = a \cdot 0 + b$

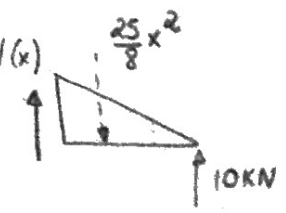
$b = 0$

$x = 4m, q(4) = 25 \therefore 25 = 4a + 0$

$a = \frac{25}{4}$

$\boxed{q(x) = \frac{25}{4} x}$

Cálculo do V(x):



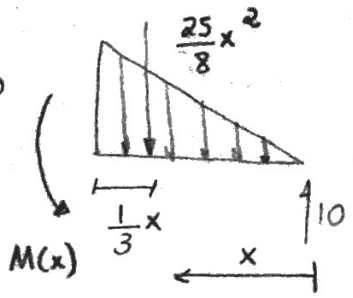
$V(x) + 10 - \frac{25}{4} x \cdot x \cdot \frac{1}{2} = 0$

$\boxed{V(x) = -10 + 3{,}125 x^2}$

Calculo de $M(x)$:

$$M(x) + 10x - \frac{25}{8}x^2 \cdot \frac{x}{3} = 0$$

$$M(x) = -10x + \frac{25x^3}{24}$$



$x = ?$ p/ $V(x) = 0$

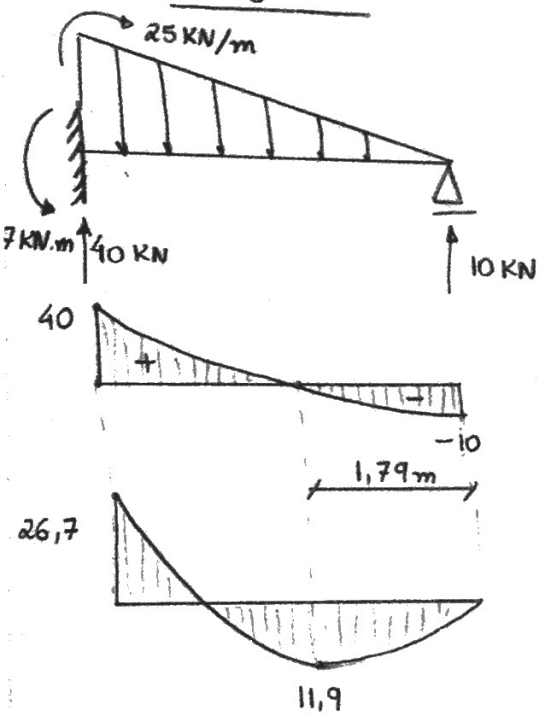
$$-10 + 3{,}125x^2 = 0$$

$$\boxed{x = 1{,}79 \text{ m}}$$

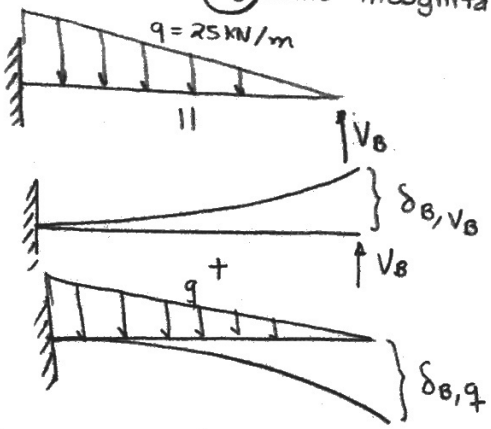
$$M(1{,}79) = -10 \cdot 1{,}79 + \frac{25 \cdot 1{,}79^3}{24}$$

$$M(1{,}79) = -11{,}9 \text{ KN.m}$$

Diagramas



1º) Escolhendo (V_B) como incógnita hiperestática

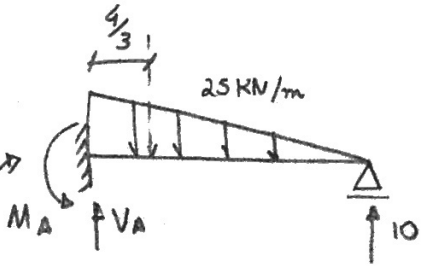


2º) Equação de compatibilidade: $\boxed{\delta_{B,V_B} + \delta_{B,q} = 0}$

$$\delta_{B,V_B} = -\frac{V_B L^3}{3EI} \quad \text{e} \quad \delta_{B,q} = \frac{q L^4}{30EI}$$

$$\frac{-V_B \cancel{L^3}}{\cancel{3}EI} + \frac{q L^{\cancel{4}\,1}}{\cancel{30}\,10 EI} = 0$$

$$V_B = \frac{qL}{10} \Rightarrow \boxed{V_B = 10 \text{ KN}}$$



$$\Sigma F_V = 0 \therefore V_A + 10 - \frac{25 \cdot 4}{2} = 0$$

$$\boxed{V_A = 40 \text{ KN}}$$

$$\Sigma M_A = 0 \therefore M_A + 10 \cdot 4 - \frac{25 \cdot 4}{2} \cdot \frac{4}{3} = 0$$

$$\boxed{M_A = 26{,}67 \text{ KN.m}}$$

Diagrama de esforços solicitantes

↳ 3º)