

O Jogo Coronel Blotto

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Introdução

O Coronel Blotto luta contra o General Vermelho por dois postos.

- ▶ Coronel Blotto tem 4 regimentos General Vermelho 3
- ▶ O resultado do jogo em cada posto é os regimentos capturados mais o posto.

Matriz do Jogo

	(3,0)	(2,1)	(1,2)	(0,3)
(4,0)	4	2	1	0
(3,1)	1	3	0	-1
(2,2)	-2	2	2	-2
(1,3)	-1	0	3	1
(0,4)	0	1	2	4

Tabela 1: A tabela de pagamento do jogo.

Precisamos aumentar a matriz de jogo para garantir que o valor de coluna seja positivo.

$$\begin{pmatrix} 4 & 2 & 1 & 0 \\ 1 & 3 & 0 & -1 \\ -2 & 2 & 2 & -2 \\ -1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{pmatrix} + 3 = \begin{pmatrix} 7 & 5 & 4 & 3 \\ 4 & 6 & 3 & 2 \\ 1 & 5 & 5 & 1 \\ 2 & 3 & 6 & 4 \\ 2 & 4 & 5 & 7 \end{pmatrix}$$

Problema Primal

maximizar: $y_1 + y_2 + y_3 + y_4$

sujeito à: $7y_1 + 5y_2 + 4y_3 + 3y_4 \leq 1$

$$4y_1 + 6y_2 + 3y_3 + 2y_4 \leq 1$$

$$y_1 + 5y_2 + 5y_3 + y_4 \leq 1$$

$$2y_1 + 3y_2 + 6y_3 + 4y_4 \leq 1$$

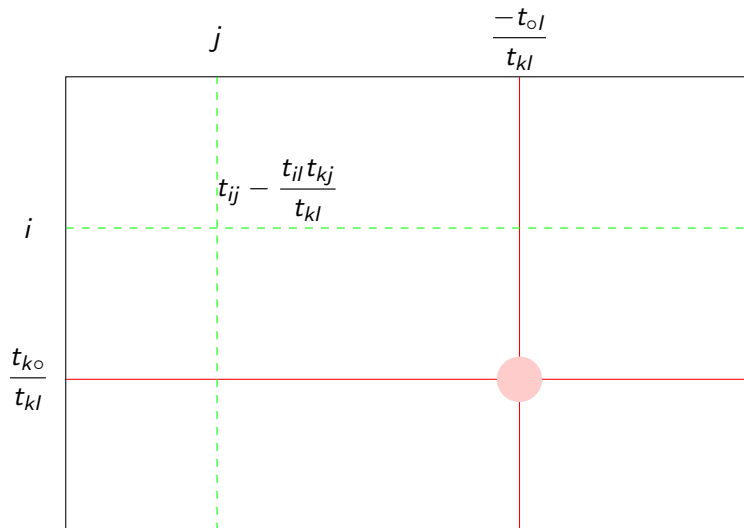
$$3y_1 + 4y_2 + 5y_3 + 7y_4 \leq 1$$

Tabelas

	y_1	y_2	y_3	y_4	-1	
x_1	7	5	4	3	1	$= -y_5$
x_2	4	6	3	2	1	$= -y_6$
x_3	1	5	5	1	1	$= -y_7$
x_4	2	3	6	4	1	$= -y_8$
x_5	3	4	5	7	1	$= -y_9$
	1	1	1	1	0	f
	x_6	x_7	x_8	x_9		

Tabela 2: A tabela inicial, pivo (1, 1).

Regra de Pivotação



	y_5	y_2	y_3	y_4	-1	
x_6	$1/7$	$5/7$	$4/7$	$3/7$	$1/7$	$= -y_1$
x_2	$-4/7$	$22/7$	$5/7$	$2/7$	$3/7$	$= -y_6$
x_3	$-1/7$	$30/7$	$31/7$	$4/7$	$6/7$	$= -y_7$
x_4	$-2/7$	$11/7$	$34/7$	$22/7$	$5/7$	$= -y_8$
x_5	$-3/7$	$13/7$	$23/7$	$40/7$	$4/7$	$= -y_9$
	$-1/7$	$2/7$	$3/7$	$4/7$	$-1/7$	f
	x_1	x_7	x_8	x_9		

Tabela 3: Segunda tabela pivo (5, 4).

	y_5	y_2	y_3	y_9	-1	
x_6	$7/40$	$23/40$	$13/40$	$-3/40$	$1/10$	$= -y_1$
x_2	$-11/20$	$61/20$	$11/20$	$-1/20$	$2/5$	$= -y_6$
x_3	$-1/10$	$41/10$	$41/10$	$-1/10$	$4/5$	$= -y_7$
x_4	$-1/20$	$11/20$	$61/20$	$-11/20$	$2/5$	$= -y_8$
x_9	$-3/40$	$13/40$	$23/40$	$7/40$	$1/10$	$= -y_4$
	$-1/10$	$1/10$	$1/10$	$-1/10$	$-1/5$	f
	x_1	x_7	x_8	x_5		

Tabela 4: Pivo: (2, 2)

	y_5	y_6	y_3	y_9	-1	
x_6	$17/61$	$-23/122$	$27/122$	$-4/61$	$3/122$	$= -y_1$
x_7	$-11/61$	$20/61$	$11/61$	$-1/61$	$8/61$	$= -y_2$
x_3	$39/61$	$-82/61$	$205/61$	$-2/61$	$16/61$	$= -y_7$
x_4	$3/61$	$-11/61$	$180/61$	$-33/61$	$20/61$	$= -y_8$
x_9	$-1/61$	$-13/61$	$63/122$	$11/61$	$7/122$	$= -y_4$
	$-5/61$	$-2/61$	$5/61$	$-6/61$	$-13/61$	f
	x_1	x_2	x_8	x_5		

Tabela 5: pivo: $(3, 3)$.

	y_5	y_6	y_7	y_9	-1	
x_6			$-27/410$		$3/410$	$= -y_1$
x_7			$-11/205$		$24/205$	$= -y_2$
x_8	$39/205$	$-2/5$	$61/205$	$-2/205$	$16/205$	$= -y_3$
x_4			$36/41$		$4/41$	$= -y_8$
x_9			$-63/410$		$7/410$	$= -y_4$
	$-4/41$	0	$-1/41$	$-4/41$	$-9/41$	f
	x_1	x_2	x_3	x_5		

Tabela 6: Tabela final.