Assíntotas $\quad$ pg 5 slides aulas 04 e 05

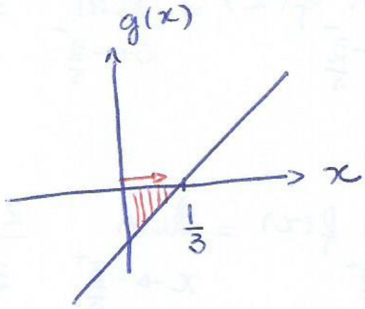
a) $f(x) = \dfrac{5x}{3x-1} \longrightarrow \neq 0 \qquad 3x - 1 \neq 0 \Rightarrow x \neq 1/3$

$$D_f = \left\{ x \in \mathbb{R} \mid x \neq \tfrac{1}{3} \right\}$$

$\hookrightarrow x = \frac{1}{3}$ é um candidato à A.V.

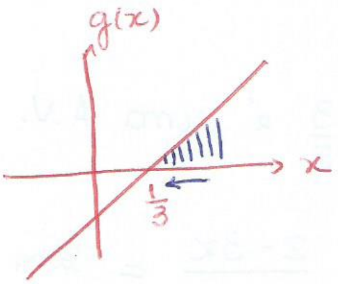
<u>A.V.</u>

$$\lim_{x \to \frac{1}{3}^-} \frac{5x}{3x-1} = \frac{5/3 \to c > 0}{0 \to g(x)\,?} \quad g(x) < 0$$



$$\lim_{x \to \frac{1}{3}^-} \frac{5x}{3x-1} = -\infty$$

$$\lim_{x \to \frac{1}{3}^+} \frac{5x}{3x-1} = \frac{5/3 \to c > 0}{0 \to g(x) = ?} \quad g(x) > 0$$



$$\lim_{x \to \frac{1}{3}^+} \frac{5x}{3x-1} = +\infty$$

$x = \frac{1}{3}$ é uma assíntota vertical

<u>A.H.</u>

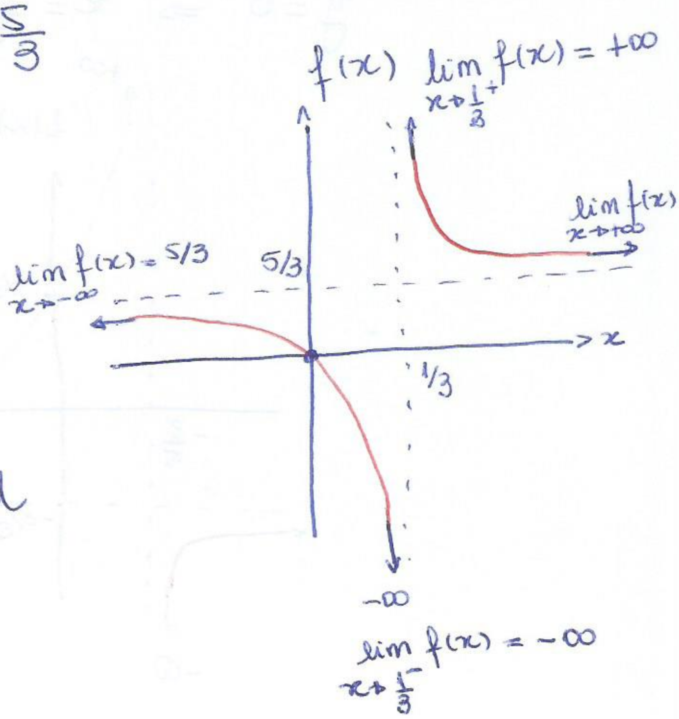
$$\lim_{x \to +\infty} \frac{5x}{3x-1} = \frac{+\infty}{+\infty}$$

$$\lim_{x \to +\infty} \frac{\frac{5x}{x}}{\frac{3x}{x} - \frac{1}{x}} = \lim_{x \to +\infty} \frac{5}{3 - \cancel{\frac{1}{x}}_{\to 0}} = \frac{5}{3}$$

$$\lim_{x \to -\infty} \frac{5x}{3x-1} = \frac{-\infty}{-\infty}$$

$$\lim_{x \to -\infty} \frac{5}{3 - \cancel{\frac{1}{x}}_{\to 0}} = \frac{5}{3}$$

$y = \frac{5}{3}$ é uma assíntota vertical



<u>Interceptos</u> $\quad x = 0 \Leftrightarrow y = 0$

b) $f(x) = \dfrac{2-3x}{3+5x} \rightarrow \neq 0 \qquad 3+5x \neq 0 \Rightarrow x \neq -\dfrac{3}{5}$

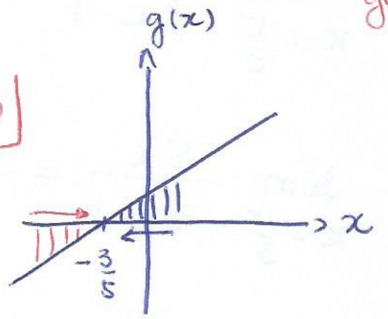
$D_f = \left\{ x \in \mathbb{R} \mid x \neq -\dfrac{3}{5} \right\}$

↳ candidato à A.V.

A.V.

$$\lim_{x \to -\frac{3}{5}^-} f(x) = \lim_{x \to -\frac{3}{5}^-} \frac{2-3x}{3+5x} = \frac{2-3\left(-\frac{3}{5}\right)}{0} = \frac{2+\frac{9}{5}}{0} = \frac{\overset{c>0}{19/5}}{\underset{g(x)\,?}{0}} = -\infty$$

$$\lim_{x \to -\frac{3}{5}^+} f(x) = \lim_{x \to -\frac{3}{5}^+} \frac{2-3x}{3+5x} = \frac{\overset{c>0}{19/5}}{\underset{g(x)\,?}{0}} = +\infty$$



$x = -\dfrac{3}{5}$ é uma A.V.

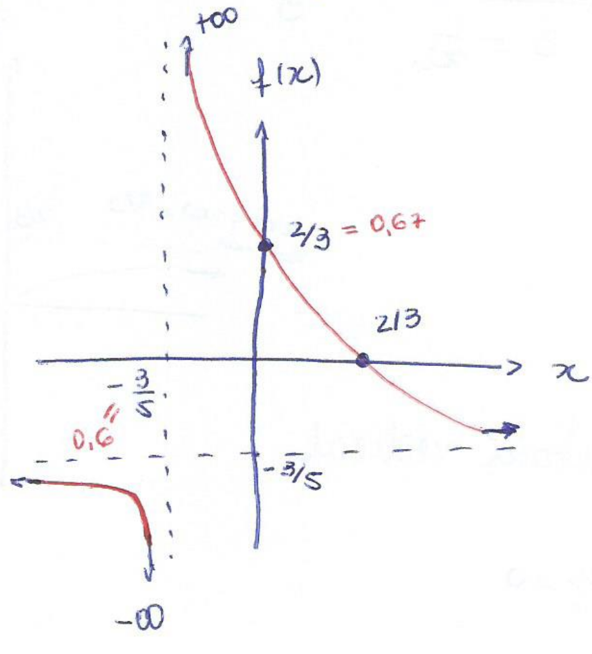
A.H.

$$\lim_{x \to +\infty} \frac{2-3x}{3+5x} = \lim_{x \to +\infty} \frac{\overset{0}{\cancel{\frac{2}{x}}} - \frac{3\cancel{x}}{\cancel{x}}}{\underset{0}{\cancel{\frac{3}{x}}} + \frac{5\cancel{x}}{\cancel{x}}} = -\frac{3}{5}$$

$$\lim_{x \to -\infty} \frac{2-3x}{3+5x} = -\frac{3}{5}$$

Interceptos $\quad x = 0 \Rightarrow y = \dfrac{2}{3}$

$y = 0 \Rightarrow x = \dfrac{2}{3}$



c) $f(x) = \dfrac{-3}{(x+2)^2}$

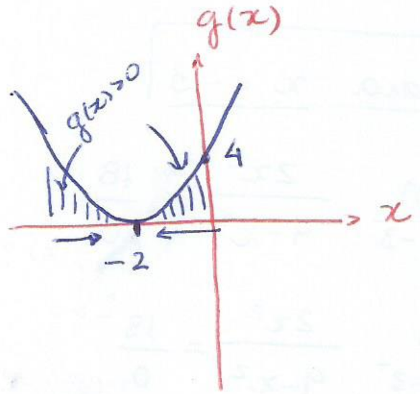
$\underbrace{(x+2)^2}_{\neq 0} \qquad (x+2)^2 \neq 0 \Rightarrow x \neq -2$

$D_f = \{x \in \mathbb{R} \mid x \neq -2\}$

↳ candidato à A.V.

A.V. $\lim\limits_{x \to -2^-} \dfrac{-3}{(x+2)^2} = \dfrac{-3}{0} \overset{c<0}{=} \boxed{-\infty}$ $g(x)$?

$\lim\limits_{x \to -2^+} \dfrac{-3}{(x+2)^2} = \dfrac{-3}{0} \overset{c<0}{=} \boxed{-\infty}$ $g(x)$?



$x = -2$ é uma A.V.

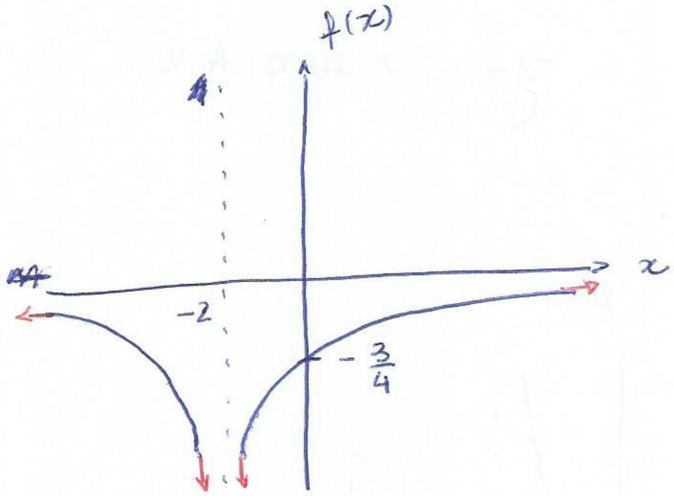
A.H. $\lim\limits_{x \to +\infty} \dfrac{-3}{(x+2)^2} = \dfrac{-3}{+\infty} = 0$ (ver P4)

$\lim\limits_{x \to -\infty} \dfrac{-3}{(x+2)^2} = \dfrac{-3}{+\infty} = 0$ (ver P4)

$y = 0$ é uma A.H.

Interceptos: $x = 0 \Rightarrow y = -3/4$

y nunca será zero (não haverá intercepto-x)



d) $f(x) = \dfrac{2x^2}{9-x^2} \to \neq 0 \qquad 9 - x^2 \neq 0 \Rightarrow x \neq 3$ e $x \neq -3$

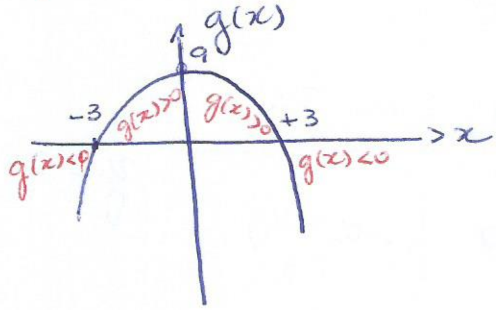
$D_f = \{ x \in \mathbb{R} \mid x \neq -3$ e $x \neq 3 \}$ → dois candidatos à A.V.

A.V. $\boxed{\text{para } x = -3}$ $\qquad$ $\boxed{\text{para } x = 3}$

$\displaystyle\lim_{x \to -3^-} \frac{2x^2}{9-x^2} = \frac{18^{\,c>0}}{0_{\,g(x)\,?}} = \underline{-\infty}$ $\qquad$ $\displaystyle\lim_{x \to 3^-} \frac{2x^2}{9-x^2} = \frac{18^{\,c>0}}{0_{\,g(x)=\,?}} = \underline{+\infty}$

$\displaystyle\lim_{x \to -3^+} \frac{2x^2}{9-x^2} = \frac{18^{\,c>0}}{0_{\,g(x)=\,?}} = \underline{+\infty}$ $\qquad$ $\displaystyle\lim_{x \to 3^+} \frac{2x^2}{9-x^2} = \frac{18^{\,c>0}}{0_{\,g(x)\,?}} = \underline{-\infty}$

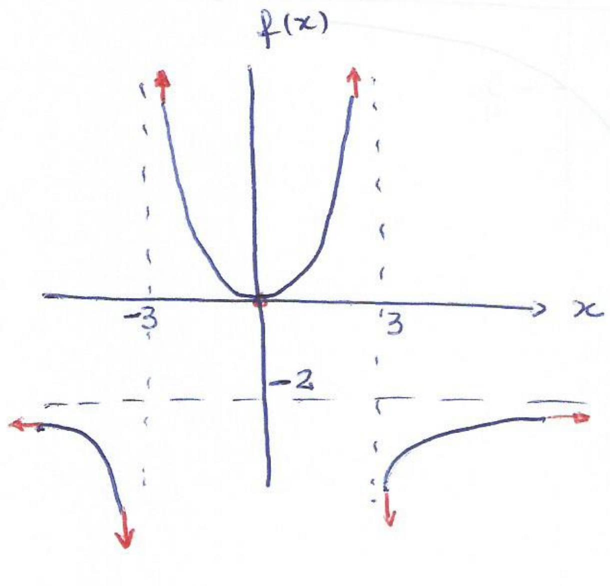
$x = -3$ e $x = 3$ são duas A.V.



A.H $\displaystyle\lim_{x \to +\infty} \frac{2x^2}{9-x^2} = \lim_{x \to +\infty} \frac{\frac{2x^2}{x^2}}{\frac{9}{x^2} - \frac{x^2}{x^2}} = \lim_{x \to +\infty} \frac{2}{\underbrace{\frac{9}{x^2}}_{0} - 1} = \underline{-2}$

$\displaystyle\lim_{x \to -\infty} \frac{2x^2}{9-x^2} = \underline{-2}$ $\qquad$ $y = -2$ é uma A.V.

Interceptos: $x = 0 \Rightarrow y = 0$
$\Leftarrow$



e) $f(x) = \dfrac{2x^2+1}{2x^2-3x} \rightarrow \neq 0$

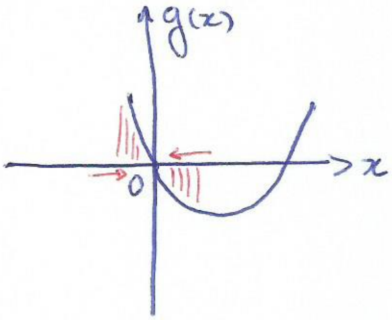
$x(2x-3)=0 \begin{cases} x=0 \\ 2x-3=0 \Rightarrow x = 3/2 \end{cases} \rightarrow$ candidatos à A.V.

$Df = \{x \in \mathbb{R} \mid x \neq 0 \text{ e } x \neq \frac{3}{2}\}$

A.V. $\boxed{\text{para } x=0}$

$$\lim_{x\to 0^-} \frac{2x^2+1}{2x^2-3x} = \frac{1^{\;c>0}}{0_{\;g(x)\,?}} = +\infty$$

$$\lim_{x\to 0^+} \frac{2x^2+1}{2x^2-3x} = \frac{1^{\;c>0}}{0_{\;g(x)\,?}} = -\infty$$

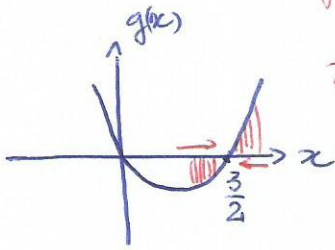


$x=0$ é uma A.V.

$\boxed{\text{para } x = \frac{3}{2}}$

$$\lim_{x\to \frac{3}{2}^-} \frac{2x^2+1}{2x^2-3x} = \frac{11/2^{\;c>0}}{0_{\;g(x)=?}} = -\infty$$

$$\lim_{x\to \frac{3}{2}^+} \frac{2x^2+1}{2x^2-3x} = \frac{11/2^{\;c>0}}{0_{\;g(x)=?}} = +\infty$$



$x = \frac{3}{2}$ é uma A.V.

A.H. $$\lim_{x\to +\infty} \frac{2x^2+1}{2x^2-3x} = \lim_{x\to+\infty} \frac{\frac{2x^2}{x^2}+\frac{1}{x^2}}{\frac{2x^2}{x^2}-\frac{3x}{x^2}} = \lim_{x\to+\infty} \frac{2+\cancel{\frac{1}{x^2}}^{\to 0}}{2-\cancel{\frac{3}{x}}_{\to 0}} = 1$$

$$\lim_{x\to -\infty} \frac{2x^2+1}{2x^2-3x} = 1$$

$y=1$ é uma A.V.

Interceptos: $x=0 \Rightarrow$ a função não está definida $\Rightarrow$ é uma A.V.

$y=0 \Rightarrow 2x^2+1=0$ nunca!