

# Cap. 10. Interiores Estelares.

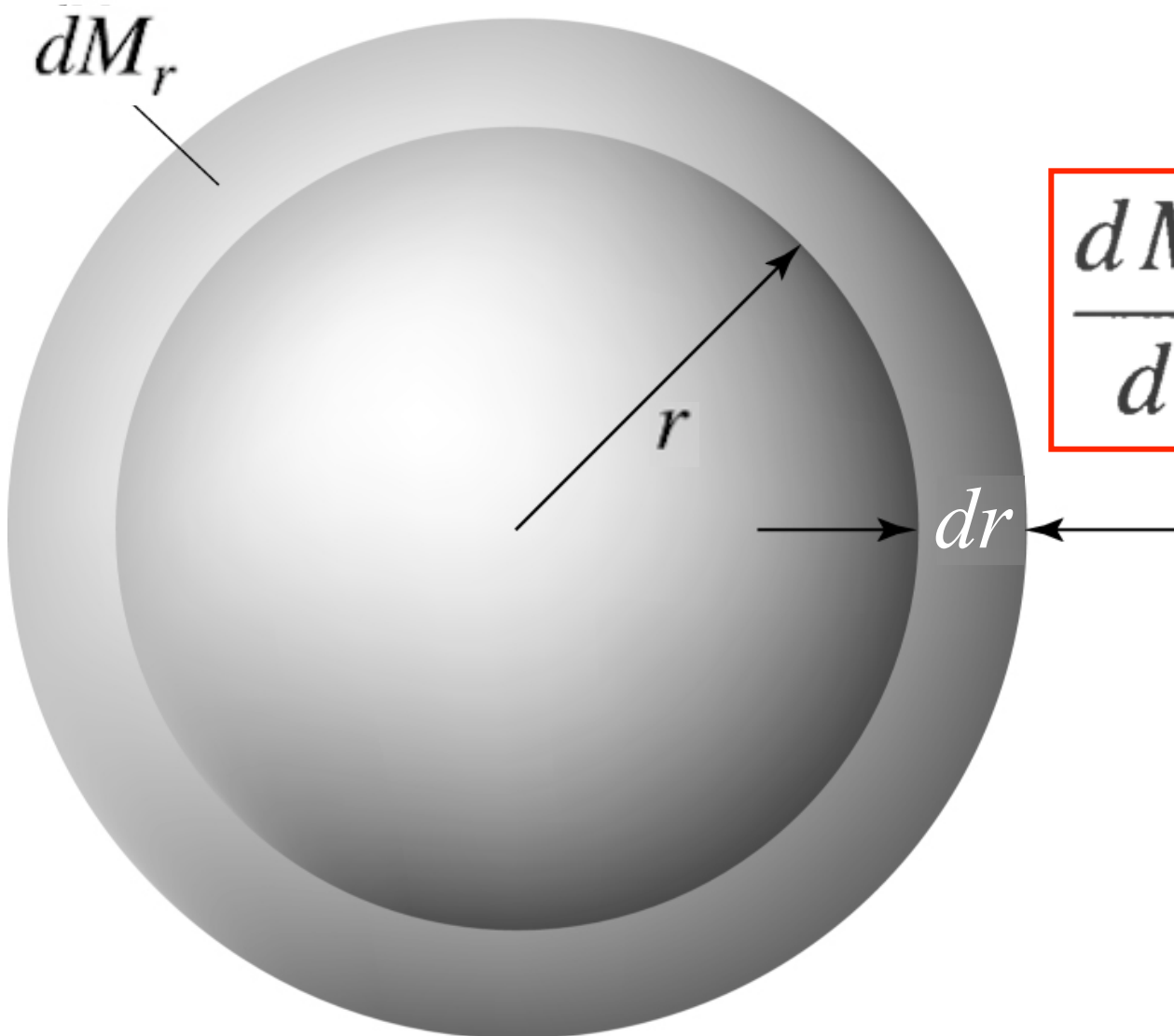
Fontes de energia.

Transporte de energia: radiação

AGA 0293, Astrofísica Estelar

Jorge Meléndez

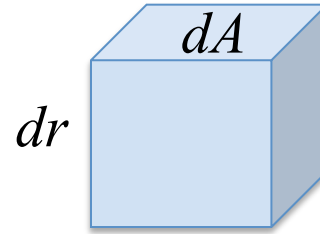
# Equação de Conservação de Massa



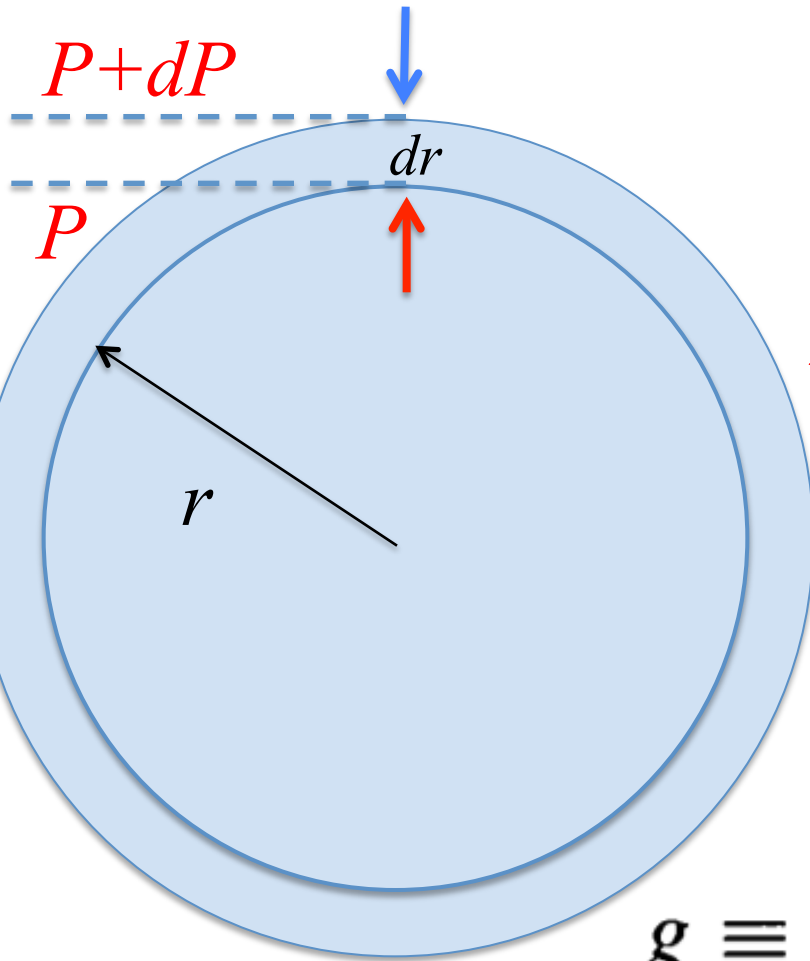
$$dM_r = \rho dV$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

# Equilíbrio hidrostático



$$dV = dA dr$$
$$dM = \rho dV$$



$$F_{\text{grav}} = -dM \times g = -\rho dV g$$

$$F_{\text{pressão}} = dP \times dA = dP dV/dr$$

$$\rightarrow dP dV/dr = -\rho dV g$$

$$dP/dr = -\rho g$$

$$g \equiv GM_r/r^2$$

$$\frac{dP}{dr} = -G \frac{M_r \rho}{r^2}$$

## 10.2 Equação de Estado da Pressão

$$P_g = nkT$$

$$P_g = \frac{\rho kT}{\bar{m}}$$

$$P_g = \frac{\rho kT}{\mu m_H}$$

$\bar{m}$  : massa média

$$\mu \equiv \frac{\bar{m}}{m_H}$$

$\mu$ : peso molecular médio  
 $m_H$ : massa H  
 $m_H = 1.673532499 \times 10^{-27}$  kg

Para um gás completamente neutro:

$$\bar{m}_n = \frac{\sum_j N_j m_j}{\sum_j N_j}$$

$m_j$ : massa  
do átomo  $j$   
 $N_j$ : número  
de átomos  $j$

$$\mu_n = \frac{\sum_j N_j A_j}{\sum_j N_j}$$

$$A_j \equiv m_j / m_H$$

Para um gás  
completamente  
ionizado:

$$\mu_i \simeq \frac{\sum_j N_j A_j}{\sum_j N_j (1 + z_j)}$$

$1 + z_j$  : número de núcleos e elétrons livres

Em função das frações de massa  $X_j$ :

$$\frac{1}{\mu_n} = \sum_j \frac{1}{A_j} X_j$$

Para um gás completamente neutro:

$$A_j \equiv m_j / m_H$$

$$\frac{1}{\mu_n} \simeq X + \frac{1}{4}Y + \left\langle \frac{1}{A} \right\rangle_n Z$$

$\langle 1/A \rangle_n$  é a média ponderada de todos os elementos mais pesados que He. Para abundâncias solares,  $\langle 1/A \rangle_n \sim 1/15.5$

Para um gás completamente ionizado:

$$\frac{1}{\mu_i} = \sum_j \frac{1 + z_j}{A_j} X_j$$

Incluindo explicitamente hidrogênio (X) e He (Y):

$$\frac{1}{\mu_i} \simeq 2X + \frac{3}{4}Y + \left\langle \frac{1 + z}{A} \right\rangle_i Z$$

If we assume that  $X = 0.70$ ,  $Y = 0.28$ , and  $Z = 0.02$ , a composition typical of younger stars, then with these expressions for the mean molecular weight,  $\mu_n = 1.30$  and  $\mu_i = 0.62$ .

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

A energia cinética média de uma partícula é  $\frac{1}{2}kT$  por grau de liberdade.

Pressão de radiação: 
$$P_{\text{rad}} = \frac{1}{3}aT^4$$

Pressão total: 
$$P_t = \frac{\rho kT}{\mu m_H} + \frac{1}{3}aT^4$$

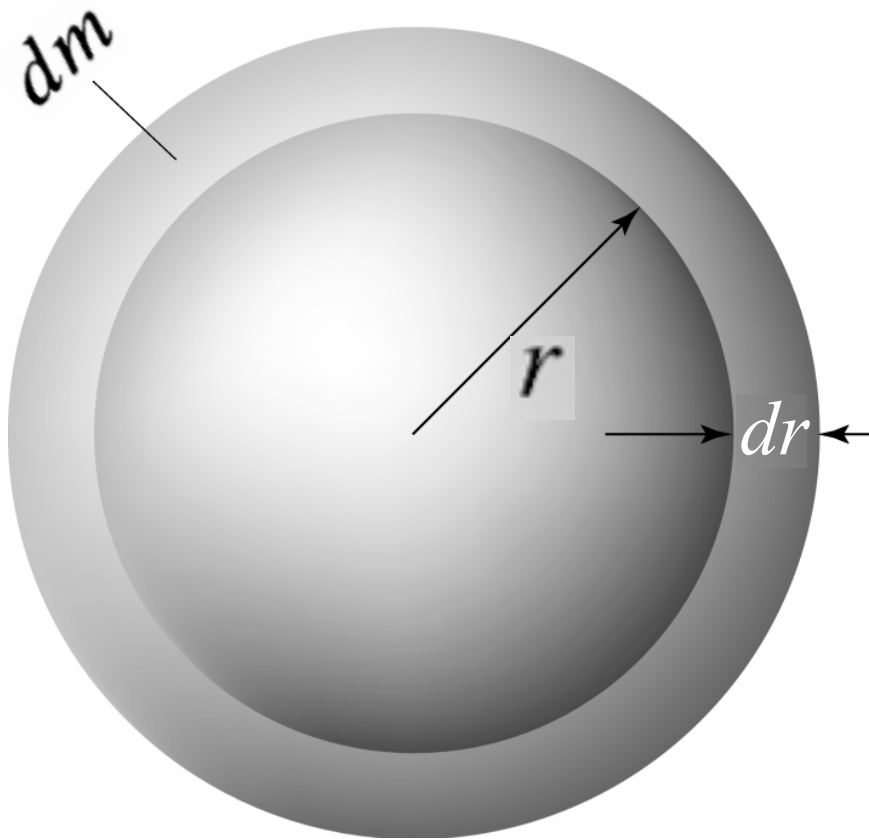
No centro do Sol,  $P_c \sim 2 \times 10^{16} \text{ N m}^{-2}$ ,  $T_c \sim 1,5 \times 10^7 \text{ K}$

# 10.3 Fontes de energia estelar

Energia potencial gravitacional:

$$U = -G \frac{Mm}{r}$$

$$dm = 4\pi r^2 \rho dr$$



$$dU_{g,i} = -G \frac{M_r dm_i}{r}$$

$$dU_g = -G \frac{M_r 4\pi r^2 \rho}{r} dr$$

$$U_g = -4\pi G \int_0^R M_r \rho r dr$$

R: raio da estrela



$$U_g = -4\pi G \int_0^R M_r \rho r dr$$

Para integrar,  
precisamos saber  
 $M_r$  ou  $\rho_r$

Em 1a aproximação, considerar uma densidade média:

$$\rho \sim \bar{\rho} = \frac{M}{\frac{4}{3}\pi R^3} \rightarrow M_r \sim \frac{4}{3}\pi r^3 \bar{\rho}$$

$$\rightarrow U_g \sim -\frac{16\pi^2}{15} G \bar{\rho}^2 R^5 \sim -\frac{3}{5} \frac{GM^2}{R}$$

Pelo virial ( $E_{\text{total}} = \frac{1}{2} E_{\text{pot}}$ ),  
→ energia disponível:

$$E \sim -\frac{3}{10} \frac{GM^2}{R}$$

Exemplo 10.3.1. Se o Sol foi inicialmente muito maior do que ele é hoje, quanta energia teria liberado pelo seu colapso?

$$E \sim -\frac{3}{10} \frac{GM^2}{R}$$

Assumindo  $R_i \gg R_{\text{sol}}$ :

$$\Delta E_g = -(E_f - E_i) \simeq -E_f \simeq \frac{3}{10} \frac{GM_{\odot}^2}{R_{\odot}} \simeq 1.1 \times 10^{41} \text{ J.}$$

Supondo luminosidade aprox. constante para o Sol:

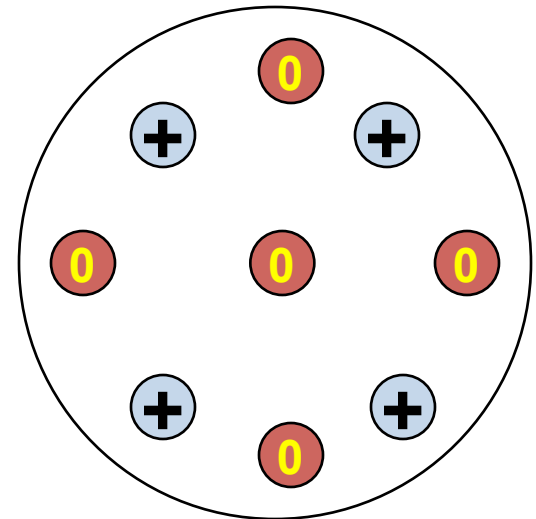
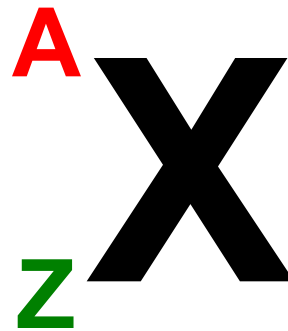
$$t_{\text{KH}} = \frac{\Delta E_g}{L_{\odot}} \sim 10^7 \text{ anos} \quad t_{\text{KH}} \text{ é a escala de tempo Kelvin-Helmholtz}$$

$t_{\text{KH}} \ll$  idade do Sistema Solar

# Representação de elemento químico X

$$A = \text{Número de núcleons} = Z + \text{Nêutrons}$$

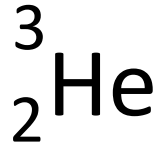
Isótopos de um elemento químico: mesmo número de prótons (Z) mas diferentes números de nêutrons  
→ diferente A.



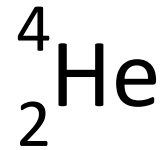
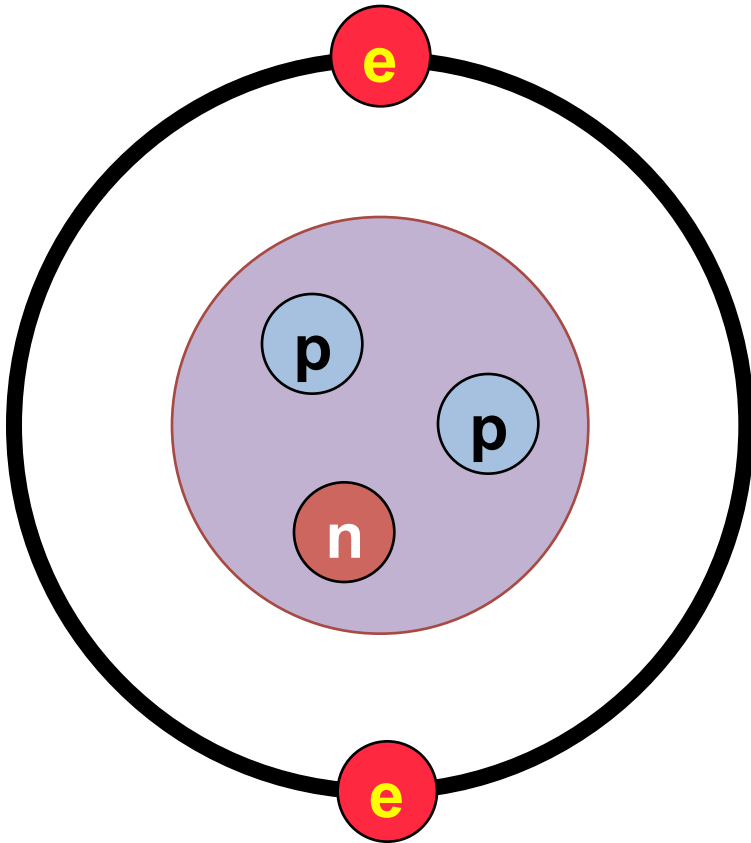
**Z : Número de Prótons**

**A: Número de massa**

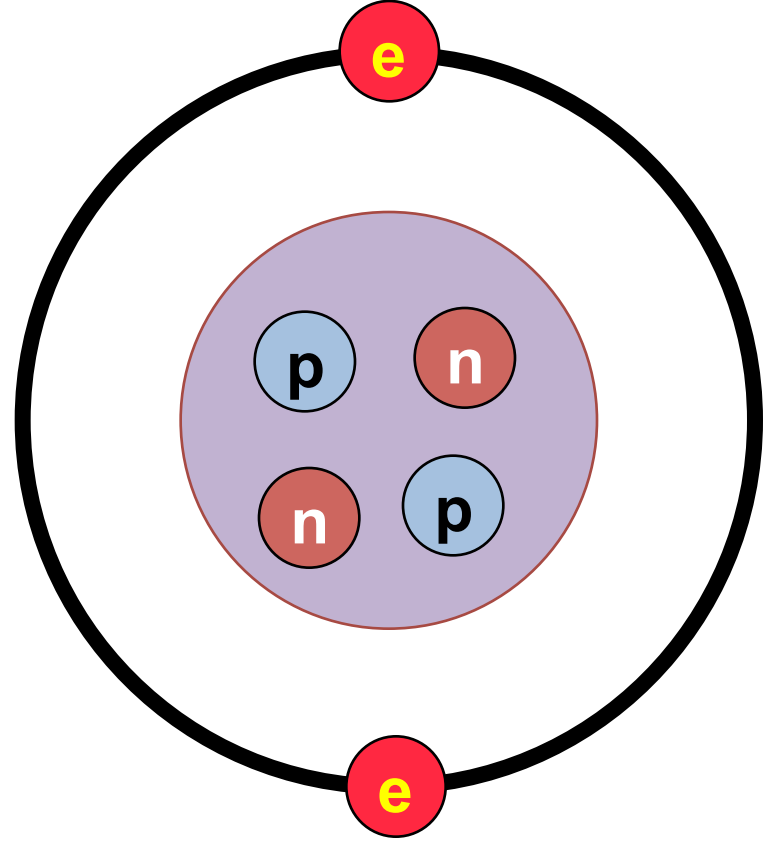
# Isótopos do Hélio



Hélio 3 (tralphium)



Hélio 4 (núcleo=partícula  $\alpha$ )



# Massa atômica

$$m_p = 1.67262158 \times 10^{-27} \text{ kg} = 1.00727646688 \text{ u}$$

$$m_n = 1.67492716 \times 10^{-27} \text{ kg} = 1.00866491578 \text{ u}$$

$$m_e = 9.10938188 \times 10^{-31} \text{ kg} = 0.0005485799110 \text{ u}$$

u: unidade de massa atômica (1/12 massa do carbono-12)

$$1 \text{ u} = 1.66053873 \times 10^{-27} \text{ kg}$$

$$E = mc^2 \rightarrow 1 \text{ u} = 931,494013 \text{ MeV} / c^2$$

Geralmente são usadas massas em repouso  $\rightarrow c^2$  é ignorado

# Fusão nuclear



Massa de 4H:  
4,03130013 u

Massa de He:  
4,02603 u

$\Delta m$ : 0,028697 u (0,7%)

$$E_b = \Delta mc^2 = 26.731 \text{ MeV}$$



b: binding

Exemplo 10.3.1. É a energia nuclear suficiente para manter o Sol brilhando durante seu tempo de vida?

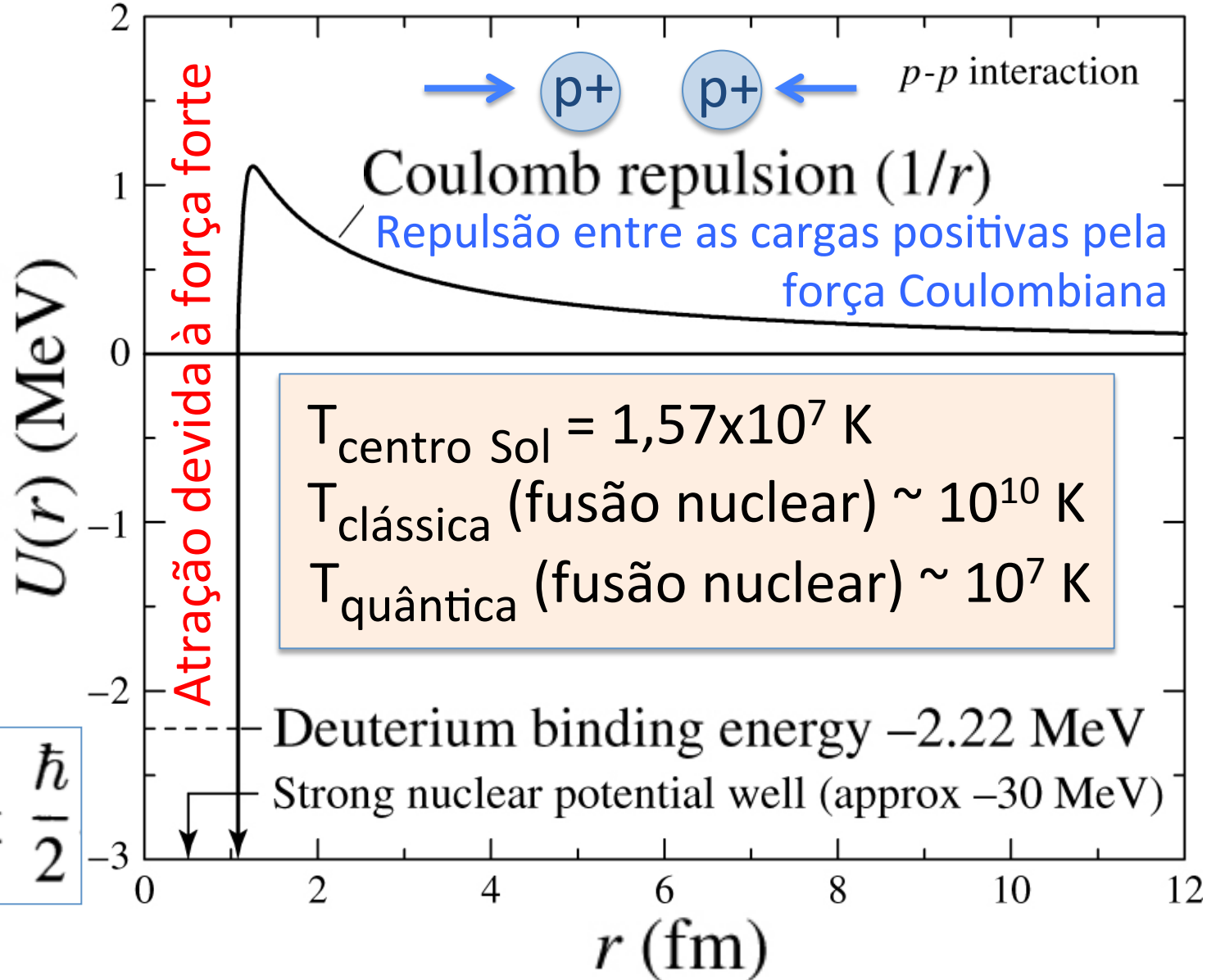
Por simplicidade, supor que o Sol é 100% hidrogênio e que somente o 10% da massa mais interna do Sol é quente o suficiente para a fusão nuclear

$$E_{\text{nuclear}} = 0.1 \times 0.007 \times M_{\odot} c^2 = 1.3 \times 10^{44} \text{ J}$$

↑  
 $\Delta m = 0,7\%$

**Escala de Tempo Nuclear:**  $t_{\text{nuclear}} = \frac{E_{\text{nuclear}}}{L_{\odot}}$   
 $\sim 10^{10}$  anos

# Tunelamento quântico (efeito túnel)



$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

$1 \text{ fm} = 10^{-15} \text{ m}$



$\mu_m$ : massa reduzida

$$\frac{1}{2} \mu_m \overline{v^2} = \frac{3}{2} k T_{\text{classical}} = \frac{1}{4\pi\epsilon_0} \frac{Z_1 Z_2 e^2}{r}$$

$$T_{\text{classical}} = \frac{Z_1 Z_2 e^2}{6\pi\epsilon_0 k r}$$

$Z_1 = Z_2 = 1$ ; raio núcleo  $\sim 1 \text{ fm} = 10^{-15} \text{ m}$

$$T_{\text{clássica}} \sim 10^{10} \text{ K}$$

Uma estimativa grosseira de T para efeito túnel:

$$\frac{1}{2} \mu_m v^2 = \frac{p^2}{2\mu_m}$$

$$\frac{1}{4\pi\epsilon_0} \frac{Z_1 Z_2 e^2}{\lambda} = \frac{p^2}{2\mu_m} = \frac{(h/\lambda)^2}{2\mu_m}$$

$$E = pc = hc/\lambda \\ \rightarrow \lambda = h/p$$

Resolver  $\lambda$  e substituir em  $T_{\text{classical}}$ :

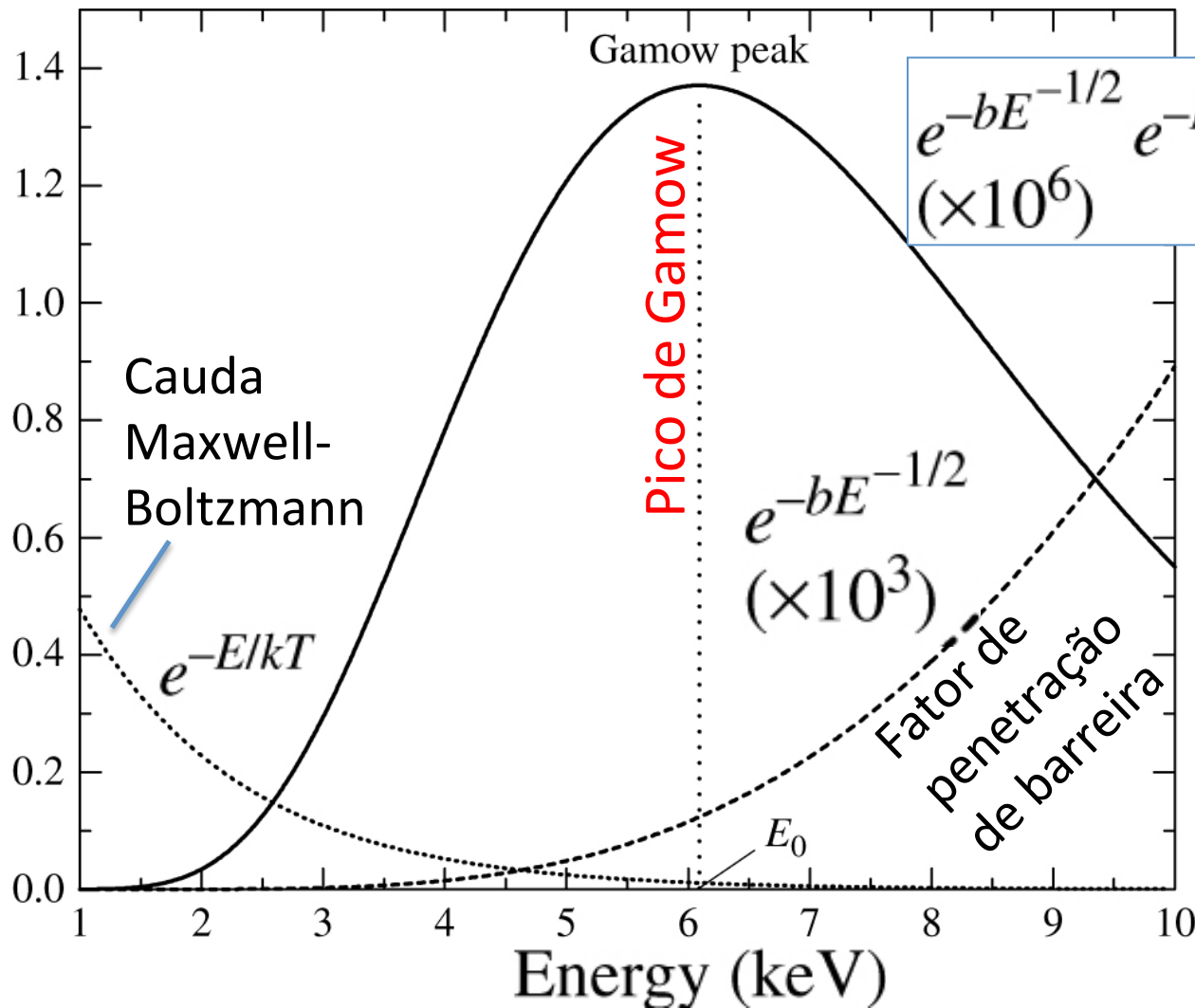
$$T_{\text{quantum}} = \frac{Z_1^2 Z_2^2 e^4 \mu_m}{12\pi^2 \epsilon_0^2 h^2 k} \sim 10^7 \text{ K}$$

Supor que p+ deve estar dentro de  $\lambda_{\text{Broglie}}$

$$\mu_m = m_p/2 \text{ and } Z_1 = Z_2 = 1$$

Número de reações nucleares (por unidade de volume e tempo):

$$r_{ix} = \left( \frac{2}{kT} \right)^{3/2} \frac{n_i n_x}{(\mu_m \pi)^{1/2}} \int_0^{\infty} S(E) e^{-bE^{-1/2}} e^{-E/kT} dE$$



$$b \equiv \frac{\pi \mu_m^{1/2} Z_1 Z_2 e^2}{2^{1/2} \epsilon_0 h}$$

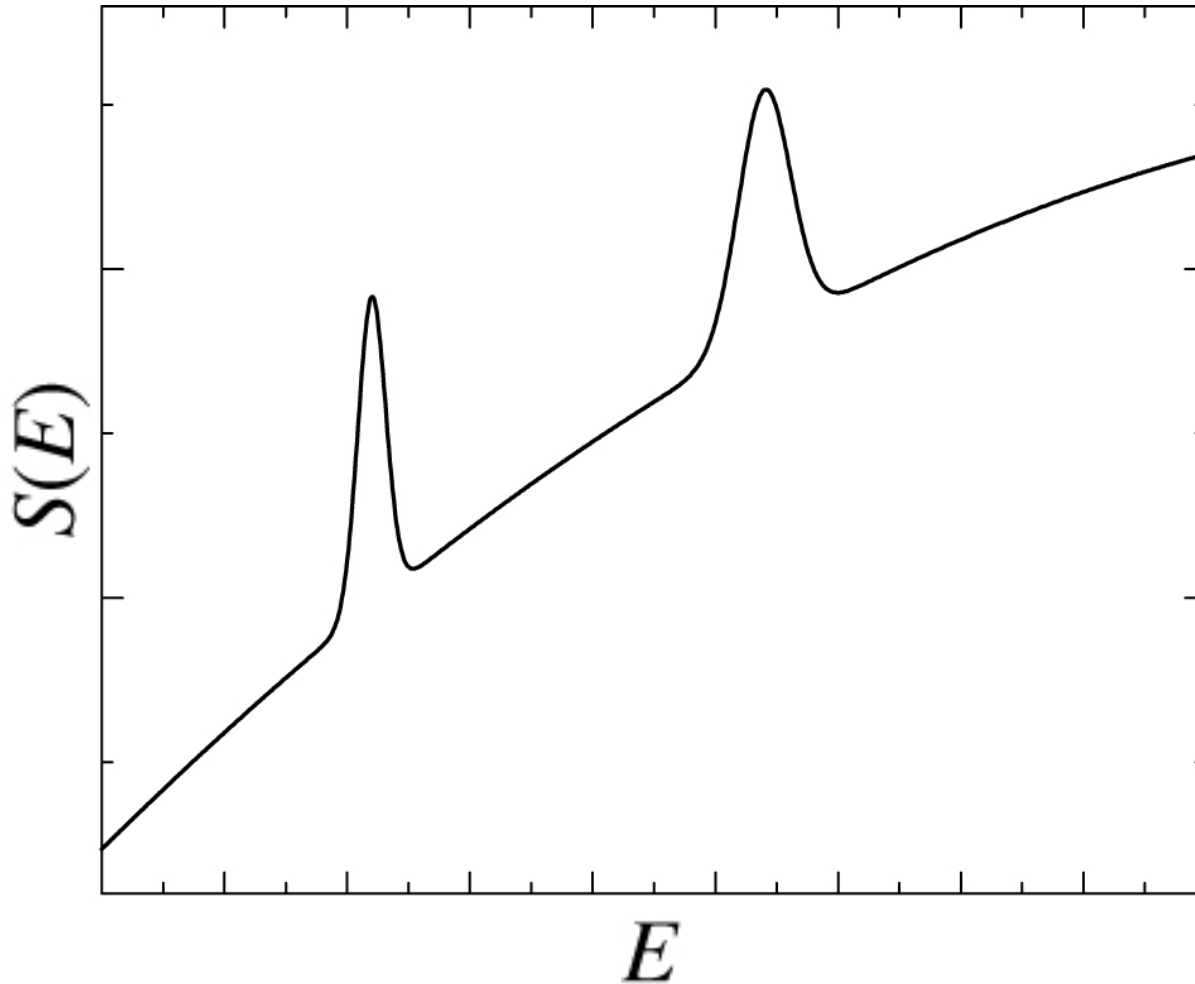
$i$ : partícula incidente  
 $x$ : partícula alvo  
 $S(E)$ : função de E

**Pico de Gamow:**

$$E_0 = \left( \frac{bkT}{2} \right)^{2/3}$$

$$K = E = \mu_m v^2 / 2$$

## Ressonância



Níveis de energia  
dentro do núcleo  
→ ressonâncias  
→ maior chance  
de reação nuclear

**FIGURE 10.7** A hypothetical example of the effect of resonance on  $S(E)$ .

# Blindagem eletrônica (*electron screening*)

$$U_{\text{eff}} = \frac{1}{4\pi\epsilon_0} \frac{Z_1 Z_2 e^2}{r} + U_s(r)$$

$$U_s(r) < 0$$

Nuvem de e- devido à ionização  
→ reduz a barreira Coulombiana

Pode aumentar as reações que produzem  
He por 10% - 50%

# Representando as taxas de reações nucleares usando leis de potência

Sem blindagem eletrônica:

$$r_{ix} \simeq r_0 X_i X_x \rho^{\alpha'} T^\beta$$

$r_0$ : constante;

$X_i, X_x$ : frações de massa das partículas;

$\alpha' \sim 2$ ;  $\beta \sim 1 - 40$

Se conhecemos a **energia liberada por reação**  $\mathcal{E}_0$

→ a energia liberada por segundo em cada quilograma:

$$\epsilon_{ix} = \left( \frac{\mathcal{E}_0}{\rho} \right) r_{ix}$$

→

$$\epsilon_{ix} = \epsilon'_0 X_i X_x \rho^\alpha T^\beta$$

Onde:  $\alpha = \alpha' - 1$

$\epsilon_{ix}$  : unidades de W kg<sup>-1</sup>

$r_{ix}$ : número total de reações por unidade de volume e tempo

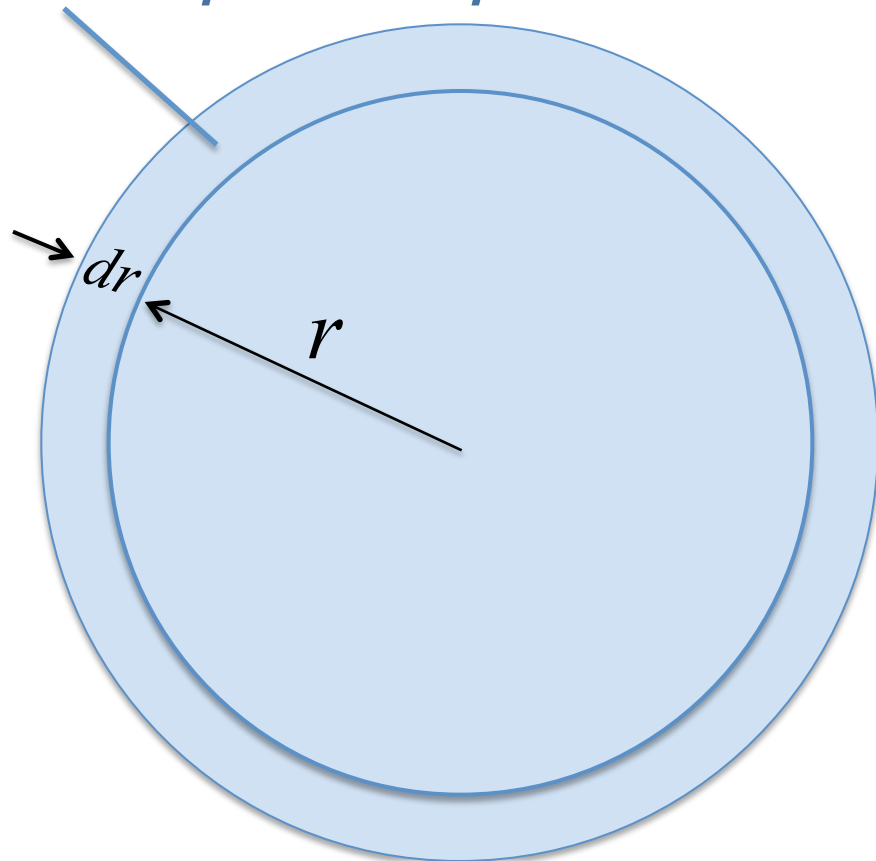
# A equação do gradiente de luminosidade

$\epsilon$  : energia total liberada por todas as reações nucleares por quilograma ( $\text{W kg}^{-1}$ )

→ uma massa  $dm$  contribui para a luminosidade em:

$$dm = \rho dV = \rho 4\pi r^2 dr$$

$$dL = \epsilon dm$$



$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$

$L_r$ : luminosidade interior a  $r$

# Nucleossíntese estelar e Leis de conservação

Reações nucleares não acontecem arbitrariamente.

Conservação:

- Carga
- número de núcleons ( $p + n$ )
- número de léptons ( $e, \nu_e$  e antipartículas)

## Três gerações de matéria (Férmions)

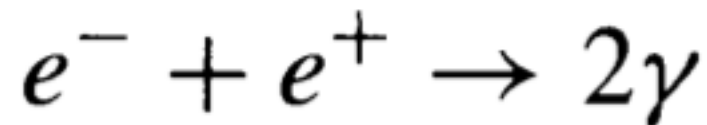
	I	II	III	<b>Bosons</b>
massa →	2.4 MeV	1.27 GeV	171.2 GeV	0
carga →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin →	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
nome →	<b>u</b> up	<b>c</b> charme	<b>t</b> top	<b><math>\gamma</math></b> fóton
<b>Quarks</b>	4.8 MeV	104 MeV	4.2 GeV	0
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	<b>d</b> down	<b>s</b> estranho	<b>b</b> bottom	<b>g</b> glúon
<b>Léptons</b>	<2.2 eV	<0.17 MeV	<15.5 MeV	91.2 GeV
	0	0	0	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	<b><math>\nu_e</math></b> elétron neutrino	<b><math>\nu_\mu</math></b> múon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b>Z<sup>0</sup></b> força fraca
	0.511 MeV	105.7 MeV	1.777 GeV	80.4 GeV
	-1	-1	-1	$\pm 1$
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	<b>e</b> elétron	<b><math>\mu</math></b> múon	<b><math>\tau</math></b> tau	<b>W<sup>±</sup></b> força fraca

**Bosons (Forças)**

# Antimatéria

- Mistura da matéria e antimatéria → aniquilamento.
- Colisão de uma partícula e antipartícula → energia

Por exemplo, colisão de elétron e antielétron (pósitron) resulta em fótons de alta energia (radiação gama  $\gamma$ ):



São necessários 2 fótons para a conservação da quantidade de movimento (*momento*)



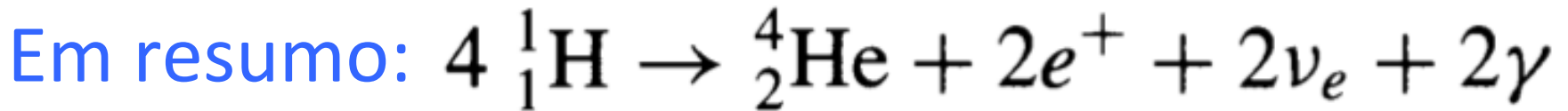
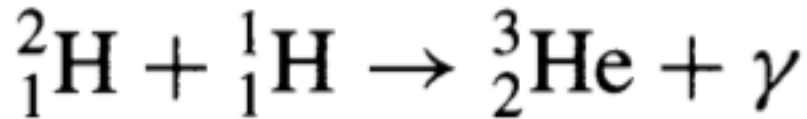
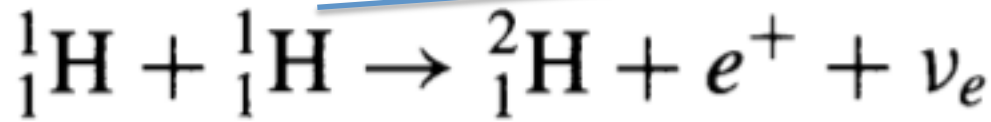
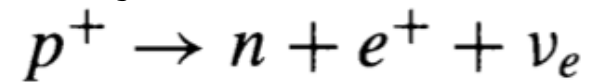
**A**  
**Z** **X**

A: Número de massa (p + n)

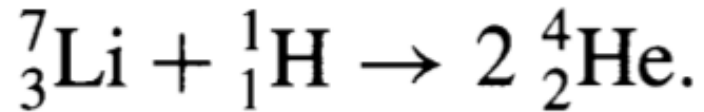
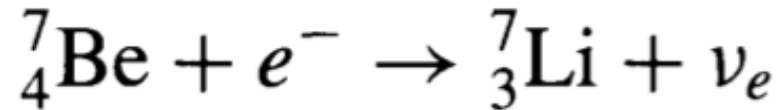
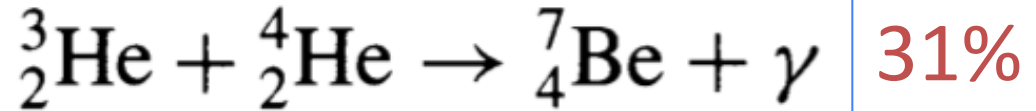
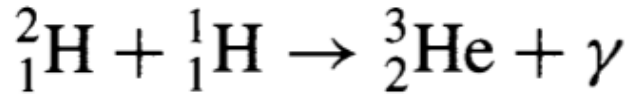
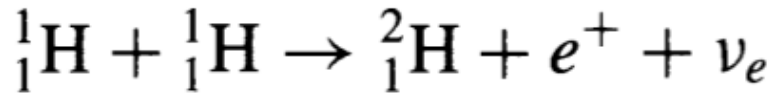
Z: Número de p (carga positiva)

## Cadeia próton-próton, PP-I

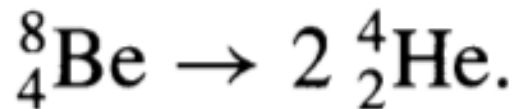
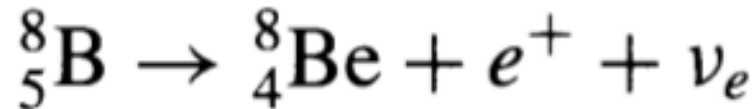
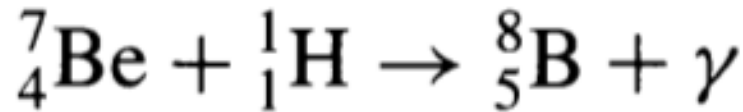
Força fraca



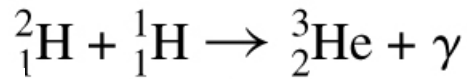
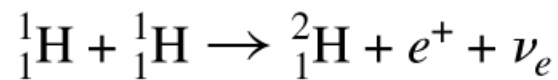
# Cadeia próton-próton, PP-II



## PP-III

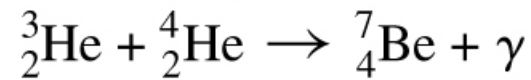
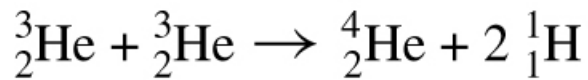


0,3%



**69%**

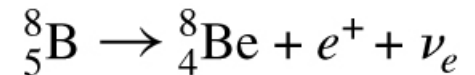
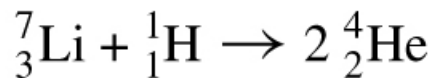
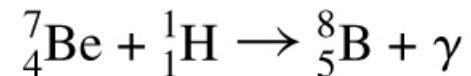
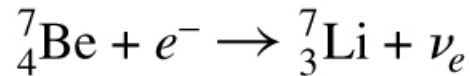
**31%**



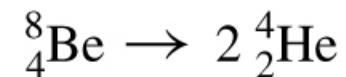
**PP-I**

**99,7%**

**0,3%**



**PP-II**



**PP-III**

# Produção de energia por toda a cadeia PP

$$\epsilon_{pp} = 0.241 \rho X^2 f_{pp} \psi_{pp} C_{pp} T_6^{-2/3} e^{-33.80 T_6^{-1/3}} \text{ W kg}^{-1}$$

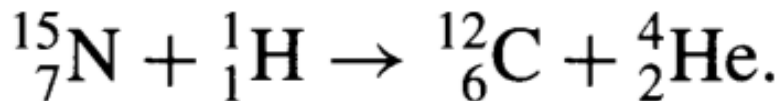
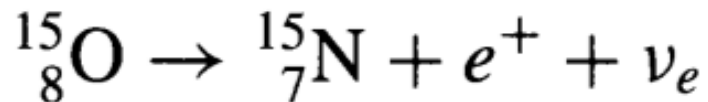
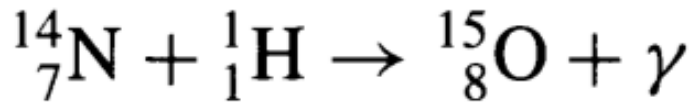
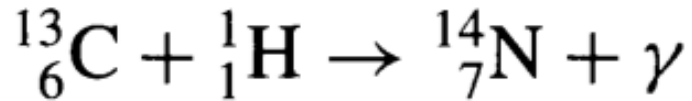
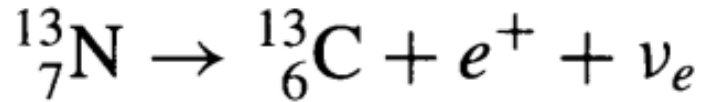
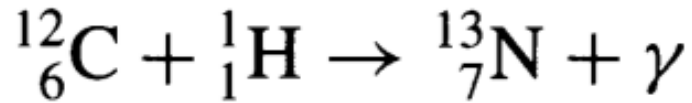
Termos de correção  $\sim 1$        $T_6 = T/10^6 \text{ K}$

Escrevendo como lei de potência para  $T = 1,5 \times 10^7 \text{ K}$ :

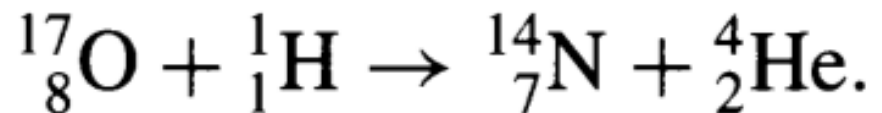
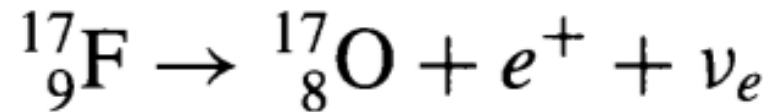
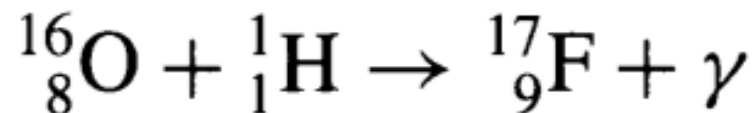
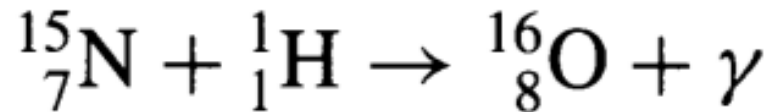
$$\epsilon_{pp} \simeq \epsilon'_{0,pp} \rho X^2 f_{pp} \psi_{pp} C_{pp} T_6^4$$

Onde:  $\epsilon'_{0,pp} = 1.08 \times 10^{-12} \text{ W m}^3 \text{ kg}^{-2}$

## O Ciclo CNO



0,04%



# Produção de energia pelo ciclo CNO

$$\epsilon_{\text{CNO}} = 8.67 \times 10^{20} \rho X X_{\text{CNO}} C_{\text{CNO}} T_6^{-2/3} e^{-152.28 T_6^{-1/3}} \text{ W kg}^{-1}$$

Fração de massa  
total CNO

Termo de  
correção

$$T_6 = T/10^6 \text{ K}$$

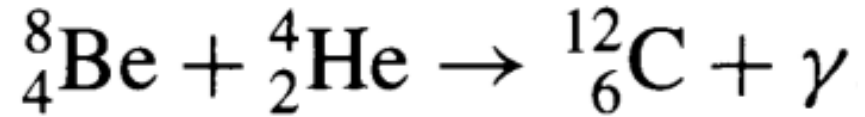
Escrevendo como lei de potência para  $T = 1,5 \times 10^7 \text{ K}$ :

$$\epsilon_{\text{CNO}} \simeq \epsilon'_{0,\text{CNO}} \rho X X_{\text{CNO}} T_6^{19.9}$$

$$\text{Onde: } \epsilon'_{0,\text{CNO}} = 8.24 \times 10^{-31} \text{ W m}^3 \text{ kg}^{-2}$$

**Dependência muito maior com a temperatura**

## Processo triplo alfa



$$\epsilon_{3\alpha} = 50.9 \rho^2 Y^3 T_8^{-3} f_{3\alpha} e^{-44.027 T_8^{-1}} \text{ W kg}^{-1}$$

$$T_8 = T/10^8 \text{ K}$$

Termo de blindagem

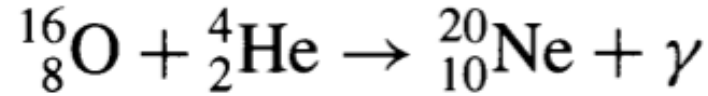
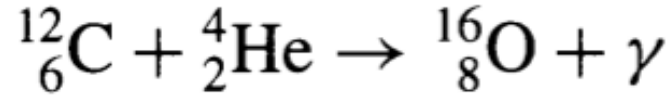
$$\epsilon_{3\alpha} \simeq \epsilon'_{0,3\alpha} \rho^2 Y^3 f_{3\alpha} T_8^{41}$$

**Ultra sensível à temperatura:**

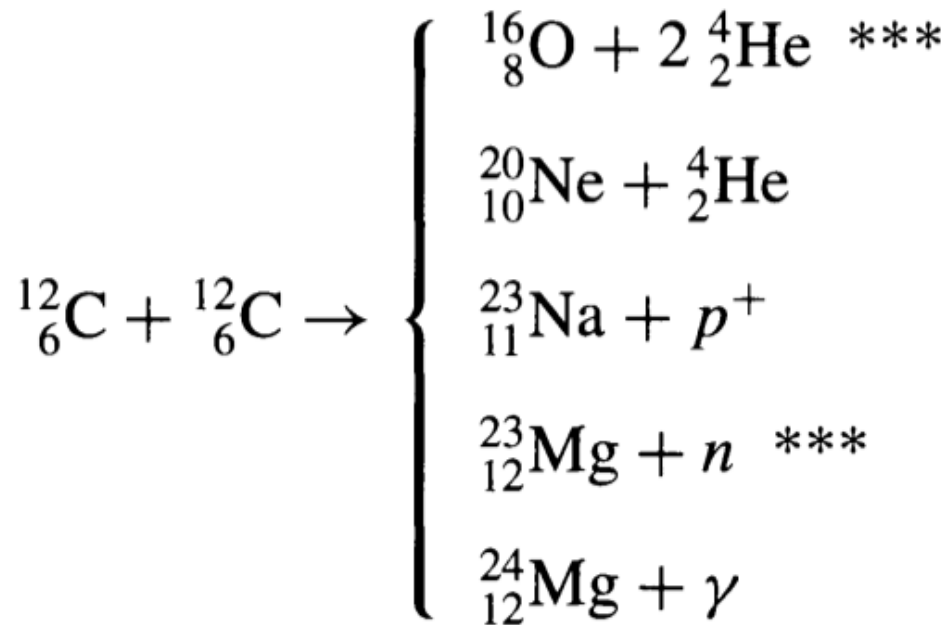
**10% em T → 50 vezes na produção de energia**

# Queima de C e O

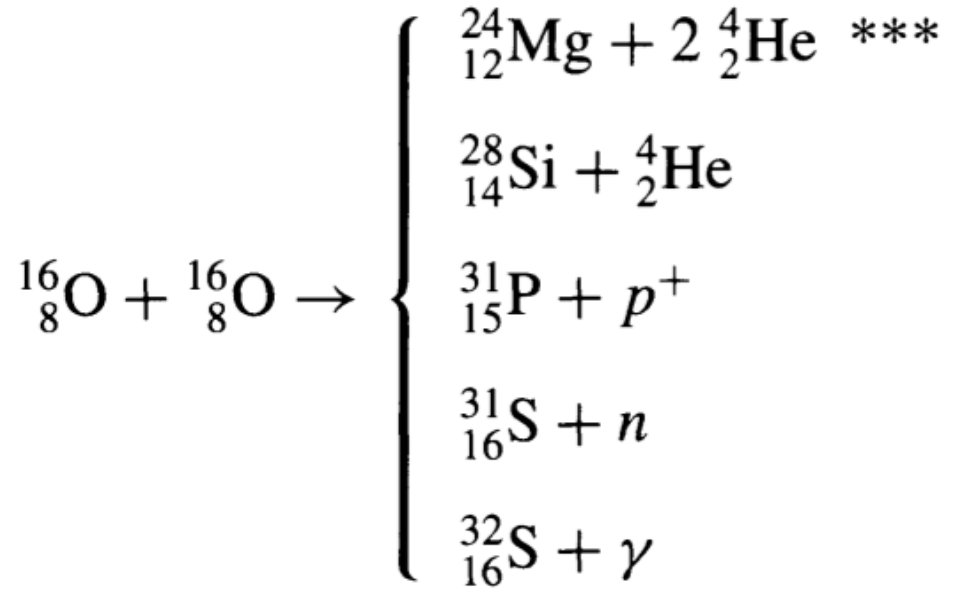
$$T \sim 10^8 \text{ K}$$



$$T \sim 6 \times 10^8 \text{ K}$$



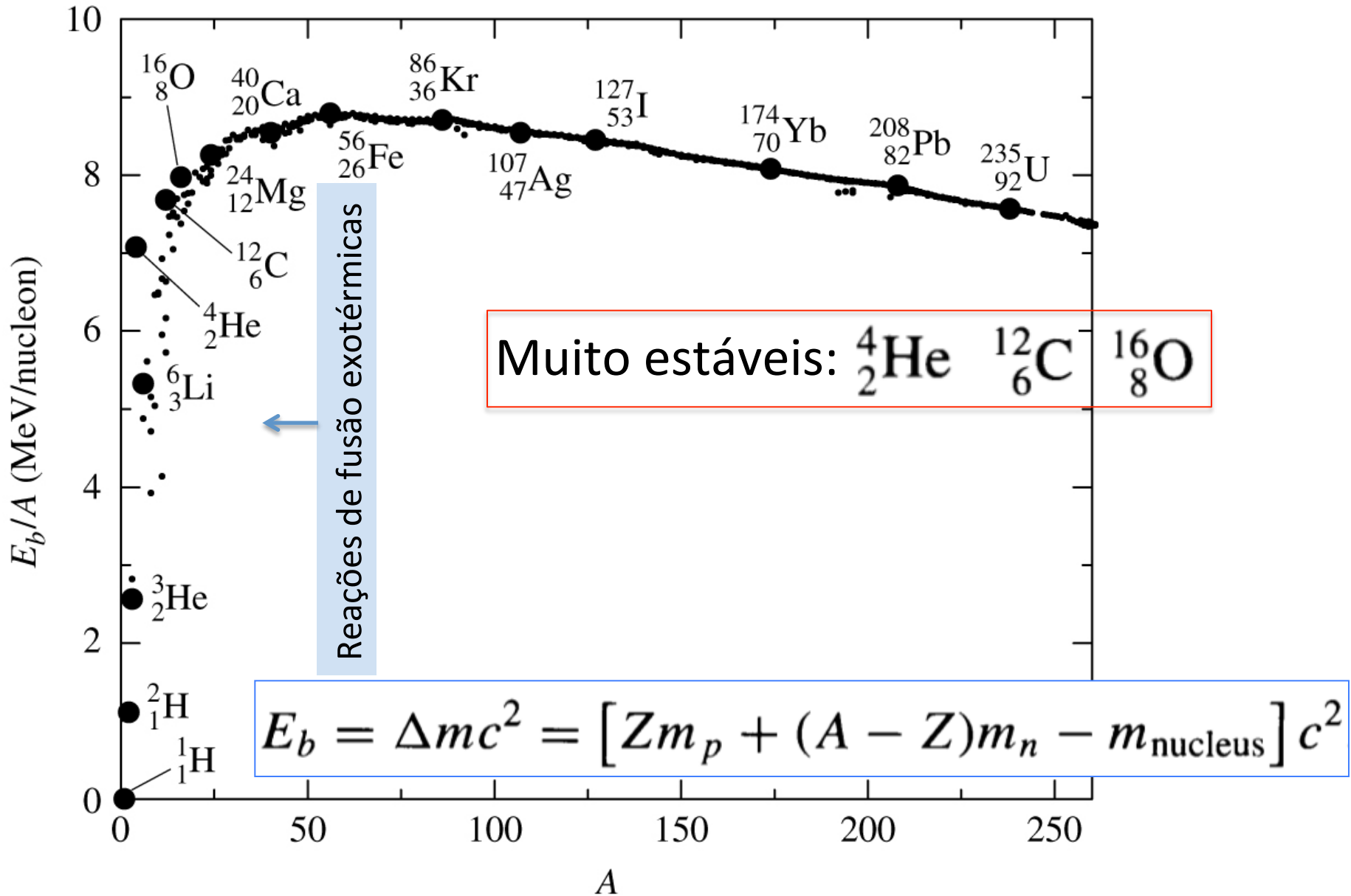
$$T \sim 10^9 \text{ K}$$



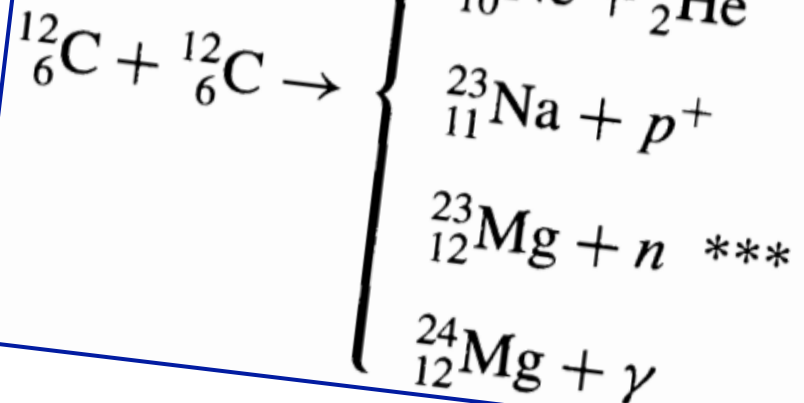
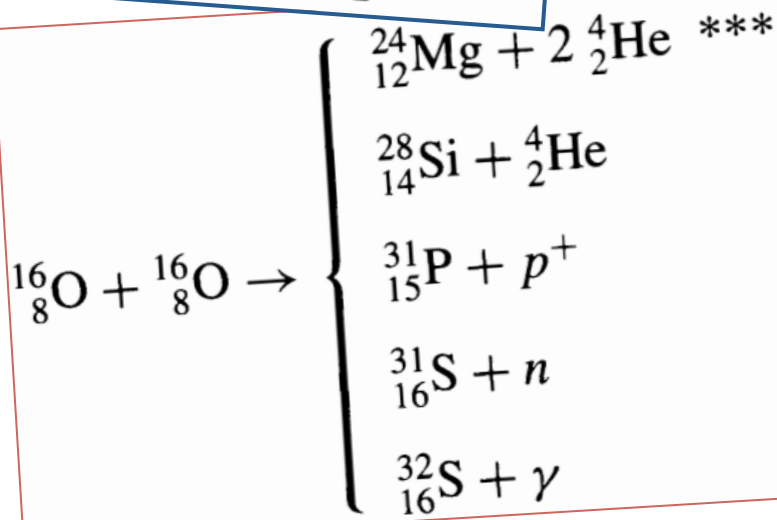
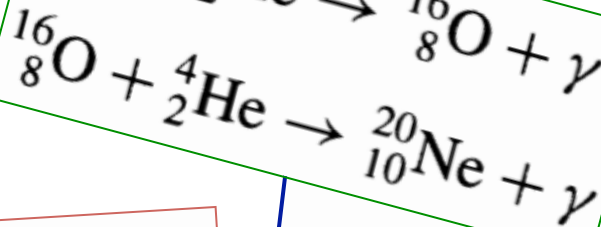
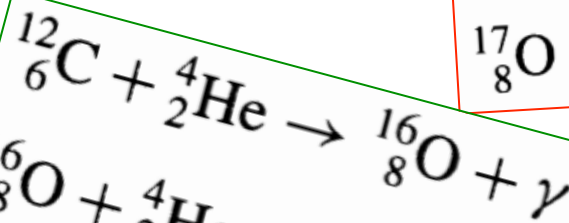
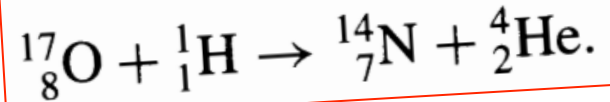
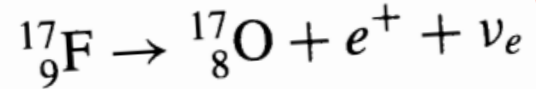
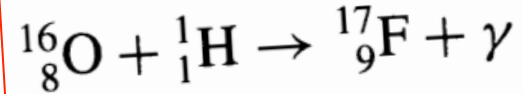
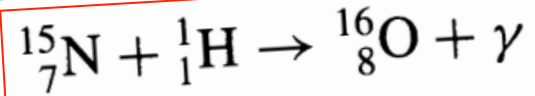
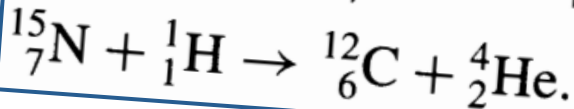
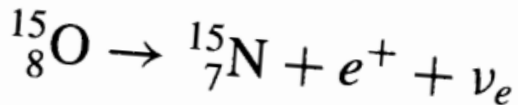
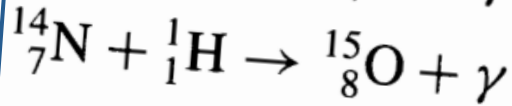
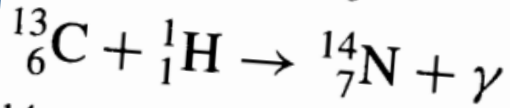
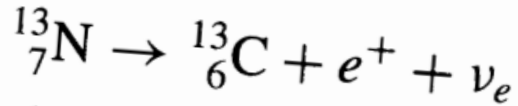
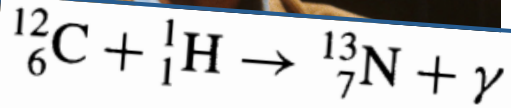
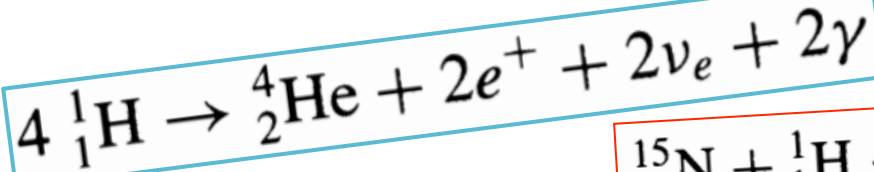
\*\*\*: endotérmica



# Energia de Ligação por Núcleon: $E_b/A$

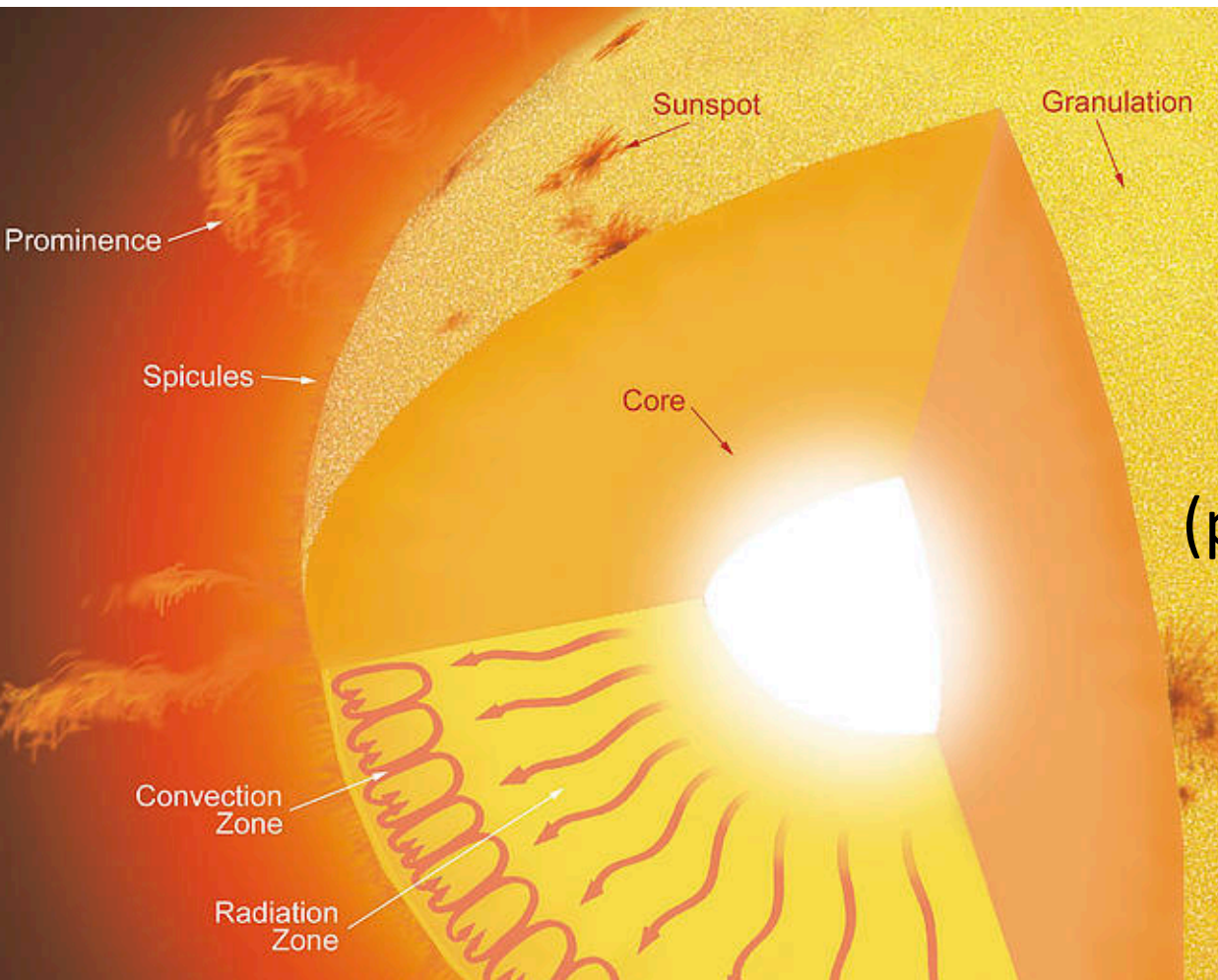


# Somos poeira de estrelas



# Transporte de energia

- Já temos 3 equações básicas ( $P_r$ ,  $M_r$ ,  $L_r$ ).
- Falta equação do transporte de energia.



- 3 mecanismos:
  - Radiação
  - Convecção
  - Condução(pouco importante na sequência principal)

# O Gradiente radiativo de temperatura

$$P_{\text{rad}} = \frac{1}{3} a T^4 \rightarrow \frac{dP_{\text{rad}}}{dr} = \frac{4}{3} a T^3 \frac{dT}{dr} \left. \vphantom{\frac{dP_{\text{rad}}}{dr}} \right\} \frac{dT}{dr} = -\frac{3}{4ac} \frac{\bar{\kappa} \rho}{T^3} F_{\text{rad}}$$

Cap. 9:  $\frac{dP_{\text{rad}}}{dr} = -\frac{\bar{\kappa} \rho}{c} F_{\text{rad}}$

Lembrando:

$$F_{\text{rad}} = \frac{L_r}{4\pi r^2} \rightarrow \frac{dT}{dr} = -\frac{3}{4ac} \frac{\bar{\kappa} \rho}{T^3} \frac{L_r}{4\pi r^2}$$

Se a opacidade aumenta ou a temperatura diminui

→ maior gradiente de T para transportar a luminosidade

# Usando Astrofísica Estelar e Twitter para divulgar Astronomia ao público



A screenshot of a Twitter profile card for Jorge Melendez. The header image shows a row of telescope domes on a hill at dusk. The profile picture is a headshot of a man with grey hair. The name 'Jorge Melendez' is in bold black text, with the handle '@DrJorgeMelendez' below it. At the bottom, three statistics are listed: 'TWEETS' with a value of 4.732, 'SEGUINDO' with a value of 180, and 'SEGUIDORES' with a value of 1.052.

TWEETS	SEGUINDO	SEGUIDORES
4.732	180	1.052

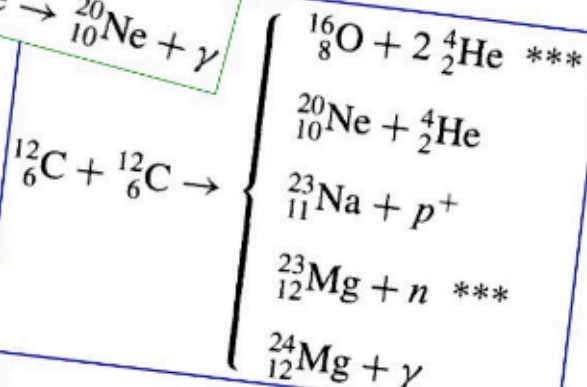
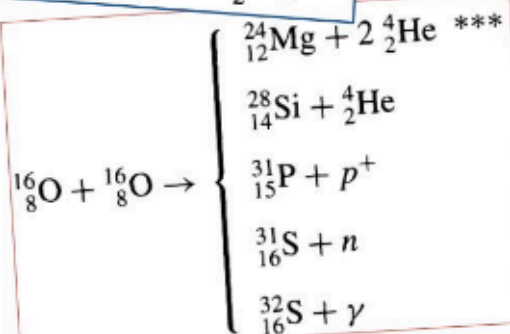
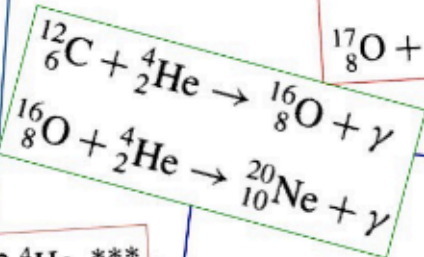
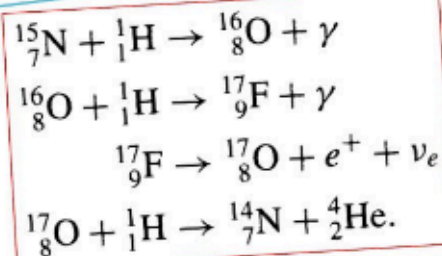
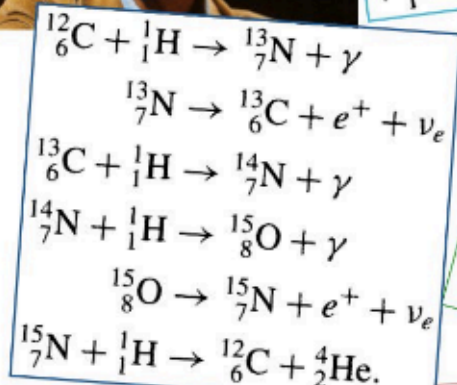
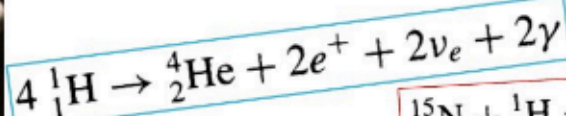


**Jorge Melendez @DrJorgeMelendez** · 1 de mai

Preparando a minha aula sobre a Formação dos Elementos Químicos no interior das estrelas, me lembrei do Carl Sagan: somos poeira de estrelas



## Somos poeira de estrelas



Impressões

3.785

Total de engajamentos:

354

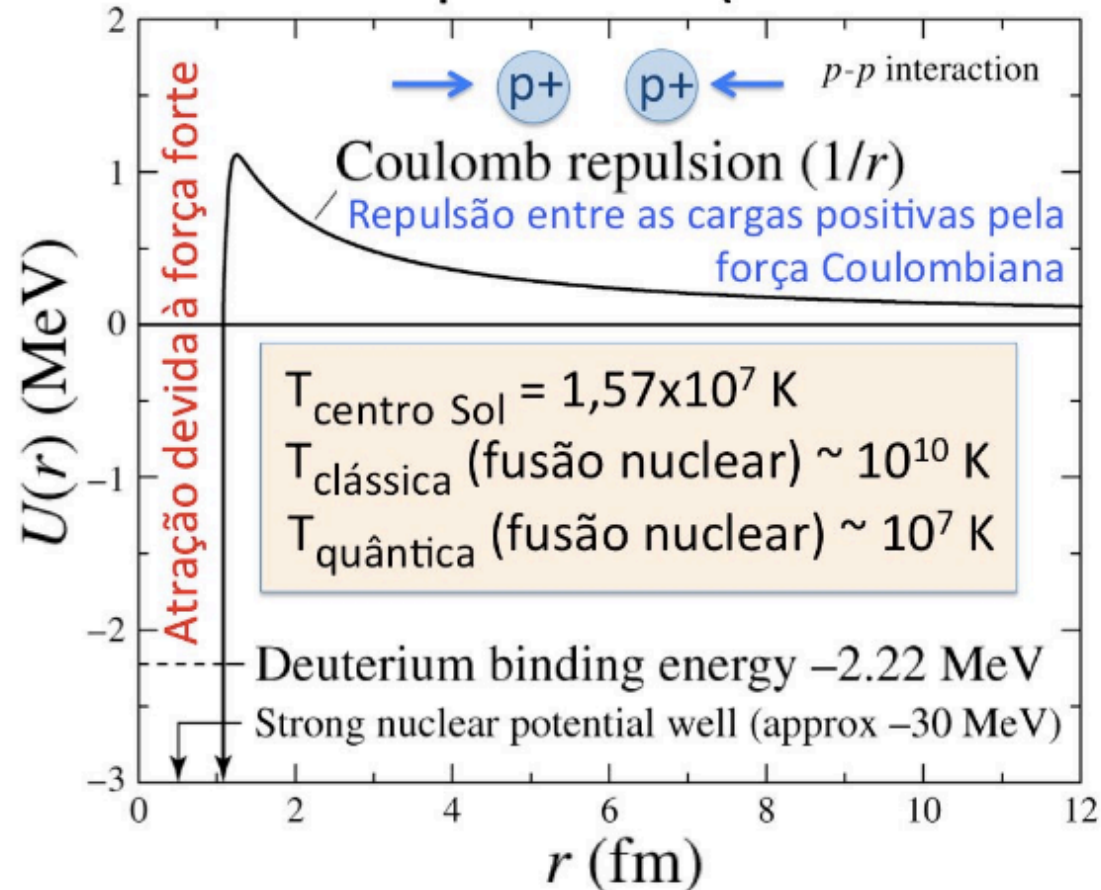




Jorge Melendez @DrJorgeMelendez · 1 de mai

O milagre da fusão nuclear no Sol: impossível pela física clássica, porem segundo a física quântica a barreira é superada por efeito túnel

## Tunelamento quântico (efeito túnel)



Impressões

2.860

Total de engajamentos:

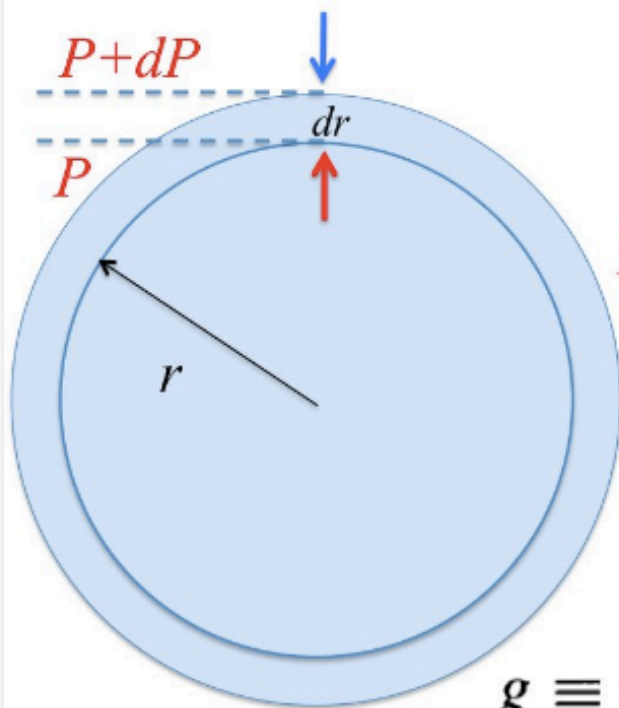
273



Jorge Melendez @DrJorgeMelendez · 30 de abr

O interior de estrelas como o Sol se encontra em equilíbrio: a força gravitacional em direção ao centro e a força de pressão para fora

## Equilíbrio hidrostático



$$F_{\text{grav}} = -dM \times g = -\rho dV g$$

$$F_{\text{pressão}} = dP \times dA = dP dV/dr$$

$$\rightarrow dP dV/dr = -\rho dV g$$

$$dP/dr = -\rho g$$

$$g \equiv GM_r/r^2 \quad \frac{dP}{dr} = -G \frac{M_r \rho}{r^2}$$

Impressões

2.594

Total de engajamentos:

324





Jorge Melendez @DrJorgeMelendez · 30 de abr

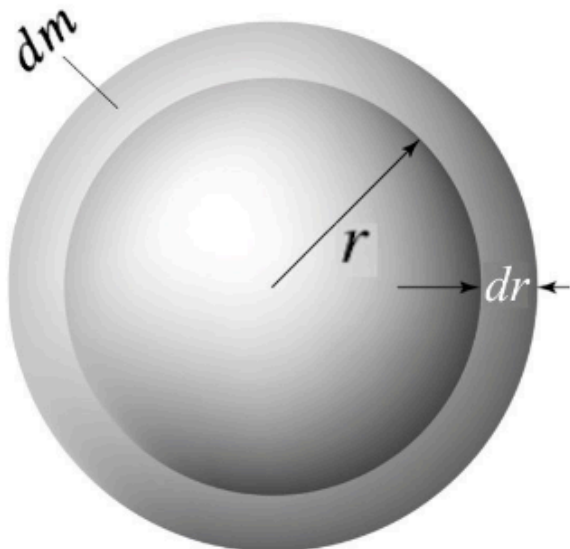
Para calcular a energia potencial gravitacional de uma estrela, podemos dividi-la em camadas esféricas e integrar

#AstrofisicaEstelar #USP

Energia potencial gravitacional:

$$U = -G \frac{Mm}{r}$$

$$dm = 4\pi r^2 \rho dr$$



$$dU_{g,i} = -G \frac{M_r dm_i}{r}$$

$$dU_g = -G \frac{M_r 4\pi r^2 \rho}{r} dr$$

$$U_g = -4\pi G \int_0^R M_r \rho r dr$$

Impressões

4.926

Total de engajamentos:

503