Journal of Hydrology, 106 (1989) 211-232
Elsevier Science Publishers B.V., Amsterdam — Printed in The Netherlands

[1]

A NEW PROCEDURE FOR GRIDDING ELEVATION AND STREAM LINE DATA WITH AUTOMATIC REMOVAL OF SPURIOUS PITS

M.F. HUTCHINSON*

CSIRO Division of Water Resources, G.P.O. Box 1666, Canberra, A.C.T. 2601 (Australia)

(Received March 28, 1988; accepted after revision July 1, 1988)

ABSTRACT

Hutchinson, M.F., 1989. A new procedure for gridding elevation and stream line data with automatic removal of spurious pits. J. Hydrol., 106: 211–232.

A morphological approach to the interpolation of regular grid digital elevation models (DEMs) from surface specific elevation data points and selected stream lines is described. The approach has given rise to a computationally efficient interpolation procedure which couples the minimization of a terrain specific roughness penalty with an automatic drainage enforcement algorithm. The drainage enforcement algorithm removes spurious sinks or pits yielding DEMs which may be used to advantage in hydrological process studies. The drainage enforcement algorithm has also been found to significantly increase the accuracy of DEMs interpolated from sparse, but well chosen, surface specific elevation data. Moreover, it facilitates the detection of errors in elevation data that would not be detected by more conventional statistical means and forms a sound physical basis for cartographic generalization.

INTRODUCTION

Regular grid or raster digital elevation models (DEMs) have become the basis for recent approaches to process modelling of the earth's surface. Of prime interest is the use of DEMs in hydrological modelling, as embodied in the Système Hydrologique Européen (Abbot et al., 1986a, b; Bathurst, 1986) and the Deterministic Site Model of the Braunschweig Research Group (Bork and Rohdenburg, 1986; Rohdenburg et al., 1986). At suitable levels of generalization DEMs also have a major role to play in geographic information systems (Evans, 1980; Berry, 1985; Wiltshire et al., 1986), particularly in the modelling of erosion (Knisel, 1980), in the classification of landforms and soils (Speight, 1974; Heerdegen and Beran, 1982; Pennock et al., 1987), in the integration of biophysical data with remotely sensed satellite data (Shelton and Estes, 1981; Jupp et al., 1986) and in the modelling of mesoscale and macroscale climatic phenomena (Tesche and Bergstrom, 1978; Hutchinson and Bischof, 1983;

*Present address: Centre for Resource and Environmental Studies, The Australian National University, G.P.O. Box 4, Canberra, A.C.T. 2601 (Australia)

0022-1694/89/$03.50

Weiringa, 1986). They have even been useful at quite coarse levels of generalization in detecting geological structures of significance for mineral exploration (Harrington et al., 1982).

Raster DEMs can be calculated directly from stereophoto maps when these are available (Kelly et al., 1977), or more recently from satellite imagery (Konecny et al., 1987), but there remains a significant role for the interpolation of DEMs from scattered point elevation data, perhaps accompanied by stream line data, particularly when the point data include surface specific points such as peaks, pits, saddles and selected points on stream lines and ridge lines. These data can be obtained by ground survey at minimal cost for the small catchment areas often used in hydrologic studies and can also be digitized at moderate cost for larger areas from existing topographic maps. At national or even continental scales, existing data banks which hold trigonometric points consisting of the principal peaks, as well as scattered ground control points and digitized stream line data, may be sufficient, especially if the interpolation technique is of high quality, to generate broad scale DEMs of sufficient accuracy to be useful in natural resource assessment and global climatic modelling (Trezise and Hutchinson, 1986; Hutchinson and Dowling, 1989).

This paper describes a morphological approach to the interpolation of digital terrain data which attempts to take into acount the special nature of terrain surfaces, and the surface specific points that can be used to sample terrain, as well as potential hydrological applications of the interpolated elevation grid. It has given rise to a procedure, first outlined by Hutchinson (1986), which can efficiently calculate raster DEMs with sensible drainage characteristics from large numbers of irregularly spaced elevation data points and stream line data. The principal innovation of the procedure is a drainage enforcement algorithm which automatically removes spurious sinks or pits from the fitted grid, in recognition of the observation that sinks are rare in nature (Mark, 1984; Band, 1986; Goodchild and Mark, 1987). The approach extends more conventional statistical approaches implicit in existing assessments of optimum sampling strategies for digital terrain modelling (Ayeni, 1982) and provides an alternative to an existing physically sound approach to digital terrain modelling using phenomenon-based data structures (Mark, 1979).

The drainage enforcement algorithm is coupled with an iterative finite difference interpolation technique which is based on minimizing a terrain specific, rotation invariant roughness penalty. This technique has its origins in the minimum curvature interpolation method of Briggs (1974) but is more computationally efficient. The roughness penalty has been tailored to yield good results in conjunction with the drainage enforcement algorithm while maintaining artifact-free behaviour of the fitted surface away from data points. The finite difference nature of the technique facilitates the monitoring of the surface characteristics of the fitted grid and permits the imposition of simple-ordered chain constraints. These constraints are the means by which the procedure enforces drainage and incorporates the ordered descent and breakline conditions implicit in stream line data.

The drainage enforcement algorithm eliminates one of the principal weaknesses of elevation grids produced by general purpose interpolation techniques which has limited their usefulness in hydrologic applications, particularly those which rely on the automatic calculation of channels, ridges and basin catchment areas (Peucker and Douglas, 1975; Mark, 1984; Band, 1986; Palacios-Vélez and Cuevas-Renaud, 1986). If spurious sinks are not removed from the DEM then programs which delineate basins and simulate drainage networks must take special action in order to produce acceptable results (Marks et al., 1984; Yuan and Vanderpool, 1986). However, eliminating sinks from an existing DEM is difficult. Manual methods can be time consuming and subject to individual operator error. On the other hand, existing automatic methods for digital filtering or smoothing of DEMs can remove most, but not all, spurious sinks only at the expense of simultaneously oversmoothing well defined surface features (Mark, 1984). Heerdegen and Beran (1982) have similarly noted the tendency of one interpolation technique to produce grids with nonsensical surface properties if one attempts to choose the grid spacing fine enough to capture known complexities in the data. The technique presented here in fact provides the "judicious filtering" called for by Band (1986).

Moreover, the imposed drainage condition has been found in practice to be a powerful condition which can significantly increase the accuracy, especially in terms of their drainage properties, of digital elevation models interpolated from sparse, but well chosen, surface-specific data sets. The size of such data sets can be at least an order of magnitude smaller than the number of points normally required to describe elevation using digital contours. This can minimize the expense of obtaining hydrologically sound digital elevation models in terms of the capture, correction and storage of primary elevation data.

An important feature of the method is its sensitivity to data errors. The drainage enforcement algorithm acts conservatively when attempting to remove sinks by not imposing drainage conditions which would contradict the elevation data by more than a user supplied elevation tolerance. Consequently, errors in both elevation and position of input elevation data can often be indicated by sinks in the final fitted grid and thence easily corrected. This is useful when processing large data sets which almost inevitably contain errors. The procedure routinely detects errors in this fashion which are small enough to remain undetected as outliers by conventional statistical methods.

The interpolation procedure and its accompanying drainage enforcement algorithm are described in detail below. A much analysed data set of Davis (1973) is used to illustrate several points and to provide a useful comparison with other interpolation methods. The paper concludes with an application to a larger data set.

It should be noted that the procedure has been designed principally to interpolate scattered surface-specific point elevation data and stream line data. It can be applied to contour elevation data with acceptable results, but it is possible to capitalize on the special nature of contour data to improve the quality of interpolation. The extension of the present technique to the interpolation of contour elevation data will be the subject of a separate study.

THE INTERPOLATION ALGORITHM

The relative merits of various general purpose interpolation methods have been discussed by a number of authors including Ripley (1981), Sibson (1981), Hutchinson (1984), Laslett et al. (1987) and others. Most existing approaches to the interpolation of irregularly distributed data can be broadly classified as having either global or local character.

The two most popular global methods are the thin plate splines of Duchon (1976) as taken up by Wahba and Wendelberger (1980) and others and the method of Kriging advocated by Matheron (1965). These techniques offer elegant solutions to the general interpolation problem. The interpolated surface normally depends on the data in a rotation invariant manner and can be made to have reasonable continuity properties. Their chief disadvantage is a computational cost proportional to n^3 where n is the number of data points, making them prohibitively expensive for very large data sets.

Local methods are usually based on partitioning the region containing the data points into small elements and fitting simple functions on each element so that the functions are continuously differentiable across the boundaries of adjoining elements. This significantly reduces the computational burden at the expense of somewhat arbitrary restrictions on the form of the fitted functions. Such techniques are also sensitive to the positions of data points, particularly when the data points are very irregularly spaced, and spurious edge effects can be generated. Recently developed local methods of Sibson (1981) and Watson and Philip (1985), which employ Dirichlet tessellations of the plane, appear to overcome at least some of these difficulties.

The iterative finite difference interpolation method adopted here has been designed to have the efficiency of a local method without sacrificing the continuity and rotation invariance of high quality global methods. Most importantly, because the fitted grid values are available at every stage during the iteration, it is relatively straightforward to monitor the drainage characteristics of the fitted grid and to impose appropriate ordered chain constraints to enforce drainage. This is not easily achieved for either the global or local methods described above. The interpolation problem is solved by minimizing a discretized rotation invariant roughness penalty which is defined in terms of first and second order partial derivatives of the fitted function. Similar approaches have been adopted by Briggs (1974), Swain (1976) and Testud and Chong (1983). An application of an early version of this technique to the interpolation of bathymetric data occurs in Torgersen et al. (1983).

The iteration technique employs a simple nested grid strategy which calculates grids at successively finer resolutions, starting from an initial coarse grid, and successively halving the grid spacing until the final user specified grid resolution is obtained. For each grid resolution, the data points are simply allocated to the nearest grid point and values at grid points not occupied by data points are calculated by Gauss–Seidel iteration with overrelaxation (Young, 1971; Golub and Van Loan, 1983) subject to the specified roughness penalty and ordered chain constraints. The iteration matrix used in the Gauss–Seidel iteration depends on the roughness penalty and is easily seen to be positive definite and symmetric for the roughness penalties considered below, thereby guaranteeing convergence. Starting values for the first coarse grid resolution are calculated from the average height of all data points while starting values for each successive finer grid are linearly interpolated from the preceding coarser grid. An empirically determined overrelaxation parameter of 1.6 has been found to give useful acceleration of convergence for the roughness penalties considered and iteration terminates for each grid resolution when the user specified maximum number of iterations (normally 25) has been reached. The computational cost of the technique is then optimal in the sense that it is essentially proportional to the number of interpolated grid points.

A TERRAIN SPECIFIC ROUGHNESS PENALTY

The roughness penalty which defines the interpolating function, f, is based on two nonnegative, rotation invariant functionals. These are defined in terms of first and second order partial derivatives of f by:

$$J_1(f) = \int (f_x^2 + f_y^2)\, dx\, dy$$

and:

$$J_2(f) \int (f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2)\, dx\, dy$$

where the range of integration is the region occupied by the fitted grid. Minimizing $J_1(f)$ in its discretized form over all suitably continuous interpolating functions f gives rise to discretized minimum potential interpolation while minimizing $J_2(f)$ gives rise to the minimum curvature interpolation of thin plate splines in the discrete form first devised by Briggs (1974). Testud and Chong (1983) have discussed these roughness penalties in the context of data smoothing.

Minimizing $J_2(f)$ is a good general purpose strategy which can give visually pleasing results as seen in Fig. 1 where it has been applied to the elevation data in table 6.4 of Davis (1973, p. 316). In terms of freedom from spurious surface features, the result displayed in Fig. 1 is clearly superior to the results of four different interpolation methods applied to the same data by Lodwick (1982) and compares favourably on the same grounds with the results of the preferred version of Kriging in fig. 4.17 of Ripley (1981, p. 64) and the local interpolation method in fig. 5 of Watson and Philip (1985, p. 322). However, minimum curvature interpolation of terrain is not ideal. Its tendency to maintain trends

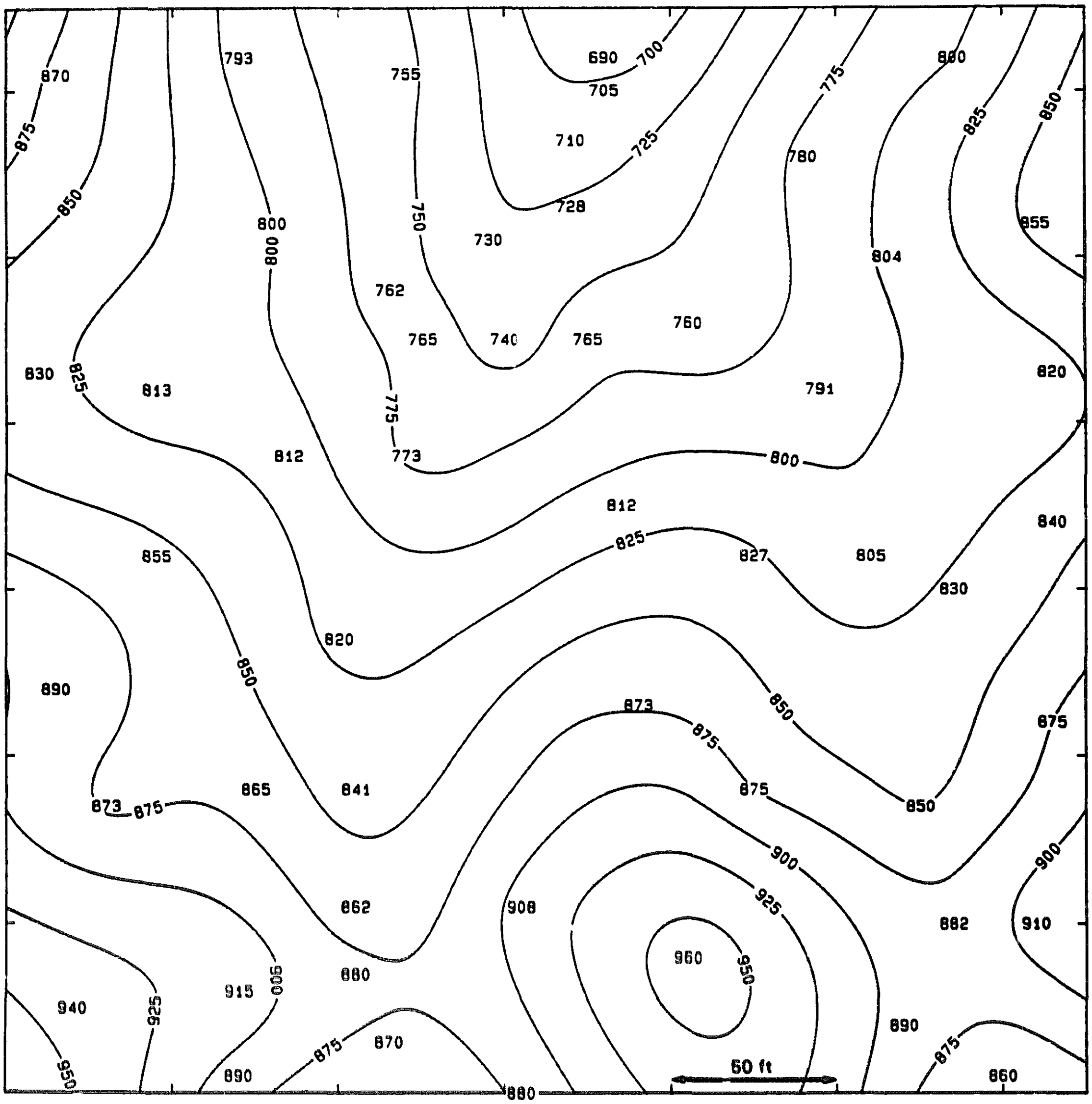

Fig. 1. Discretized minimum curvature interpolation of the point elevation data of Davis (1973, table 6.4). Elevations in feet.

away from data points can generate spurious overshoot and undershoot in regions containing closely spaced data points with large variations in elevation. Though not in evidence here, this phenomenon is well documented, particularly in the context of univariate minimum curvature interpolation (De Boor, 1978; Fritsch and Carlson, 1980). Moreover, a closer look at Fig. 1 reveals more subtle shortcomings. The real pattern of stream lines and ridges, as illustrated in Fig. 8 and in fig. 6.10 of Davis (1973, p. 323), is not accurately represented in Fig. 1 despite the lack of any spurious sinks.

Consideration of the statistical nature of natural terrain by Mandelbrot (1982), using the theory of fractals (see also Goodchild and Mark, 1987), has led Frederiksen et al. (1985) to suggest that a more appropriate roughness penalty should lie somewhere between J_1 (f) and J_2 (f). This would allow the fitted surface to follow more closely the sharp changes in terrain often associated with ridges and streams.

The result of minimizing $J_1(f)$ is shown in Fig. 2. This discretized minimum potential surface is less sensitive to local trends in the data, displaying a characteristically flatter appearance away from the data points, of which most are represented as sharp local maxima and minima. It should be noted that minimizing $J_1(f)$ is only well defined in its discretized form, since it can be shown that the corresponding continuous interpolation problem does not have a unique solution amongst continuous functions. Moreover, the discretized solution is sensitive to the grid spacing, with local maxima and minima becoming increasingly sharp as the grid spacing decreases. While this is undesirable in general, the important feature here is that the local minima occur at data points on stream lines and the local maxima occur at data points on ridges and peaks.

An empirically determined compromise between the minimum curvature and minimum potential roughness penalties is illustrated in Fig. 3 which has been obtained by minimizing the discretized form of:

$$J(f) = 0.5\,h^{-2}\,J_1(f) + J_2(f)$$

over all suitably continuous interpolating functions f where h is the grid spacing. The effect of this roughness penalty is to modify the usual finite difference recurrence formula for minimizing $J_2(f)$ away from data points (see Briggs, 1974; Testud and Chong, 1983) in a way which is independent of the grid spacing h. It has been confirmed empirically that the resulting interpolated surface is also insensitive to the grid spacing. This surface maintains trends away from data points in a similar fashion to minimum curvature interpolation but still identifies most of the points on stream lines as sinks. It also identifies breaks in slope corresponding to data points on ridges more sharply. If the sinks, which have been identified in Fig. 3 by circles, were to be linked in a sensible fashion to form stream lines, as indicated by the dashed lines in Fig. 3, then the drainage pattern of this landscape would have been effectively recovered from a very small data set. Moreover the surface specific nature of the data points would have been automatically recovered without the need for explicitly identifying their nature in the data. This is precisely what is achieved by the drainage enforcement algorithm described below.

THE DRAINAGE ENFORCEMENT ALGORITHM

The drainage enforcement algorithm attempts to remove all sink points in the fitted grid which have not been identified as such in input sink data. The essence of the algorithm is to recognize that each spurious sink is surrounded by a drainage divide containing at least one saddle point. If the sink is associated with an elevation data point then the lowest such saddle, provided it is not also associated with an elevation data point, identifies the region of the grid which is modified in order to remove the spurious sink. If on the other hand

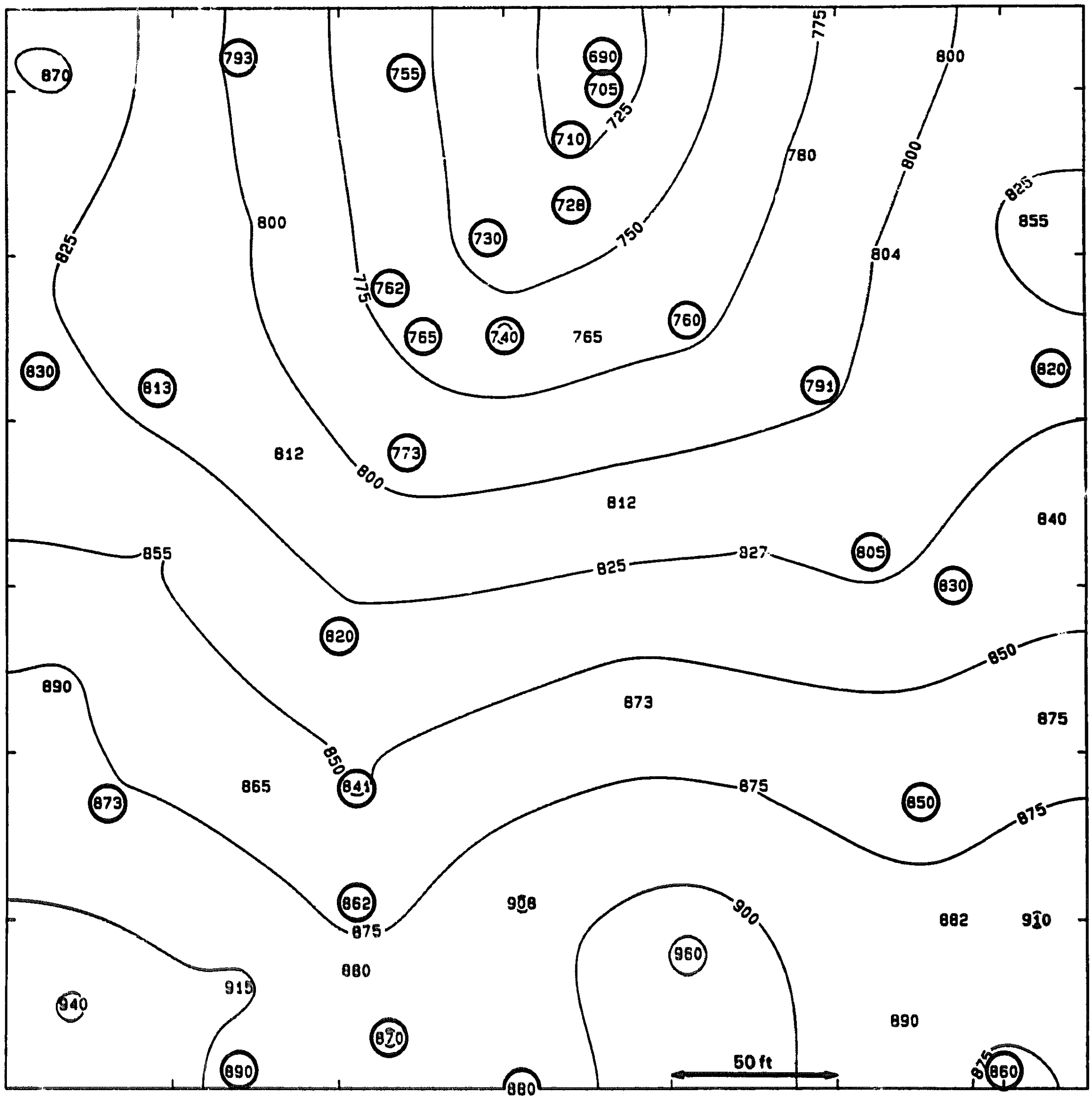

Fig. 2. Discretized minimum potential interpolation of the point elevation data of Davis (1973, table 6.4). Sinks are denoted by circles.

the lowest saddle point is associated with an elevation data point but the sink is not, then the sink and its immediate neighbours are raised above the height of the data point saddle. If neither the sink nor the lowest saddle are associated with elevation data points then grid points in the neighbourhood of both the sink and the lowest saddle are modified to ensure drainage. Finally, if both sink point and saddle point are associated with elevation data points, then a choice is made, depending on a user-supplied tolerance, between enforcing drainage and maintaining fidelity to the data. This last situation arises when calculating generalized (coarse resolution) DEMs.

The action of this drainage enforcement algorithm is conceptually similar to the action taken by the basin delineation and drainage network simulation

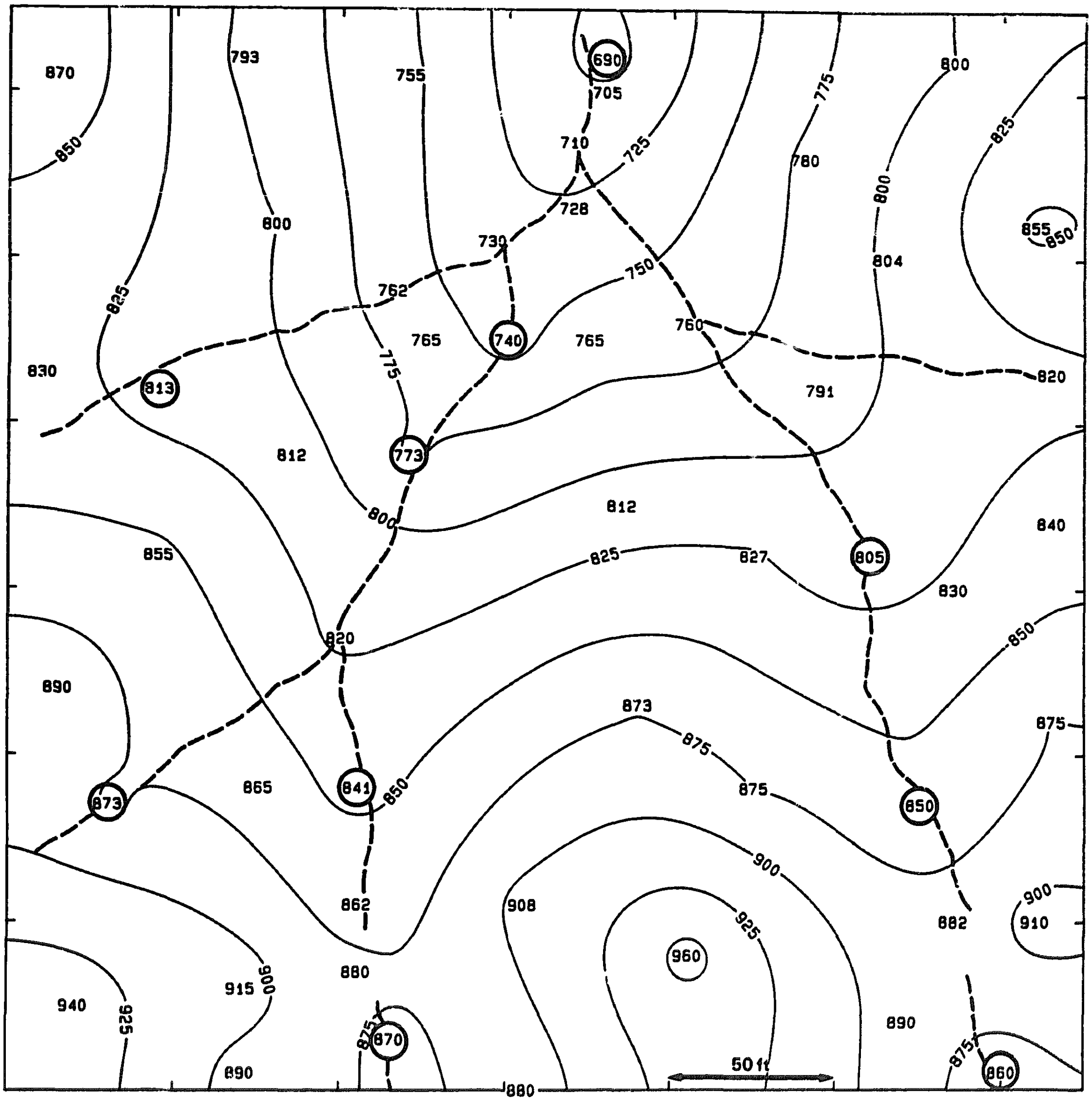

Fig. 3. Interpolation of the point elevation data of Davis (1973, table 6.4) by minimizing the roughness penalty $J(f)$. Sinks are denoted by circles. True stream lines are indicated by dashed lines.

programs of Marks et al. (1984) and Yuan and Vanderpool (1986) to overcome the problem of spurious sinks. It is also related to the methods of cartographic generalization suggested initially by Warntz (1975) and taken up by Pfaltz (1976) and Wolf (1984). In fact the approach suggested here of maintaining connected drainage patterns provides a more secure physical basis for cartographic generalization than the partially lexicographic approach adopted by these authors.

The drainage enforcement algorithm proceeds concurrently with the iterative interpolation algorithm described in the preceding section. For each grid resolution, after the first very coarse resolution, the grid is periodically inspected (normally after every five Gauss–Seidel iterations) for sinks and their accompanying saddle points. These are found by comparing the height of each grid point with the height of each of its eight immediate neighbours (cf. Peucker and Douglas, 1975). A sink point is characterized by having an elevation no higher than the elevation of each of its eight immediate neighbours while a grid point is a saddle point if it has at least two neighbours strictly higher than itself interleaved by neighbours no higher than itself when moving in a clockwise or an anticlockwise direction through the eight immediate neighbours surrounding the grid point. Saddles are associated with sink points by searching in each of the two, three or four possible steepest downhill directions away from each saddle point until a sink or an edge of the grid is found. This is illustrated in Fig. 4 where the saddle points associated with the sink point S 1 are the points A, B, C, D and E.

Ordered chain conditions effecting drainage clearance are then applied to the grid by inserting ordered chains which lead from each spurious sink point, via the lowest associated saddle point, to a data point or existing ordered chain on the other side of the saddle, provided this does not lead to an elevation conflict exceeding a user supplied tolerance. The action of the ordered chains is to impose linear descent, to within a small tolerance, between successive elevation data points down the entire length of the chain.

Thus, in the example of Fig. 4, the lowest saddle associated with the sink S 1 is the point D. Since this saddle point is not associated with an elevation data point, and it leads to the sink point S 2 which is strictly lower than S 1, an ordered chain is inserted from S 1 to S 2 via D as shown in Fig. 5. Each ordered chain in Fig. 5 is made up of two flow lines leading from the lowest saddle associated with each sink point in Fig. 4. The procedure for detecting sinks and saddles and inserting ordered chain conditions is reasonably efficient, requiring less computer time than the basic interpolation algorithm, especially since it is only enacted once every five Gauss–Seidel iterations.

The action of the drainage enforcement algorithm is modified in practice by the systematic application of three user-supplied elevation tolerances. These tolerances allow the strength of drainage enforcement to be adjusted in relation to the accuracy and density of the input elevation data as well as the level of generalization required. Their detailed action has undergone considerable development and testing with data sets of varying densities and accuracies at a variety of scales. The aim has been to achieve the strongest possible drainage enforcement without making serious errors in the placement of drainage lines. The action of the tolerances is most critical when the input data are limited in terms of accuracy or density, especially when calculating generalized DEMs for large areas (Hutchinson and Dowling, 1989). Their action naturally becomes less critical as the accuracy and density of the input data improve. When the tolerances have been set appropriately, the sink points not cleared by the program are normally those associated with genuine sinks, with significant elevation errors in input data, or, with areas where the input data are not of sufficient density to reliably resolve the drainage characteristics of the fitted grid.

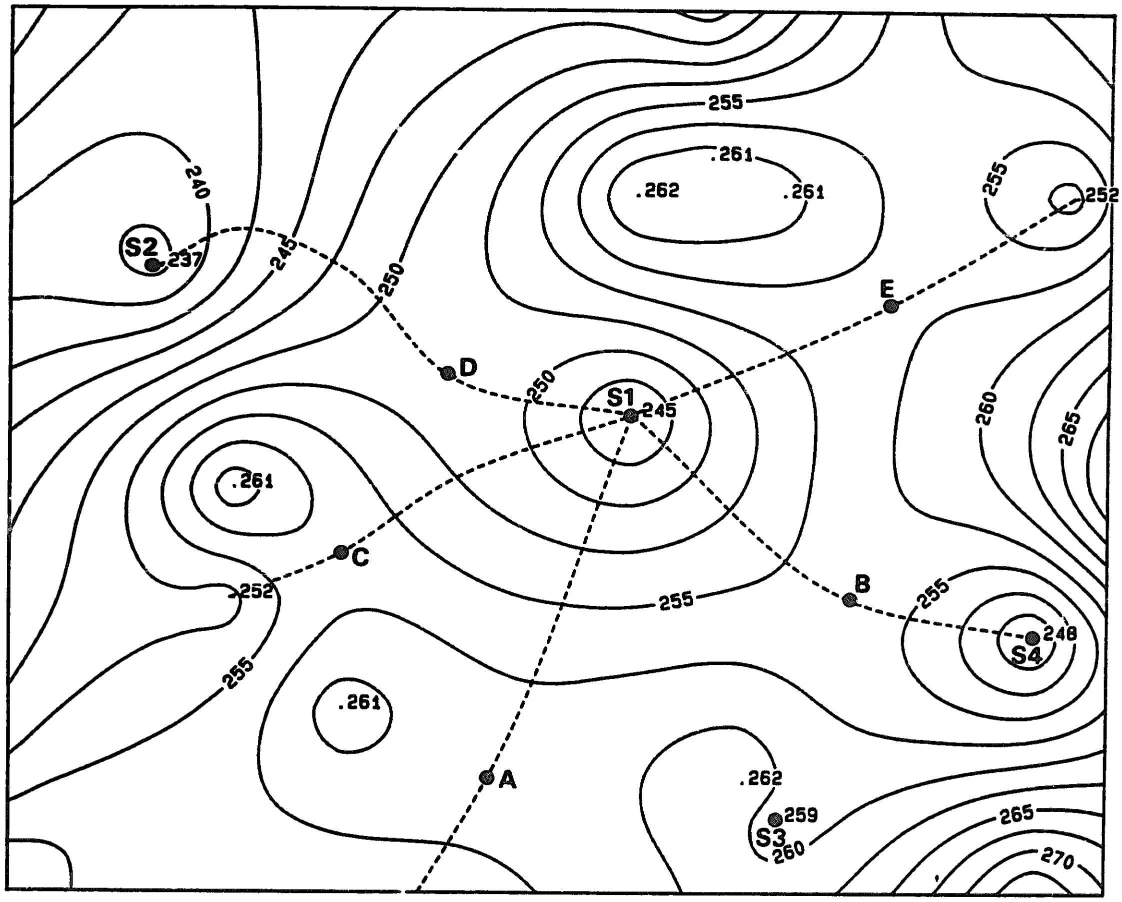

Fig. 4. Example showing how the saddle points A, B, C, D, E are associated with the sink point $S1$ via flow lines which are indicated by dashed lines. Additional sink points are denoted by $S2$, $S3$, $S4$. Data points are indicated by their height in metres.

The first user-supplied tolerance is essentially a measure of the elevation accuracy of the data. Elevation differences between data points not exceeding this tolerance are judged to be insignificant with respect to drainage. Thus data points which block drainage by no more than this tolerance are removed. When data points are not sufficiently dense to accurately resolve drainage, this tolerance may be increased to yield a generalized but sensible drainage pattern at the expense of fidelity of the fitted surface to the elevation data. The first tolerance is also used when searching for possible clearances for sinks, to slightly favour those saddle points which are not associated with elevation data points over saddle points which are associated with elevation data points. This is based on the understanding that the height of the grid at a data point saddle is more reliable than the height at a nondata point saddle.

The second and third elevation tolerances play a more technical role in limiting various searching operations by the procedure in order to increase efficiency and to prevent nonsensical drainage clearances, particularly when data are sparse. The second tolerance is a measure of local relief which is most naturally set to the contour interval when gridding contour data. It limits the

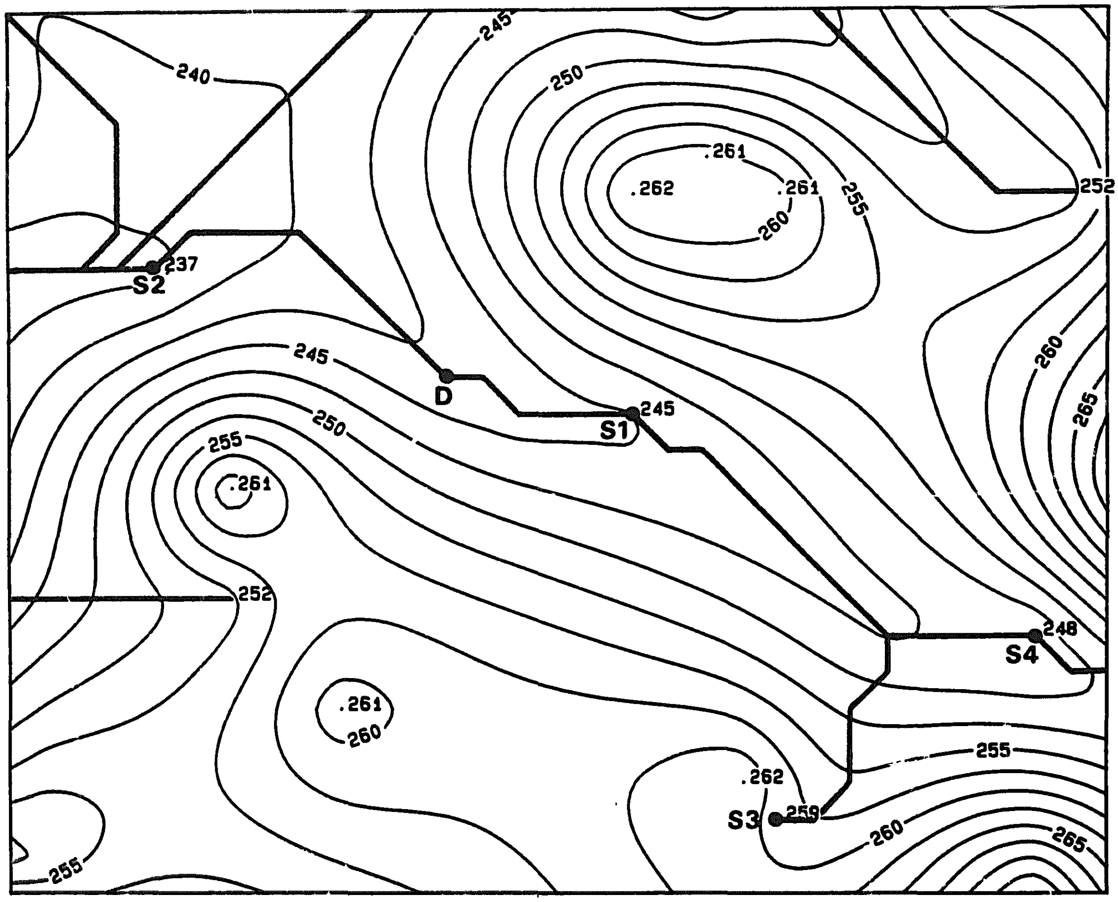

Fig. 5. The result of drainage enforcement applied to the example of Fig. 4. Piecewise linear lines indicate inferred drainage lines.

height above each data point sink of data point saddles which may be considered as possible exits from the sink. This can remove from consideration certain data point saddles, even though they may be the lowest saddle associated with a particular sink, in order to allow drainage clearance via a higher non-data point saddle. If the second tolerance is set to a large value then the procedure acts conservatively when attempting to remove sinks because more data point saddles will be considered. The third elevation tolerance is simply used as a final check to prevent drainage clearances which would entail very large changes to the grid. It is only active when the elevation data are very sparse or contain large errors in elevation.

Two additional features of the drainage enforcement algorithm merit comment. The first is that spurious sinks are sorted by elevation and cleared in order of increasing elevation. This facilitates the searching operations required to associate saddle points with sinks and improves the placement of ordered chains which clear higher sinks, particularly in their lower reaches where they normally join existing ordered chains. This is illustrated in Fig. 5 where the sink point S 3 has not been immediately cleared to the lower sink point S 4, since this point was first cleared to S 1. A subsequent enactment of the drainage enforcement algorithm cleared S 3 to the ordered chain leading from S 4 to S 1. The second feature is that ordered chains clearing sinks are always extended beyond any lower data points encountered until they meet an existing ordered chain, a sink or an edge of the grid. This ensures that a connected drainage pattern is obtained.

The result of applying the drainage enforcement algorithm in conjunction with the roughness penalty $J(f)$ to the data of Davis is shown in Fig. 6. The inferred drainage pattern and associated contours are remarkably similar to the actual drainage pattern and the contours shown in Fig. 8 and in fig. 6.10 of Davis (1973, p. 323). The remaining sinks at the top and bottom of Fig. 6 illustrate the essentially non-local nature of the drainage enforcement algorithm. They would be removed if additional data beyond the extremes of the figure were available.

THE INCORPORATION OF STREAM LINE DATA

Drainage enforcement can also be obtained by incorporating stream line data. This can be useful when more accurate placement of streams is required than can be calculated automatically by the procedure. It can also be used to remove sinks which would not otherwise be removed by the automatic drainage enforcement algorithm. This is in fact the recommended way to correct remaining drainage anomalies in elevation grids calculated by the procedure. All elevation data points which conflict with strict descent down each stream line are removed, with all conflicts which exceed the third user specified elevation tolerance being flagged for possible correction of errors. The incorporation of actual stream line data also provides a more reliable way of ensuring sensibly generalized elevation models.

The method of incorporating stream lines and their associated side conditions into a grid is illustrated in Fig. 7. First, the data stream line, which flows from the bottom to the top of the figure as indicated by the dashed line, is approximated by straight line segments, indicated by heavy lines. These have been generalized so that successive segments change in direction by at most 45°. Thus, the segments CX and XY, which were an initial representation of a portion of the stream, have been replaced by the single segment CY. This minimizes the number of straight line segments required to represent each stream line and removes unnecessarily sharp bends from its representation on the grid. Side conditions are then added, as indicated by the light line segments. These normally lie at 45° to the prevailing direction of the stream line except at bends, the point Y in this example, where the side condition represented by the segment XY is obtained as the uphill extension from the lower stream line segment YF. Note that each grid point can have a number of adjoining upper stream segments but at most one adjoining lower stream segment.

The action of these straight line segments on the interpolation conditions which arise from minimizing the prescribed roughness penalty is then as follows. For points such as B, with one lower adjoining segment and at least

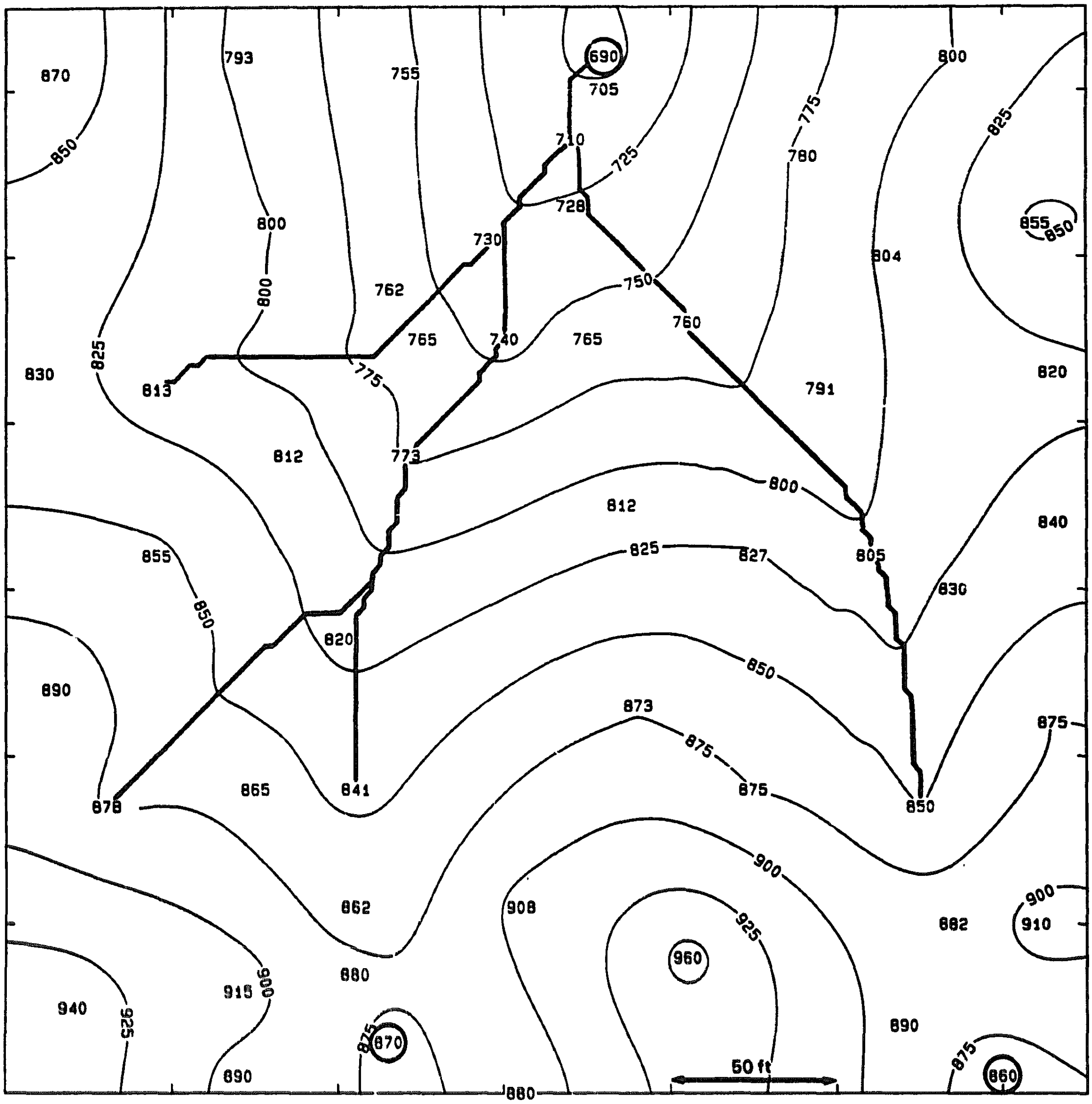

Fig. 6. Interpolation of the point elevation data of Davis (1973, table 6.4) as in Fig. 3 and employing the drainage enforcement algorithm. Sinks are denoted by circles. Piecewise linear lines indicate inferred drainage lines.

one upper adjoining segment, the height at B is constrained so that descent is (approximately) uniform from the lowest of A1, A2 and A3 down to C. For points such as E, with one lower adjoining segment and no upper adjoining segment, the height is similarly constrained between D (the uphill extension of EF) and F. These actions maintain linear descent between elevation data points down each stream line and ensure that each stream line acts as a breakline for the interpolation conditions. This in turn ensures that each stream line lies at the bottom of its accompanying valley. Side conditions are not set for data points beside streams whose elevations are more than the third elevation tolerance below the height of the stream. These conflicts are also flagged for possible correction of errors.

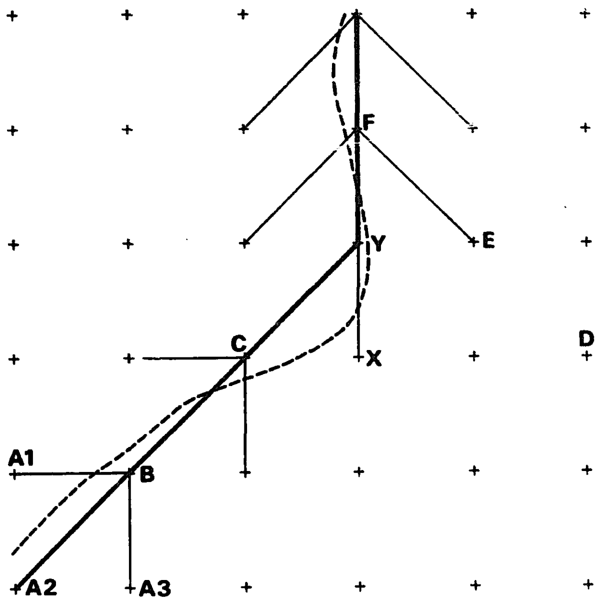

Fig. 7. Example showing how a stream line is incorporated into a grid with associated side conditions.

The stream lines in fig. 6.6 of Davis (1973, p. 312) were digitized and added to the point elevation data to yield Fig. 8. The straight line segments describing the stream lines and their associated side conditions are also shown. A modest improvement to the modelled drainage pattern has been obtained in this case. Stream line data are best used in practice to define the major drainage lines associated with high order streams, leaving the drainage enforcement algorithm to define the lower order stream lines to a level of detail that is controlled mainly by the amount of available data and the specified resolution of the DEM. This eliminates the need for digitizing the large number of lines associated with low order streams which may not exist in mapped form.

A LARGER EXAMPLE

An application to a larger data set from an area in central Queensland, Australia at 23°S and 143°E is now described. Figure 9 shows the result of fitting a minimum curvature surface to this rather sparse data set. In contrast to Fig. 1, in this case there are many remaining sinks which obscure the underlying drainage pattern. The drainage enforcement algorithm, in combination with the terrain specific roughness penalty $J(f)$, resolves all of the apparent drainage anomalies as illustrated in Fig. 10 and the resulting drainage pattern, though generalized because of the sparseness of the data, is in good agreement with the actual drainage pattern shown in Fig. 11. The grid was fitted to data extending beyond the limits of the figure so that no edge anomalies are apparent.

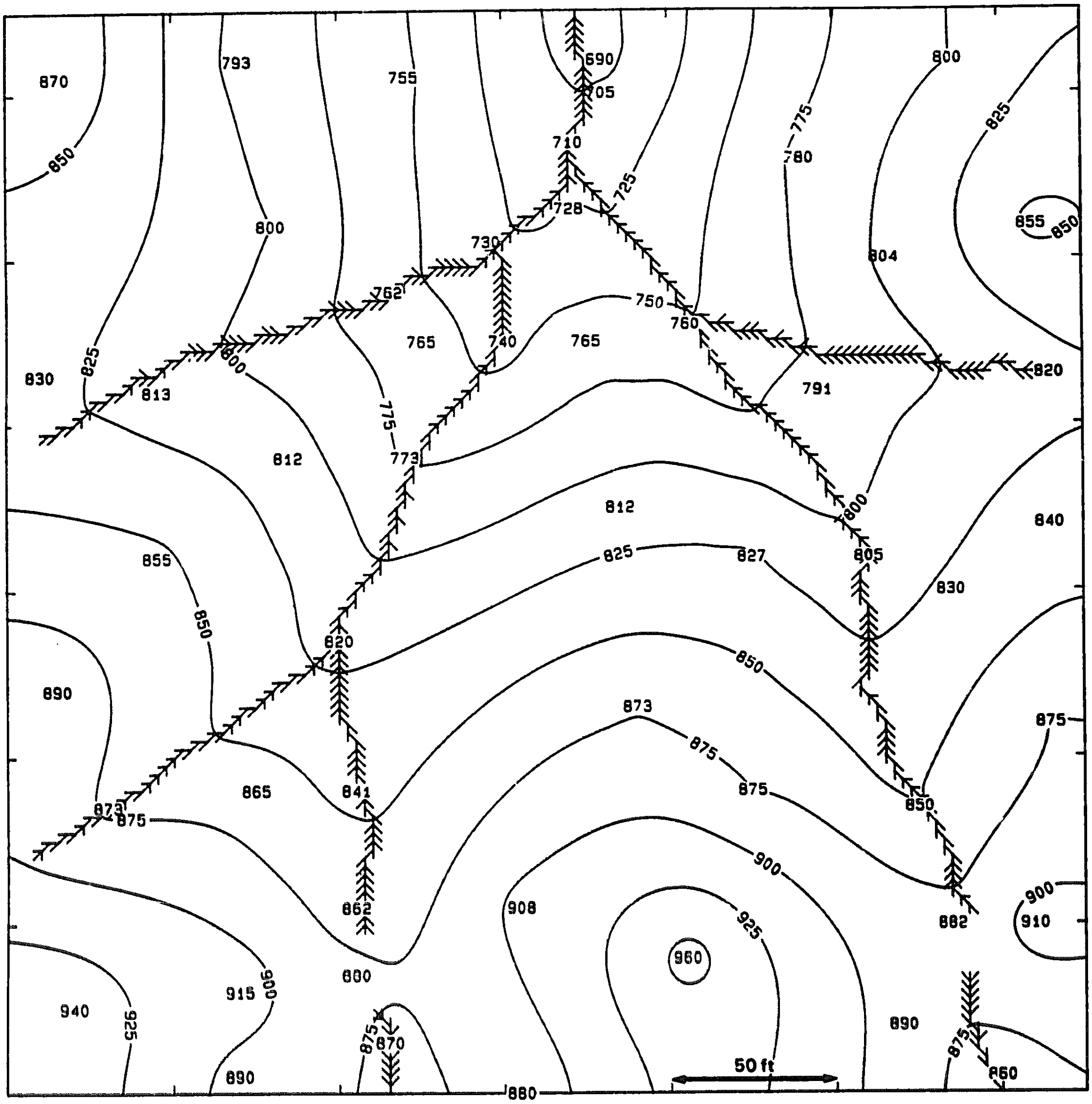

Fig. 8. Interpolation of the point elevation data of Davis (1973, table 6.4) as in Fig. 6 and incorporating stream line data with associated side conditions.

DISCUSSION

An effective procedure for calculating digital elevation models with sensible drainage properties from comparatively small sets of surface specific point elevation data and stream lines has been described. Digital elevation models calculated by this procedure may be used to advantage in hydrological process studies. The procedure embodies a morphological approach to digital elevation modelling, since the interpolation algorithm and the accompanying drainage enforcement algorithm are defined directly in terms of morphological properties of the fitted surface. The success of the technique has also been assessed primarily in terms of morphological properties of the fitted surface.

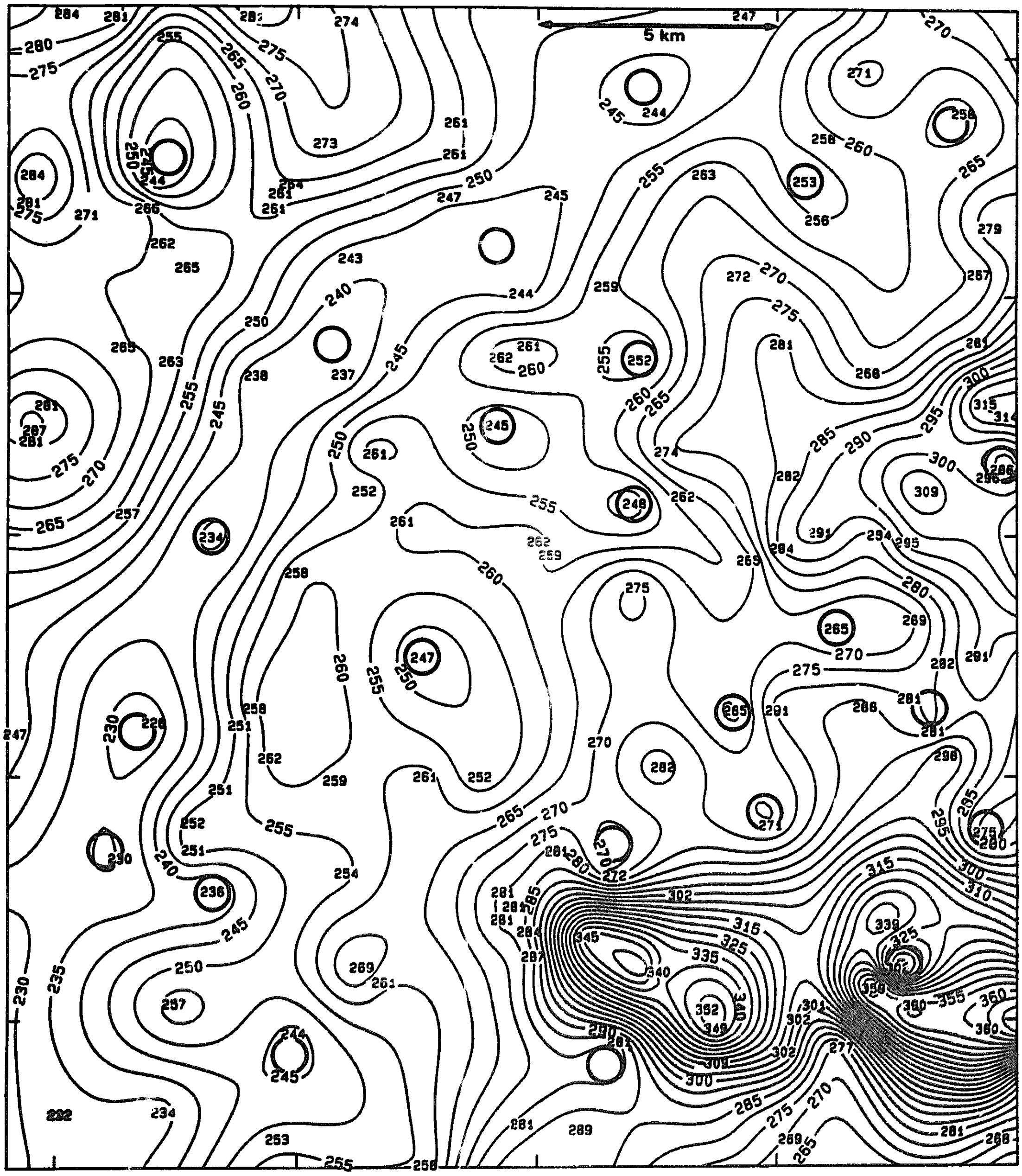

Fig. 9. Minimum curvature interpolation of scattered point elevation data in central Queensland, Australia, 23°S, 143°E. Sinks are denoted by circles. Elevations in metres.

The drainage enforcement algorithm plays an essential role in the interpolation process. Minimization of a variety of general roughness penalties as a basis for interpolation has been seen to be inadequate without the imposition of localized constraints as calculated by the drainage enforcement algorithm or obtained from stream line data. Moreover, the drainage enforcement algorithm provides a sound basis for cartographic generalization and displays significant advantages over conventional statistical techniques for extracting information from and detecting errors in spatially distributed elevation data.

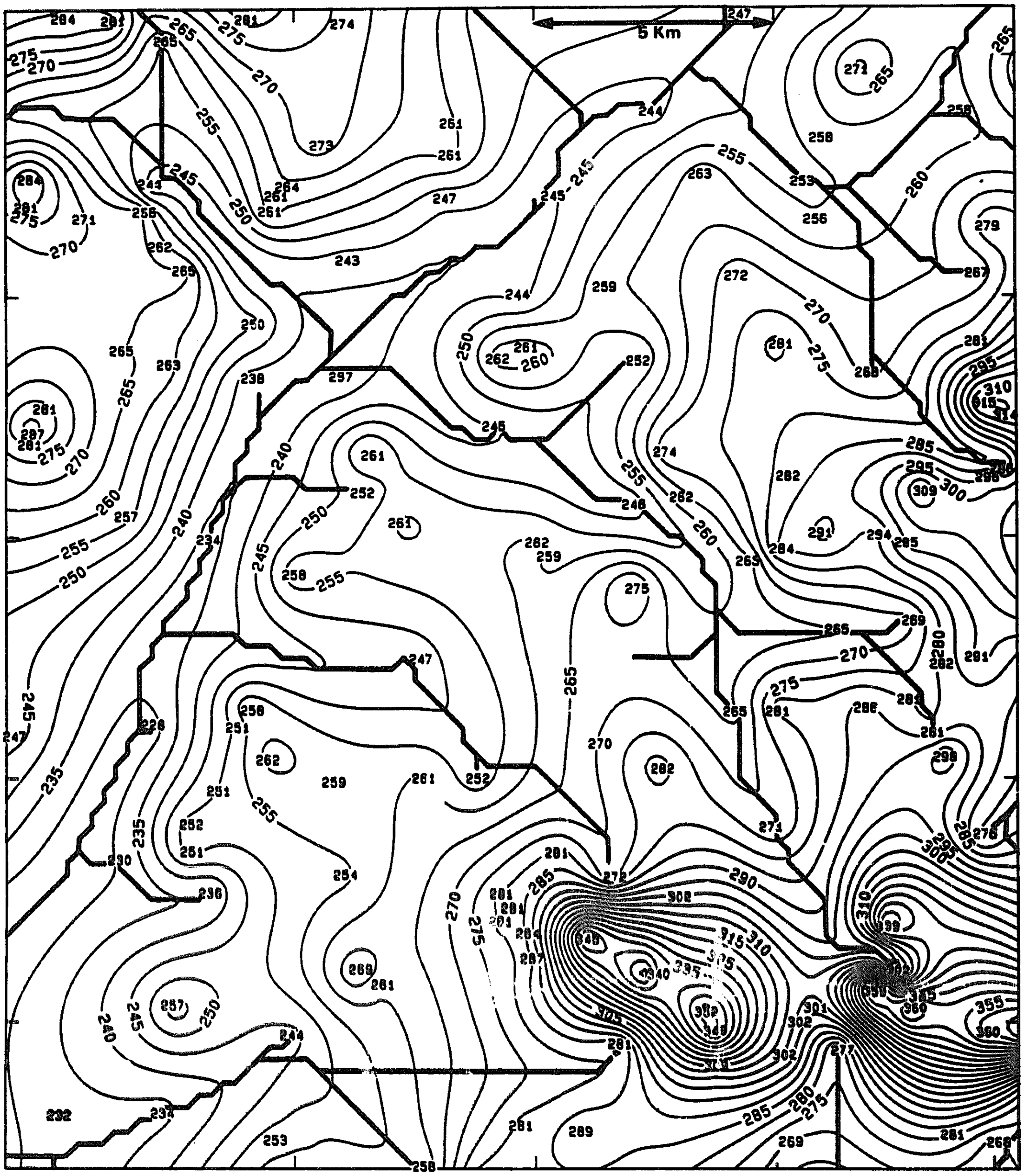

Fig. 10. Interpolation of the point elevation data used in Fig. 9 by minimizing the roughness penalty $J(f)$ and employing the drainage enforcement algorithm. Piecewise linear lines indicate inferred drainage lines.

The relative weighting of the minimum potential and minimum curvature roughness penalties has been empirically determined to produce good results in combination with the drainage enforcement algorithm when applied to surface specific point elevation data. It is feasible that different weightings would be better suited to different types of terrain and/or different data point sampling strategies, however good results have been obtained with this relative weighting for both arid and humid areas in Australia. The choice of roughness penalty naturally becomes less critical as data density increases.

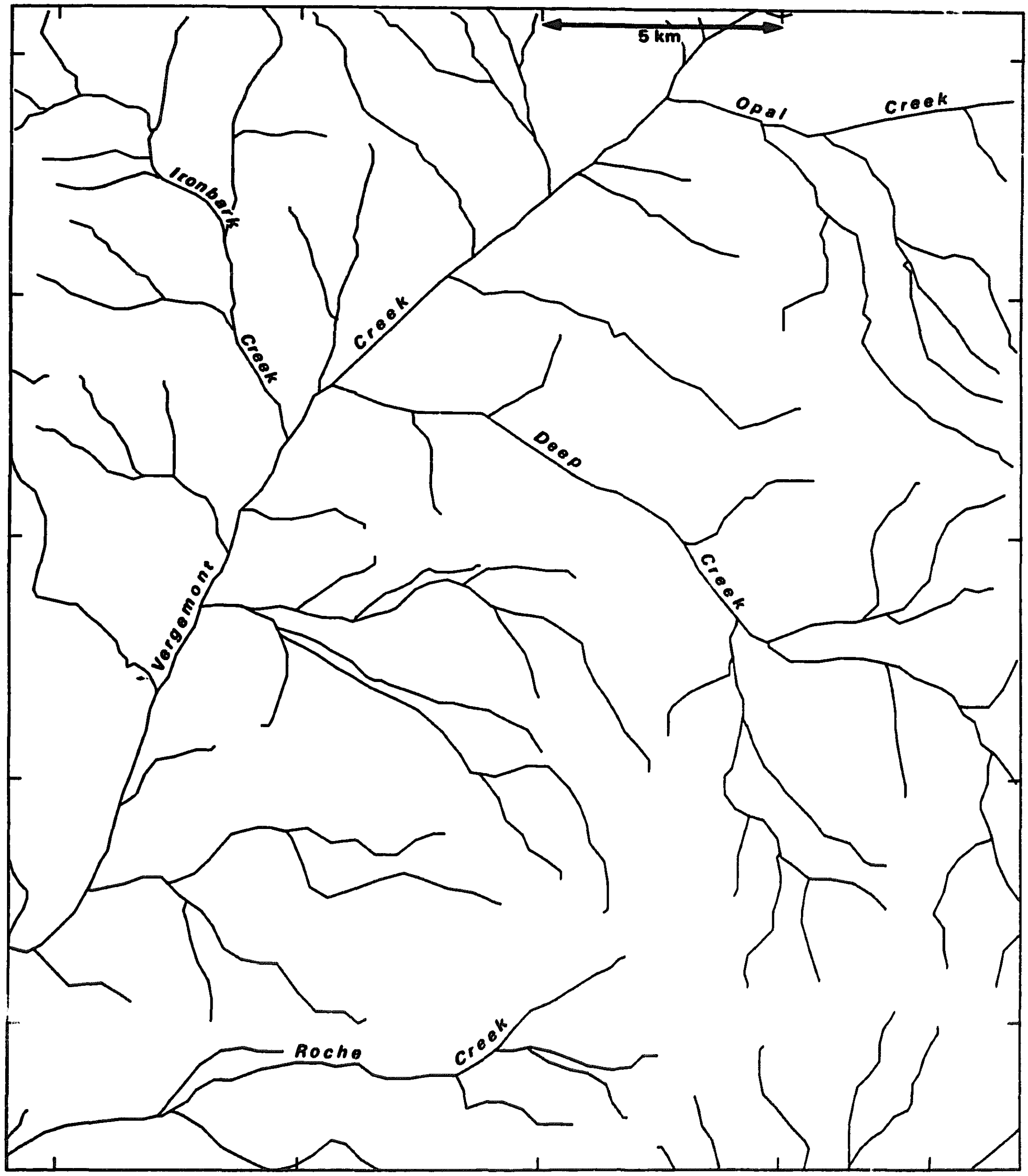

Fig. 11. The drainage pattern for the area of Fig. 9 as digitized from a 1:250 000 topographic map.

The procedure can in principle be applied to data at any scale, the only limits being the availability of sufficient data and practical limitations on the size of the interpolated grid. The procedure has been incorporated by the Australian Division of National Mapping into its production of a national digital topographic database at a nominal scale of 1:1 million (Trezise and Hutchinson, 1986). It is also being used routinely by researchers in the hydrologic modelling of small catchments (Moore et al., 1988).

The interpolation procedure is computationally optimal in the sense that computer time is essentially proportional to the number of interpolated grid points. However, it is anticipated that a suitable multigrid technique (Fulton et al., 1986) will yield a significant reduction in the computer time required. Work is also in progress to extend the procedure to the optimal interpolation of contour data.

ACKNOWLEDGMENTS

The Australian Division of National Mapping (NATMAP) provided the elevation data used to produce Figs 9, 10. I wish to thank H.A. Nix, T.I. Dowling and J.G. Speight from the former CSIRO Division of Water and Land Resources, A.L. Clarke and P.H. Trezise from NATMAP and K. Kubic for their support, stimulating feedback and patience during the considerable period it has taken to develop the digital elevation modelling procedure described above.

REFERENCES

Abbott, M.B., Bathurst, J.C., Cunge, J.A., Connell, P.E. and Rasmussen, J., 1986a. An introduction to the European Hydrological System — Système Hydrologique Européen, "SHE", 1: History and philosophy of a physically-based, distributed modelling system. J.Hydrol., 87: 45–59.

Abbott, M.B., Bathurst, J.C., Cunge, J.A., Connell, P.E. and Rasmussen, J., 1986b. An introduction to the European Hydrological System — Système Hydrologique Européen, "SHE", 2: Structure of a physically-based, distributed modelling system. J. Hydrol., 87: 61–77.

Ayeni, O.O., 1982. Optimum sampling for digital terrain models: a trend towards automation. Photogramm. Eng. Remote Sensing, 48(11): 1687–1694.

Band, L.E., 1986. Topographic partition of watersheds with digital elevation models. Water Resour. Res., 22(1): 15–24.

Bathurst, J.C., 1986. Physically-based, distributed modelling of an upland catchment using the Système Hydrologique Européen. J. Hydrol., 87: 79–102.

Berry, J.K., 1985. Computer-assisted map analysis: fundamental techniques. Computer Graphics Conference, 1985, Natl. Comput. Graphics Assoc., Fairfax, Va.

Bork, H.-R. and Rohdenburg, H., 1986. Transferable parameterization methods for distributed hydrological and agroecological models. Catena, 13: 99–117.

Briggs, I.C., 1974. Machine contouring using minimum curvature. Geophysics, 39: 39–48.

Davis, J.C., 1973. Statistics and Data Analysis in Geology. Wiley, New York, N.Y.

De Boor, C., 1978. A Practical Guide to Splines. Springer, New York, N.Y.

Duchon, J., 1976. Interpolation des fonctions de deaux variables suivant le principe de la flexion des plaques minces. R.A.I.R.O. Anal. Numer., 10: 5–12.

Evans, I.S., 1980. An integrated system of terrain analysis and slope mapping. Z. Geomorphol. N.F. Suppl.-Bd. 36: 274–295.

Frederiksen, P., Jacobi, O. and Kubik, K., 1985. A review of current trends in terrain modeling I.T.C. Journal, 1985: 101–106.

Fritsch, F.N. and Carlson, R.E., 1980. Monotone piecewise cubic interpolation. SIAM J. Numer. Anal., 17(2): 238–246.

Fulton, S.R., Ciesielski, P.E. and Schubert, W.H., 1986. Multigrid methods for elliptic problems: a review. Mon. Weather Rev., 114: 943–959.

Golub, G.H. and Van Loan, C.F., 1983. Matrix Computations. Johns Hopkins University, Baltimore.

Goodchild, M.F. and Mark, D.M., 1987. The fractal nature of geographic phenomena. Ann. Assoc. Am. Geogr., 77(2): 265–278.

Harrington, H.J., Simpson, C.J. and Moore, R.F., 1982. Analysis of continental structures using a digital terrain model (DTM) of Australia. Bur. Miner. Resour. J. Aust. Geol. Geophys., 7: 68–72.

Heerdegen, R.G. and Beran, M.A., 1982. Quantifying source areas through land surface curvature and shape. J. Hydrol., 57: 359–373.

Hutchinson, M.F., 1984. A summary of some surface fitting and contouring programs for noisy data. CSIRO Div. Math. Stat. Consult. Rep., ACT 84/6, 24 pp.

Hutchinson, M.F., 1986. Fitting surfaces to terrain data. CSIRO Div. Water Land Resour., Res. Rep. 1983-85: 63–64.

Hutchinson, M.F. and Bischof, R.J., 1983. A new method for estimating the spatial distribution of mean seasonal and annual rainfall applied to the Hunter Valley, New South Wales. Aust. Meteorol. Mag., 31: 179–184.

Hutchinson, M.F and Dowling, T.I., 1989. A new digital elevation model for Australia. CRES Pap. Australian National University, Canberra, A.C.T.

Jupp, D.B.L., Hutchinson, M.F., Hutton, P.G. and Kalma, J.D.K., 1986. Models and data integration in thermal data: a comparison between sensors and scales. Proc. First Aust. AVHRR Conf., Perth, W. A., CSIRO Div. Groundwater Res., pp. 210–220.

Kelly, R.E., McConnell, P.R.H. and Mildberger, S.J., 1977. The Gestalt photomapping system. Photogramm. Eng. Remote Sensing, 43(11): 1407–1417.

Knisel, W.G. (Editor), 1980. CREAMS — A field scale model for chemicals, runoff and erosion from agricultural management systems. USDA Conserv. Res. Rep. 26, 643 pp.

Konecny, G., Lohmann, P., Engel, H. and Kruck, E., 1987. Evaluation of SPOT imagery on analytical photogrammetric instruments. Photogramm. Eng. Remote Sensing, 53(9): 1223–1230.

Laslett, G.M., McBratney, A.B., Pahl, P.J. and Hutchinson, M.F., 1987. Comparison of several spatial prediction methods for soil pH. J. Soil Sci., 38: 325–341.

Lodwick, G.D., 1982. A comparison of four digital terrain modelling packages. Proc. Harvard Computer Graphics Week, Harvard University.

Mandelbrot, B.B., 1982. The Fractal Geometry of Nature. Freeman, San Francisco, Calif.

Mark, D.M., 1979. Phenomenon-based data-structuring and digital terrain modelling. Geo-proc., 1: 27–36.

Mark, D.M., 1984. Automated detection of drainage networks from digital elevation models. Cartographica, 21: 168–178.

Marks, D., Dozier, J. and Frew, J., 1984. Automated basin delineation from digital elevation data. Geo-proc., 2: 299–311.

Matheron, G., 1965. Les Variables Regionalisées et Leur Estimation. Masson, Paris.

Moore, I.D., O'Loughlin, E.M. and Burch, G.J., 1988. A contour-based topographic model for hydrological and ecological applications. Earth Surf. Processes Landform 13(4): 305–320.

Palacios-Vélez, O.L. and Cuevas-Renaud, B., 1986. Automated river-course, ridge and basin delineation from digital elevation data. J. Hydrol., 86: 299–314.

Pennock, D.J., Zebarth, B.J. and de Jong, E., 1987. Landform classification and soil distribution in hummocky terrain, Saskatchewan, Canada. Geoderma, 40: 297–315.

Peucker, T.K. and Douglas, D.H., 1975. Detection of surface-specific points by local parallel processing of discrete terrain elevation data.Comput. Graph. Image Proc., 4: 375–387.

Pfaltz, J.L., 1976. Surface networks. Geogr. Ann., 8(1): 77–93.

Ripley, B., 1981. Spatial Statistics. Wiley, New York, N.Y.

Rohdenburg, H., Diekkrüger, B. and Bork, H.-R., 1986. Deterministic hydrological site and catchment models for the analysis of agroecosystems. Catena, 13: 119–137.

Shelton, R.L. and Estes, J.E., 1981. Remote sensing and geographic information systems: an unrealized potential. Geo-proc., 1: 395–420.

Sibson, R., 1981. A brief description of natural neighbourhood interpolation. In: V. Barnett (Editor), Interpreting Multivariate Data, Wiley, Chichester, pp. 21–53.

Speight, J.G., 1974. A parametric approach to landform regions. Inst. Brit. Geogr. Spec. Publ., 7: 213–230.

Swain, C.J., 1976. A FORTRAN IV program for interpolating irregularly spaced data using the difference equations for minimum curvature. Comput. Geosci., 1: 231–240.

Tesche, T.W. and Bergstrom, R.W., 1978. Use of digital terrain data in meteorological and air quality modelling. Photogramm. Eng. Remote Sensing, 44(12): 1549–1559.

Testud, J. and Chong, M., 1983. Three-dimensional wind field analysis from dual-doppler radar data. Part I: Filtering and differentiating the raw data. J. Clim. Appl. Meteorol., 22: 1204–1215.

Torgersen, T., Hutchinson, M.F., Searle, D.E. and Nix, H.A., 1983. General bathymetry of the Gulf of Carpentaria and the quaternary physiography of Lake Carpentaria. Palaeogeogr., Palaeoclimatol., Palaeoecol., 41: 207–225.

Trezise, P.H. and Hutchinson, M.F., 1986. The development of a national digital elevation model. Proc. 6th Aust. Cartogr. Conf. (Austra Carto Two), Melbourne, 1985, Aust. Inst. Cartogr.

Wahba, G. and Wendelberger, J., 1980. Some new mathematical methods for variational objective analysis using splines and cross-validation. Mon. Weather Rev., 108: 1122–1143.

Warntz, W., 1975. Stream ordering and contour mapping. J. Hydrol., 25: 209–227.

Watson, D.F. and Philip, G.M., 1985. A refinement of inverse distance weighted interpolation. Geo-proc., 2: 315–327.

Weiringa, J., 1986. Roughness-dependent geographical interpolation of surface wind speed averages. Q. J. R. Meteorol. Soc., 112: 867–889.

Wiltshire, S.E., Morris, D.G. and Beran, M.A., 1986. Digital data capture and automated overlay analysis for basin characteristic calculation. Cartogr. J., 23: 60–65.

Wolf, G.W., 1984. A mathematical model of cartographic generalization. Geo-proc., 2: 271–286.

Young, D.M., 1971. Iterative Solution of Large Linear Systems. Academic Press, New York, N.Y.

Yuan, L.-P. and Vanderpool, N.L., 1986. Drainage network simulation. Comput. Geosci., 12(5): 653–665.