4300303
Eletromagnetismo I
Segundo Semestre 2014

Prova Substitutiva
Gabarito

1. a) Como o fluxo através do circuito formado pela barra e resistor começa a diminuir, a corrente tem que circular no sentido de que o campo por ela produzido aumente o fluxo. Portanto a corrente irá circular no sentido indicado na figura.

b) $$\mathcal{E} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} + \oint (\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$\vec{v} = 0; \quad \frac{\partial\vec{B}}{\partial t} = -\frac{B_0}{T}\hat{i}_z \quad \therefore \quad \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} = -\frac{B_0}{T}dS \quad \therefore \quad \mathcal{E} = \frac{B_0}{T} a z_0$$

$$\therefore \boxed{I = \frac{B_0}{RT} a z_0}$$

$$\vec{f} = I\vec{l}\times\vec{B} = Ia(-\hat{i}_x)\times B_0\hat{i}_z \quad \therefore \quad \vec{f} = IaB_0\hat{i}_y = \frac{B_0^2a^2z_0}{RT}\hat{i}_y$$

Para que a barra suba, esta força tem que ser maior que a da gravidade;

$$|\vec{f}| \geq |\vec{f}_g| \quad \therefore \quad \boxed{\frac{B_0^2a^2z_0}{RT}\geq mg}$$

c) $$\mathcal{E} = -\int\frac{\partial\vec{B}}{\partial t}\cdot d\vec{S} + \oint(\vec{v}\times\vec{B})\cdot d\vec{l} = +\frac{B_0}{T}a(z-z_0) + \int_0^a (v\hat{i}_y\times B\hat{i}_z)\cdot(-dx\,\hat{i}_x)$$

$$\therefore \mathcal{E} = +\frac{B_0}{T}a(y-z_0) - v\,B_0\left(1-\frac{t}{T}\right)a \quad \therefore \quad \boxed{I = \frac{B_0 a}{RT}\left[y - z_0 - v(T-t)\right]}$$

d) $$\vec{f} = I\vec{l}\times\vec{B} = \frac{B_0^2a^2}{RT}\left[y - z_0 - v(T-t)\right]\hat{i}_y$$

$$m\frac{d\vec{v}}{dt} = \vec{f} + \vec{f}_g \quad \therefore \quad \boxed{m\frac{dv}{dt} = \frac{B_0 a}{RT}\left[y - z_0 - v(T-t)\right] - mg}$$

2. a) $\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$

$\vec{r} = 0;\ \vec{r}' = x'\hat{i}_x + a\hat{i}_y;\ d\vec{l}' = dx'\,\hat{i}_x$

$\therefore d\vec{l}'\times(\vec{r}-\vec{r}') = +a\,\hat{i}_z \quad \therefore \vec{B}_1(0) = \frac{\mu_0 I a}{4\pi}\int_{-\infty}^{0} \frac{dx'}{[a^2+x'^2]^{3/2}}\,\hat{i}_z$

$\therefore \vec{B}_1(0) = \frac{\mu_0 I a}{4\pi}\left[\frac{x'}{a^2\sqrt{a^2+x'^2}}\right]_{-\infty}^{0}\hat{i}_z$

$\therefore \boxed{\vec{B}_1(0) = \frac{\mu_0 I}{4\pi a}\,\hat{i}_z}$

b) $\vec{r} = 0;\ \vec{r}' = a\,\hat{e}_r;\ d\vec{l}' = a\,d\theta'\,\hat{i}_\theta$

$\therefore d\vec{l}'\times(\vec{r}-\vec{r}') = -a^2 d\theta'\,\hat{i}_\theta\times\hat{i}_r = a^2 d\theta'\,\hat{i}_z$

$\therefore \vec{B}_2(0) = \frac{\mu_0 I a^2}{4\pi}\int_{-\pi/2}^{0}\frac{d\theta'}{a^3}\,\hat{i}_z \quad \therefore \boxed{\vec{B}_2(0) = \frac{\mu_0 I}{8a}\,\hat{i}_z}$

c) $\vec{r} = 0;\ \vec{r}' = a\,\hat{e}_x + y'\,\hat{e}_y;\ d\vec{l}' = dy'\,\hat{e}_y$

$\therefore d\vec{l}'\times(\vec{r}-\vec{r}') = a\,dy'\,\hat{i}_z \quad \therefore \vec{B}_3 = \frac{\mu_0 I a}{4\pi}\int_0^{\infty}\frac{dy'}{[a^2+y'^2]^{3/2}} \quad \therefore \boxed{\vec{B}_3 = \frac{\mu_0 I}{4\pi a}\,\hat{i}_z}$

$\boxed{\vec{B}_T = \frac{\mu_0 I}{2\pi a}\left[1+\frac{\pi}{4}\right]\hat{i}_z}$

3. a) $\nabla\cdot\vec{B} = \nabla\cdot(\nabla\times\vec{A}) = 0$

$\nabla\times\vec{E} = -\nabla\times\frac{\partial\vec{A}}{\partial t} \quad \therefore \nabla\times\left(\vec{E}+\frac{\partial\vec{A}}{\partial t}\right) = 0 \quad \therefore \vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t}$

b) $\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \quad \therefore \nabla\times(\nabla\times\vec{B}) = \mu_0\vec{J} + \mu_0\epsilon_0\left[-\nabla\frac{\partial\phi}{\partial t} - \frac{\partial^2\vec{A}}{\partial t^2}\right]$

$\therefore \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A} = \mu_0\vec{J} + \mu_0\epsilon_0\left[-\nabla\frac{\partial\phi}{\partial t} - \frac{\partial^2\vec{A}}{\partial t^2}\right]$

3. a) $\nabla \cdot \vec{B} = \nabla \cdot (\nabla \times \vec{A}) = 0$

$\nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{A}}{\partial t} \therefore \nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0 \therefore \vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$ (3)

b) $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \therefore \nabla \times (\nabla \times \vec{A}) = \mu_0 \vec{J} + \mu_0 \epsilon_0 \left[-\nabla \frac{\partial \phi}{\partial t} - \frac{\partial^2 \vec{A}}{\partial t^2}\right]$

$\therefore \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \left[-\nabla \frac{\partial \phi}{\partial t} - \frac{\partial^2 \vec{A}}{\partial t^2}\right]$

$\therefore \nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} + \nabla\left[\nabla \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}\right]$

$\therefore$ Impondo $\nabla \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial \phi}{\partial t} = 0 \Rightarrow \nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$

c) Para fontes constantes no tempo, o potencial vetor é dado por

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d\tau'$$

Se a fonte variar no tempo, a informação de que ela está variando na posição $\vec{r}\,'$ se propaga com a velocidade da luz até a posição $\vec{r}$, levando um tempo $|\vec{r} - \vec{r}\,'|/c$. Portanto, o potencial medido na posição $\vec{r}$, no instante t, é correspondente ao valor que a fonte tinha na posição $\vec{r}\,'$ no instante retardado $t - \frac{|\vec{r} - \vec{r}\,'|}{c}$.

//

4. a) $R_s + T_s = 1$; $R_s = \frac{1}{3} T_s \Rightarrow R_s = \frac{1}{4} \therefore r_s = \frac{1}{2} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$

$\therefore n_1 \cos\theta_1 = 3 n_2 \cos\theta_2 \therefore \left(\frac{n_1}{3n_2}\right)^2 (1 - \sin^2\theta_1) = 1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_1$

$\therefore \sin^2\theta_1 = \frac{1 - \frac{n_1^2}{9n_2^2}}{\frac{8n_1^2}{9n_2^2}} = \frac{3}{4} \therefore \sin\theta_1 = \frac{\sqrt{3}}{2}$ $\boxed{\therefore \theta_1 = \frac{\pi}{3}}$

b) $E'_{0s} = r_s |E_{0s}| \therefore$

4.b) $E_{1S}' = r_S E_1$ $\therefore$ $\boxed{E_{1S}' = \dfrac{E_1 S}{2}}$; $E_{2S} = t_S E_{1S}$

$$t_S = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = \frac{2\cancel{n_1 \cos\theta_1}}{\cancel{n_1\cos\theta_1}\left(1 + \dfrac{n_2\cos\theta_2}{n_1\cos\theta_1}\right)}$$

Como do item a) temos:

$$\frac{n_2 \cos\theta_2}{n_1 \cos\theta_1} = \frac{1}{3}$$

$$\therefore \quad t_S = \frac{2}{1+\frac{1}{3}} = \frac{2}{4/3} \Rightarrow \boxed{t_S = \frac{3}{2}}$$

Daí:

$$\boxed{E_{2S} = \frac{3}{2} E_1 S}$$