ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

USP

Análise Dinâmica: método de Gibbs-Appell

Tarcisio A. H. Coelho

2015

Análise Dinâmica: método de Gibbs-Appell

Método de Newton-Euler:

- separação entre os elos da cadeia cinemática
- na formulação consideram-se as forças e momentos reativos
- no. de equações = λ.N

Método de Gibbs-Appel:

- não existe a necessidade da separação dos elos
- forças e momentos reativos não são considerados
- no. de equações = Mobilidade

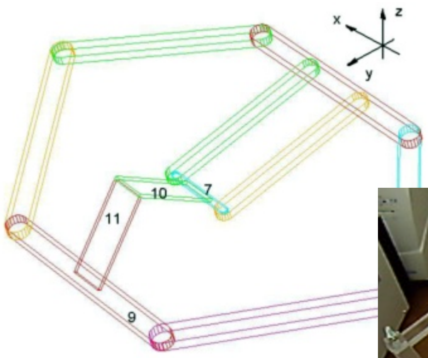
Método de Gibbs-Appell na forma proposta por KANE

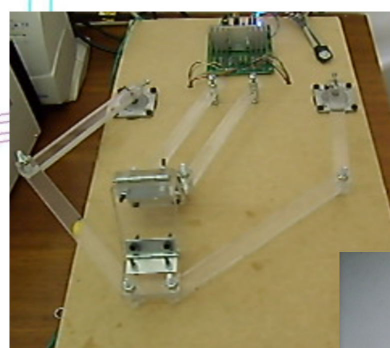
- Professor Emeritus. Stanford University. (1961-2012)

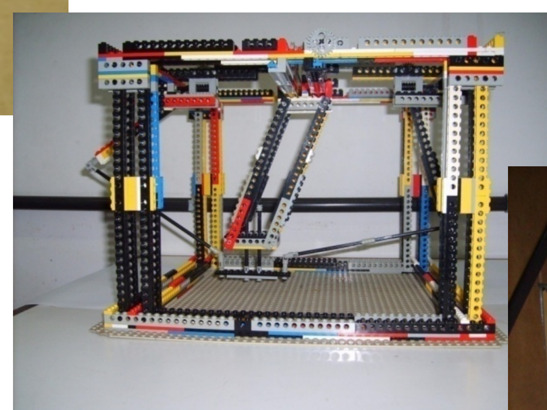
- Kane, T.R.; Levinson, D.A. *Dynamics, theory and applications.* McGraw-Hill, 1985

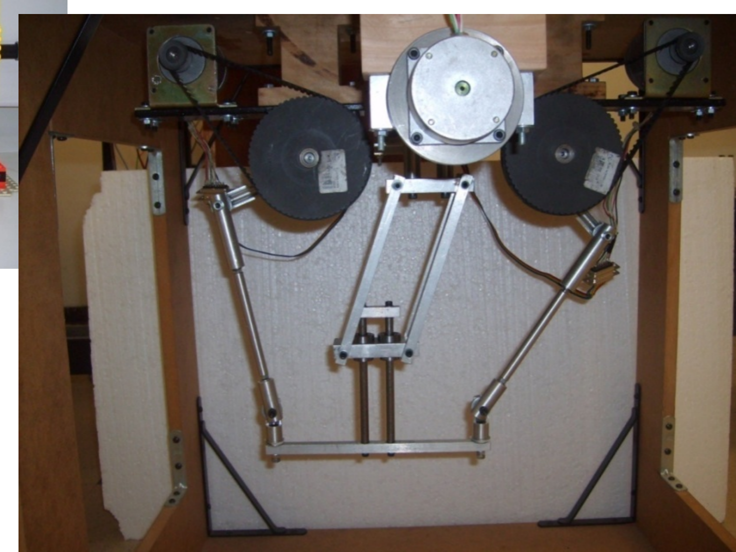
- Radestky, P. *The man who mastered motion* **Science**. May, 1986

Robô Manipulador – 3 gdl









vídeo

Método de Gibbs-Appell na forma proposta por KANE

$$F_i + F_i^* = 0 \qquad (i = 1, \ldots, n)$$

sendo

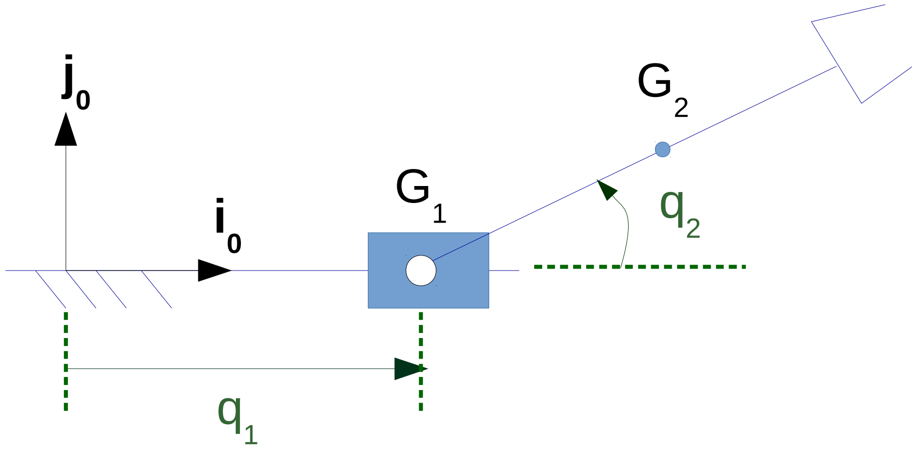
$$F_i = \sum_{k=1}^{N} \left(\sum \mathbf{F}_{\text{ativa,k}} \cdot \frac{\partial \mathbf{V}_{\mathbf{G_k}}}{\partial u_i} + \sum \mathbf{M}_{\text{ativa,k}} \cdot \frac{\partial \vec{\Omega}_k}{\partial u_i} \right)$$

$$F_i^* = -\sum_{k=1}^{N} \left(m_k \mathbf{a}_{\mathbf{G_k}} \cdot \frac{\partial \mathbf{V}_{\mathbf{G_k}}}{\partial u_i} + (I_k \dot{\vec{\Omega}}_k + \vec{\Omega}_k \wedge I_k \vec{\Omega}_k) \cdot \frac{\partial \vec{\Omega}_k}{\partial u_i} \right)$$

Análise Dinâmica: método de Gibbs-Appell

$$\boldsymbol{e}_{nx1} = \boldsymbol{0}_{nx1}$$

no. de velocidades generalizadas
ou coordenadas generalizadas



n = 2

N = 2

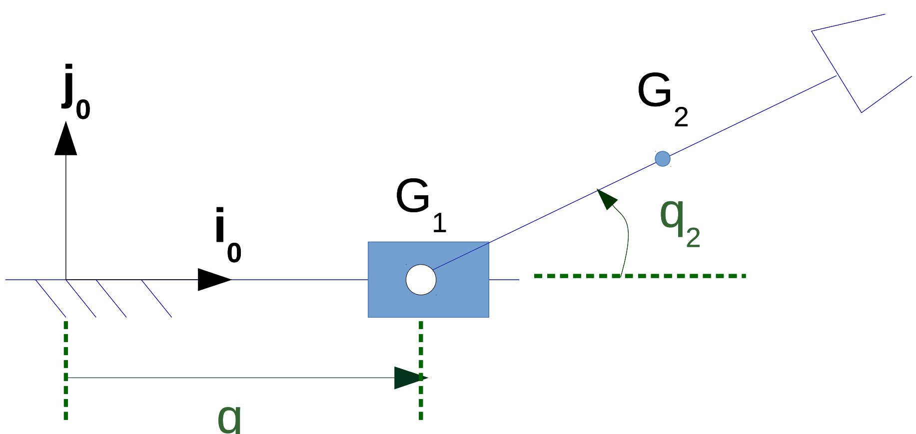
número de elos
móveis

Análise Dinâmica: método de Gibbs-Appell

$$\boldsymbol{e}_{2\boldsymbol{x}1}=\boldsymbol{0}_{2\boldsymbol{x}1}$$

$$e_i=\sum_{k=1}^{N}\left(\sum_{\square}\vec{F}_{at,k}-m\,\vec{a}_{G_k}\right).\left(\frac{\partial\vec{V}_{G_K}}{\partial\dot{q}_i}\right)+\left[\sum_{\square}\vec{M}_{at,k}-\left(I_k\dot{\vec{\omega}}_k+\vec{\omega}_k\times\left(I\,\vec{\omega}_k\right)\right)\right].\left(\frac{\partial\vec{\omega}_K}{\partial\dot{q}_i}\right)$$

$$1\leqslant i\leqslant 2$$



n = 2

Análise Dinâmica: método de Gibbs-Appell

elo 1

$$e_i=\boxed{\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_i}\right)+\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I\vec{\omega}_1\right)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_i}\right)}\ +$$

$$\left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_i}\right)+\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I\vec{\omega}_2\right)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_i}\right)$$

$$1\leqslant i\leqslant 2 \quad \Rightarrow \quad \vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_i = \left(\sum_{\square} \vec{F}_{at,1} - m\,\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_i}\right) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I\,\vec{\omega}_1\right)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_i}\right) +$$

$$\boxed{\left(\sum_{\square} \vec{F}_{at,2} - m\,\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_i}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I\,\vec{\omega}_2\right)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_i}\right)}$$

elo 2

$$1 \leqslant i \leqslant 2 \quad \Rightarrow \quad \vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1 = \left(\sum \vec{F}_{at,1} - m\,\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1}\right) + \left[\sum \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I\,\vec{\omega}_1\right)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1}\right) +$$

$$\left(\sum \vec{F}_{at,2} - m\,\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I\,\vec{\omega}_2\right)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$e_2 = \left(\sum \vec{F}_{at,1} - m\,\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_2}\right) + \left[\sum \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I\,\vec{\omega}_1\right)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_2}\right) +$$

$$\left(\sum \vec{F}_{at,2} - m\,\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I\,\vec{\omega}_2\right)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1=\left(\sum_{\square}\vec{F}_{at,1}-m\vec{a}_{G_1}\right).\left(\frac{\partial\vec{V}_{G_1}}{\partial\dot{q}_1}\right)+\left[\sum_{\square}\vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I\vec{\omega}_1\right)\right)\right].\left(\frac{\partial\vec{\omega}_1}{\partial\dot{q}_1}\right)+$$

$$\left(\sum_{\square}\vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(\frac{\partial\vec{V}_{G_2}}{\partial\dot{q}_1}\right)+\left[\sum_{\square}\vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I\vec{\omega}_2\right)\right)\right].\left(\frac{\partial\vec{\omega}_2}{\partial\dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$e_2=\left(\sum_{\square}\vec{F}_{at,1}-m\vec{a}_{G_1}\right).\left(\frac{\partial\vec{V}_{G_1}}{\partial\dot{q}_2}\right)+\left[\sum_{\square}\vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I\vec{\omega}_1\right)\right)\right].\left(\frac{\partial\vec{\omega}_1}{\partial\dot{q}_2}\right)+$$

$$\left(\sum_{\square}\vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(\frac{\partial\vec{V}_{G_2}}{\partial\dot{q}_2}\right)+\left[\sum_{\square}\vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I\vec{\omega}_2\right)\right)\right].\left(\frac{\partial\vec{\omega}_2}{\partial\dot{q}_2}\right)$$

Análise Dinâmica: método de Gibbs-Appell

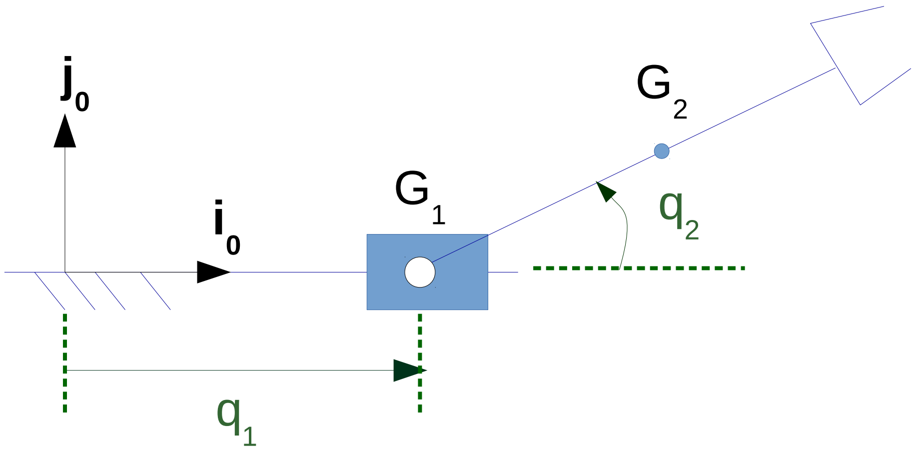
$$e_1 = \left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right) \cdot \left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I \vec{\omega}_1)\right)\right] \cdot \left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1}\right) +$$

$$\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right) \cdot \left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I \vec{\omega}_2)\right)\right] \cdot \left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{V}_{G_1} = \dot{q}_1 \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1} = \boldsymbol{i_0}$$



Análise Dinâmica: método de Gibbs-Appell

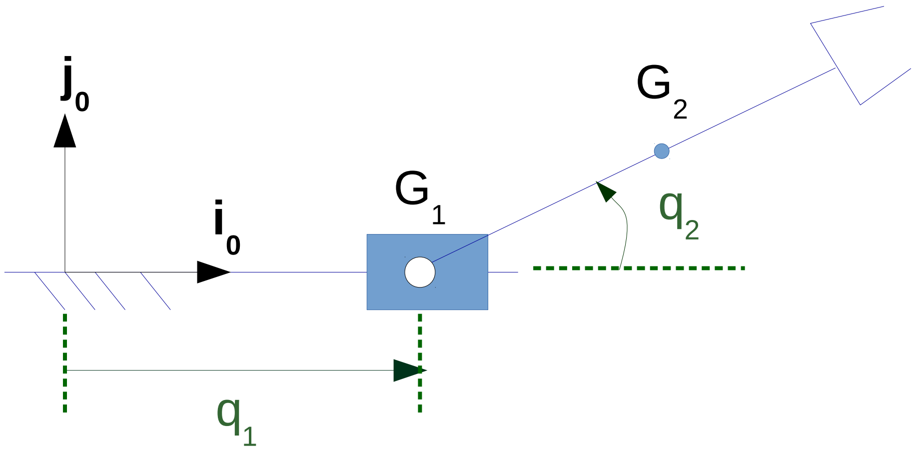
$$e_1=\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right).\left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1}\right)+\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I\vec{\omega}_1)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1}\right)+$$

$$\left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right)+\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times(I\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{\omega}_1=\mathbf{0}$$

$$\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1}=\mathbf{0}$$

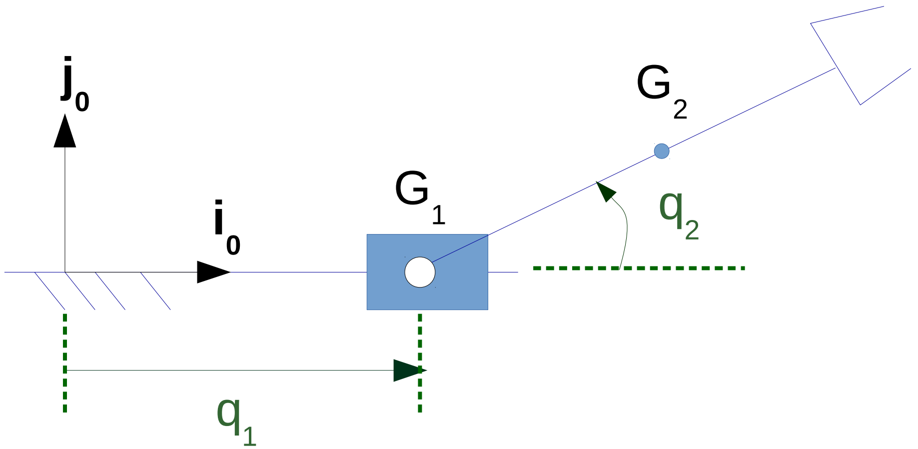


Análise Dinâmica: método de Gibbs-Appell

$$e_1 = \left(\sum_{\square} \vec{F}_{at,1} - m\,\vec{a}_{G_1}\right).(\mathbf{i_0}) + \left[\sum_{\square} \vec{M}_{at,1} - \left(I_1\,\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\,\vec{\omega}_1)\right)\right].(\mathbf{0}) \quad +$$

$$\left(\sum_{\square} \vec{F}_{at,2} - m\,\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right) + \left[\sum_{\square} \vec{M}_{at,2} - \left(I_2\,\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\,\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$



Análise Dinâmica: método de Gibbs-Appell

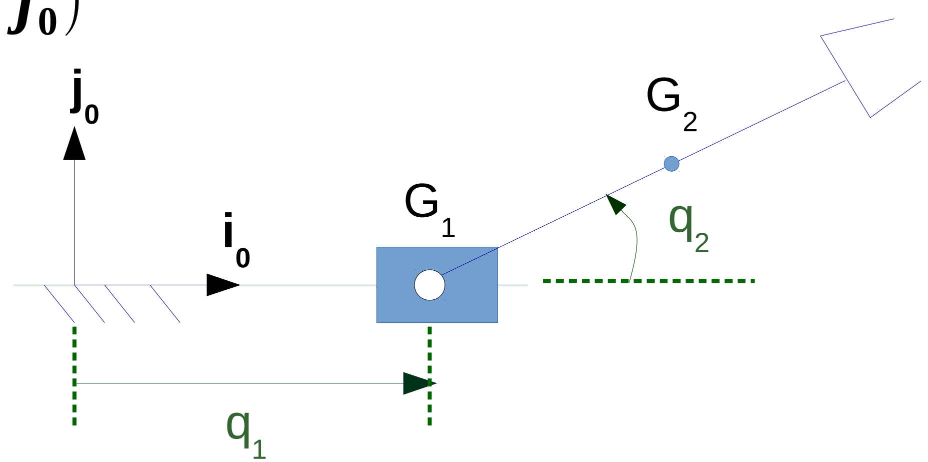
$$e_1 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\boldsymbol{i_0}) + \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1)\right)\right].(\boldsymbol{0}) \quad +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{V}_{G_2} = \dot{q}_1 \boldsymbol{i_0} + \dot{q}_2 L\left(-sen\, q_2 \boldsymbol{i_0} + \cos q_2 \boldsymbol{j_0}\right)$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1} = \boldsymbol{i_0}$$



Análise Dinâmica: método de Gibbs-Appell

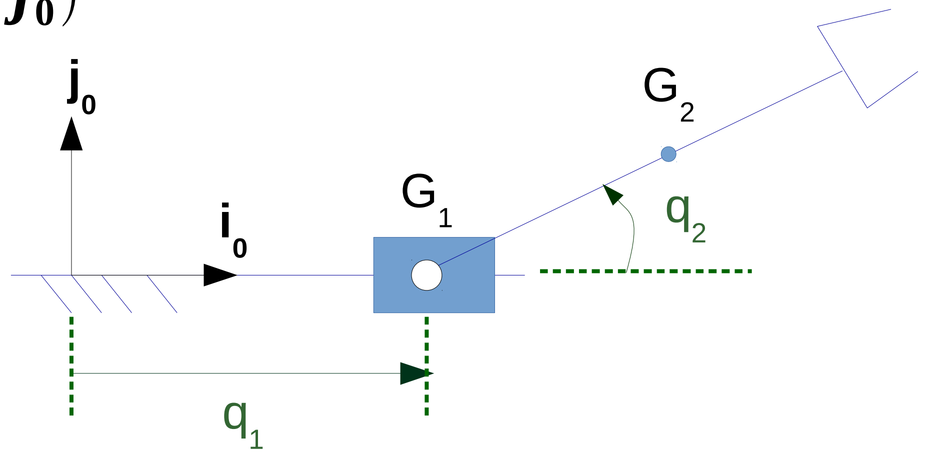
$$e_1=\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right).(\boldsymbol{i_0})+\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I\,\vec{\omega}_1)\right)\right].(\boldsymbol{0}) \quad +$$

$$\left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right).(\boldsymbol{i_0})+\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times(I\,\vec{\omega}_2)\right)\right].\left(\frac{\partial\vec{\omega}_2}{\partial\dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{V}_{G_2}=\dot{q}_1\boldsymbol{i_0}+\dot{q}_2L\left(-sen\,q_2\boldsymbol{i_0}+\cos q_2\boldsymbol{j_0}\right)$$

$$\frac{\partial\vec{V}_{G_2}}{\partial\dot{q}_1}=\boldsymbol{i_0}$$



Análise Dinâmica: método de Gibbs-Appell

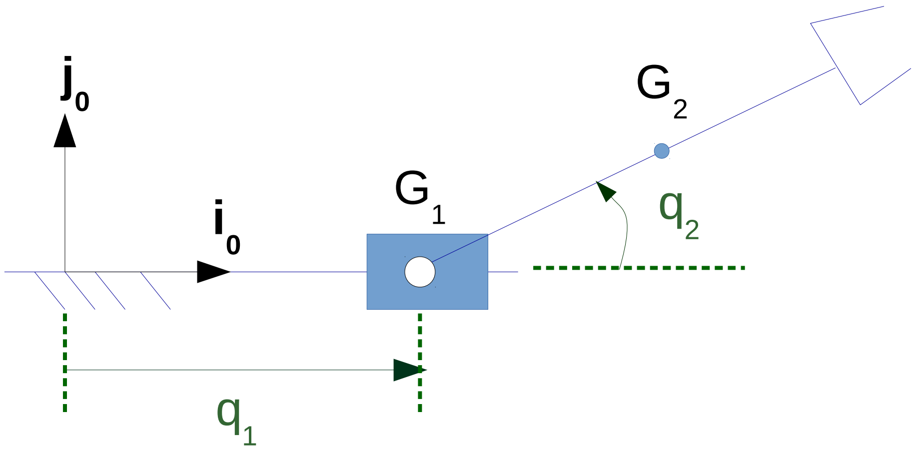
$$e_1 = \left(\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\boldsymbol{i_0}) + \left[\sum_{\square} \vec{M}_{at,1} - (I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1))\right].(\boldsymbol{0}) \quad +$$

$$\left(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}\right).(\boldsymbol{i_0}) + \left[\sum_{\square} \vec{M}_{at,2} - (I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2))\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1}\right)$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{\omega}_2 = \dot{q}_2 \boldsymbol{k_0}$$

$$\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1} = \boldsymbol{0}$$



Análise Dinâmica: método de Gibbs-Appell

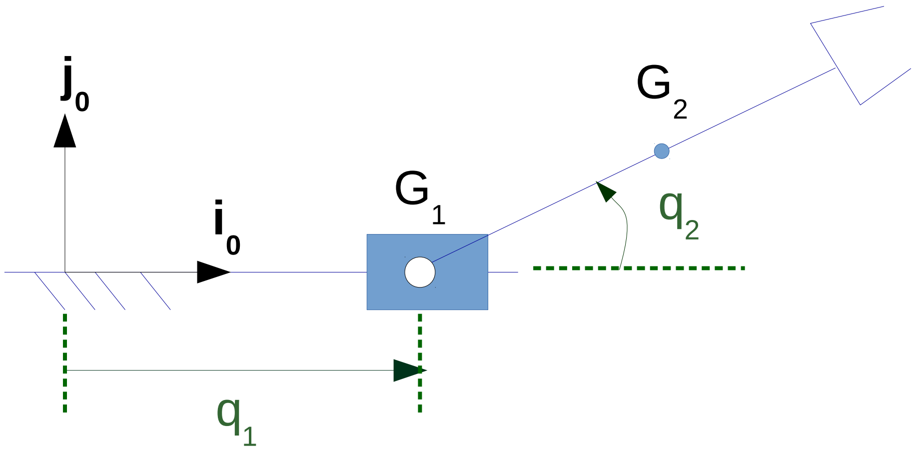
$$e_1 = (\sum_{\square} \vec{F}_{at,1} - m\vec{a}_{G_1}).(\boldsymbol{i_0}) + [\sum_{\square} \vec{M}_{at,1} - (I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1))].(\boldsymbol{0}) \quad +$$

$$(\sum_{\square} \vec{F}_{at,2} - m\vec{a}_{G_2}).(\boldsymbol{i_0}) + [\sum_{\square} \vec{M}_{at,2} - (I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2))].(\boldsymbol{0})$$

Derivadas parciais em relação a $\dot{q}_1$

$$\vec{\omega}_2 = \dot{q}_2 \boldsymbol{k_0}$$

$$\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1} = \boldsymbol{0}$$



Análise Dinâmica: método de Gibbs-Appell

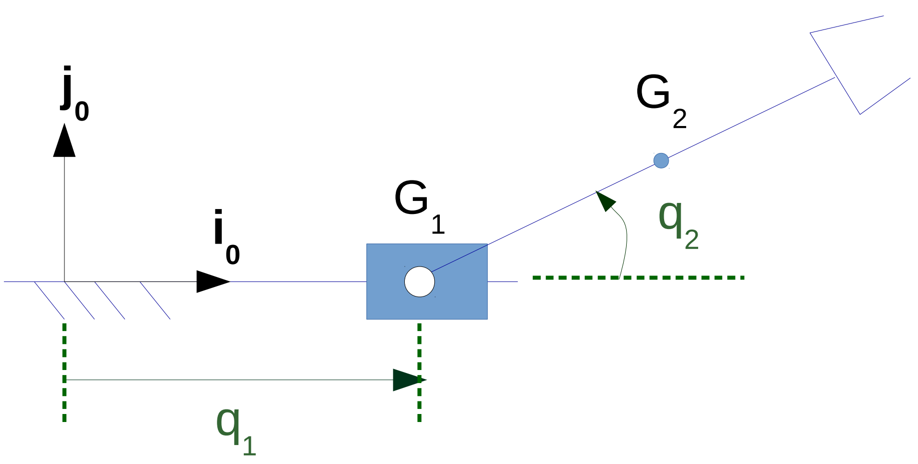
$$e_2 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right) . \left(\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_2}\right) + \left[\sum \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1)\right)\right] . \left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_2}\right) +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right) . \left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2)\right)\right] . \left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{V}_{G_1} = \dot{q}_1 \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_2} = \mathbf{0}$$



Análise Dinâmica: método de Gibbs-Appell

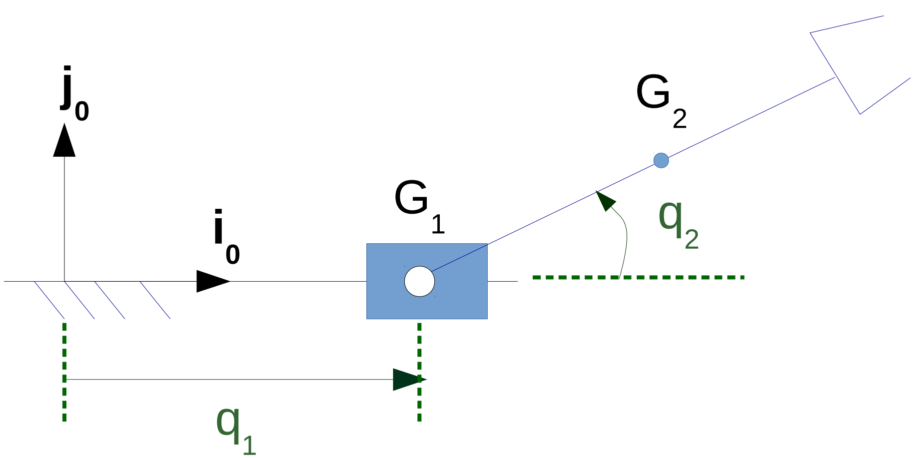
$$e_2 = \left(\sum \vec{F}_{at,1} - m\,\vec{a}_{G_1}\right).(\mathbf{0}) + \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\,\vec{\omega}_1)\right)\right].\left(\frac{\partial \vec{\omega}_1}{\partial \dot{q}_2}\right) +$$

$$\left(\sum \vec{F}_{at,2} - m\,\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\,\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{V}_{G_1} = \dot{q}_1 \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_2} = \mathbf{0}$$



Análise Dinâmica: método de Gibbs-Appell

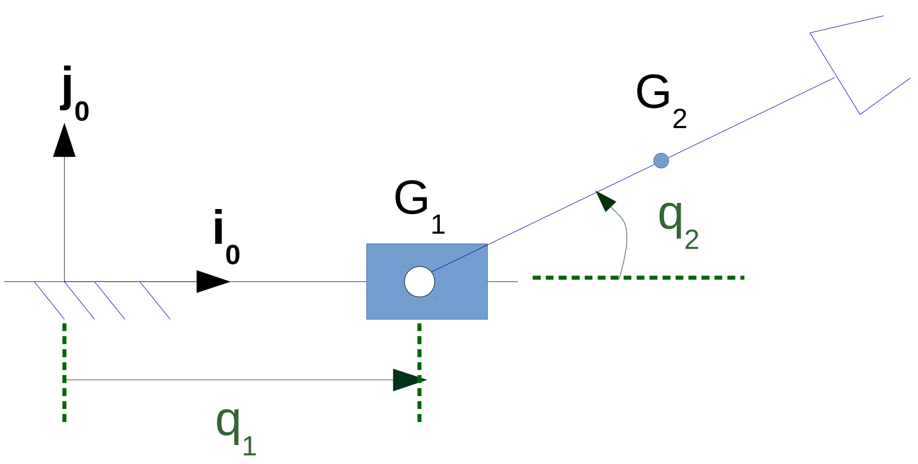
$$e_2=\left(\sum \vec{F}_{at,1}-m\,\vec{a}_{G_1}\right).(\mathbf{0})+\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I\,\vec{\omega}_1)\right)\right].\left(\frac{\partial\,\vec{\omega}_1}{\partial\,\dot{q}_2}\right) \quad +$$

$$\left(\sum \vec{F}_{at,2}-m\,\vec{a}_{G_2}\right).\left(\frac{\partial\,\vec{V}_{G_2}}{\partial\,\dot{q}_2}\right)+\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times(I\,\vec{\omega}_2)\right)\right].\left(\frac{\partial\,\vec{\omega}_2}{\partial\,\dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{\omega}_1=\mathbf{0}$$

$$\frac{\partial\,\vec{\omega}_1}{\partial\,\dot{q}_2}=\mathbf{0}$$



Análise Dinâmica: método de Gibbs-Appell

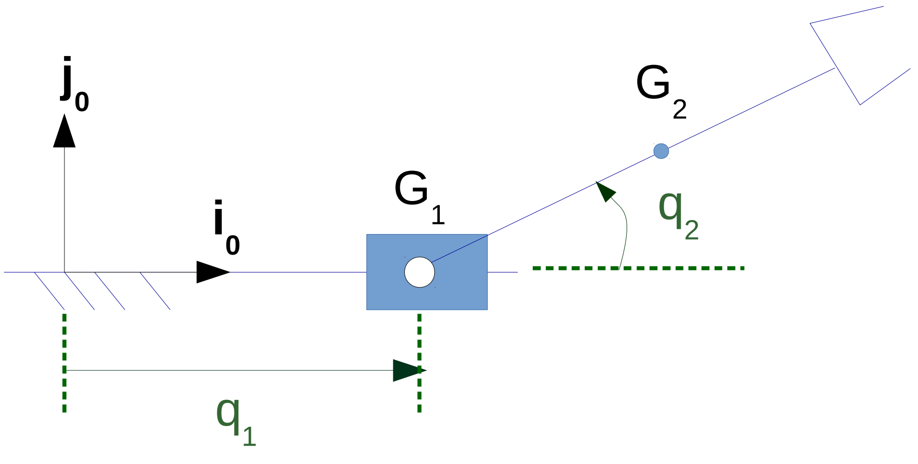
$$e_2 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right).(\mathbf{0}) + \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I\vec{\omega}_1)\right)\right].(\mathbf{0}) +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{\omega}_1 = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_1}{\partial \dot{q}_2} = \mathbf{0}$$



Análise Dinâmica: método de Gibbs-Appell

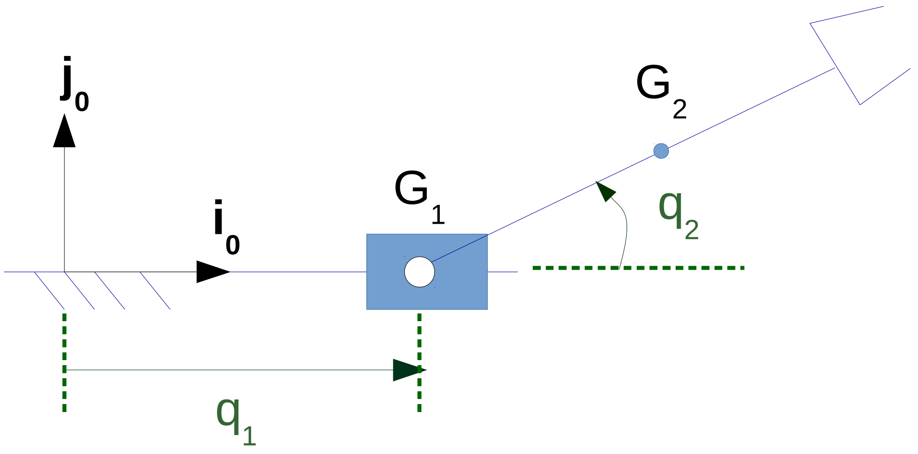
$$e_2=\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right).(\mathbf{0})+\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I\,\vec{\omega}_1)\right)\right].(\mathbf{0})\quad +$$

$$\left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}\right)+\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times(I\,\vec{\omega}_2)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{V}_{G_2}=\dot{q}_1\,\boldsymbol{i_0}+\dot{q}_2\,L\left(-sen\,q_2\,\boldsymbol{i_0}+\cos q_2\,\boldsymbol{j_0}\right)$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2}=L\left(-sen\,q_2\,\boldsymbol{i_0}+\cos q_2\,\boldsymbol{j_0}\right)$$



Análise Dinâmica: método de Gibbs-Appell

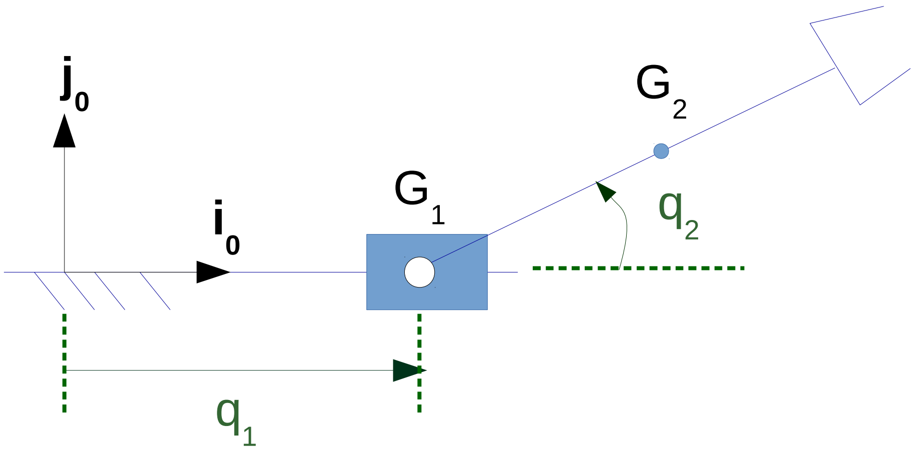
$$e_2 = \left(\sum \vec{F}_{at,1} - m\,\vec{a}_{G_1}\right).\left(\mathbf{0}\right) + \left[\sum \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I_1 \vec{\omega}_1\right)\right)\right].\left(\mathbf{0}\right) +$$

$$\left(\sum \vec{F}_{at,2} - m\,\vec{a}_{G_2}\right).\left(L\left(-sen\,q_2\,\boldsymbol{i_0} + \cos q_2\,\boldsymbol{j_0}\right)\right) + \left[\sum \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I_2 \vec{\omega}_2\right)\right)\right].\left(\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{V}_{G_2} = \dot{q}_1\,\boldsymbol{i_0} + \dot{q}_2\,L\left(-sen\,q_2\,\boldsymbol{i_0} + \cos q_2\,\boldsymbol{j_0}\right)$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2} = L\left(-sen\,q_2\,\boldsymbol{i_0} + \cos q_2\,\boldsymbol{j_0}\right)$$



Análise Dinâmica: método de Gibbs-Appell

$$e_2 = \Big(\sum_{\square} \vec{F}_{at,1} - m\,\vec{a}_{G_1}\Big).(\mathbf{0}) + \Big[\sum_{\square} \vec{M}_{at,1} - \big(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)\big)\Big].(\mathbf{0}) +$$

$$\Big(\sum_{\square} \vec{F}_{at,2} - m\,\vec{a}_{G_2}\Big).\big(L(-sen\,q_2\,\boldsymbol{i_0} + \cos q_2\,\boldsymbol{j_0})\big) + \Big[\sum_{\square} \vec{M}_{at,2} - \big(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2\vec{\omega}_2)\big)\Big].(\boldsymbol{k_0})$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right] \begin{bmatrix} \Big(\sum_{\square} \vec{F}_{at,1} - m\,\vec{a}_{G_1}\Big) \\ \Big[\sum_{\square} \vec{M}_{at,1} - \big(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1)\big)\Big] \\ \Big(\sum_{\square} \vec{F}_{at,2} - m\,\vec{a}_{G_2}\Big) \\ \Big[\sum_{\square} \vec{M}_{at,2} - \big(I_2\dot{\vec{\omega}}_1 + \vec{\omega}_2 \times (I_2\vec{\omega}_2)\big)\Big] \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_2=\left(\sum_{\square}\vec{F}_{at,1}-m\vec{a}_{G_1}\right).(\mathbf{0})+\left[\sum_{\square}\vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right].(\mathbf{0})\quad+$$

$$\left(\sum_{\square}\vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(L\left(-sen\,q_2\,\boldsymbol{i}_{\mathbf{0}}+\cos q_2\,\boldsymbol{j}_{\mathbf{0}}\right)\right)+\left[\sum_{\square}\vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right].(\boldsymbol{k}_{\mathbf{0}})$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e}=\begin{bmatrix}e_1\\ e_2\end{bmatrix}=\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & \boxed{0} & -L\,senq_2 & L\,cosq_2 & 1\end{array}\right]\begin{bmatrix}\left(\sum_{\square}\vec{F}_{at,1}-m\vec{a}_{G_1}\right)\\ \left[\sum_{\square}\vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right]\\ \left(\sum_{\square}\vec{F}_{at,2}-m\vec{a}_{G_2}\right)\\ \left[\sum_{\square}\vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right]\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_2=\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right).(\mathbf{0})+\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right].(\mathbf{0})\quad +$$

$$\left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(L\left(-sen\,q_2\,\boldsymbol{i_0}+\cos q_2\,\boldsymbol{j_0}\right)\right)+\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right].(\boldsymbol{k_0})$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e}=\begin{bmatrix}e_1\\ e_2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1\end{bmatrix}\begin{bmatrix}\left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right)\\ \left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right]\\ \left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right)\\ \left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right]\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_2=\left(\sum \vec F_{at,1}-m\,\vec a_{G_1}\right).\left(\mathbf{0}\right)+\left[\sum \vec M_{at,1}-\left(I_1\dot{\vec\omega}_1+\vec\omega_1\times\left(I_1\vec\omega_1\right)\right)\right].\left(\mathbf{0}\right)+$$

$$\left(\sum \vec F_{at,2}-m\,\vec a_{G_2}\right).\left(L\left(-sen\,q_2\,\boldsymbol{i_0}+\cos q_2\,\boldsymbol{j_0}\right)\right)+\left[\sum \vec M_{at,2}-\left(I_2\dot{\vec\omega}_2+\vec\omega_2\times\left(I_2\vec\omega_2\right)\right)\right].\left(\boldsymbol{k_0}\right)$$

Derivadas parciais em relação a $\dot q_2$

$$\vec e=\begin{bmatrix}e_1\\ e_2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & \boxed{1}\end{bmatrix}\begin{bmatrix}\left(\sum \vec F_{at,1}-m\,\vec a_{G_1}\right)\\ \left[\sum \vec M_{at,1}-\left(I_1\dot{\vec\omega}_1+\vec\omega_1\times\left(I_1\vec\omega_1\right)\right)\right]\\ \left(\sum \vec F_{at,2}-m\,\vec a_{G_2}\right)\\ \left[\sum \vec M_{at,2}-\left(I_2\dot{\vec\omega}_1+\vec\omega_2\times\left(I_2\vec\omega_2\right)\right)\right]\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1=\left(\sum_{\square}\vec{F}_{at,1}-m\vec{a}_{G_1}\right).\left(\boldsymbol{i_0}\right)+\left[\sum_{\square}\vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I\,\vec{\omega}_1\right)\right)\right].\left(\mathbf{0}\right)\;+$$

$$\left(\sum_{\square}\vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(\boldsymbol{i_0}\right)+\left[\sum_{\square}\vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I\,\vec{\omega}_2\right)\right)\right].\left(\mathbf{0}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e}=\begin{bmatrix}e_1\\ e_2\end{bmatrix}=\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1\end{array}\right]\begin{bmatrix}\left(\sum_{\square}\vec{F}_{at,1}-m\vec{a}_{G_1}\right)\\ \left[\sum_{\square}\vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right]\\ \left(\sum_{\square}\vec{F}_{at,2}-m\vec{a}_{G_2}\right)\\ \left[\sum_{\square}\vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right]\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1 = \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right).\left(\boldsymbol{i_0}\right) + \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I\,\vec{\omega}_1\right)\right)\right].\left(\mathbf{0}\right) +$$

$$\left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right).\left(\boldsymbol{i_0}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I\,\vec{\omega}_2\right)\right)\right].\left(\mathbf{0}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right] \begin{bmatrix} \left(\sum \vec{F}_{at,1} - m\vec{a}_{G_1}\right) \\ \left[\sum \vec{M}_{at,1} - \left(I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I_1\vec{\omega}_1\right)\right)\right] \\ \left(\sum \vec{F}_{at,2} - m\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2} - \left(I_2\dot{\vec{\omega}}_1 + \vec{\omega}_2 \times \left(I_2\vec{\omega}_2\right)\right)\right] \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1=\left(\sum_{\square}\vec{F}_{at,1}-m\vec{a}_{G_1}\right).\left(\boldsymbol{i_0}\right)+\left[\sum_{\square}\vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I\,\vec{\omega}_1\right)\right)\right].\left(\mathbf{0}\right)\ +$$

$$\left(\sum_{\square}\vec{F}_{at,2}-m\vec{a}_{G_2}\right).\left(\boldsymbol{i_0}\right)+\left[\sum_{\square}\vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I\,\vec{\omega}_2\right)\right)\right].\left(\mathbf{0}\right)$$

Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e}=\begin{bmatrix}e_1\\e_2\end{bmatrix}=\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&0&0&-L\,senq_2&L\,cosq_2&1\end{array}\right]\begin{bmatrix}\left(\sum_{\square}\vec{F}_{at,1}-m\vec{a}_{G_1}\right)\\\left[\sum_{\square}\vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right]\\\left(\sum_{\square}\vec{F}_{at,2}-m\vec{a}_{G_2}\right)\\\left[\sum_{\square}\vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right]\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$e_1 = \left(\sum \vec{F}_{at,1} - m\,\vec{a}_{G_1}\right).\left(\boldsymbol{i_0}\right) + \left[\sum \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I\,\vec{\omega}_1\right)\right)\right].\left(\mathbf{0}\right) \; +$$

$$\left(\sum \vec{F}_{at,2} - m\,\vec{a}_{G_2}\right).\left(\boldsymbol{i_0}\right) + \left[\sum \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times \left(I\,\vec{\omega}_2\right)\right)\right].\left(\mathbf{0}\right)$$

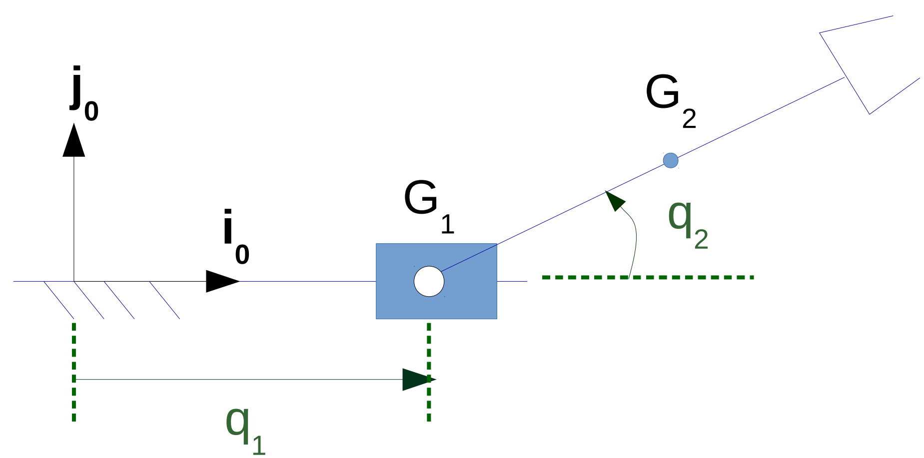
Derivadas parciais em relação a $\dot{q}_2$

$$\vec{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right] \begin{bmatrix} \left(\sum \vec{F}_{at,1} - m_1 \vec{a}_{G_1}\right) \\ \left[\sum \vec{M}_{at,1} - \left(I_1 \dot{\vec{\omega}}_1 + \vec{\omega}_1 \times \left(I_1 \vec{\omega}_1\right)\right)\right] \\ \left(\sum \vec{F}_{at,2} - m_2 \vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2} - \left(I_2 \dot{\vec{\omega}}_1 + \vec{\omega}_2 \times \left(I_2 \vec{\omega}_2\right)\right)\right] \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\begin{bmatrix} \left(\sum \vec{F}_{at,1}-m\vec{a}_{G_1}\right) \\ \left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right] \\ \left(\sum \vec{F}_{at,2}-m\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right] \end{bmatrix}$$

$$\vec{F}_{at,1}-m\vec{a}_{G_1}$$

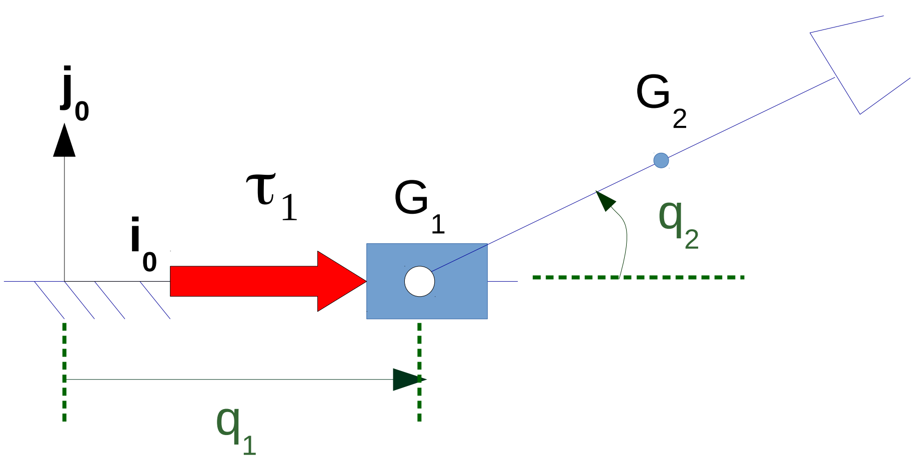


Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\begin{bmatrix} \boxed{\left(\sum \vec{F}_{at,1}-m_1\vec{a}_{G_1}\right)} \\ \left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right] \\ \hline \left(\sum \vec{F}_{at,2}-m_2\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_1+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right] \end{bmatrix}$$

$$\vec{F}_{at,1}-m_1\vec{a}_{G_1}$$

$$\boxed{\tau_1 \boldsymbol{i_0}}$$

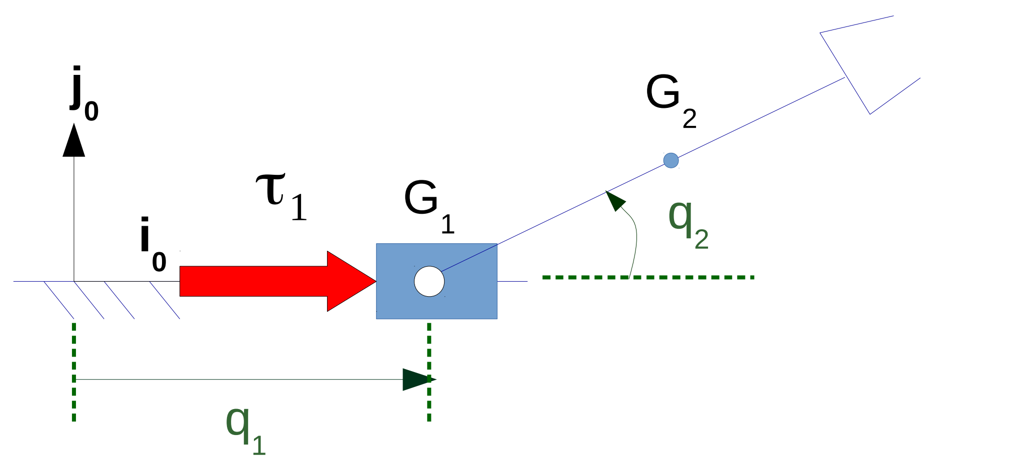


Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right]\left[\begin{array}{c} \left(\sum \vec{F}_{at,1}-m_1\vec{a}_{G_1}\right) \\ \left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right] \\ \hline \left(\sum \vec{F}_{at,2}-m_2\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right] \end{array}\right]$$

$$\vec{F}_{at,1}-m_1\vec{a}_{G_1}$$

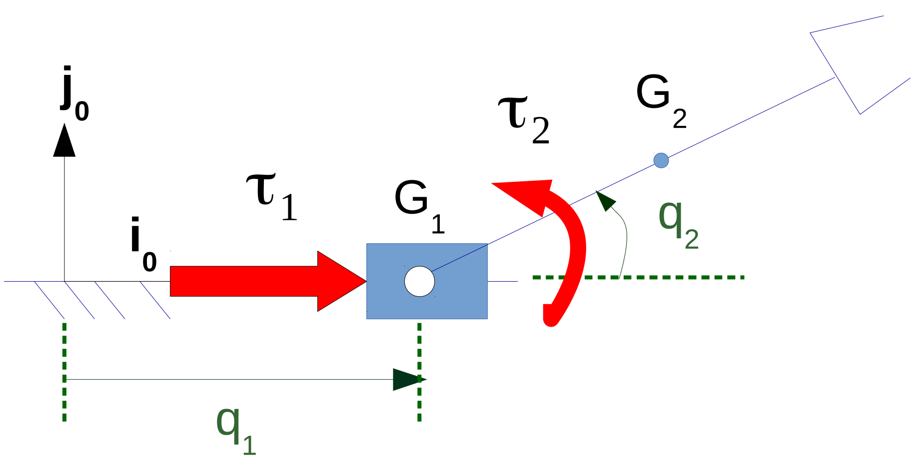
$$\vec{a}_{G_1}=\ddot{q}_1\,\boldsymbol{i_0}$$



Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\begin{bmatrix} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ \left[\sum \vec{M}_{at,1}-(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I_1\vec{\omega}_1))\right] \\ \left(\sum \vec{F}_{at,2}-m_2\vec{a}_{G_2}\right) \\ \left[\sum \vec{M}_{at,2}-(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times(I_2\vec{\omega}_2))\right] \end{bmatrix}$$

$$\left[\underbrace{\sum \vec{M}_{at,1}}_{-\tau_2 \boldsymbol{k_0}}-\underbrace{(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times(I_1\vec{\omega}_1))}_{\mathbf{0}}\right]$$

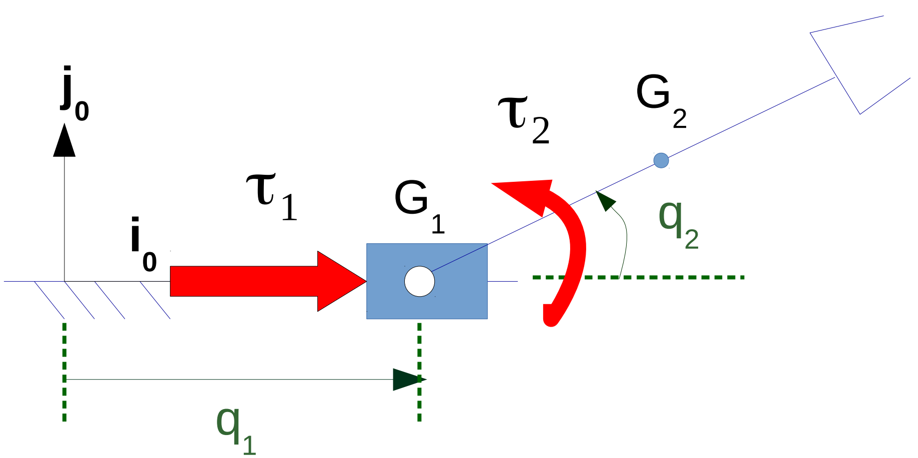


Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\begin{bmatrix} (\tau_1 - m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline (\sum \vec{F}_{at,2} - m_2\vec{a}_{G_2}) \\ [\sum \vec{M}_{at,2} - (I_2\dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2\vec{\omega}_2))] \end{bmatrix}$$

$$[\sum \vec{M}_{at,1} - (I_1\dot{\vec{\omega}}_1 + \vec{\omega}_1 \times (I_1\vec{\omega}_1))]$$

$$\underbrace{\sum \vec{M}_{at,1}}_{-\tau_2 \boldsymbol{k}_0} \qquad \underbrace{\vec{\omega}_1 \times (I_1\vec{\omega}_1)}_{\mathbf{0}}$$



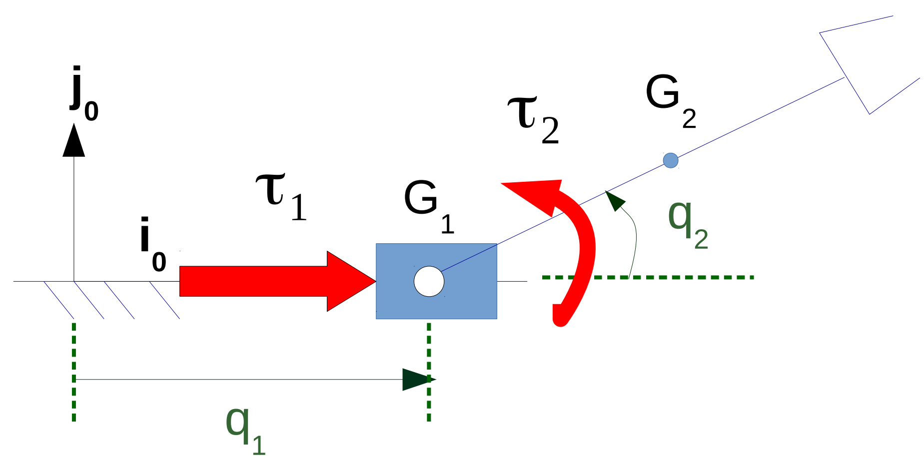
Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right]\left[\begin{array}{c} (\tau_1 - m_1 \ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline (\sum \vec{F}_{at,2} - m_2 \vec{a}_{G_2}) \\ \boxed{[\sum \vec{M}_{at,2} - (I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2 \vec{\omega}_2))]} \end{array}\right]$$

$$[\sum \vec{M}_{at,2} - (I_2 \dot{\vec{\omega}}_2 + \vec{\omega}_2 \times (I_2 \vec{\omega}_2))]$$

$\tau_2 \boldsymbol{k_0}$

$-I_2 \ddot{q}_2 \boldsymbol{k_0}$



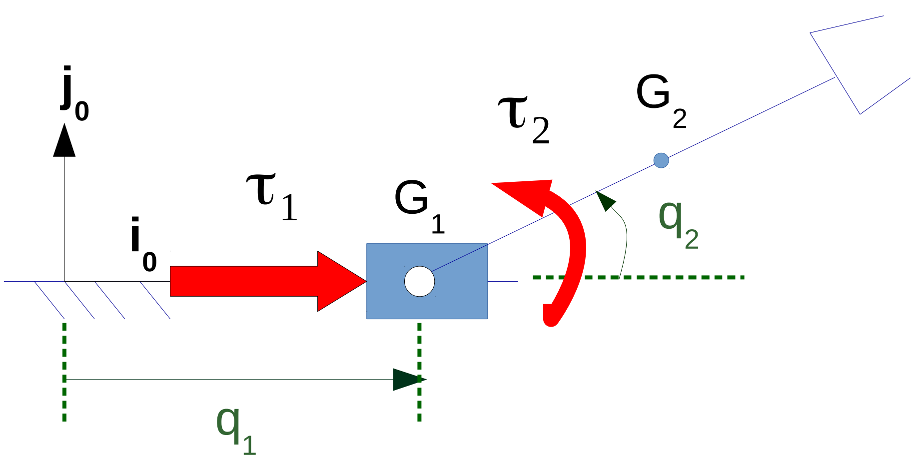
Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\begin{bmatrix} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline (\sum \vec{F}_{at,2}-m_2\vec{a}_{G_2}) \\ \tau_2-I_2\ddot{q}_2 \end{bmatrix}$$

$$[\sum \vec{M}_{at,2}-(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times(I_2\vec{\omega}_2))]$$

$$\tau_2\boldsymbol{k_0}$$

$$-I_2\ddot{q}_2\boldsymbol{k_0}$$

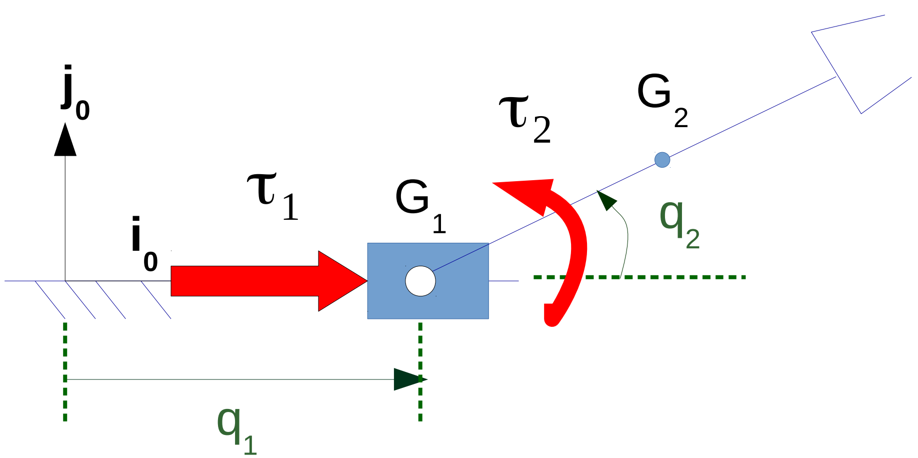


Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right]\begin{bmatrix} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline \left(\sum \vec{F}_{at,2}-m_2\vec{a}_{G_2}\right) \\ \tau_2-I_2\ddot{q}_2 \end{bmatrix}$$

$$\vec{F}_{at,2}-m_2\vec{a}_{G_2}$$

$$\boxed{\mathbf{0}}$$



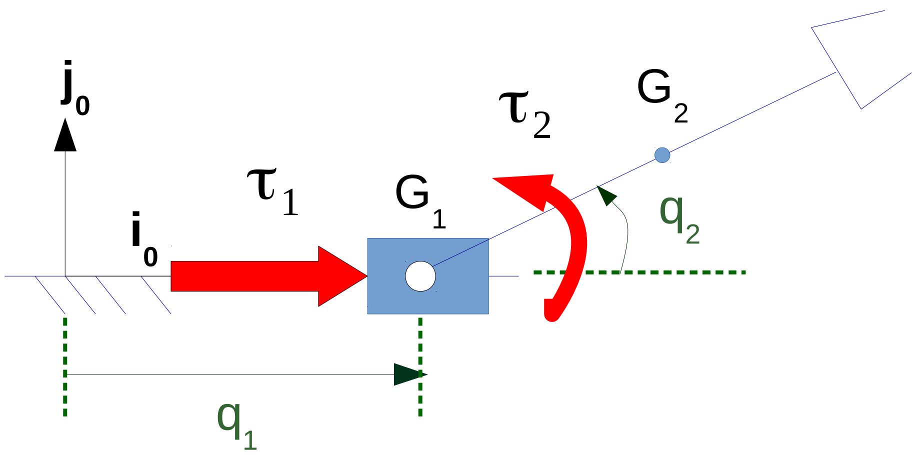
$$\vec{a}_{G_2}=\ddot{q}_1\,\boldsymbol{i_0}+\ddot{q}_2\,L\left(-sen\,q_2\,\boldsymbol{i_0}+\cos q_2\,\boldsymbol{j_0}\right)-\dot{q}_2{}^2\,L\left(\cos q_2\,\boldsymbol{i_0}+sen\,q_2\,\boldsymbol{j_0}\right)$$

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\begin{bmatrix} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline -m_2[\ddot{q}_1-L(\ddot{q}_2 senq_2+\dot{q}_2{}^2 cosq_2)] \\ -m_2[L(\ddot{q}_2 cosq_2-\dot{q}_2{}^2 senq_2)] \\ \tau_2-I_2\ddot{q}_2 \end{bmatrix}$$

$$\vec{F}_{at,2}-m_2\vec{a}_{G_2}$$

$$\mathbf{0}$$



$$\vec{a}_{G_2}=\ddot{q}_1\boldsymbol{i_0}+\ddot{q}_2 L(-sen\,q_2\boldsymbol{i_0}+\cos q_2\boldsymbol{j_0})-\dot{q}_2{}^2 L(\cos q_2\boldsymbol{i_0}+sen\,q_2\boldsymbol{j_0})$$

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1 \end{array}\right]\left[\begin{array}{c} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline -m_2[\ddot{q}_1-L(\ddot{q}_2\,senq_2+\dot{q}_2{}^2\,cosq_2)] \\ -m_2[L(\ddot{q}_2\,cosq_2-\dot{q}_2{}^2\,senq_2)] \\ \tau_2-I_2\dot{\omega}_2 \end{array}\right]$$

$$\begin{bmatrix} \tau_1-m_1\ddot{q}_1-m_2[\ddot{q}_1-L(\ddot{q}_2\,senq_2+\dot{q}_2{}^2\,cosq_2)] \\ m_2[\ddot{q}_1-L(\ddot{q}_2\,senq_2+\dot{q}_2{}^2\,cosq_2)]\,Lsenq_2-m_2[L(\ddot{q}_2\,cosq_2-\dot{q}_2{}^2\,senq_2)]\,Lcosq_2+\tau_2-I_2\ddot{q}_2 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}=\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -L\,sen q_2 & L\,cos q_2 & 1 \end{array}\right]\left[\begin{array}{c} (\tau_1-m_1\ddot{q}_1) \\ 0 \\ -\tau_2 \\ \hline -m_2[\ddot{q}_1-L(\ddot{q}_2\,sen q_2+\dot{q}_2^{\,2}\,cos q_2)] \\ -m_2[L(\ddot{q}_2\,cos q_2-\dot{q}_2^{\,2}\,sen q_2)] \\ \tau_2-I_2\dot{\omega}_2 \end{array}\right]$$

$$\begin{bmatrix} \tau_1-m_1\ddot{q}_1-m_2[\ddot{q}_1-L(\ddot{q}_2\,sen q_2+\dot{q}_2^{\,2}\,cos q_2)] \\ m_2[\ddot{q}_1-L(\ddot{q}_2\,sen q_2+\dot{q}_2^{\,2}\,cos q_2)]\,L sen q_2-m_2[L(\ddot{q}_2\,cos q_2-\dot{q}_2^{\,2}\,sen q_2)]\,L cos q_2+\tau_2-I_2\ddot{q}_2 \end{bmatrix}$$

$$\begin{bmatrix} \tau_1-(m_1+m_2)\ddot{q}_1+(m_2 L\,sen q_2)\ddot{q}_2+(m_2 L\,cos q_2)\dot{q}_2^{\,2} \\ m_2 L sen q_2\,\ddot{q}_1-(m_2 L^2+I_2)\ddot{q}_2+\tau_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}=\begin{bmatrix}e_1\\ e_2\end{bmatrix}=\left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -L\,senq_2 & L\,cosq_2 & 1\end{array}\right]\left[\begin{array}{c}(\tau_1-m_1\ddot{q}_1)\\ 0\\ -\tau_2\\ \hline -m_2[\ddot{q}_1-L(\ddot{q}_2 senq_2+\dot{q}_2^{\,2}cosq_2)]\\ -m_2[L(\ddot{q}_2 cosq_2-\dot{q}_2^{\,2}senq_2)]\\ \tau_2-I_2\dot{\omega}_2\end{array}\right]$$

$$\begin{bmatrix}\tau_1-(m_1+m_2)\ddot{q}_1+(m_2 L\,senq_2)\ddot{q}_2+(m_2 L\,cosq_2)\dot{q}_2^{\,2}\\ m_2 Lsenq_2\ddot{q}_1-(m_2 L^2+I_2)\ddot{q}_2+\tau_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

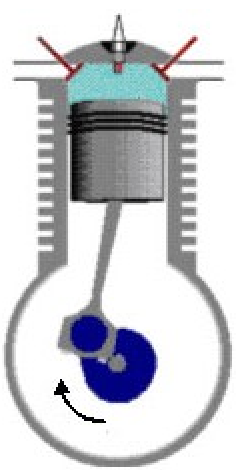
$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\begin{bmatrix}(m_1+m_2)\ddot{q}_1-(m_2 L\,senq_2)\ddot{q}_2-(m_2 L\,cosq_2)\dot{q}_2^{\,2}\\ -m_2 Lsenq_2\ddot{q}_1+(m_2 L^2+I_2)\ddot{q}_2\end{bmatrix}$$

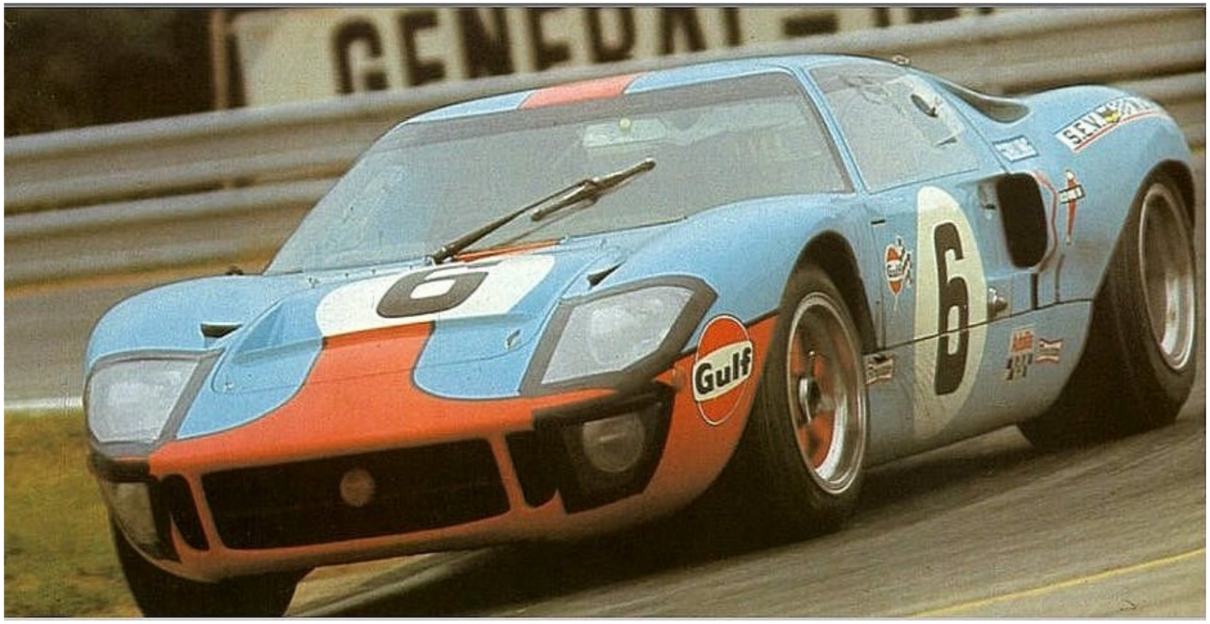
Tipos de carregamentos sobre o mecanismo

Motor de combustão interna　　　Ford GT 40 Mk I

Rotação 5000 rpm = 523,6 rad/s (torque máximo 447 Nm)
Curso 73mm, potência 385 HP

Aceleração pistão　$A\,\omega^2$ = 10.000 m/s^2 = 1000 G　$\longrightarrow$　Predomínio das forças de inércia





Tipos de carregamentos sobre o mecanismo

Veloc. Máx. = 10 m/s

Acel. Máx. = 150 m/s^2 = 15 G

Predomínio das forças de inércia

Tipos de carregamentos sobre o mecanismo

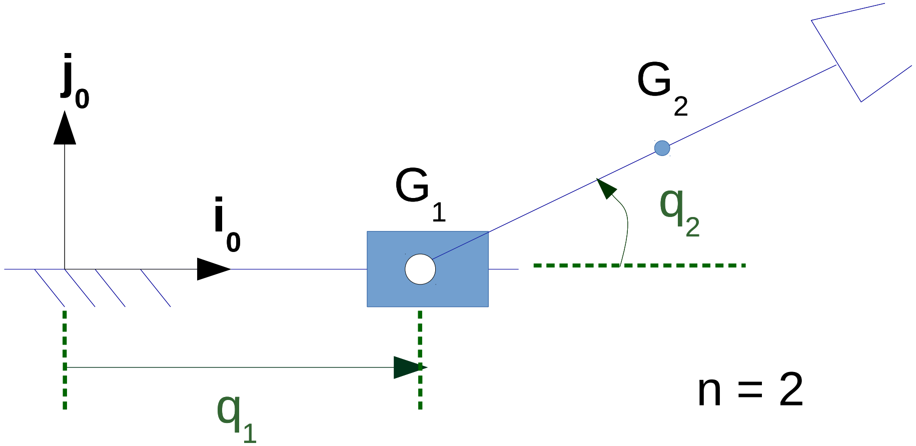
Predomínio da força de campo gravitacional

Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}_{nx1} = D_{nx\lambda N}\,\boldsymbol{f}_{\boldsymbol{\lambda N x 1}} = \mathbf{0}_{nx1}$$

$$D_{nx\lambda N} = \left[\begin{array}{cc|cc|c|cc}
\frac{\partial \mathbf{v}_{G_1}^T}{\partial u_1} & \frac{\partial \omega_1^T}{\partial u_1} & \frac{\partial \mathbf{v}_{G_2}^T}{\partial u_1} & \frac{\partial \omega_2^T}{\partial u_1} & \cdots & \frac{\partial \mathbf{v}_{G_N}^T}{\partial u_1} & \frac{\partial \omega_N^T}{\partial u_1} \\
\frac{\partial \mathbf{v}_{G_1}^T}{\partial u_2} & \frac{\partial \omega_1^T}{\partial u_2} & \frac{\partial \mathbf{v}_{G_2}^T}{\partial u_2} & \frac{\partial \omega_2^T}{\partial u_2} & \cdots & \frac{\partial \mathbf{v}_{G_N}^T}{\partial u_2} & \frac{\partial \omega_N^T}{\partial u_2} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{\partial \mathbf{v}_{G_1}^T}{\partial u_n} & \frac{\partial \omega_1^T}{\partial u_n} & \frac{\partial \mathbf{v}_{G_2}^T}{\partial u_n} & \frac{\partial \omega_2^T}{\partial u_n} & \cdots & \frac{\partial \mathbf{v}_{G_N}^T}{\partial u_n} & \frac{\partial \omega_N^T}{\partial u_n}
\end{array}\right]$$

$$u_i = \dot{q}_i$$

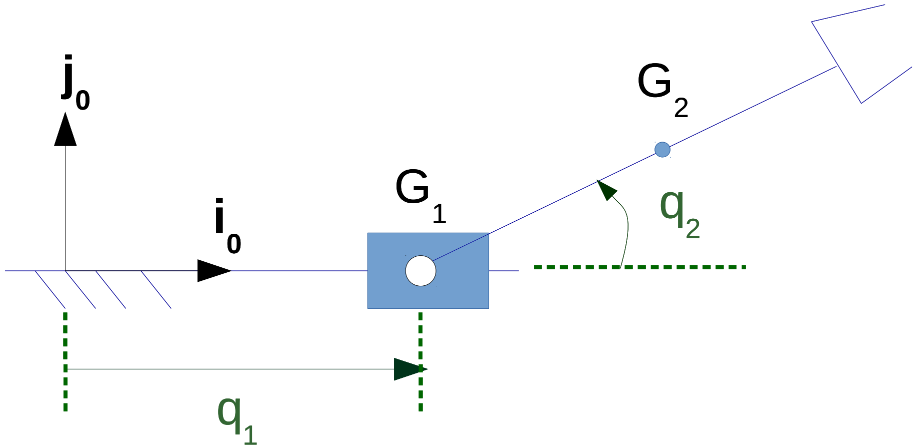


Análise Dinâmica: método de Gibbs-Appell

$$\vec{e}_{nx1} = D_{nx\lambda N} \boldsymbol{f}_{\boldsymbol{\lambda} N x 1} = \mathbf{0}_{nx1}$$

$$\mathbf{f}_{\lambda N x 1} = \begin{bmatrix} \sum \mathbf{F}_{at,1} - m_1 \mathbf{a}_{G_1} \\ \sum \mathbf{M}_{at,1} - (I_k \dot{\omega}_1 + \omega_1 \times (I_1 \omega_1)) \\ \sum \mathbf{F}_{at,2} - m_2 \mathbf{a}_{G_2} \\ \sum \mathbf{M}_{at,2} - (I_2 \dot{\omega}_2 + \omega_2 \times (I_2 \omega_2)) \\ \vdots \\ \sum \mathbf{F}_{at,N} - m_N \mathbf{a}_{G_N} \\ \sum \mathbf{M}_{at,N} - (I_N \dot{\omega}_N + \omega_N \times (I_N \omega_N)) \end{bmatrix}$$

Válida para mecanismos de CADEIA ABERTA !!



Análise Dinâmica: método de Gibbs-Appell

$$C_{mxn}^{\,T}\vec{e}_{nx1}=C_{mxn}^{\,T}D_{nx\lambda N}\boldsymbol{f}_{\boldsymbol{\lambda N x 1}}=\boldsymbol{0}_{mx1}$$

m = Mobilidade

Válida para mecanismos de CADEIA FECHADA !!

Análise Dinâmica: método de Gibbs-Appell

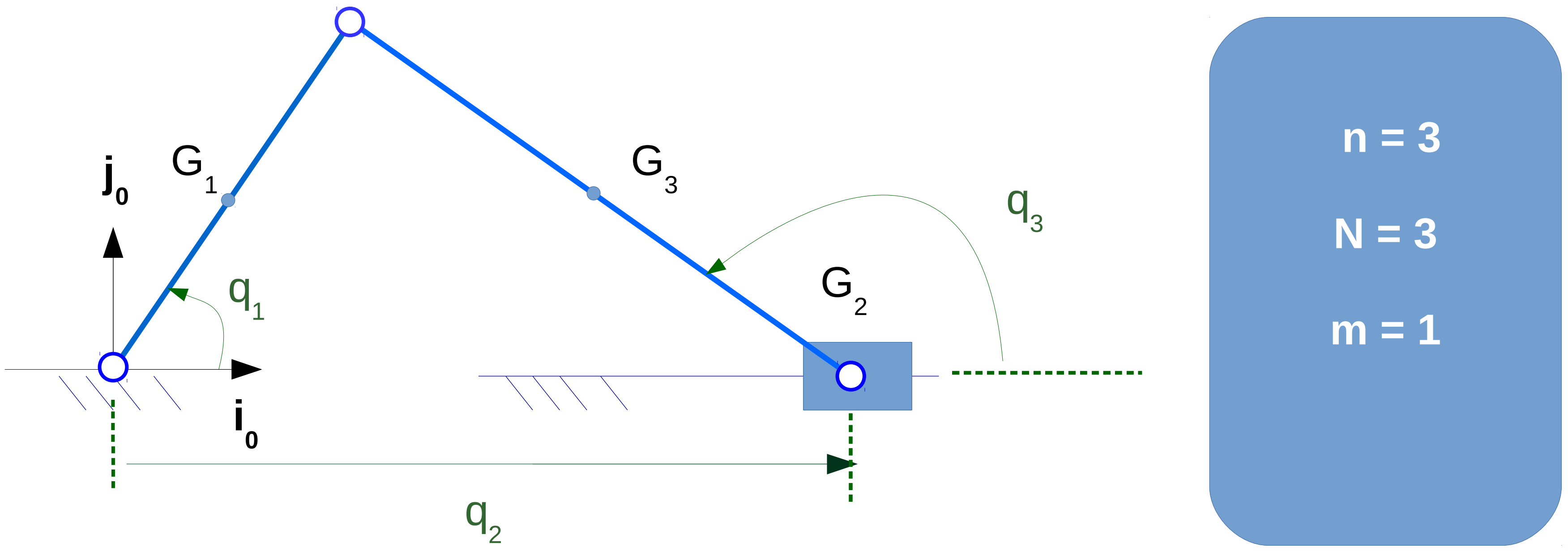
$$C_{mxn}{}^{T}\vec{e}_{nx\,1}=C_{mxn}{}^{T}D_{nx\,\lambda\,N}\boldsymbol{f}_{\boldsymbol{\lambda}\,\boldsymbol{N}\,\boldsymbol{x}\,\boldsymbol{1}}=\boldsymbol{0}_{mx\,1}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \\ u_{m+1} \\ \vdots \\ u_n \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{1}_{mxm} \\ A_{n-mxm} \end{bmatrix}}_{C_{nxm}} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$$u_i=\dot{q}_i$$

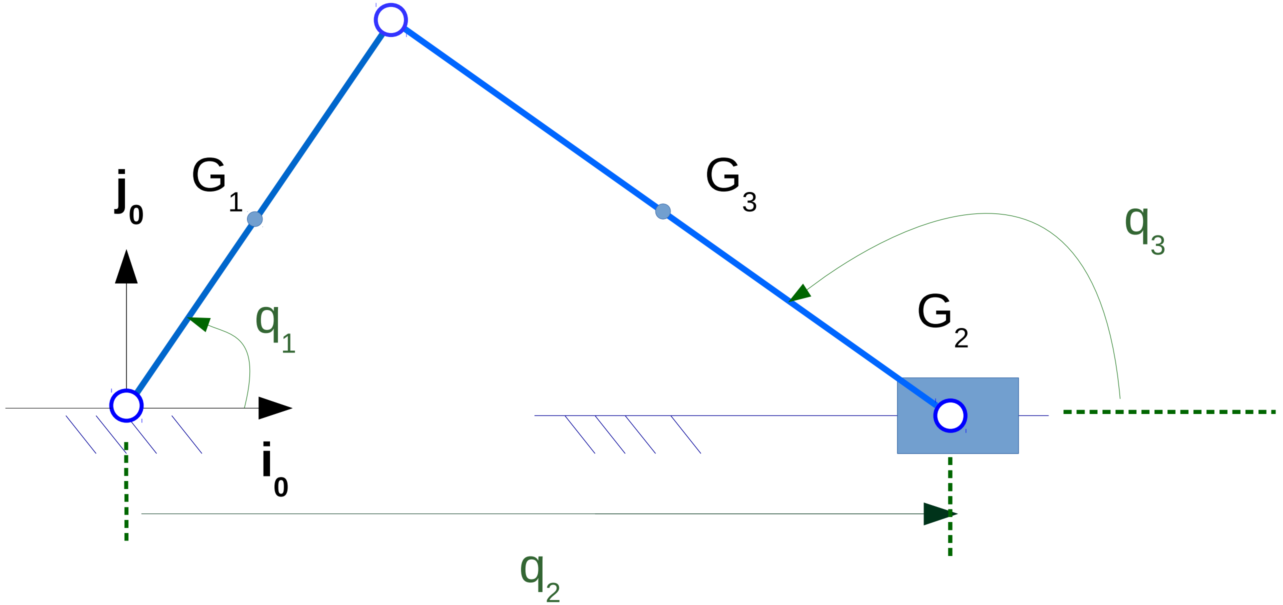
Análise Dinâmica: método de Gibbs-Appell

$$C_{mxn}{}^{T}\vec{e}_{nx1}=C_{mxn}{}^{T}D_{nx\lambda N}\boldsymbol{f}_{\boldsymbol{\lambda N x 1}}=\boldsymbol{0}_{mx1}$$



Análise Dinâmica: método de Gibbs-Appell

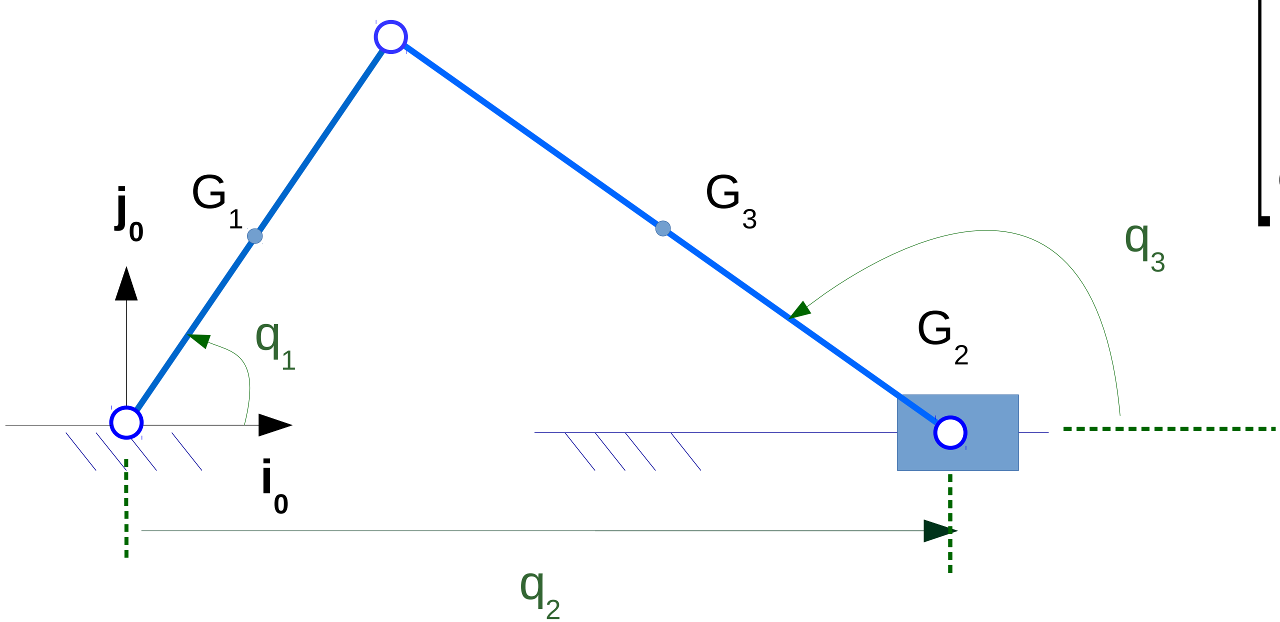
$$C_{mxn}^{\ T}\vec{e}_{nx1}=C_{mxn}^{\ T}D_{nx\lambda N}\boldsymbol{f}_{\boldsymbol{\lambda N x 1}}=\boldsymbol{0}_{mx1}$$



$C_{nxm} = C_{3x1}$

Análise Dinâmica: método de Gibbs-Appell

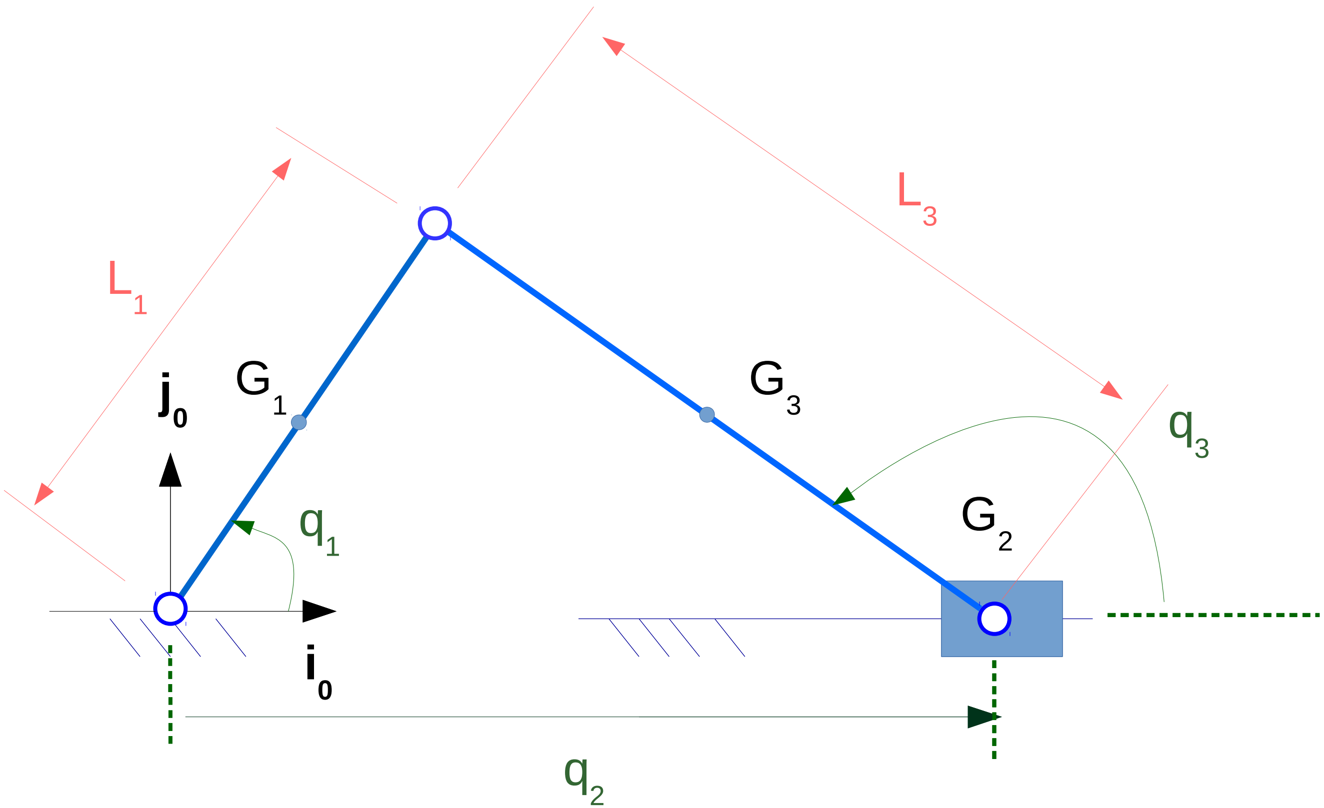
$$C_{mxn}^{T}\vec{e}_{nx1}=C_{mxn}^{T}D_{nx\lambda N}\boldsymbol{f}_{\boldsymbol{\lambda N x 1}}=\mathbf{0}_{mx1}$$



$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix}=\underbrace{\begin{bmatrix}1\\ A_{2x1}\end{bmatrix}}_{C_{3x1}}\begin{bmatrix}\dot{q}_1\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

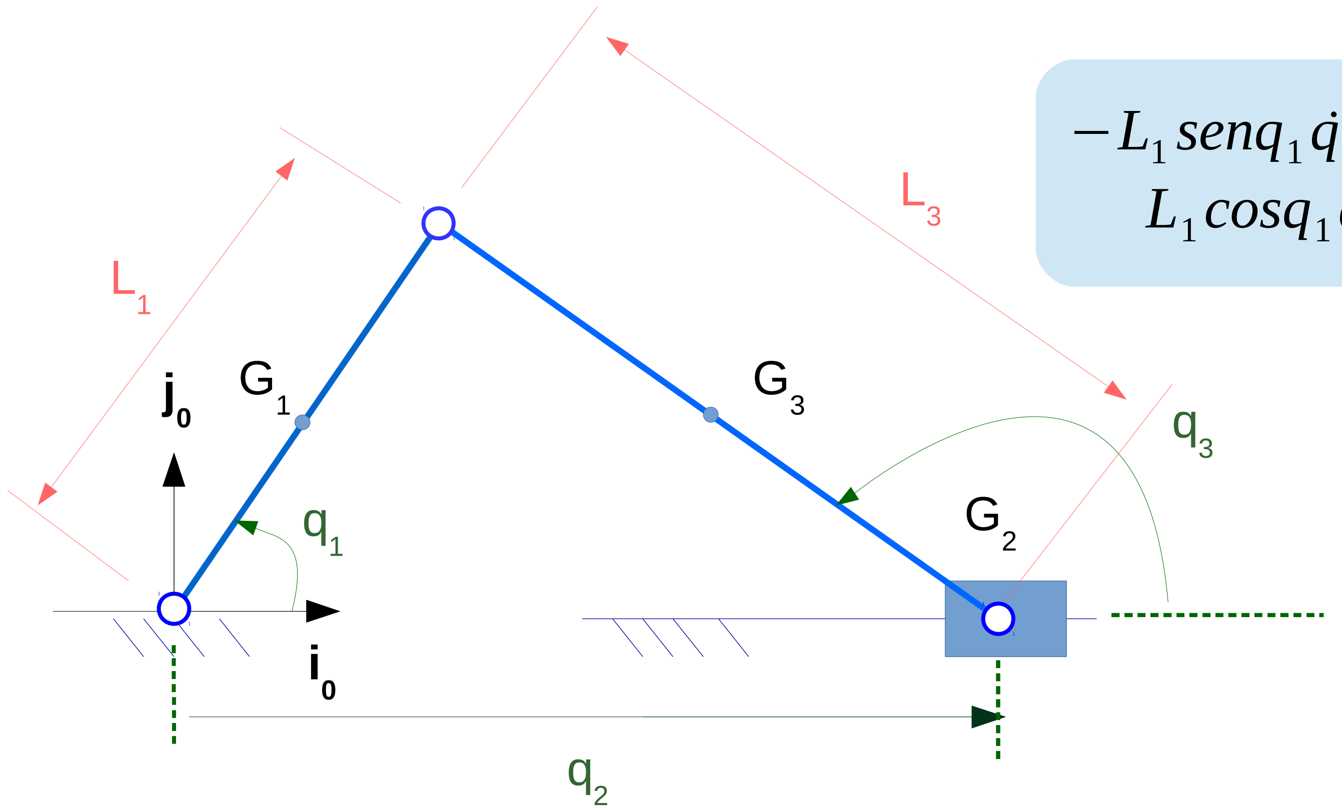
$$L_1 cosq_1 = q_2 + L_3 cosq_3$$
$$L_1 senq_1 = L_3 senq_3$$



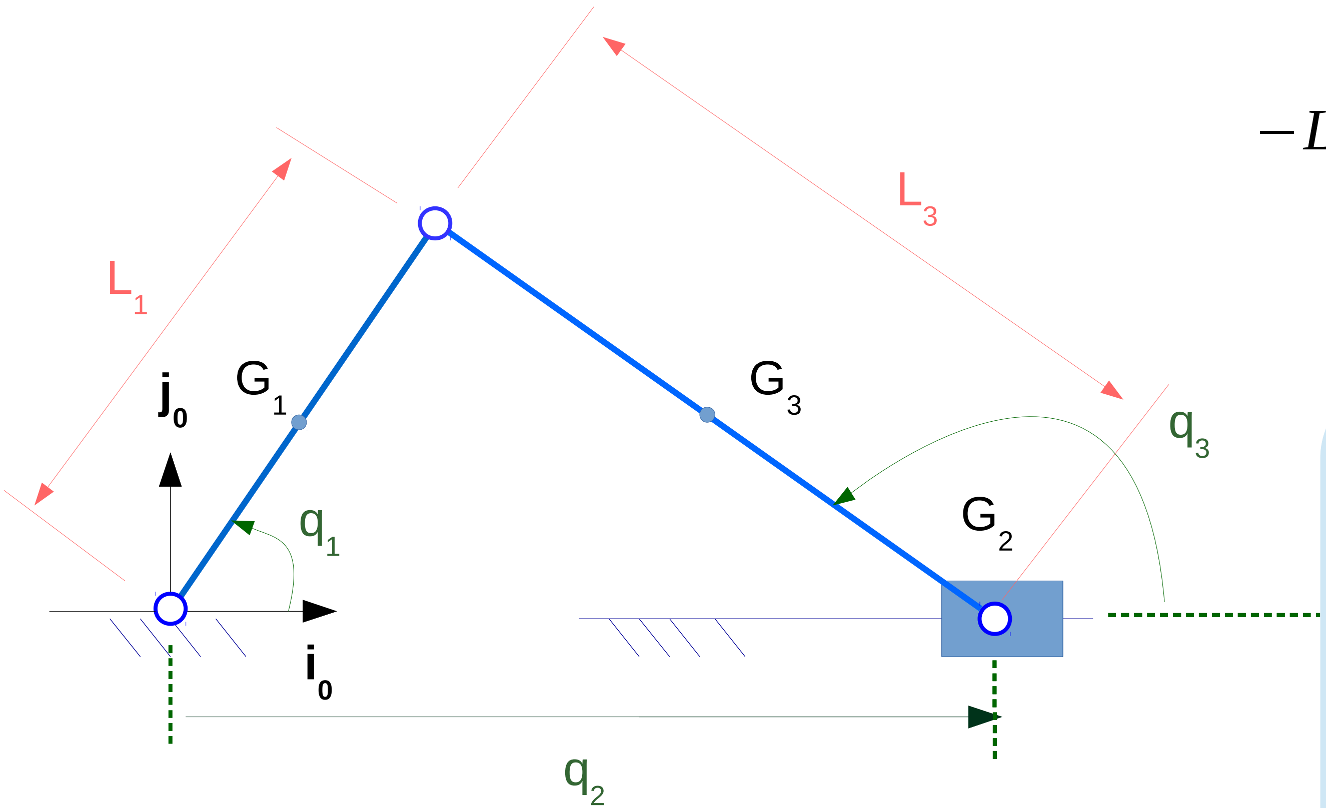
Análise Dinâmica: método de Gibbs-Appell

$$L_1 cosq_1 = q_2 + L_3 cosq_3$$
$$L_1 senq_1 = L_3 senq_3$$

$$-L_1 senq_1 \dot{q}_1 = \dot{q}_2 - L_3 senq_3 \dot{q}_3$$
$$L_1 cosq_1 \dot{q}_1 = L_3 cosq_3 \dot{q}_3$$



Análise Dinâmica: método de Gibbs-Appell



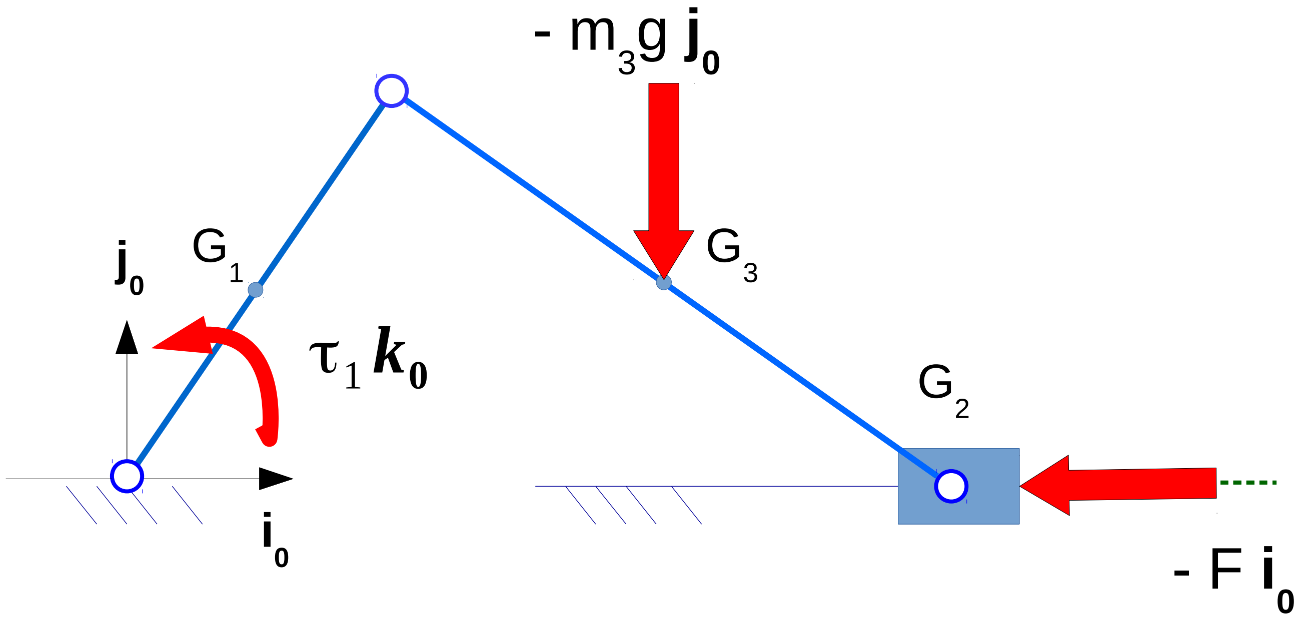
$$L_1 cosq_1 = q_2 + L_3 cosq_3$$
$$L_1 senq_1 = L_3 senq_3$$

$$-L_1 senq_1 \dot{q}_1 = \dot{q}_2 - L_3 senq_3 \dot{q}_3$$
$$L_1 cosq_1 \dot{q}_1 = L_3 cosq_3 \dot{q}_3$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 1 \\ L_1 \dfrac{sen(q_3 - q_1)}{cosq_3} \\ \left(\dfrac{L_1}{L_3}\right)\left(\dfrac{cosq_1}{cosq_3}\right) \end{bmatrix} [\dot{q}_1]$$

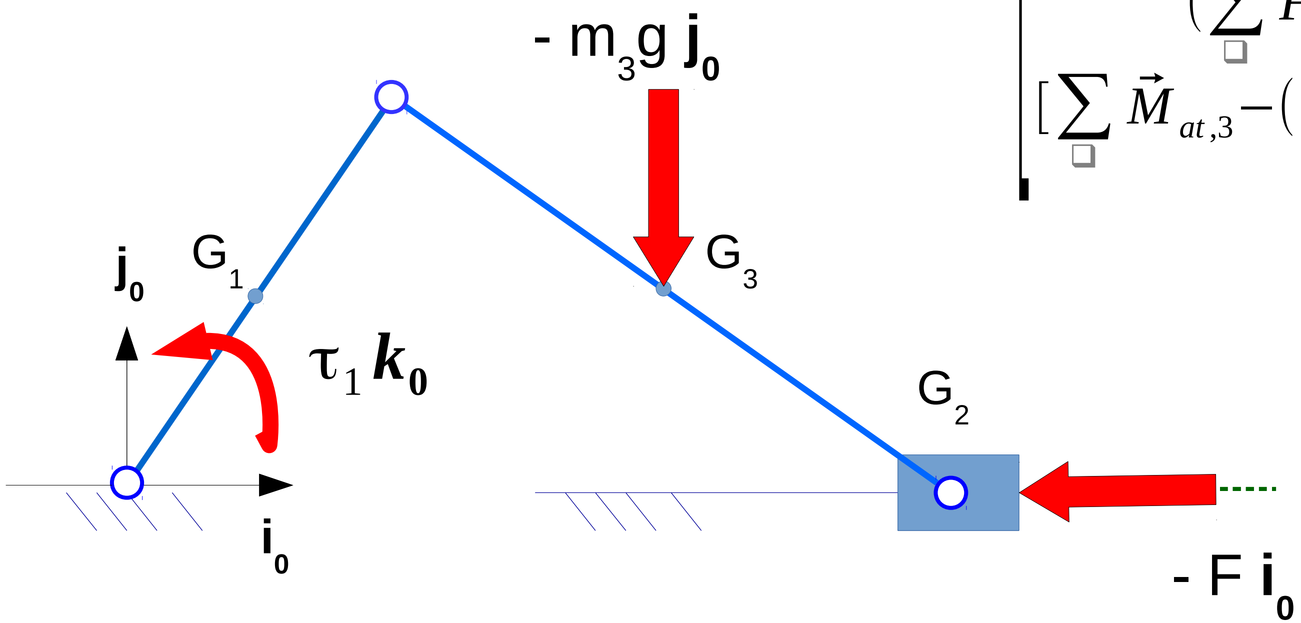
Análise Dinâmica: método de Gibbs-Appell

$$C_{1x3}{}^{T}\vec{e}_{3x1}=C_{1x3}{}^{T}D_{3x9}\boldsymbol{f}_{\mathbf{9x1}}=0_{1x1}$$



Análise Dinâmica: método de Gibbs-Appell

$$
f_{9x1}=\begin{bmatrix}
\left(\sum \vec{F}_{at,1}-m_1\vec{a}_{G_1}\right)\\
\left[\sum \vec{M}_{at,1}-\left(I_1\dot{\vec{\omega}}_1+\vec{\omega}_1\times\left(I_1\vec{\omega}_1\right)\right)\right]\\
\left(\sum \vec{F}_{at,2}-m_2\vec{a}_{G_2}\right)\\
\left[\sum \vec{M}_{at,2}-\left(I_2\dot{\vec{\omega}}_2+\vec{\omega}_2\times\left(I_2\vec{\omega}_2\right)\right)\right]\\
\left(\sum \vec{F}_{at,3}-m_3\vec{a}_{G_3}\right)\\
\left[\sum \vec{M}_{at,3}-\left(I_3\dot{\vec{\omega}}_3+\vec{\omega}_2\times\left(I_3\vec{\omega}_3\right)\right)\right]
\end{bmatrix}=\begin{bmatrix}0\\0\\\tau_1\\-F\\0\\0\\0\\-m_3 g\\0\end{bmatrix}
$$

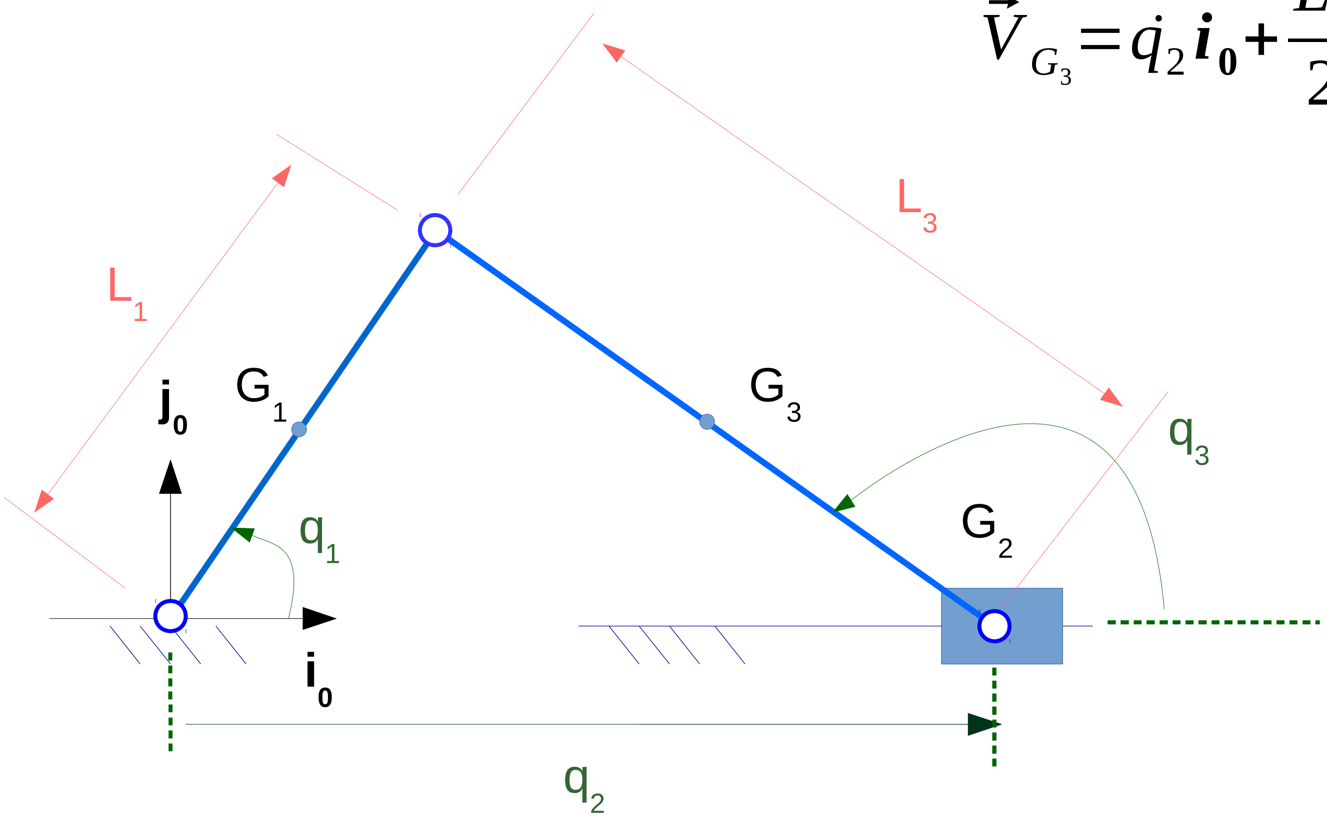


Análise Dinâmica: método de Gibbs-Appell

$$\vec{V}_{G_1} = \frac{L_1}{2}\dot{q}_1(-sen q_1\,\boldsymbol{i_0} + cos q_1\,\boldsymbol{j_0})$$

$$\vec{V}_{G_2} = \dot{q}_2\,\boldsymbol{i_0}$$

$$\vec{V}_{G_3} = \dot{q}_2\,\boldsymbol{i_0} + \frac{L_3}{2}\dot{q}_3(-sen q_3\,\boldsymbol{i_0} + cos q_3\,\boldsymbol{j_0})$$



$$\vec{\omega}_1 = \dot{q}_1\,\boldsymbol{k_0}$$

$$\vec{\omega}_2 = \boldsymbol{0}$$

$$\vec{\omega}_3 = \dot{q}_3\,\boldsymbol{k_0}$$

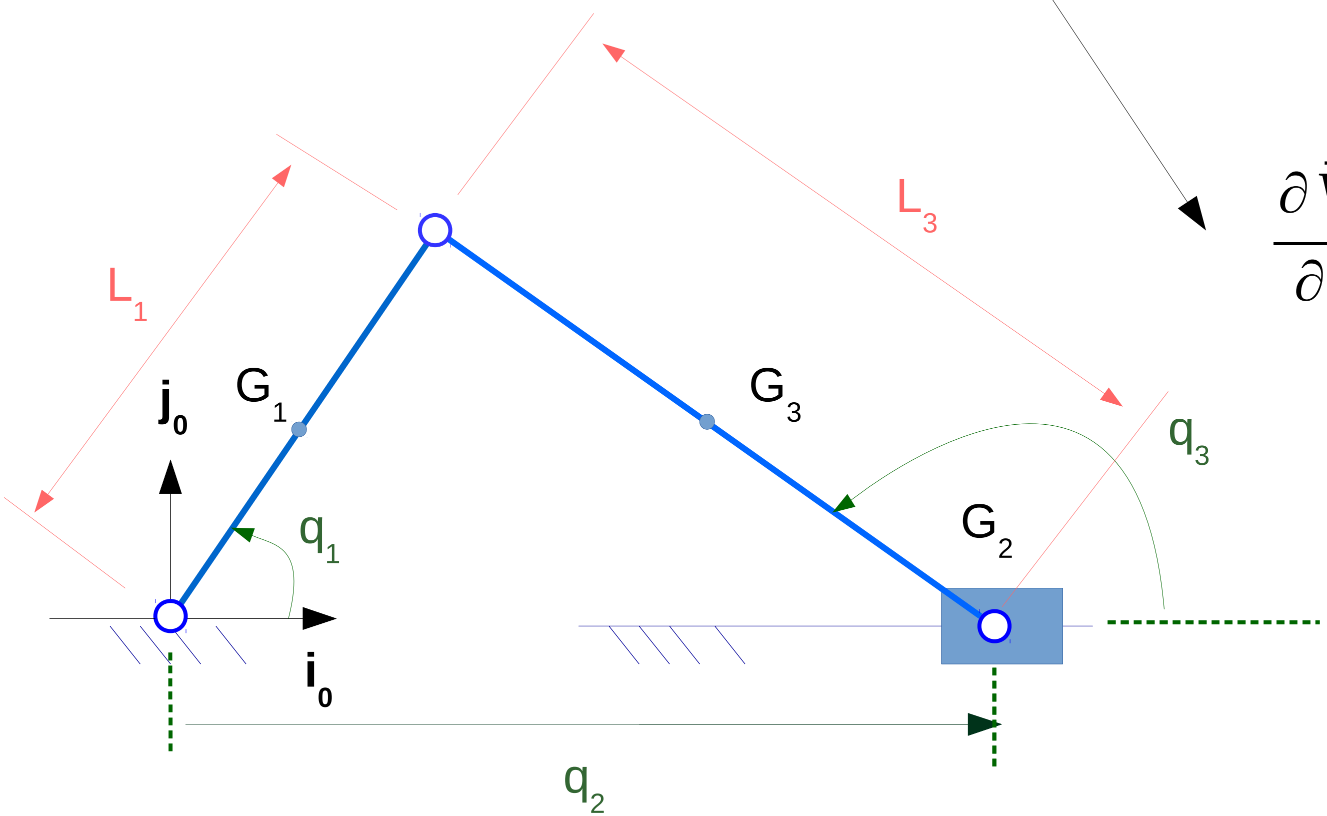
Análise Dinâmica: método de Gibbs-Appell

$$\vec{V}_{G_1} = \frac{L_1}{2}\dot{q}_1(-senq_1\,\boldsymbol{i_0} + cosq_1\,\boldsymbol{j_0})$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_1} = \frac{L_1}{2}(-senq_1\,\boldsymbol{i_0} + cosq_1\,\boldsymbol{j_0})$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_2} = \mathbf{0}$$

$$\frac{\partial \vec{V}_{G_1}}{\partial \dot{q}_3} = \mathbf{0}$$



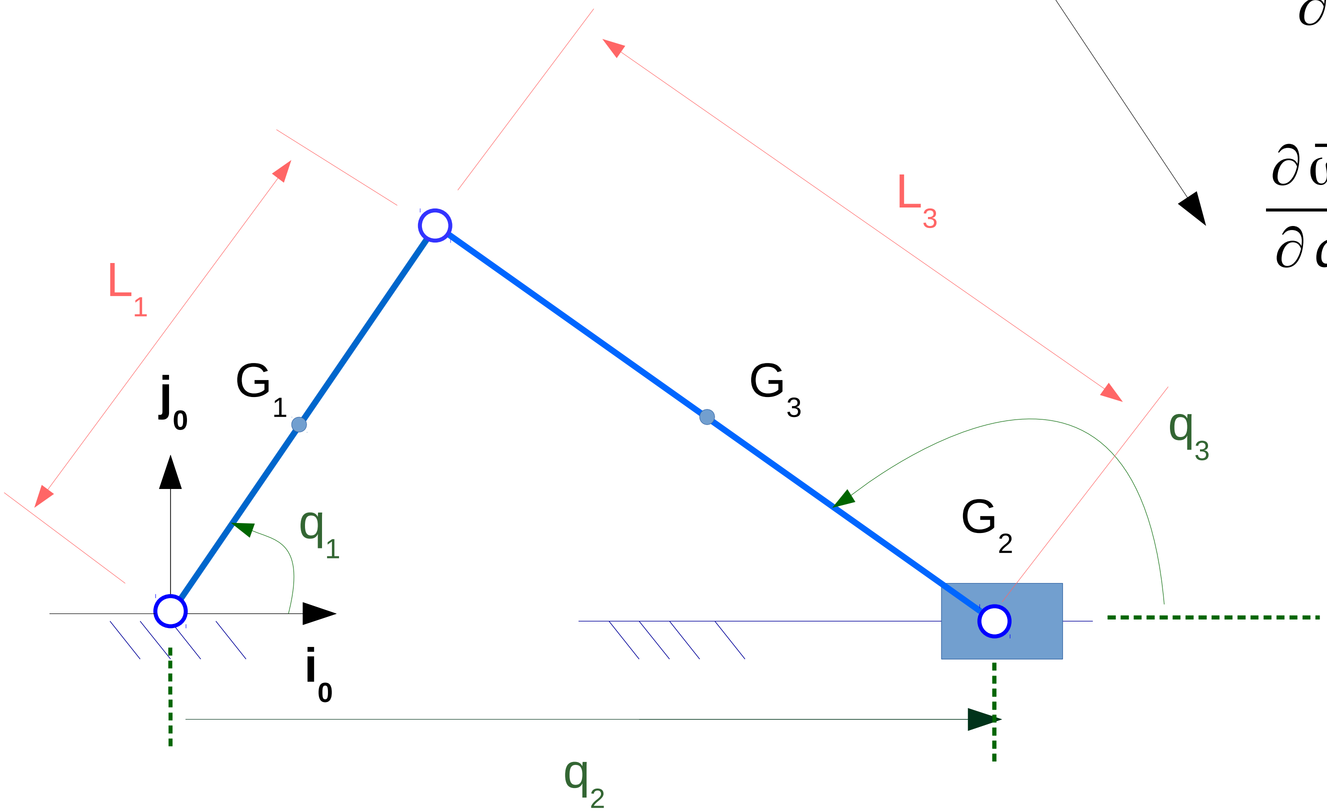
Análise Dinâmica: método de Gibbs-Appell

$$\vec{\omega}_1 = \dot{q}_1 \boldsymbol{k_0}$$

$$\frac{\partial \vec{\omega}_1}{\partial \dot{q}_1} = \boldsymbol{k_0}$$

$$\frac{\partial \vec{\omega}_1}{\partial \dot{q}_2} = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_1}{\partial \dot{q}_3} = \mathbf{0}$$



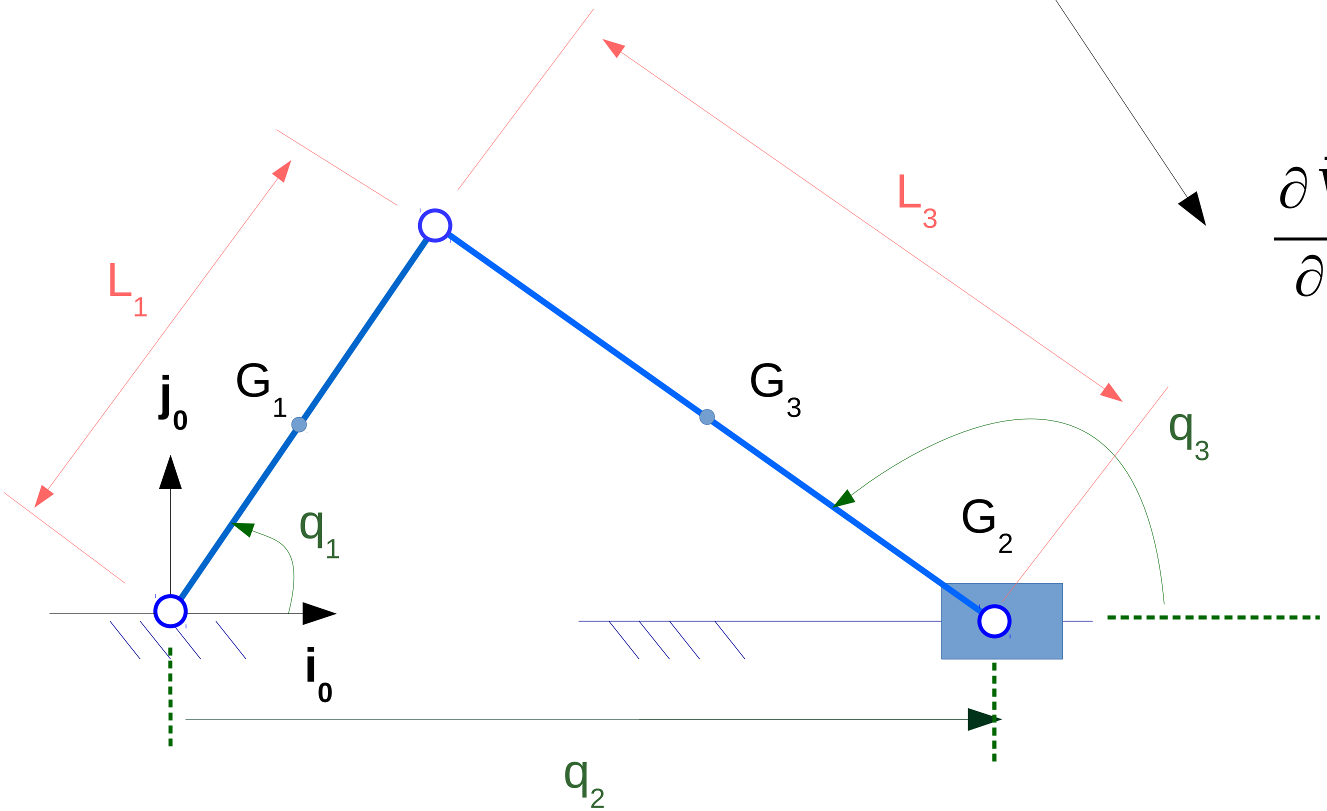
Análise Dinâmica: método de Gibbs-Appell

$$\vec{V}_{G_2} = \dot{q}_2 \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_1} = \mathbf{0}$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_2} = \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_2}}{\partial \dot{q}_3} = \mathbf{0}$$



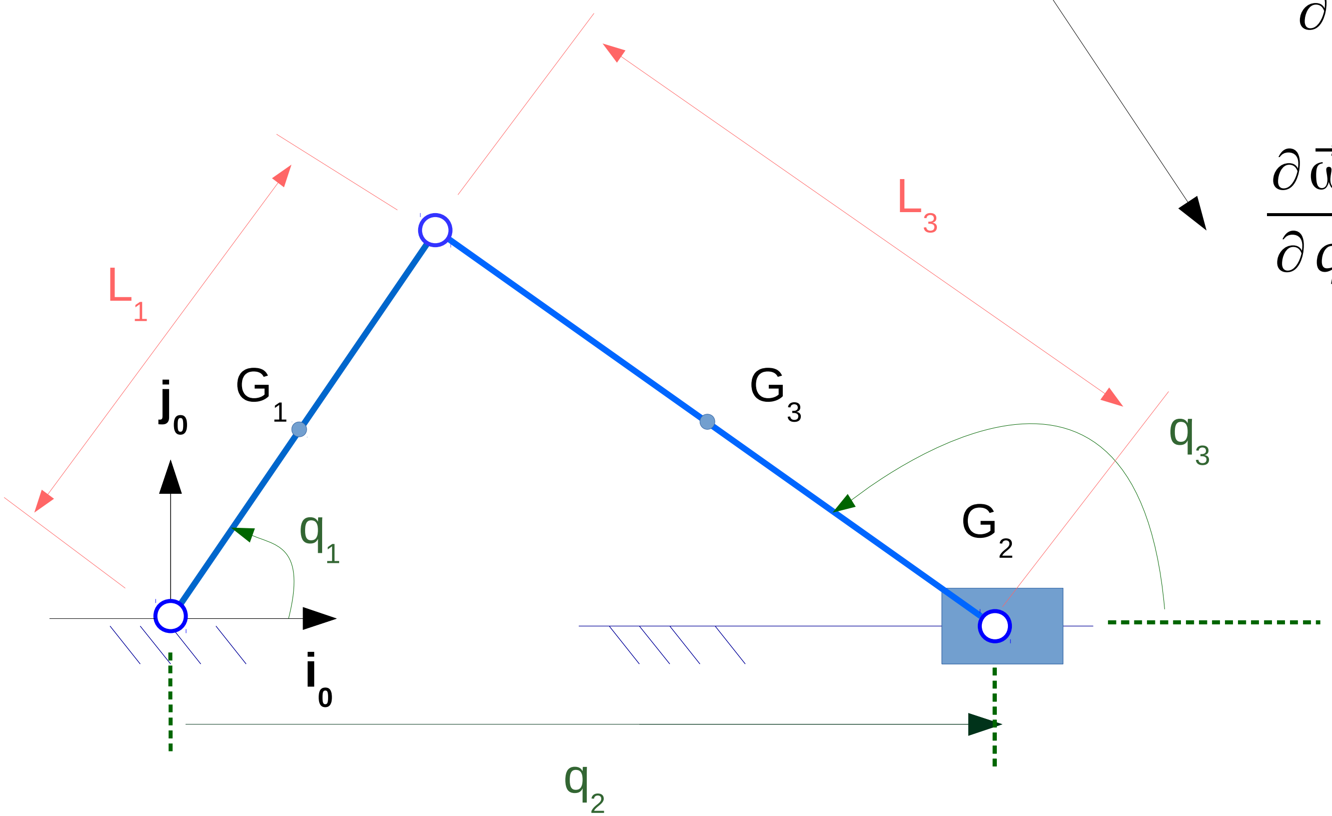
Análise Dinâmica: método de Gibbs-Appell

$$\vec{\omega}_2 = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_2}{\partial \dot{q}_1} = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_2}{\partial \dot{q}_2} = \mathbf{0}$$

$$\frac{\partial \vec{\omega}_2}{\partial \dot{q}_3} = \mathbf{0}$$



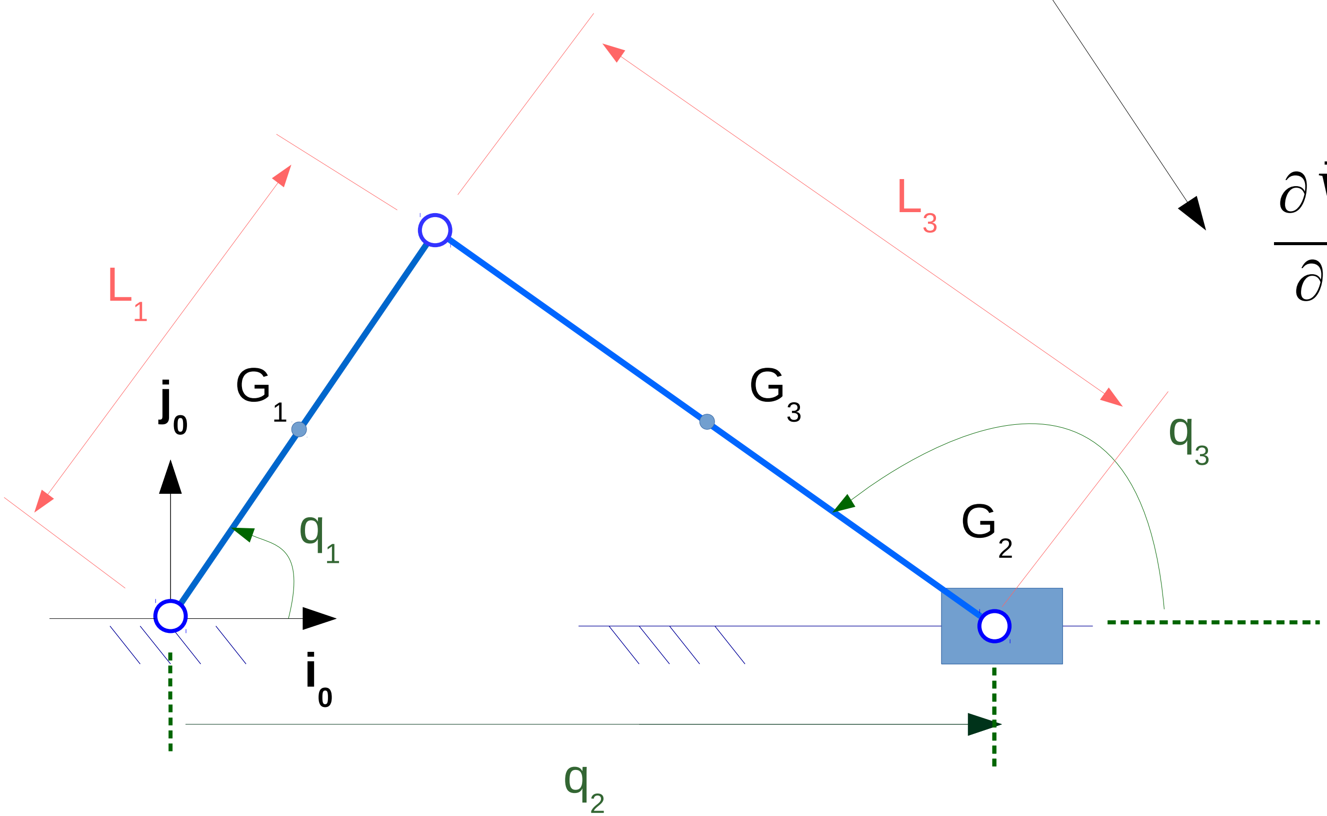
Análise Dinâmica: método de Gibbs-Appell

$$\vec{V}_{G_3} = \dot{q}_2 \boldsymbol{i_0} + \frac{L_3}{2} \dot{q}_3 (-senq_3 \boldsymbol{i_0} + cosq_3 \boldsymbol{j_0})$$

$$\frac{\partial \vec{V}_{G_3}}{\partial \dot{q}_1} = \boldsymbol{0}$$

$$\frac{\partial \vec{V}_{G_3}}{\partial \dot{q}_2} = \boldsymbol{i_0}$$

$$\frac{\partial \vec{V}_{G_3}}{\partial \dot{q}_3} = \frac{L_3}{2} (-senq_3 \boldsymbol{i_0} + cosq_3 \boldsymbol{j_0})$$



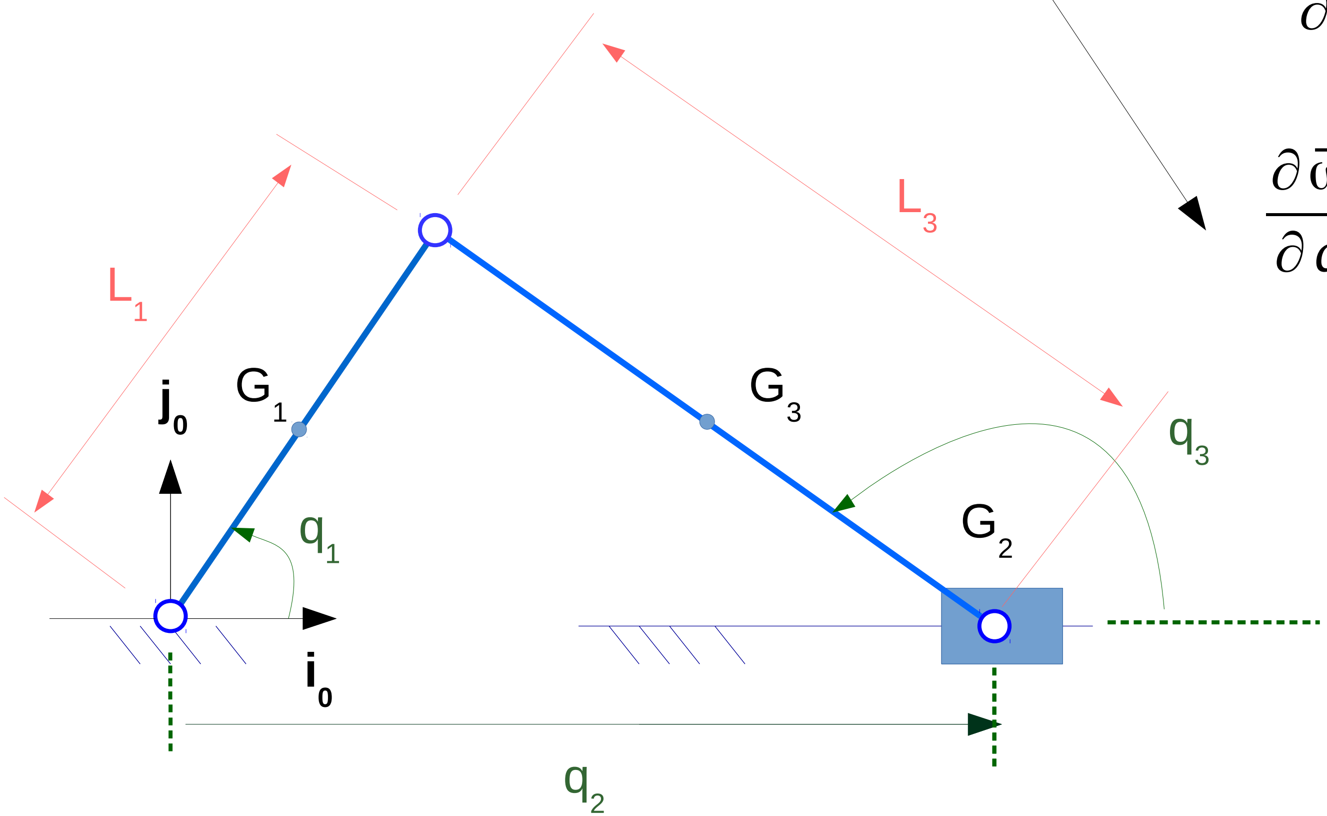
Análise Dinâmica: método de Gibbs-Appell

$$\vec{\omega}_3 = \dot{q}_3 \boldsymbol{k_0}$$

$$\frac{\partial \vec{\omega}_3}{\partial \dot{q}_1} = \boldsymbol{0}$$

$$\frac{\partial \vec{\omega}_3}{\partial \dot{q}_2} = \boldsymbol{0}$$

$$\frac{\partial \vec{\omega}_3}{\partial \dot{q}_3} = \boldsymbol{k_0}$$



Análise Dinâmica: método de Gibbs-Appell

$$D_{3x9}=\left[\begin{array}{ccc|ccc|ccc} -\frac{L_1}{2}senq_1 & \frac{L_1}{2}cosq_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{L_3}{2}senq_3 & \frac{L_3}{2}cosq_3 & 1 \end{array}\right]$$

Análise Dinâmica: método de Gibbs-Appell

$$
\begin{bmatrix}
-\frac{L_1}{2}senq_1 & \frac{L_1}{2}cosq_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{L_3}{2}senq_3 & \frac{L_3}{2}cosq_3 & 1
\end{bmatrix}
\begin{bmatrix}
0 \\ 0 \\ \tau_1 \\ -F \\ 0 \\ 0 \\ 0 \\ -m_3 g \\ 0
\end{bmatrix}
$$

$$
D\boldsymbol{f} = \begin{bmatrix}
\tau_1 \\
-F \\
-\left(m_3 \frac{L_3}{2} g\right) cosq_3
\end{bmatrix}
$$

Análise Dinâmica: método de Gibbs-Appell

$$C^T D\boldsymbol{f} = \begin{bmatrix} 1 & L_1 \dfrac{sen(q_3 - q_1)}{cosq_3} & (\dfrac{L_1}{L_3})(\dfrac{cosq_1}{cosq_3}) \end{bmatrix} \begin{bmatrix} \tau_1 \\ -F \\ -(m_3 \dfrac{L_3}{2} g) cosq_3 \end{bmatrix} = 0$$

$$C^T D\boldsymbol{f} = \tau_1 - F L_1 \frac{sen(q_3 - q_1)}{cosq_3} - (m_3 \frac{L_3}{2} g) cosq_3 (\frac{L_1}{L_3})(\frac{cosq_1}{cosq_3}) = 0$$

Análise Dinâmica: método de Gibbs-Appell

$$C^T D\boldsymbol{f} = \tau_1 - F L_1 \frac{sen(q_3 - q_1)}{cos q_3} - (m_3 \frac{L_3}{2} g) cos q_3 (\frac{L_1}{L_3})(\frac{cos q_1}{cos q_3}) = 0$$

$$C^T D\boldsymbol{f} = \tau_1 - F L_1 \frac{sen(q_3 - q_1)}{cos q_3} - (m_3 \frac{L_1}{2} g) cos q_1 = 0$$

$$\tau_1 = F L_1 \frac{sen(q_3 - q_1)}{cos q_3} + (m_3 \frac{L_1}{2} g) cos q_1$$

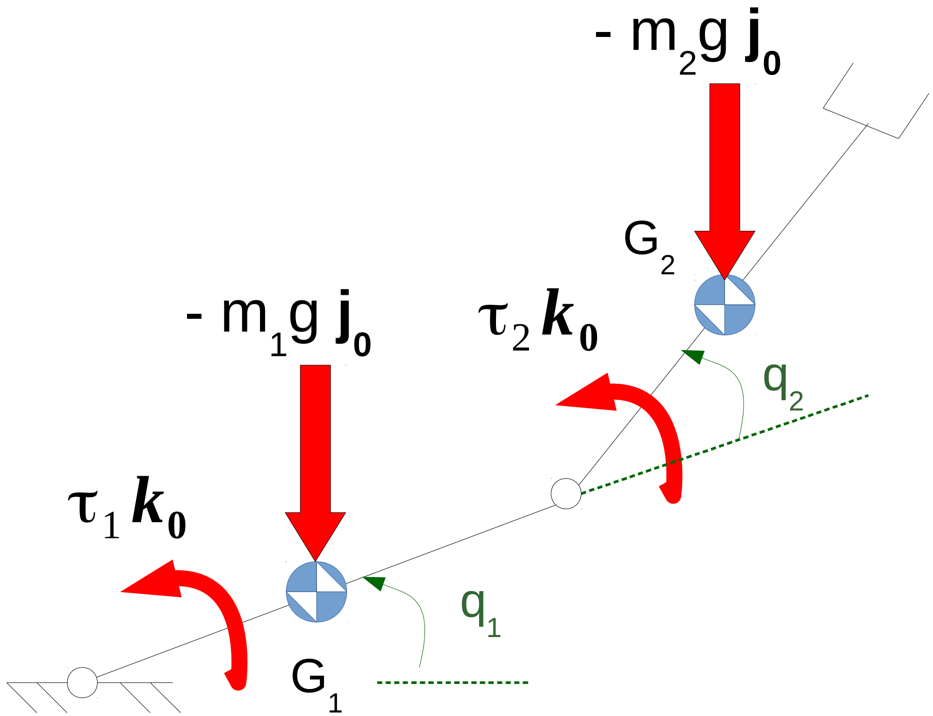
Análise Dinâmica: método de Gibbs-Appell

$\lambda = 3$

$N = 2, \;\; M = 2$

$n = 6 \;\; (\omega_1, \;\; \omega_2, V_{G1x}, V_{G1y}, V_{G2x}, V_{G2y})$

Redundância

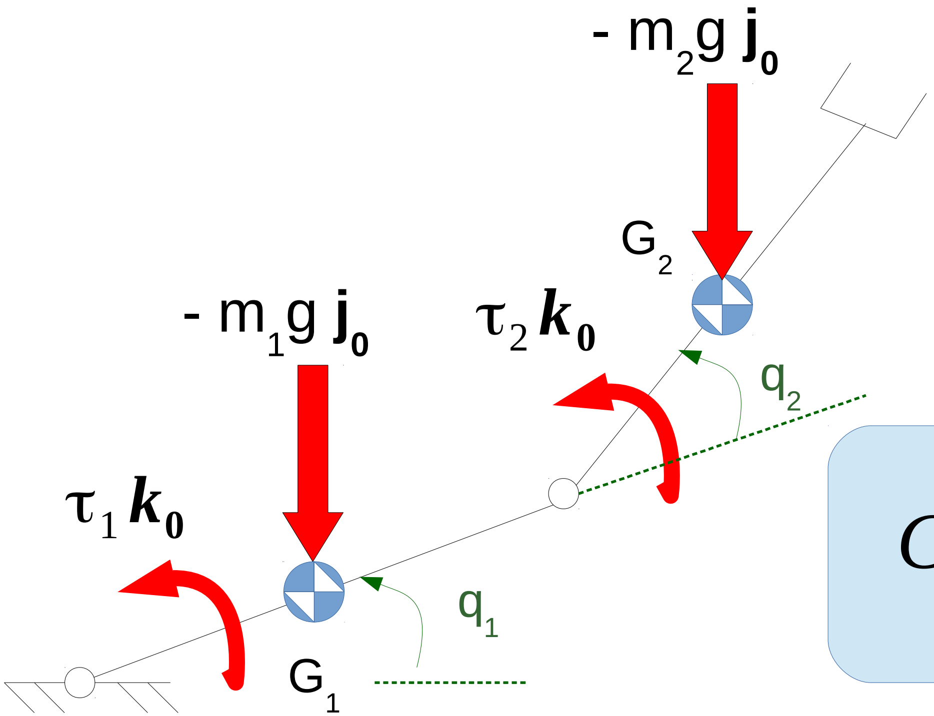


Análise Dinâmica: método de Gibbs-Appell

$\lambda = 3$

$N = 2, \; M = m = 2$

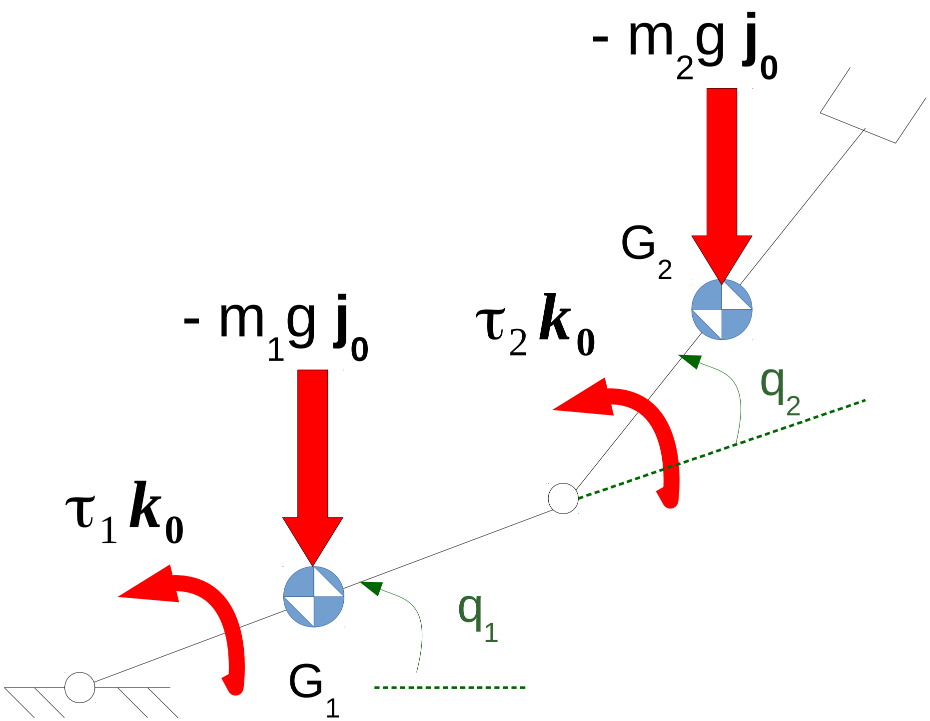
$n = 6 \; (\omega_1, \; \omega_2, V_{G1x}, V_{G1y}, V_{G2x}, V_{G2y})$



$$C_{2x6}^{\;T} \vec{e}_{6x1} = C_{2x6}^{\;T} D_{6x6} \boldsymbol{f}_{\boldsymbol{6x1}} = 0_{2x1}$$

Análise Dinâmica: método de Gibbs-Appell

$$C_{2x6}^{\;T}\vec{e}_{6x1}=C_{2x6}^{\;T}D_{6x6}\boldsymbol{f}_{\boldsymbol{6x1}}=0_{2x1}$$



$$\begin{bmatrix}\omega_1\\ \omega_2\\ V_{G1x}\\ V_{G1y}\\ V_{G2x}\\ V_{G2y}\end{bmatrix}=\overbrace{\begin{bmatrix}1 & 0\\ 0 & 1\\ -(L/2)s_1 & 0\\ (L/2)c_1 & 0\\ -Ls_1 & -(L/2)s_{12}\\ Lc_1 & (L/2)c_{12}\end{bmatrix}}^{\boldsymbol{C}_{\boldsymbol{6x2}}}\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}$$

$$\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$C_{2x6}{}^{T}\vec{e}_{6x1}=C_{2x6}{}^{T}D_{6x6}\boldsymbol{f}_{\boldsymbol{6x1}}=0_{2x1}$$

$$\boldsymbol{f}_{\boldsymbol{6x1}}=\begin{bmatrix} -m_1\dot{V}_{G1x} \\ -m_1\dot{V}_{G1y}-m_1 g \\ \tau_1-\tau_2-I_1\dot{\omega}_1 \\ \hline -m_2\dot{V}_{G2x} \\ -m_2\dot{V}_{G2y}-m_2 g \\ \tau_2-I_2\dot{\omega}_2 \end{bmatrix}$$

Análise Dinâmica: método de Gibbs-Appell

$$
D_{6x6} = \begin{array}{c}
\begin{array}{cccc} \dfrac{\partial \vec{V}_{G_1}}{\partial u_i} & \dfrac{\partial \vec{\omega}_1}{\partial u_i} & \dfrac{\partial \vec{V}_{G_2}}{\partial u_i} & \dfrac{\partial \vec{\omega}_2}{\partial u_i} \end{array} \\
\begin{array}{c}
\left[\begin{array}{ccc|ccc}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{array} \right]
\end{array}
\begin{array}{l}
\omega_1 \\ \omega_2 \\ V_{G1x} \\ V_{G1y} \\ V_{G2x} \\ V_{G2y}
\end{array} \\
\begin{array}{cc} \underbrace{\qquad\qquad\qquad}_{\text{Elo 1}} & \underbrace{\qquad\qquad\qquad}_{\text{Elo 2}} \end{array}
\end{array}
$$

u_i

Análise Dinâmica: método de Gibbs-Appell

$$C_{2x6}^{T}\vec{e}_{6x1}=C_{2x6}^{T}D_{6x6}\boldsymbol{f}_{\mathbf{6x1}}=0_{2x1}$$

$$\begin{bmatrix}\tau_1-\tau_2-I_1\dot{\omega}_1+(L/2)s_1m_1\dot{V}_{G1x}-(L/2)c_1(m_1\dot{V}_{G1y}+m_1g)+Ls_1m_2\dot{V}_{G2x}-Lc_1(m_2\dot{V}_{G2y}+m_2g)\\ \tau_2-I_2\dot{\omega}_2+(L/2)s_{12}m_2\dot{V}_{G2x}-(L/2)c_{12}(m_2\dot{V}_{G2y}+m_2g)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

Modelagem Dinâmica