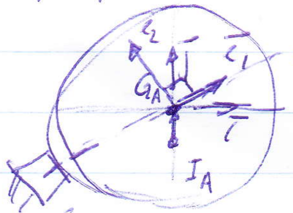
PME 2352 P1 05/09/2014

1ª Q) Observando que as pás são ortogonais entre si e que se escolhermos versores ortogonais $\vec{e}_1, \vec{e}_2$ nas direções das pás ① e ② deveremos facilitar a solução final, representaremos os vetores girantes, em geral, nesse sistema. Os versores $\vec{i}$ e $\vec{j}$ são definidos com $\vec{i}$ na direção do ... no instante do pulso e $\vec{j}$ na que vem 90° após.

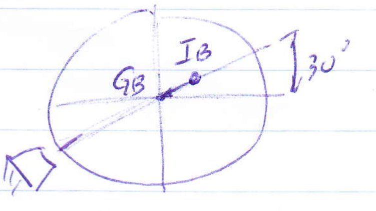
a) $\begin{cases} \vec{\delta}_{A orig} = 0{,}2\vec{j} = 0{,}2\angle 90^\circ \\ \delta_{B orig} = -0{,}1\angle 30^\circ \end{cases}$

nas direções $\vec{e}_1$ e $\vec{e}_2$ teríamos $\vec{\delta}_{A orig} = 0{,}2\angle 60^\circ = 0{,}1\,\vec{e}_1 + 0{,}178\,\vec{e}_2$

$\vec{\delta}_{B orig} = -0{,}1\,\vec{e}_1$



plano A



Plano B

b) $\vec{\delta}_{A orig+t} = 0{,}18\,\vec{e}_1 + 0{,}05\,\vec{e}_2$ abcissa 3 ms corresponde a $\vec{e}_1$ e 12 ms a $\vec{e}_2$

$\vec{\delta}_{B orig+t} = -0{,}18\,\vec{e}_1 + 0{,}05\,\vec{e}_2$

$$\begin{cases} \vec{\delta}_{A orig+t} - \vec{\delta}_{A orig} = 0{,}08\,\vec{e}_1 - 0{,}12\,\vec{e}_2 = \alpha_{CA}\cdot m_t\cdot\vec{e}_2 + \alpha_{DA}\cdot m_t\cdot\vec{e}_1 \\ \vec{\delta}_{B orig+t} - \vec{\delta}_{B orig} = -0{,}08\,\vec{e}_1 + 0{,}05\,\vec{e}_2 = \alpha_{CB}\cdot m_t\cdot\vec{e}_2 + \alpha_{DB}\cdot m_t\cdot\vec{e}_1 \end{cases}$$

Observar que a massa de teste em C foi colocada na pá ② e em D na ①

$$\boxed{\begin{array}{ll} \alpha_{CA} = \frac{-0{,}12}{10} = -0{,}012\ mm/g & \alpha_{DA} = 0{,}008\ mm/g \\ \alpha_{CB} = 0{,}005\ mm/g & \alpha_{DB} = -0{,}008\ mm/g \end{array}}$$

c) $\begin{pmatrix} \alpha_{CA} & \alpha_{DA} \\ \alpha_{CB} & \alpha_{DB} \end{pmatrix} \begin{pmatrix} m_C\vec{e}_C \\ m_D\vec{e}_D \end{pmatrix} = -\begin{pmatrix} \vec{\delta}_{A orig} \\ \vec{\delta}_{B orig} \end{pmatrix} \therefore \begin{pmatrix} -0{,}012 & 0{,}008 \\ 0{,}005 & -0{,}008 \end{pmatrix} \begin{pmatrix} m_C\vec{e}_C \\ m_D\vec{e}_D \end{pmatrix} = \begin{pmatrix} -0{,}1\vec{e}_1 - 0{,}178\vec{e}_2 \\ 0{,}1\vec{e}_1 \end{pmatrix}$

$\begin{pmatrix} -1{,}2 & 0{,}8 \\ 0{,}5 & -0{,}8 \end{pmatrix} \begin{pmatrix} m_C\vec{e}_C \\ m_D\vec{e}_D \end{pmatrix} = \begin{pmatrix} -10\vec{e}_1 - 17{,}8\vec{e}_2 \\ 10\vec{e}_1 \end{pmatrix} \therefore m_C\vec{e}_C = \dfrac{\begin{vmatrix} -10\vec{e}_1 - 17{,}8\vec{e}_2 & 0{,}8 \\ 10\vec{e}_1 & -0{,}8 \end{vmatrix}}{1{,}2\cdot 0{,}8 - 0{,}5\cdot 0{,}8} =$

$m_C\vec{e}_C = \dfrac{8\vec{e}_1 + 13{,}9\vec{e}_2 - 8\vec{e}_1}{} = 24{,}7$ g na direção de $\vec{e}_2$ ou (adicionar)

Plano C: retirar 24,7 g na pá ④

$m_D\vec{e}_D = \dfrac{\begin{vmatrix} -1{,}2 & -10\vec{e}_1 - 17{,}8\vec{e}_2 \\ 0{,}5 & 10\vec{e}_1 \end{vmatrix}}{0{,}56} = \dfrac{-12\vec{e}_1 + 5\vec{e}_1 + 8{,}7\vec{e}_2}{0{,}56} = -12{,}5\,\vec{e}_1 + 15{,}5\,\vec{e}_2 \therefore$

Plano D: retirar 12,5 g na pá ① e 15,5 g na pá ④

d) ISO G6,3 , $n_{op} = 3600$ r.p.m $\therefore \omega_{op} = 377$ rad/s

$e_{ad} \cdot \omega_{op} \leq 6{,}3$ mm/s $\therefore e_{ad} \leq 0{,}0167$ mm

$M \cdot e_{ad} = U_{ad} = 10000 \times 0{,}0167 = 167$ g.mm

$U_{ad_C} = 84$ g.mm $\quad U_{ad_D} = 84$ g.mm

2ª Q)

$\omega_{bal} = \frac{2\pi}{0{,}036} = 174{,}5$ rad/s

a) $F_A = 20\vec{j}$

$\vec{F_B} = -15N \angle 30° = -13\vec{i} - 7{,}5\vec{j}$

$\vec{F_{res}} = \vec{F_A} + \vec{F_B} = -13\vec{i} + 12{,}5\vec{j}$; $\vec{F_{res}} + \vec{F_E} = \vec{0}$

$\vec{F_E} = 13\vec{i} - 12{,}5\vec{j} = 18N \angle -44°$ ∴ $18 = m_E R_E \cdot \omega_{bal}^2$ ∴ $m_E = 9{,}9 \times 10^{-3}$ kg

Precisamos retirar 9,9g a 136° para anular a resultante

b) Efeito de $\vec{F_E}$ nas forças dos mancais

$\vec{F_{EA}} = \vec{F_E} \cdot \frac{(L-d)}{L} = \frac{3}{5}\vec{F_E}$

$\vec{F_{EB}} = \vec{F_E} \frac{d}{L} = \frac{2}{5}\vec{F_E}$

Forças remanescentes nos mancais

$\vec{F_A'} = \vec{F_A} + \vec{F_{EA}} = 20\vec{j} + 7{,}8\vec{i} - 7{,}5\vec{j} = 7{,}8\vec{i} + 12{,}5\vec{j} = 14{,}7N \angle 58°$

$\vec{F_B'} = \vec{F_B} + \vec{F_{EB}} = -13\vec{i} + 12{,}5\vec{j} + 5{,}2\vec{i} - 5\vec{j} = -7{,}8\vec{i} - 12{,}5\vec{j} = -14{,}7N \angle 58°$

Observe que a resultante é nula.

$$\vec{F_C}\begin{cases} \vec{F_{CA}} = \vec{F_C} \cdot \frac{L-a}{L} = \frac{4}{5}\vec{F_C} \\ \vec{F_{CB}} = \vec{F_C} \cdot \frac{a}{L} = \frac{1}{5}\vec{F_C} \end{cases} \qquad \vec{F_D}\begin{cases} \vec{F_{DA}} = \vec{F_D} \cdot \frac{b}{L} = \frac{1}{10}\vec{F_D} \\ \vec{F_{DB}} = \vec{F_D} \cdot \frac{L-b}{L} = \frac{9}{10}\vec{F_D} \end{cases}$$

mas $\begin{cases} \vec{F_{CA}} + \vec{F_{DA}} + \vec{F_{A'}} = \vec{0} \\ \vec{F_{CB}} + \vec{F_{DB}} + \vec{F_{B'}} = \vec{0} \end{cases}$

$$\begin{bmatrix} 4/5 & 1/10 \\ 1/5 & 9/10 \end{bmatrix} \begin{pmatrix} \vec{F_C} \\ \vec{F_D} \end{pmatrix} = \begin{pmatrix} -7{,}8\vec{i} - 12{,}5\vec{j} \\ 7{,}8\vec{i} + 12{,}5\vec{j} \end{pmatrix}$$

$$\vec{F_C} = \frac{\begin{vmatrix} -7{,}8\vec{i} - 12{,}5\vec{j} & 0{,}1 \\ 7{,}8\vec{i} + 12{,}5\vec{j} & 0{,}9 \end{vmatrix}}{0{,}8 \times 0{,}9 - 0{,}2 \times 0{,}1} = \frac{-7{,}8\vec{i} - 12{,}5\vec{j}}{0{,}7} = -11{,}1\vec{i} - 17{,}9\vec{j} = 21{,}0N \angle 238{,}2°$$

$21 = m_C R_C \omega_{bal}^2$ ∴ $\boxed{m_C = \frac{21}{0{,}04 \cdot 174{,}5^2} = 17{,}2 \times 10^{-3} \text{ kg} \quad \text{retirar a } 58{,}2°}$

$\vec{F_D} = \frac{7{,}8\vec{i} + 12{,}5\vec{j}}{0{,}7} = 21N \angle 58{,}2°$

$\boxed{m_D = 17{,}2 \text{ g p/ retirar a } 238{,}2°}$

c) ISO G 6.3 e $n_{op} = 5000$ r.p.m. ∴ $\omega_{op} = 523{,}6$ rad/s

$e_{ad}\cdot\omega_{op} \le 6{,}3$ mm/s ∴ $e_{ad} \le 0{,}012$ mm

$M\cdot e_{ad} = 20\times 12 = 240$ g·mm | Sugerimos tolerância de 80 g·mm para cada plano.

3ª Q

$$\left.\begin{cases} F_{in} = M\cdot(x+\delta)\cdot\omega^2 \\ F_{el} = 2k_m\cdot x \end{cases}\right\} \Rightarrow \frac{x}{\delta} = \frac{M\omega^2}{2k_m - M\omega^2} = \frac{\left(\frac{\omega}{\omega_{cr}}\right)^2}{1-\left(\frac{\omega}{\omega_{cr}}\right)^2} \quad \text{com } \omega_{cr} = \sqrt{\frac{2k_m}{M}}$$

a) $\omega_{op} = \frac{40000}{60}\cdot 2\pi = 4189$ rad/s

$\omega_{cr_{min}} = \frac{4189}{5} = 837{,}8$ rad/s ∴ $\omega_{cr_{min}}^2\cdot M = 2\cdot k_m$ ∴ $\boxed{k = 702000\ N/m}$

$n_{cr_{min}} = \frac{40000}{5} = 8000$ r.p.m

$K = 2\times 10^6$ N/m ∴ $\omega_{cr_{max}} = \sqrt{\frac{2K}{M}} = 1414$ rad/s $= 13500$ r.p.m

b) $$\left.\frac{x}{\delta}\right)_{min} = \frac{\left(\frac{n}{n_{cr_{min}}}\right)^2}{\left(\frac{n}{n_{cr_{min}}}\right)^2 - 1}$$ (estamos acima da crítica mais baixa)

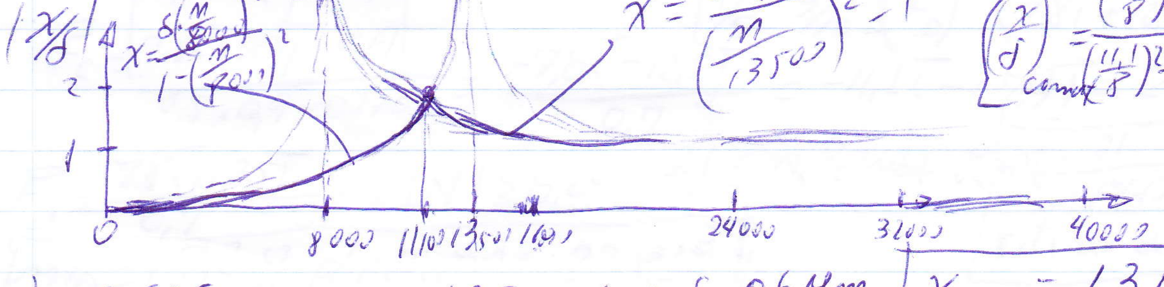
$$\left.\frac{x}{\delta}\right)_{max} = \frac{\left(\frac{n}{n_{cr_{max}}}\right)^2}{1-\left(\frac{n}{n_{cr_{max}}}\right)^2}$$ (estamos abaixo da crítica mais alta)

Para a rotação de comutação $\left.\frac{x}{\delta}\right)_{min} = \left.\frac{x}{\delta}\right)_{max}$

$$\frac{\left(\frac{n}{8000}\right)^2}{\left(\frac{n}{8000}\right)^2 - 1} = \frac{\left(\frac{n}{13500}\right)^2}{1-\left(\frac{n}{13500}\right)^2} \therefore \frac{1}{1-\left(\frac{8000}{n}\right)^2} = \frac{1}{\left(\frac{13500}{n}\right)^2 - 1}$$

$$\left(\frac{13500}{n}\right)^2 + \left(\frac{8000}{n}\right)^2 = 2 \therefore n^2 = \frac{8000^2 + 13500^2}{2} = 0$$

$\boxed{n_{comut} = 11100 \text{ r.p.m}}$



$$\boxed{\left(\frac{x}{\delta}\right)_{comut} = \frac{\left(\frac{11{,}1}{8}\right)^2}{\left(\frac{11{,}1}{8}\right)^2 - 1} = 2{,}1}$$

c) ISO G2,5 ⟶ $e_{ad}\cdot\omega_{op} \le 2{,}5$ mm/s ∴ $\delta = 0{,}6\ \mu m$ $\boxed{x_{comut} = 1{,}3\ \mu m}$

1ª Questão: balanceamento por retirada de massa
balanceadora de mancais flexíveis

$M = 10\ kg$

$n_0 = 3600\ rpm$

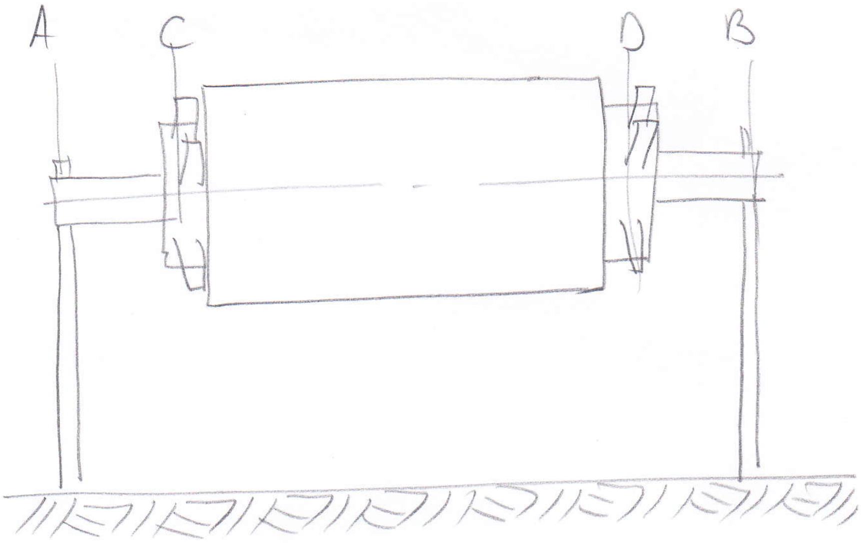
$m_t = 10\ g$ $\begin{cases} \text{na pá 2 no plano C} \\ \text{na pá 1 no plano D} \end{cases}$

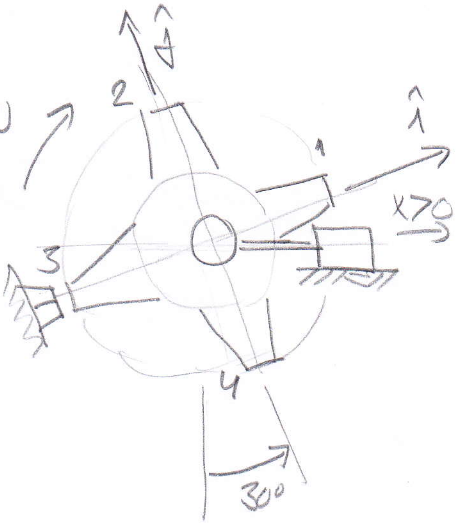
a) $\vec{\delta}_A = ?$ $\vec{\delta}_B = ?$

b) $\alpha_{ij} = ?$

c) m_C e m_D

d) $U_{I,Adm} = ?$, $U_{II,Adm} = ?$ p/ ISO G6,3





$$\vec{\delta}_{A0} = 0{,}1\,\hat{\imath} + 0{,}173\,\hat{j} = 0{,}2 \angle 60°$$

$$\vec{\delta}_{B0} = -0{,}1\,\hat{\imath} + 0{,}0\,\hat{j} = 0{,}1 \angle 180°$$

$$\vec{\delta}_{A} = +0{,}14\,\hat{\imath} + 0{,}05\,\hat{j} = 0{,}15 \angle 20°$$

$$\vec{\delta}_{B} = -0{,}18\,\hat{\imath} + 0{,}05\,\hat{j} = 0{,}19 \angle 165°$$

$$\vec{\delta}_{At} = \vec{\delta}_A - \vec{\delta}_{A0} = 0{,}04\,\hat{\imath} - 0{,}123\,\hat{j} = 0{,}129 \angle -72°$$

$$\vec{\delta}_{Bt} = \vec{\delta}_B - \vec{\delta}_{B0} = -0{,}08\,\hat{\imath} + 0{,}05\,\hat{j} = 0{,}094 \angle 148°$$

$$\vec{m}_{Ct} = 10\,g \angle 90^{\circ} = 0\hat{i} + 10\hat{j}$$

$$\vec{m}_{Dt} = 10\,g \angle 0^{\circ} = 10\hat{i} + 0\hat{j}$$

$$\underbrace{\begin{bmatrix} \alpha_{AC} & \alpha_{AD} \\ \alpha_{BC} & \alpha_{BD} \end{bmatrix}}_{A} \begin{Bmatrix} \vec{m}_{Ct} \\ \vec{m}_{Dt} \end{Bmatrix} = \begin{Bmatrix} \vec{\delta}_{At} \\ \vec{\delta}_{Bt} \end{Bmatrix}$$

$$\begin{bmatrix} \alpha_{AC} & \alpha_{AD} \\ \alpha_{BC} & \alpha_{BD} \end{bmatrix} \begin{Bmatrix} 0\hat{i} + 10\hat{j} \\ 10\hat{i} + 0\hat{j} \end{Bmatrix} = \begin{Bmatrix} 0{,}04\hat{i} - 0{,}123\hat{j} \\ -0{,}08\hat{i} + 0{,}05\hat{j} \end{Bmatrix}$$

$$\alpha_{AC} \cdot 10\hat{j} + \alpha_{AD}\, 10\hat{i} = 0{,}04\hat{i} - 0{,}123\hat{j}$$

$$\alpha_{AC} = -0{,}0123\ g/mm \qquad \alpha_{AD} = 0{,}004\ g/mm$$

$$\alpha_{BC}\, 10\hat{j} + \alpha_{BD}\, 10\hat{i} = -0{,}08\hat{i} + 0{,}05\hat{j}$$

$$\alpha_{BC} = 0{,}005\ g/mm \qquad \alpha_{BD} = -0{,}008\ g/mm$$

c) $$A = \begin{bmatrix} -0{,}0123 & 0{,}004 \\ 0{,}005 & -0{,}008 \end{bmatrix} = \frac{1}{1000} \begin{bmatrix} -12{,}3 & 4 \\ 5 & -8 \end{bmatrix}$$

$$|A| = \frac{1}{10^6} \left(-12{,}3(-8) - 20 \right) = 78{,}4 \cdot 10^{-6}$$

$$A^{-1} = \frac{10^6}{78{,}4} \frac{1}{1000} \begin{bmatrix} -8 & -4 \\ -5 & -12{,}3 \end{bmatrix} = \frac{10^3}{78{,}4} \begin{bmatrix} -8 & -4 \\ -5 & -12{,}3 \end{bmatrix}$$

$$\begin{Bmatrix} \vec{m}_C \\ \vec{m}_D \end{Bmatrix} = A^{-1} \begin{Bmatrix} \vec{\delta}_{A0} \\ \vec{\delta}_{B0} \end{Bmatrix}$$

$$\begin{Bmatrix} \vec{m}_C \\ \vec{m}_D \end{Bmatrix} = \frac{10^3}{78,4} \begin{bmatrix} -2 & -4 \\ -5 & -12,3 \end{bmatrix} \begin{Bmatrix} 0,1\,\hat{\imath} + 0,173\,\hat{\jmath} \\ -0,1\,\hat{\imath} + 0,0\,\hat{\jmath} \end{Bmatrix}$$

$$\begin{Bmatrix} \vec{m}_C \\ \vec{m}_D \end{Bmatrix} = \frac{10^3}{78,4} \begin{Bmatrix} -0,4\,\hat{\imath} - 1,384\,\hat{\jmath} \\ 0,73\,\hat{\imath} - 0,860\,\hat{\jmath} \end{Bmatrix} = \begin{Bmatrix} -5,10\,\hat{\imath} - 17,7\,\hat{\jmath} \\ 9,31\,\hat{\imath} - 11,0\,\hat{\jmath} \end{Bmatrix}$$

$\vec{m}_C = 18,4 \angle \underline{-106°}$ excesso de massa

$\vec{m}_D = 14,4 \angle \underline{-49,8°}$ excesso de massa

$\therefore$ remover $\left.\begin{array}{l} 5,10\text{ g da pá 3} \\ 17,7\text{ g da pá 4} \end{array}\right\}$ plano C //

remover $\left.\begin{array}{l} 9,31\text{ g da pá 1} \\ 11,0\text{ g da pá 4} \end{array}\right\}$ plano D //

d) ISO G 6,3

$n_0 = 3600\text{ rpm} \Rightarrow f_0 = 60\text{ Hz} \Rightarrow \omega_0 = 377\text{ rad/s}$ //

$n = 6,3\text{ mm/s} \geq \omega_0\, e_{adm}$

$e_{adm} \leq 0,0167\text{ mm}$

$U_{adm} = e_{adm} M = 0,167\text{ kg mm}$

$U_{I\,adm} = U_{II\,adm} = 0,0836\text{ kg mm} = 83,6\text{ g mm}$ //