



Twelve-Tone Invariants as Compositional Determinants

Author(s): Milton Babbitt

Source: *The Musical Quarterly*, Vol. 46, No. 2, Special Issue: Problems of Modern Music. The Princeton Seminar in Advanced Musical Studies, (Apr., 1960), pp. 246-259

Published by: Oxford University Press

Stable URL: <http://www.jstor.org/stable/740374>

Accessed: 18/04/2008 06:10

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=oup>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

---

JSTOR is a not-for-profit organization founded in 1995 to build trusted digital archives for scholarship. We enable the scholarly community to preserve their work and the materials they rely upon, and to build a common research platform that promotes the discovery and use of these resources. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

# TWELVE-TONE INVARIANTS AS COMPOSITIONAL DETERMINANTS

By MILTON BABBITT

**A**T the present moment, when many of the jagged edges of abruptness have been smoothed by time and practice, there are those who — presumably in the spirit of mediation and moderation — would minimize, not so much Schoenberg's achievement as a composer, as the degree to which the twelve-tone system is genuinely "revolutionary" in its nature and implications, the degree to which it imposes new demands of perception and conception upon the composer and listener, and — therefore — the degree to which it admits of further and extensive exploration and discovery.

Such an attitude does a disservice not only to Schoenberg, but to the cause of understanding, particularly since it so often involves the invocation of the alleged historical-analogical origins of the operations of the system, along with conjectures as to Schoenberg's mode of and motivation for arriving at the system. However intriguing such conjectures may be, they are as irrelevant as they are futile; however pedagogically convenient and intuitively suggestive a quasi-genetic approach may be, eventually it succeeds only in obscuring both the character of the system and the profound differences between the twelve-tone system and those musical systems in which the "historical forerunners" of the twelve-tone operations appear. The crucial point here is that these "forerunners" are not independent and fundamental structural determinants, but means of immediate procedure, neither necessarily present nor, if present, of more than local significance and influence.

Therefore it is appropriate to precede even so informal a discussion as the one to follow with the reminder that the twelve-tone system, like any formal system whose abstract model is satisfactorily formulable, can be characterized completely by stating its elements, the stipulated

relation or relations among these elements, and the defined operations upon the so-related elements. Such a characterization, though explicitly presented in verbal form at the earliest stage of the twelve-tone development, is likewise easily and explicitly inferable as the maximum procedural intersection among the “classical” twelve-tone works of Schoenberg, Webern, and Berg.

If the elements of the twelve-tone pitch system are, indeed, “traditional” ones, both insofar as they are pitch classes with class membership defined by octave equivalence, and as there are twelve such pitch classes—corresponding to the chromatically equal-tempered quantization of the frequency continuum—even here essential deviations must be noted. In the twelve-tone system there is a one-to-one correlation between pitch notation and presented pitch, as opposed to the many-to-one correlation of triadic-tonal music; there can be no such distinctions as those between explicit and functional “dissonance,” or between enharmonically identical “consonance” and “dissonance.” The independent assumption of octave equivalence has been a frequent point of attack upon the system, particularly by those who assert that the corresponding assumption in the tonal system serves to define classes of equivalent function; it need be answered only that, similarly, this assumption in the twelve-tone system serves to define classes of equivalent order position.

It is in the definition of relations among the elements that the system diverges significantly from systems of the past, for relations are defined entirely by the imposition of a total linear ordering upon the pitch classes, thus defining a twelve-tone “set” (designated: *S*). The ordering employed, in any given work, is inferable from—at most—all of the compositional presentations of the set (and its transformations), and not necessarily from any one compositional presentation. By introducing this principle as the basis of relationship, Schoenberg not only effected a fusion of the general systematic constraint with the contextually defined property—for, although the principle of formation is defined for all sets, the specific pitch class relations defined by a set are uniquely associated with it and its transformations—but established the means of a permutational musical system, as opposed to the combinational systems of the past. Given a collection of available elements, the choice of a sub-collection of these as a referential norm provides a norm that is distinguishable by content alone; such a system, and the traditional tonal system is such, is therefore combinational. But if the referential norm is the totality of elements, there is but one such norm in terms of content, and deviations from this norm cannot exist within

the system. But if an ordering is imposed upon this totality, and taken as a norm, this norm is so distinguished, in the case of twelve pitch class elements, from the  $12!$ <sup>1</sup> — 1 other possible orderings, that is, other possible permutations.

Any consideration of the operations of the system must proceed from an awareness of their permutational nature. As a simple example: transposition, excepting the identity transposition, in a combinational system results in the adjoining of pitches which are not present in the original collection, and thus establishes a new sub-collection; transposition of a set results only in a permutation of the elements. Also, compositional transposition, traditionally, implies contour preservation, a consideration that is, literally, meaningless in defining transposition as a twelve-tone operation, since contour is a function of the registral specification of the elements, and registral choice is as undefined by the structure of a set as is duration, intensity, timbre, or any of the other attributes necessarily associated with a compositional representation of a set; as a result, a set cannot be stated in musical notation without the additional qualification that each pitch sign be taken to signify the total pitch class a member of which it denotes. Since such a qualification only too easily leads to but another confusion of systematic principle with compositional permissive (“a tone may be stated in any octave”), it is both safer and more efficient to represent a twelve-tone set in numerical notation, by an ordered number couple succession, the first member of the couple signifying order position in S (“order number”), the second signifying the “pitch number” of the pitch class. The initial pitch class of S is denoted by the couple (0,0), and is taken as the origin of the coordinate system for both order and pitch numbers, both of which range over the integers 0 — 11 inclusive, each integer appearing once and only once as an order number and a pitch number. In the case of order numbers, this represents the fact that twelve and only twelve pitch classes are involved: in the case of pitch numbers, this is the arithmetical analogue of octave equivalence (congruence mod. 12).<sup>2</sup> In this notation, the set of the Schoenberg Third String Quartet, with registral representation chosen arbitrarily,

Ex. 1



is represented: 0,0; 1,9; 2,8; 3,2; 4,5; 5,10; 6,11; 7,4; 8,3; 9,6; 10,1;

<sup>1</sup>  $12! = (12 \times 11 \times 10 \times 9 \dots \times 1)$ .

<sup>2</sup> Two numbers, a and b, are said to be “congruent mod. 12” if, and only if,  $a-b = k \cdot 12$  where k is an integer (including zero).

**Representação numérica: o 1º número indica a posição na série, o segundo indica a classe de altura (a partir da PC inicial).**

11,7. Thus, the succession of differences derived by the subtraction (mod. 12) of a set number from the following set number is the ordered interval succession determined by a set, and each of these interval numbers denotes, accordingly, a class of intervals.

The operation of transposition applied to any set can be represented by adding (mod. 12) an integer, 0—11, to each pitch number of the set. Thus, if (a,b) is the couple signifying an element of S, then the transposition is represented by (a,b+t), with t termed the “transposition number.” Thus, the transpositional operation (designated: T) is conveniently regarded as an operation on, a permutation of, pitch numbers; for any specified set, it could equally well be regarded as effecting a permutation of order numbers, but the previous characterization corresponds more appropriately to the general conception of transposition.

The totality of twelve transposed sets associated with a given S constitutes a permutation group<sup>3</sup> of order 12; as such it is closed, disjoint with regard to any other collection of sets T derived from a set whose intervallic succession differs from that of any member of this totality. Thus, it constitutes a combinational collection of sets within, not only the totality of all possible sets, but the totality of sets derivable from a given S by the operations of the system.

By virtue of the group structure that it generates, and the additional properties that derive from the commutative and transitive nature of this group, a multitude of attributes necessarily associated with T can be formally deduced. The musical relevance of these attributes can be decided only empirically, of course, but it is my purpose here to examine only a few of those operational invariants (properties of a set

<sup>3</sup> A “group” is a system whose elements (denoted a, b, c . . .), an operation (denoted \*), and an equivalence relation (denoted =) satisfy the following properties:

1. Closure: If a, b are elements of the system, then a\*b is an element of the system.
2. Associativity: If a, b, c are elements of the system, then (a\*b)\*c = a\*(b\*c).
3. Existence of an identity: There is an element of the system, e, such that, for each element of the system (say, d), d\*e = e\*d = d.
4. Existence of an inverse: For each element of the system (say, d), there exists an element of the system, d-, such that d\*d- = d-\*d = e.

In interpreting the twelve-tone system as a group, the elements of the group are twelve-tone sets, represented as permutations of pitch or order numbers; the operation is the multiplication of permutations. “S” is the identity element. The “order” of a group is the number of elements of the group.

In addition, the groups presented here have the property of “commutativity”: if a, b are elements of the system, then a\*b = b\*a.

that are preserved under the operation, as well as those relationships between a set and the so-operationally transformed set that inhere in the operation) which may be termed "musical invariants," requiring for their aural recognition merely the ability to perceive pitch class identity and non-identity, and interval class identity and non-identity.

The familiar invariant associated with  $T$  is that of preservation of the interval number succession. For all its obviousness, it appears a powerfully cohesive property in the light of the total non-invariance of pitch classes with regard to order; that is, no order, pitch number couple remains fixed under  $T$ . Since each  $t$  produces a total derangement of the set elements, and the identical intervallic succession, neither of these properties can serve as the bases of differentiation, in the search for possible criteria for the compositional hierarchization of transpositions. Similarly, every value of  $t$  defines a regular permutation, but an invariant basis for differentiation appears with the recognition that complementary  $t$ 's (numbers whose sum is  $0, \text{ mod. } 12$ ), and only such  $t$ 's, produce inverse permutations, of equal order. Beyond the immediate boundary conditions on the intervallic structure of a set inferrable from this fact, a consequential musical property follows from the further fact that inverse permutations produce the same number of order inversions. This measure of the extent of order rearrangement of the pitch classes can be described most easily by, for the moment, regarding  $T$  as an operation on the order numbers of  $S$ ; an order inversion is each relation among pairs of order numbers that violates the normal ascending relation among order numbers in  $S$ . For example, in the set of Ex. 1, the application of, let us say,  $t = 4$  to the set produces the following order number succession: 7, 10, 0, 9, 1, 3, 8, 2, 11, 5, 4, 6. The complementary  $t$  ( $t = 8$ ) produces the order number succession: 2, 4, 7, 5, 10, 9, 11, 0, 6, 3, 1, 8. The number of order inversions produced by each is 32.

Complementary  $t$ 's produce the same number of pitch adjacencies with regard to  $S$ , both ordered adjacencies and reversed adjacencies. (This condition of adjacency is imposed merely in the light of the simplest compositional exploitation of this property, which is immediately extensible to pairs of pitch classes associated with any distribution of order numbers.) If a set possesses successive pitch classes represented by pitch numbers  $a$  and  $b$ , and successive pitch classes represented by pitch numbers  $c$  and  $d$  ( $c$  may or may not be equal to  $b$ , and similarly for  $d$  and  $a$ ), and if  $b - a = d - c$ , then there is a  $t$  such that  $a + t = c$ , and  $b + t = d$ , so that under  $t$ ,  $a$  and  $b$  are associated with the original

order numbers of *c* and *d*, and it then follows that under  $12 - t$ , *c* and *d* are associated with the original order numbers of *a* and *b*. So, too, for reversed adjacencies, represented in the set by complementary intervals. The intervallic structure of *S*, then, determines the number of adjacencies preserved under a particular *t* and its complement, since this number is a function of the multiplicity of and relative pitch placement of the identical and complementary interval numbers in *S*. Consider the set of Ex. 1: the interval succession determined by disjunct dyads is 9,6,5,5,3,6. The interval between the identical 5's is 6, between the complementary intervals 9 and 3 is also 6, and interval 6 is its own complement. So, under the application of  $t = 6$ :

Ex. 2



But, in addition, the pattern of intersection in terms of order numbers of 3a with regard to 1 is: 0,1,2,5; the pattern of intersection of 1 with regard to 3b is similarly: 0,1,2,5. The actual pitch classes involved, naturally, are not necessarily identical, and are not in the present case. This operational invariant resulting from complementary transposition is not only of obvious rhythmic and functional significance compositionally, but of essential systematic consequence in the theory of general combinatoriality, aggregate structure, and the resultant means of hierarchization of set segments.

The importance of transpositional complementation alone would serve to suggest the systematic operation of inversion (designated: *I*), which is definable as complementation mod. 12 of each pitch number of *S*, as opposed to complementation of the *t* applied to all pitch numbers of *S*. Given a set element (*a*,*b*), *I* transforms it into (*a*, $12 - b$ ), or, more generally (*a*, $(12 - b) + t$ ), since *T* is applicable uniquely to the inverted set; with relation to the complex of sets generated by *T*, the inverted set assumes the local role of *S*. *I* and *T* commute only to within complementation; therefore, the order of operations must be specified, and I shall assume throughout this discussion that *T* is applied after *I* (*IT*).

At this point, it is appropriate to consider comparable definitions of the remaining operations of the system, which reveal that retrogression (designated: *R*) can be regarded as affecting complementation of order numbers: (*a*,*b*) is transformed into ( $11 - a$ , *b*); therefore *RI* (or the reverse, since the operations commute) merely involves the simultaneous application of both complementation operations: (*a*,*b*) is transformed into ( $11 - a$ ,  $12 - b$ ). As in the case of *I*, *t*'s are applicable to the set numbers.

The presentation of the permutations of *S* defined by these operations, mod. transposition (in other words, this representation is independent of the operation of transposition), in the usual group multiplication table, with multiplication permutations as the group operation, and *S* denoting the identity permutation:<sup>4</sup>

<i>S</i>	<i>I</i>	<i>R</i>	<i>RI</i>
<i>I</i>	<i>S</i>	<i>RI</i>	<i>R</i>
<i>R</i>	<i>RI</i>	<i>S</i>	<i>I</i>
<i>RI</i>	<i>R</i>	<i>I</i>	<i>S</i>

<sup>4</sup> The group table is read by choosing an element in the first column and the element with which it is to be multiplied in the first row; the result of this multiplication is found at the intersection of the row of the first and the column of the second.



reveals, first of all, that this collection of permutations is an instance of a group, and as such possesses the property of closure, thus assuring another combinational aspect of the system in the large, since the collection of sets determined by these permutations is disjunct with regard to any so determined collection of sets one of whose sets is not a member of this collection, and is identical with a collection one of whose members is.

The revealed symmetry properties indicate the hazardous connotations of such terms as "basic" or "original" set to denote other than a set form norm decided upon purely on the basis of contextual considerations (temporal priority, for example), since such terms cannot designate any attribute of set structure in a general sense, either from a standpoint of internal properties or of relation to other set forms. Similarly, the fact that the period of each of the permutations is 2 should, in itself, dispel once and for all those futile attempts to "equate" these operations with tonal functions.

A vast literature of group theory supplies necessary properties of such a structure, and it is not without extra-musical interest that this particular group of permutations is an instance of a familiar group structure, the so-called "four-group."

The twelve-tone system, as system, is indeed "simple." It is simple in its principles of formation and transformation, but enormously complex and deep in its ramifications, in the necessary inferences that can be drawn from these principles, for it is of the formal model of which it is an exemplification that Hermann Weyl has said: "From these insignificant looking assumptions springs an abundance of profound relationships; and mathematics offers an astounding variety of different interpretations of this simple axiom system."<sup>5</sup>

Inversion, in the traditional sense, implies inversion of contour. In the twelve-tone system inversion, like transposition, cannot be characterized in terms of registral considerations, but merely as that permutation of pitch class numbers (or, for particular purposes, of order numbers) which results from the substitution of complementary pitch numbers in  $S$ ; it follows that there is an accompanying substitution of a succession of complementary intervals for the interval succession of  $S$ .

Even more, perhaps, than in the case of  $T$ ,  $I$  must derive its "justification" from its associated musical invariants. It must be emphasized

<sup>5</sup> *Philosophy of Mathematics and Natural Science*, Princeton, 1949, p. 28.

that, although invariants are associated necessarily with the operations in question, the degree to which they are projected explicitly in compositional terms depends upon the emphasis they receive from other musical components: rhythm, dynamics, register, phrasing, timbre, etc. Conversely, the desire for the compositional exploitation of these fundamental properties may be regarded as, at least, a partial determinant of the compositional characteristics imposed on these components.

Consider the simplest inversional invariant: if  $(a,b)$  of  $S$  is transformed by  $IT$  into the corresponding order element  $(a,(12 - b) + t)$ , where  $(12 - b) + t$  may or may not be equal to  $b$ , then, corresponding to  $(c,(12 - b) + t)$  of  $S$  is  $(c,b)$ . This property, as may be observed by reference to our examples, is possessed also by transposition when  $t = 6$ ; it holds for all  $t$ 's applied to the inversion.

Again, the cyclic representation of the pitch class permutations effected by  $IT$  shows that all even  $t$ 's produce similar permutations of six cycles of two elements each (thus, regular permutations), while odd  $t$ 's produce similar permutations of five two-element cycles and two unit cycles. So, although complementary  $t$ 's still produce similar permutations, this is merely because they are either both even or both odd. Odd values of  $t$ , then, determine six dyadic pitch classes between elements of the same order number in  $I$  related sets, and even values of  $t$  determine five such dyadic classes, and two single-element classes. These latter represent set elements whose order number, pitch number couple remains unchanged under  $IT$ . It is for this reason that a necessary condition for hexachordal inversional combinatoriality is that the sum of the set numbers of the same order number in the  $I$  related sets be odd. The pitch number of an element so fixed is equal to one half of  $t$ ; thus, the two such fixed elements associated with a given even  $t$  are unique, and are tritone related (since  $12/2 = 6$ ).

In addition to this partition of the elements of inversionally related sets into pitch classes, the  $IT$  operation also effects a categorization into interval classes. Since the intervals between pitch classes of the same order number in  $I$  related sets are either all even or all odd, and — thus — each interval occurs exactly twice, it can be shown that pitch classes in  $S$  whose pitch numbers differ by 6 are associated with the same interval determined by the element of the same order number in the  $I$  related set.

The largely "note against note" presentation of the canon in the second movement of Webern's *Variations for Piano* places these char-

acteristics in the foreground. The initially stated, I related forms of the set (Ex. 4a):

Ex. 4

The image shows three musical examples, (a), (b), and (c), each consisting of two staves. (a) and (b) show a sequence of notes in two staves, with (b) starting with a key signature change to one flat. (c) shows a sequence of notes in two staves with dashed lines connecting corresponding notes between the two staves, illustrating a permutation.

since  $t = 2$  (taking the lower of the two sets as the reference “prime” set), hold (1,1) and (11,7) fixed. Since the first hexachord of S contains no tritone related pitches, there is no repetition of pitch dyads formed by elements of the same order number, so that, if the first succession of six dyads is numbered 1,2,3,4,5,6, then the following succession is a permutation of these: 6,4,1,5,3,2. As in the Schoenberg example, although in a different manner, complexes of pitch elements become, themselves, subjected to serial permutation.

The continuation of this canon demonstrates the compositional use of another invariant. By choosing, as the initial pitches (or, as any pitches of the same order number, since the sums of the pitch numbers of such pairs are equal) of the T forms of the S and the I related set, elements the sum of whose pitch numbers, with regard to the original reference point “g#” = (0,0), is equal to the original  $t = 2$ , the pitch dyads resulting are identical with those created in the first inversive juxtaposition. The recurrence of repeated “a’s” is merely one manifestation of this general property. See Ex. 4b. Therefore, for each first element of a set, there is one and only one choice of the first element of the I related set which holds the pitch dyads so fixed with regard to a pre-defined norm. (In the light of the previous discussion, it is of interest to point out that an equivalent statement of this condition is that the initial I related forms be transposed by complementary values of  $t$ .) Webern chooses four of these twelve possibilities to determine the pitch levels of the successive sections of the movement; the first and last pitches of the I related sets which provide the pitch content of the third and fourth sections of the work are shown in Ex. 4c. The first

movement of Webern's *Quartet*, Op. 22, employs the same procedure in a more elaborate and extended manner.

The more common and traditional procedure of "totally transposing" such a section — in my terms, the applying of the same  $t$  to both simultaneously stated forms — preserves the interval succession but not (except the single case where  $t = 6$ ) the fixed pitch dyads: the procedure under discussion here permutes the interval succession while retaining the pitch content of the dyads.

If we number the dyads of the first section from 1 - 12 inclusive, the second section yields a permutation corresponding exactly to the permutation of order numbers that results from applying  $t = 5$  to the upper set, or — equivalently —  $t = 7$  to the lower set, and — of course — correspondingly with the third and fourth sections. Thus, all the properties associated with the application of  $T$  to  $S$  are translatable into properties of permutations of dyads between  $I$  related sets.

Webern's particular choice of transpositions appears to be related primarily to concerns of compositional duration and external design. The transpositional choice for the second section makes possible a final dyad for this section which is pitch identical with the initial dyad of the movement, but with pitch components reversed as to set membership. The repeat of these first two sections is founded on this identity. The third section is transpositionally determined by exact analogy with the second section, through the "double function" of the final dyad of the preceding section. But the continuation of this basis of choice, since the interval between the first and last elements of the set is prime to 12, would carry the work through all possible twelve jointly determined transpositions before returning to the first dyad in its original disposition. So, the fourth section employs the principle of interchange, already introduced into the work by the repeat of the first two sections. The fourth section thus effects a return to the first dyad of the movement, while the repeat of the third and fourth sections as a unit results in another interchange, necessarily the exact reverse of the interchange resulting from the first repeat.

Closely related to these invariants is the property: if  $(a,b)$  and  $(a + 1,c)$  are two successive elements of  $S$  (the provision of succession is, actually, unnecessary, but is introduced here for purposes of simplicity), and  $(d,e)$  is an element of an  $I$  related set (where  $d$  may or may not be identical with  $a$  or  $a + 1$ , and  $e$  may or may not be identical with  $b$  or  $c$ ), then the intervallic succession  $b - e, c - e$  is



set when it is realized that adjacent pitch elements of a set become elements of the same order number in I related sets, when these sets are so chosen that the sum of the pitch numbers of any two elements of the same order number is equal to the sum of the pitch numbers of the originally adjacent elements.

Space does not permit a consideration of invariance under R and RI. However, it must be pointed out that the traditional conception of retrogression as effecting the temporal reversal of pitches constitutes neither a meaningful description nor a "justification" for its position in the twelve-tone system. For, even with registral considerations disregarded, this characteristic is associated with but one transpositional level of the retrograde; but all transpositional levels of the retrograde present the intervallic succession of the *inversion* in reversed order, while — necessarily — the retrograde-inversion forms present the intervallic succession of the *prime* (S) in reversed order. Thus, the RI forms, often regarded as the aurally most unrealistic transformations, because the operation is viewed as applied to pitch succession rather than to interval succession, require for the perception of their relation to S merely the ability to identify interval classes. In this important sense, the RI forms can be regarded as the most closely related to S, and are so employed often by Schoenberg in his compositional, "thematic" presentation of successive set forms: see, for example, the Variations for Orchestra, the third movement of the Fourth String Quartet, and the Piano Concerto. In the pitch class order, interval class order duality between retrograde and retrograde-inverted related sets reside many of the most important properties of such transformations.

Even so incomplete and informal a discussion of so small a number of the invariants attending the operations of the system indicates, I hope, something of the essential importance of this subject, analytically in the "rational reconstruction" of compositions, and compositionally in comprehending and mastering the materials of the system. If I have led the discussion more in the direction of those aspects which suggest the "macrocosmically" combinational features of this basically permutational system, I could have — with equal appropriateness and the same means — examined the "microcosmically" combinational features (particularly what are termed in group theory "imprimitive systems"), set structure (particularly with regard to redundancy properties), combinatoriality, generalized partitioning, derivation, sequence theory, and related questions.

Certainly, any conjectures about "generalized" serialism must con-

front the problem as to whether such alleged generalizations result in a maintenance, an increase, or a decrease, of the number and scope of such invariants, and whether the apparent "freedom" of such "generalizations" does not, in a deeper sense, reduce structural resources rather than augment them.

Likewise, I would insist that a necessary condition for the application of the permutational operations of the twelve-tone system to orderable non-pitch elements is that the rules of correlation be so arrived at that these invariants, which are necessary consequences of the pitch class nature of the system, be susceptible to musically meaningful interpretations in these other, perhaps significantly dissimilar, domains.

In conclusion, I can state only, without hoping to have done more than intimate the bases for such a statement, that an "exhaustion" of the resources of the twelve-tone system in the relevant future is not only unforeseeable, but unthinkable. I trust I have begun to document, and will be given the opportunity in the future to further document, the statement that, in its vastness of structural means, its flexibility, and its precision, the twelve-tone system cedes nothing to any musical system of the past or present that has engaged the mind of musical man.