

## Lecture 3 (Part 1) Physics 4213/5213

### 1 Fundamental QED Feynman Diagram

The most fundamental process in QED, is give by the definition of how the field interacts with matter—with point charges, these can be either fermions or bosons. This interaction is defined in term of the field  $A^\mu$ , which is given by  $A^\mu = J^\mu/q^2$  and recalling that  $q^2$  is the change in the 4-momenta of the charged particle and is therefore the 4-momenta carried by the photon. This most basic interaction is given by the current  $J^\mu$  due to the charged particle and the photon that carries 4-momenta  $q^2$  (see fig. 1). From this one diagram all other process can be built.

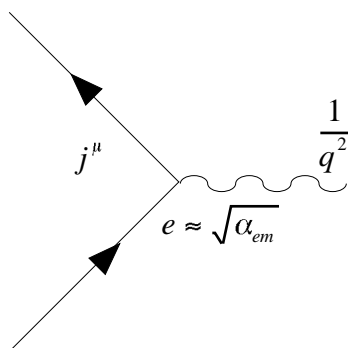


Figure 1: The basic electromagnetic interaction in terms of a Feynman Diagram. All other possible QED processes are built from this diagram.

### 2 Electron-Electron Scattering

The simplest QED process is the scattering of two electrons off each other. This process is built out of the the fundamental diagram being applied twice; once for each of the two electrons. This process represents an electron emitting a photon and an electron absorbing the photon—that is the transfer of 4-momenta between the two electrons (see fig. 2. (It is not important which electron emits the photon and which absorbs it, as the diagram represents both possibilities.)

The next question of interest is, what is the mass of the virtual photon—stated differently what is the 4-momenta transfer between the two electrons. (*The square of the 4-momenta is the mass squared of the photon.* The process that this diagram represents is elastic scattering, where the energy of the two electrons does not change, but the direction does change when viewed in the center of mass frame. This implies that 3-momentum has been exchange, therefore  $\Delta E = 0$  and  $\Delta p \geq 0$ . This leads to  $q^2 = (\Delta E)^2 - (\Delta p)^2 = -\Delta p$ , therefore the has a mass given by  $q^2 \leq 0$ . So that the photon is spacelike or in the limit of no scattering lightlike (on-shell). (As an aside,  $q^2$  is an invariant and therefore spacelike as viewed from all frames.) Further, since all elastic scattering diagrams look the same, these diagrams all represent spacelike events.

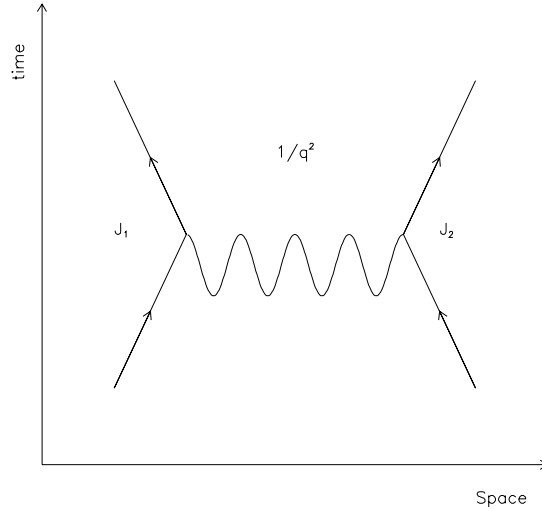


Figure 2: This Feynman diagram represents the scattering of two electrons. It is built by using the basic QED diagram twice.

### 3 Electron-Positron Scattering

Taking the Feynman diagram of figure 2 and rotating it by  $90^\circ$  gives a particle traveling forward in time and one traveling backward in time (see fig. 3). The particle that travels backward is interpreted as a forward going anti-particle. Therefore this process describes an electron and a positron annihilating creating a virtual photon and then producing an electron and a positron. This is not the complete story, the electron and positron could simply scatter (see fig. 3). This process would be indistinguishable to any observer from the annihilation process in that an electron and positron are seen in the initial and final states. Therefore, the amplitude and cross section must be composed of both diagrams when calculating to second order. This is to be compared to electron-electron scattering where only one term contributes, this is the origin of difference between having an attractive and a repulsive force for like and unlike charges—the difference is not obvious in the diagrams themselves, other than to say that the annihilation diagram might cause attraction, the full calculation needs to be carried out to show that one set leads to attraction while the other leads to repulsion.

The Feynman diagram given by the scattering process is treated the same as in electron electron scattering. That is, when viewed from the center of mass frame, only momentum is exchanged making the photon spacelike. The second process (annihilation diagram) is very different. First of all the  $e^+$  and  $e^-$  travel to a common point and convert into a photon. In the center of mass

frame the photon has zero momentum, but has an energy given by the sum of the energy of each particle  $E_\gamma = E_{e^+} + E_{e^-} = 2E_e$ . Since the photon is at rest, its mass is equal to the center of mass energy or twice the energy of one of the particles:  $q^2 = 2E_e = \sqrt{s}$ . This quantity is always positive, making the photon timelike. Notice that the photon can decay to any pair of particle anti-particle pair that has a mass that is less than or equal to the mass of the photon. Another point, as long as all quantum numbers are conserved, a single particle can be created if the mass is exactly equal to the total center of mass energy.

### 3.1 Particle Production Through $e^+e^-$ Collisions

As just stated, the annihilation diagram can be used to produce any pair of particles that have a mass less than or equal to the center of mass energy of the electron and positron—*note each particle has a mass that is half the mass of the center of mass energy*. Examples of this are the production of muon pairs, tau pairs and quark pairs, in fact any pair of particles that couple to a photon:

$$e^+ + e^- \rightarrow \mu^+ + \mu^- \quad e^+ + e^- \rightarrow \tau^+ + \tau^- \quad e^+ + e^- \rightarrow q + \bar{q} \quad (1)$$

All these process are described to lowest order by one Feynman diagram given in figure 4.

Another possible process is the production of a single massive particle. This can only occur kinematically if the center of mass energy is equal to the mass of the particle. Additionally, the particle produced must have the quantum numbers of the photon, such as zero charge (*charge conservation*), spin one (*angular momentum conservation*), zero lepton number and baryon number (*lepton and baryon number conservation*), plus addition internal quantum number that will be discussed later. Examples of single particles produced this way are:

$$e^+ + e^- \rightarrow J/\psi \quad e^+ + e^- \rightarrow \Upsilon \quad (2)$$

## 4 Higher Order Corrections

Having just shown that the electron-positron scattering is the sum of two indistinguishable Feynman diagrams to second order, the next question to ask is are there other diagrams that contribute. Again to contribute, the external lines have to be the same but the internal processes can be different. For simplicity only electron-electron scattering will be considered. The first possibility is to take the internal photon have it create an electron-positron pair that then annihilates and creates the photon again (see fig 5). The process of adding loops can be repeated an infinite number of times.

This process will effectively change the charge; the value of the coupling constant. The reason for this is that the produced pairs shield the charge with the positrons pulled toward the central electron and the electron repelled—this is the same situation as the polarization that is encounter in dielectric media:

$$F \propto \frac{q_{\text{eff}}}{r^2}, \quad q_{\text{eff}} = \frac{q}{\epsilon}. \quad (3)$$

Therefore the closer to the charge that is approached, the large will be the measured charge of the electron. The value measured in low energy laboratory experiments is  $\alpha_{\text{em}} = \frac{1}{137}$ . In high energy experiments the exchanged photon will have a shorted wavelength  $\lambda \approx \frac{1}{p}$  therefore it probes

a shorter distance (closer to the electron). This leads to the expectation that the charge seen is larger. In fact the charge measured when electron and positrons scatter off each at a center of mass energy of 80 GeV is 1.066 times larger, or stated differently  $\alpha_{\text{em}}(80 \text{ GeV}) = \frac{1}{128}$ . This is in fact predicted by QED.

The loop diagram given above is not the only correction that effects the fundamental properties of the electron (or any other particle). Figure 6 affects the mass, while figure 7 affects the dipole moment. The mass effect can be seen from the fact that the emitted and re-absorbed photon causes fluctuations in the electron mass. These photons can have any energy and so the probability for these terms to occur is infinite. This infinity is taken care of by stating that the bare mass of the electron is infinite and the infinite correction subtracts off from the bare mass giving a finite mass; this being the mass seen in experiments.

Figure 7 has the same problem but it causes a spin flip between incoming and outgoing electrons. Spin flips are connected to the magnetic dipole moment, therefore this term causes a finite change from the expected value. This effect has been measured for both electrons and for muons. The measured value of the magnetic dipole moment of muon and that predicted by QED agree to 10 significant figures, making QED the best theory in existence.

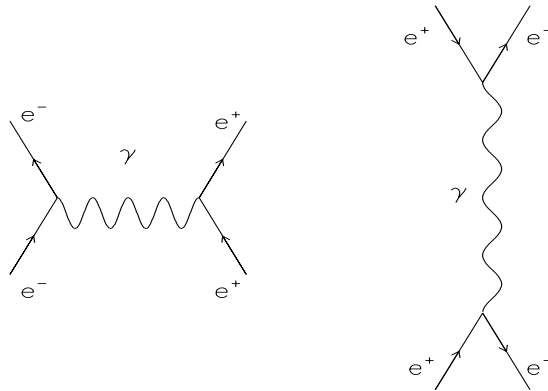


Figure 3: Lowest order Feynman diagrams describing  $e^+ + e^- \rightarrow e^+ + e^-$  scattering.

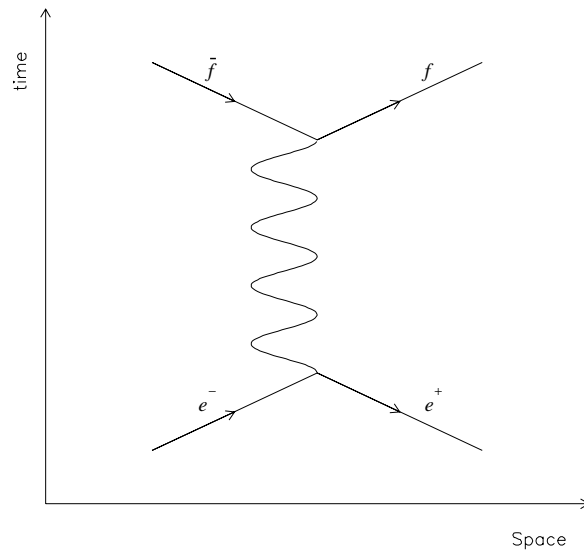


Figure 4: The annihilation diagram for electrons and positrons to other fermions. This is the only two vertex diagram for this process.

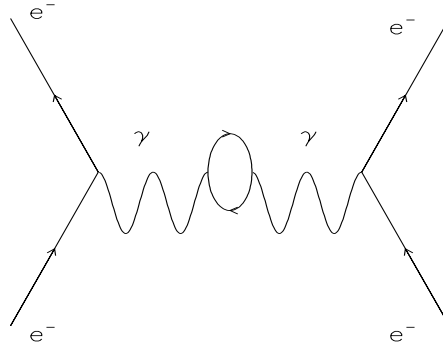


Figure 5: A possible fourth order Feynman diagram describing  $e^- + e^- \rightarrow e^- + e^-$  scattering.

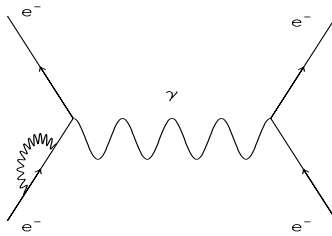


Figure 6: A possible fourth order Feynman diagram describing  $e^- + e^- \rightarrow e^- + e^-$  scattering. This diagram effects the mass of the electron.

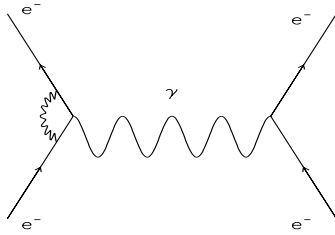


Figure 7: A possible fourth order Feynman diagram describing  $e^- + e^- \rightarrow e^- + e^-$  scattering. This diagram effects the magnetic dipole moment of the electron.

## 5 Outline

1.  $e^+ + e^- \rightarrow e^+ + e^-$  scattering
  - (a) Particles that travel backwards in time.
  - (b) Two second order processes.
2. Higher order electromagnetic corrections
  - (a) Loop corrections and infinities.
  - (b) Bremstr. and infinities.
  - (c) Renormalization and defining masses and charges.