Introduction to Nuclear and Particle Physics
8.701

Lecture 10
QED II

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Outline

- EM basics
- The QED Feynman rules
- QED example processes
  - A) Electron-Muon scattering
  - B) Electron-electron scattering (Moeller scattering)
  - C) Electron-positron scattering (Bhabha scattering)
  - D) Photon-electron scattering (Compton scattering)
  - E) Electron-positron (Fermion pair production)
- Casimir trick and trace theorems
- Renormalization

EM basics

Review

- Maxwell equations

(1) $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$

(2) $\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$

(3) $\vec{\nabla} \cdot \vec{B} = 0$

(4) $\vec{\nabla} \times \vec{B} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J}$

- Field strength tensor: Anti-symmetry 2nd rank tensor in terms of E and B

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Four-vector:

$$J^\mu = (c\rho, \vec{j})$$

Continuity equation:

$$\partial_\mu J^\mu = 0$$

Formulation of inhomogeneous Maxwell equations:

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu$$

EM basics

Scalar and vector potential

- The 3rd Maxwell is equivalent to the statement for the magnetic field:

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

- The 2nd Maxwell takes the following form:

$$\vec{\nabla} \times \left(\vec{E} + \frac{1}{c} \frac{\partial \vec{A}}{\partial t} \right) = 0$$

- The electric field can be written as follows:

$$\vec{E} = -\vec{\nabla} V - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}$$

- Relativistic notation:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$A^\mu = (V, \vec{A})$$

Defect of potential formulation:
⇒ V and A are not uniquely determined!

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{E} = -\vec{\nabla} V - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}$$

New possible potential:

$$A'_\mu = A_\mu + \partial_\mu \lambda$$

Change of potential: Gauge transformation: Lorentz condition

$$\partial_\mu A^\mu = 0$$

EM basics

Photon field

- Inhomogeneous Maxwell equations

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu (\partial_\mu A^\mu) = \frac{4\pi}{c} J^\nu$$

- With Lorentz condition:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) A^\nu = \frac{4\pi}{c} J^\nu$$

Photon field in QED

- Free photon case:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) A^\nu = 0$$

Constrain: Massless particle: Photon

$$p^\mu p_\mu = 0 \quad E = |\vec{p}|c$$

- Solution:

$$A^\mu(x) = a e^{-(i/\hbar) p \cdot x} \epsilon^\mu(x)$$

Polarization vector

EM basics

Polarization vector

- Lorentz condition requires:

$$p^{\mu}\epsilon_{\mu} = 0$$

- In the Coulomb gauge, i.e.: $\vec{\nabla} \cdot \vec{A} = 0$

$$\epsilon^0 = 0 \qquad \vec{\epsilon} \cdot \vec{p} = 0$$

- The polarization three-vector is perpendicular to the direction of propagation: Free photon is transversely polarized: Coulomb gauge = Transverse gauge

- Two linear independent three-vectors:

$$\vec{\epsilon}_1 = (1, 0, 0) \qquad \vec{\epsilon}_2 = (0, 1, 0)$$

Two linear independent three-vectors perpendicular to p!

- Note:
  - Massless particle: Only two spin states (helicity +1 or -1) regardless of its spin, except spin 0 with only one spin state!
  - Massive particle: 2s+1 spin states (3 for spin 1 particle!)

QED Feynman rules

Lagrangian in QED

- Sum of Dirac free field (charged particles) and interaction term:

$$\mathcal{L}_{QED} = \mathcal{L}_{free} + \mathcal{L}_{interaction} =$$

$$= i\hbar c\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\bar{\psi}\psi - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} - q(\bar{\psi}\gamma^{\mu}\psi)A_{\mu}$$

Compare this to:

$$\mathcal{L}_{EM} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} - \frac{1}{c}J^{\mu}A_{\mu}$$

Here: $J^{\mu} = cq(\bar{\psi}\gamma^{\mu}\psi)$

- Local gauge invariance
  - L_{QED} can be formally obtained from L_{free} with the following substitution: $\mathcal{D}_{\mu} = \partial_{\mu} + i\frac{q}{\hbar c}A_{\mu}$
  - With this substitution, L_{free} is invariant under the following local gauge transformation: U(1)

    $$\psi \rightarrow e^{i\theta(x)}\psi$$

  - With the above substitution of the partial derivative, we convert a global invariant Lagrangian into a locally invariant Lagrangian. This is called minimal coupling rule!

QED Feynman rules

General considerations: Electron/Positron

- Relativistic quantities

$$E = \sqrt{m^2c^4 + \vec{p}^2c^2} \qquad p = (E/c, \vec{p}))$$

- Electron and positron spinor

$$\psi(x) = ae^{-(i/\hbar)p\cdot x}u^s(p) \qquad \psi(x) = ae^{(i/\hbar)p\cdot x}v^s(p)$$

- Dirac equation for electron and positrons

$$(\gamma^\mu p_\mu - mc)u = 0 \quad (\gamma^\mu p_\mu + mc)v = 0$$

Adjoint: $\left\{ \begin{array}{l} \bar{u}(\gamma^\mu p_\mu - mc) = 0 \\ \bar{v}(\gamma^\mu p_\mu + mc) = 0 \end{array} \right.$

- Properties among electron and positron spinors
  - Orthogonal: $\bar{u}^{(1)} \cdot u^{(2)} = 0 \qquad \bar{v}^{(1)} \cdot v^{(2)} = 0$
  - Normalization: $\bar{u}u = 2mc \quad \bar{v}v = -2mc$
  - Completeness: $\sum_{s=1,2} u^{(s)}\bar{u}^{(s)} = (\gamma^\mu p_\mu + mc) \quad \sum_{s=1,2} v^{(s)}\bar{v}^{(s)} = (\gamma^\mu p_\mu - mc)$

QED Feynman rules

General considerations: Photon

- Relativistic quantities

$$p = (E/c, \vec{p})) \qquad E = |\vec{p}|c$$

- Photon field

$$A^{\mu}(x) = a e^{-(i/\hbar)p\cdot x}\epsilon_s^{\mu}(x) \qquad s = 1, 2$$

- Lorentz gauge $\epsilon^{\mu}p_{\mu} = 0$

$$\epsilon^{\mu^*}\epsilon_{\mu} = 1 \qquad \epsilon_{(1)}^{\mu^*}\epsilon_{\mu(2)} = 0$$

$$\vec{\epsilon}_1 = (1, 0, 0)$$

$$\vec{\epsilon}_2 = (0, 1, 0)$$

- Coulomb gauge

$$\epsilon^0 = 0 \qquad \vec{\epsilon}\cdot\vec{p} = 0$$

$$\sum_{s=1,2}(\epsilon_{(s)})_i(\epsilon^*_{(s)})_j = \delta_{ij} - \hat{p}_i\hat{p}_j$$

$$i, j = 1, ..., 4$$

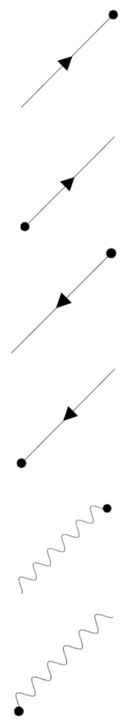
QED Feynman rules

1 Notation

- Label incoming and outgoing four-momenta: $p_1, p_2, \ldots, p_n$ and the corresponding spins: $s_1, s_2, \ldots, s_n$
- Assign arrows:
  - External lines: Electron, positron
  - Internal lines: Preserve direction of flow

2 External lines

- Electron:
  - Incoming: u
  - Outgoing: $\bar{u}$
- Positron:
  - Incoming: $\bar{v}$
  - Outgoing: v
- Photon:
  - Incoming: ϵ^{μ}
  - Outgoing: ϵ^{μ^*}



3 Vertex factor $ig_e\gamma^{\mu}$ $\quad g_e = \sqrt{4\pi\alpha}$

$$\not{a} = a^{\mu}\gamma_{\mu}$$

4 Propagator

- Electron/Positron $\dfrac{i(\gamma^{\mu}q_{\mu} + mc)}{q^2 - m^2c^2}$
- Photon $-\dfrac{ig_{\mu\nu}}{q^2}$

QED Feynman rules

- 5 Conservation of energy and momenta

$$(2\pi)^4\delta^4(k_1 + k_2 + k_3)$$

- 6 Integrate over internal lines

$$\frac{d^4q}{(2\pi)^4}$$

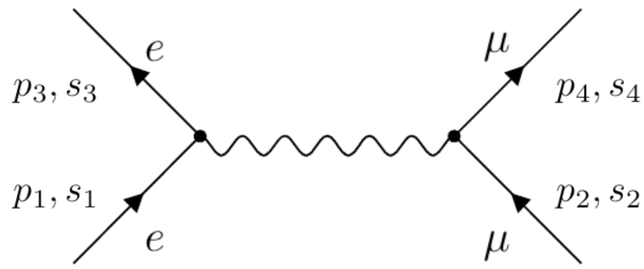
- 7 Cancel the delta function
  - The result will include a factor: $(2\pi)^4\delta^4(p_1 + p_2 + ... - p_n)$

    Corresponding to overall energy-momentum conservation. Cancel this factor and what remains is -iM!

- 8 Antisymmetrization
  - Include a minus sign between diagrams that differ only in the interchange of two incoming (or outgoing) electrons (or positrons), or of an incoming electron with an outgoing positron (or vice versa).

A) Electron-muon scattering

- Feynman graph



- Application of Feynman rules:

$$(2\pi)^4 \int [\bar{u}^{(s_3)}(p_3)(ig_e\gamma^\mu)u^{(s_1)}(p_1)] \frac{-ig_{\mu\nu}}{q^2} [\bar{u}^{(s_4)}(p_4)(ig_e\gamma^\nu)u^{(s_2)}(p_2)]$$

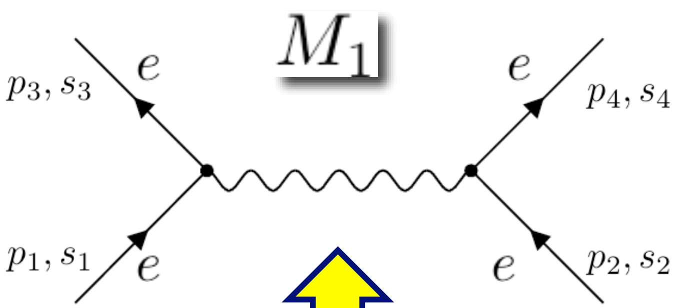
$$\times\delta^4(p_1 - p_3 - q)\delta^4(p_2 + q - p_4)d^4q$$

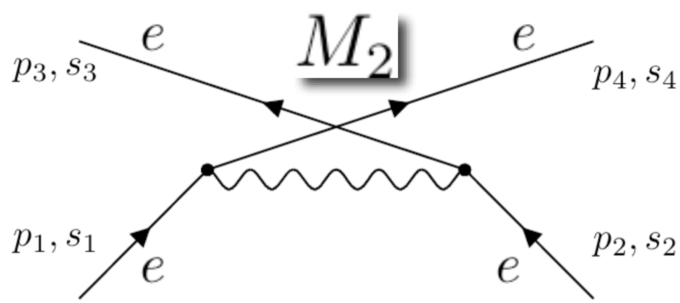
- Integration over q yields:

$$\mathcal{M} = -\frac{g_e^2}{(p_1 - p_3)^2}[\bar{u}^{(s_3)}(p_3)\gamma^\mu u^{(s_1)}(p_1)][\bar{u}^{(s_4)}(p_4)\gamma_\mu u^{(s_2)}(p_2)]$$

QED example processes

- B) Electron-electron scattering (Moeller scattering)





$$\mathcal{M} = -\frac{g_e^2}{(p_1 - p_3)^2}[\bar{u}^{(s_3)}(p_3)\gamma^\mu u^{(s_1)}(p_1)][\bar{u}^{(s_4)}(p_4)\gamma_\mu u^{(s_2)}(p_2)]$$

Interchange: p_3, s_3 p_4, s_4

$$+\frac{g_e^2}{(p_1 - p_4)^2}[\bar{u}^{(s_4)}(p_4)\gamma^\mu u^{(s_1)}(p_1)][\bar{u}^{(s_3)}(p_3)\gamma_\mu u^{(s_2)}(p_2)]$$

QED example processes

B) Electron-electron scattering (Moeller scattering)

- Cross-section calculation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2(2E^2-m^2)^2}{4E^2(E^2-m^2)^2}\left[\frac{4}{\sin^4\theta} - \frac{3}{\sin^2\theta} + \frac{(E^2-m^2)^2}{(2E^2-m^2)^2}\left(1+\frac{4}{\sin^2\theta}\right)\right]$$

Scaling in: $s = 4E^2$

- Relativistic limit: $m/E \to 0$

Rutherford term!

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2}\left(\frac{1}{\sin^4\theta/2} + \frac{1}{\cos^4\theta/2} + 1\right)$$

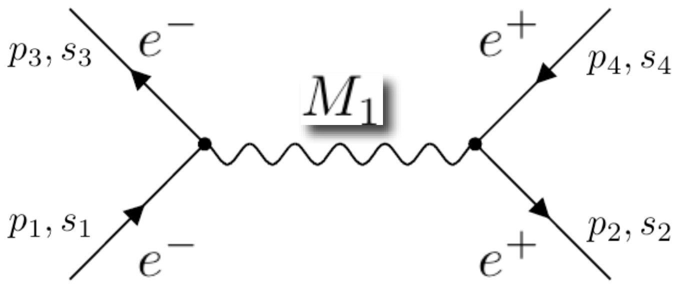
- Non-relativistic limit: $E^2 \simeq m^2, \quad v^2 = (E^2-m^2)/E^2$

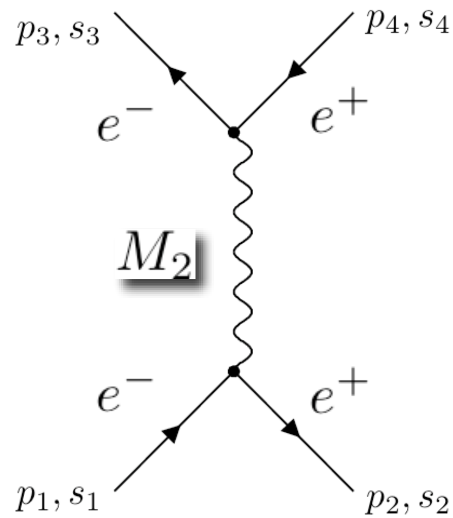
Rutherford term!

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha}{m}\right)^2 \frac{1}{16v^4}\left(\frac{1}{\sin^4\theta/2} + \frac{1}{\cos^4\theta/2} - \frac{1}{\sin^2\theta/2\cos^2\theta/2}\right)$$

QED example processes

- C) Electron-Positron scattering (Bhabha scattering)





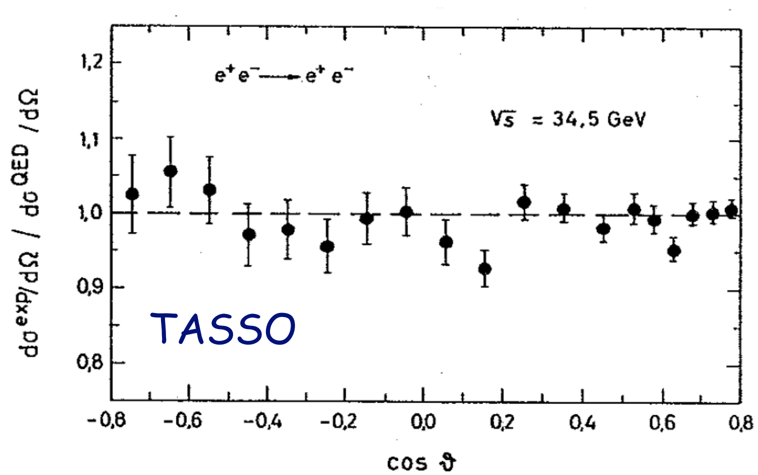
$$\mathcal{M}_1 = -\frac{g_e^2}{(p_1 - p_3)^2}[\bar{u}^{(s_3)}(p_3)\gamma^\mu u^{(s_1)}(p_1)][\bar{v}^{(s_2)}(p_2)\gamma_\mu v^{(s_4)}(p_4)]$$

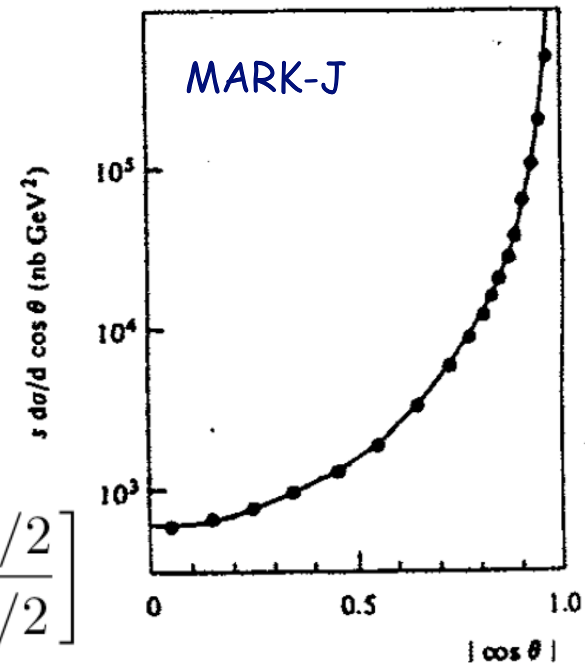
$$\mathcal{M}_2 = -\frac{g_e^2}{(p_1 + p_2)^2}[\bar{u}^{(s_3)}(p_3)\gamma^\mu v^{(s_4)}(p_4)][\bar{v}^{(s_2)}(p_2)\gamma_\mu u^{(s_1)}(p_1)]$$

QED example processes

C) Electron-Positron scattering (Bhabha scattering)

- Comparison of Bhabha scattering results at PETRA (DESY) and QED calculations





Scaling in: $s = 4E^2$

- Relativistic limit:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8E^2}\left[\frac{1+\cos^4\theta/2}{\sin^4\theta/2} + \frac{1}{2}(1+\cos^2\theta) - 2\frac{\cos^4\theta/2}{\sin^2\theta/2}\right]$$

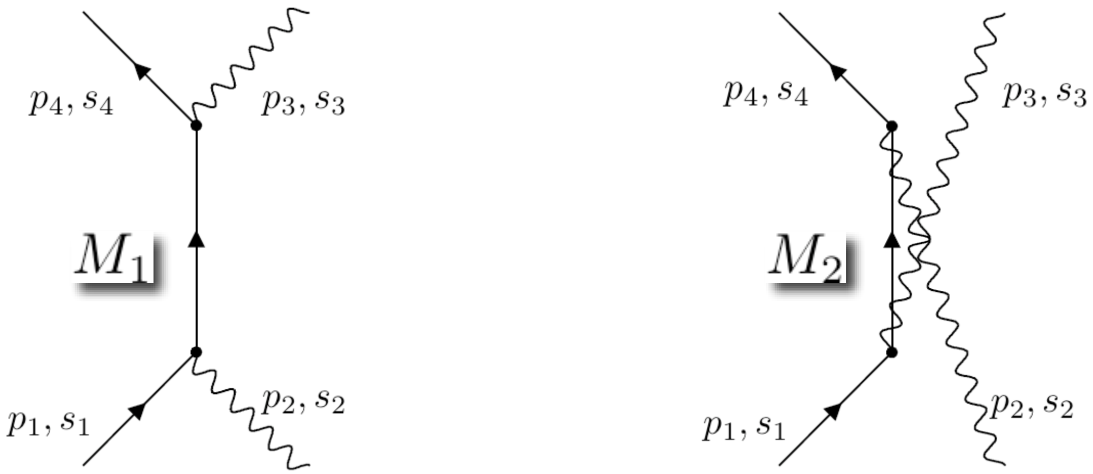
- Non-relativistic limit:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha}{m}\right)^2 \frac{1}{16v^4\sin^4\theta/2}$$

Rutherford term!

QED example processes

D) Photon-electron scattering (Compton scattering)



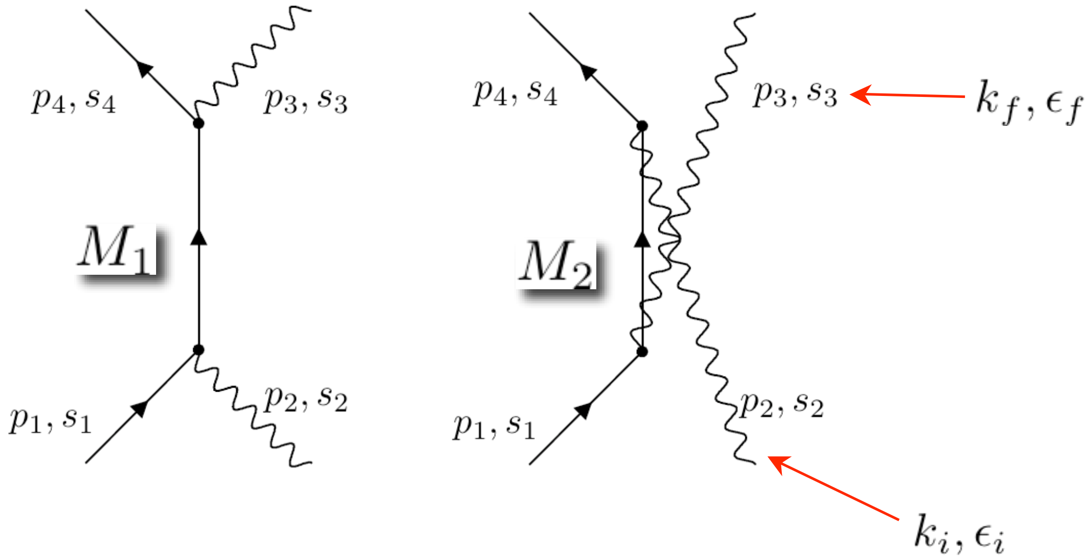
- Amplitudes:

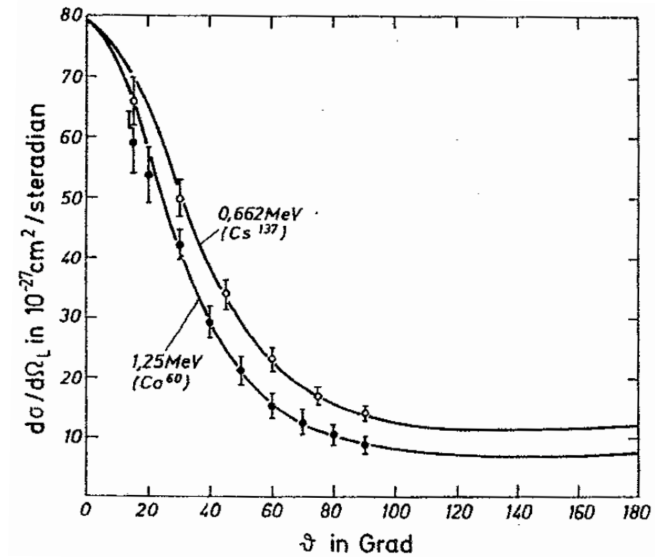
$$\mathcal{M}_1 = \frac{g_e^2}{(p_1 - p_3)^2 - m^2c^2}\left[\bar{u}(4)\not{\epsilon}(2)(\not{p}_1 - \not{p}_3 + mc)\not{\epsilon}(3)^* u(1)\right]$$

$$\mathcal{M}_2 = \frac{g_e^2}{(p_1 + p_2)^2 - m^2c^2}\left[\bar{u}(4)\not{\epsilon}(3)^*(\not{p}_1 + \not{p}_2 + mc)\not{\epsilon}(2)^* u(1)\right]$$

QED example processes

- D) Photon-electron scattering (Compton scattering)





Hofstadter 1949 and
Bernstein 1956

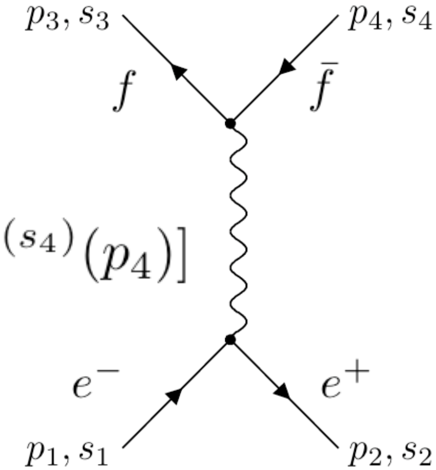
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4m^2}\left(\frac{k_f}{k_i}\right)^2\left[\frac{k_f}{k_i} + \frac{k_i}{k_f} + 4(\epsilon_f \cdot \epsilon_i)^2 - 2\right]$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{m^2}(\epsilon_f \cdot e_i)^2$$

$$k_f = \frac{k_i}{1 + (k_i/m)(1 - \cos\theta)}$$

QED example processes

- E) Electron-positron (Fermion-pair production)



Charge of fermion pair!

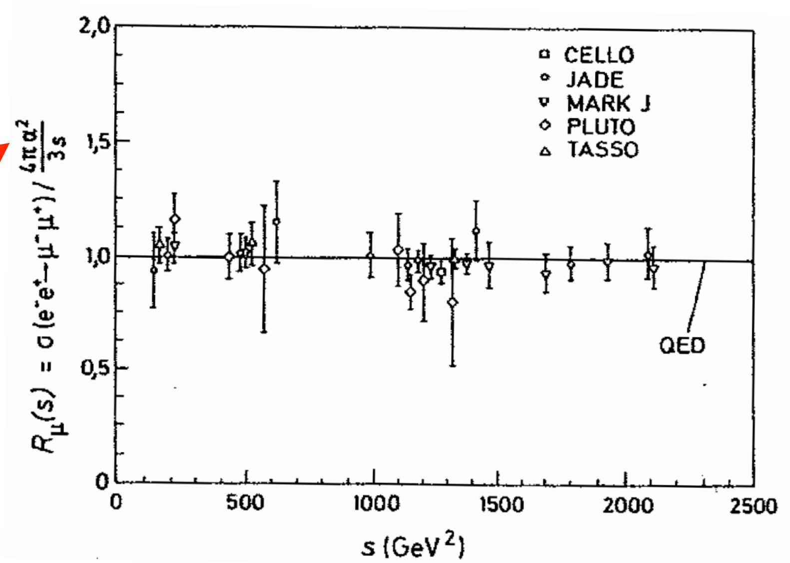
$$\mathcal{M} = \frac{Qg_e^2}{(p_1+p_2)^2}[\bar{v}^{(s_2)}(p_2)\gamma^\mu u^{(s_1)}(p_1)][\bar{u}^{(s_3)}(p_3)\gamma_\mu v^{(s_4)}(p_4)]$$

$$\frac{d\sigma}{d\Omega} = N_c\frac{\alpha^2}{4s}\beta[1+\cos^2\theta+(1-\beta^2)\sin^2\theta]Q^2$$

Number of colors: : N_c=3

$$\sigma = N_c\frac{4\pi\alpha^2}{3s}Q^2 = N_c\frac{86.8Q^2\ nb}{s}$$

$$s = 4E^2 \qquad Q = 1$$



Casimir trick and trace theorems

Casmir trick

- See Latex document
- Result:

$$\sum_{all\ spins} [\bar{u}(a)\Gamma_1 u(b)][\bar{u}(a)\Gamma_2 u(b)]^* = Tr[\Gamma_1(\not{p}_b + m_b c)\bar{\Gamma}_2(\not{p}_a + m_a c)]$$

No spinors on left side!

Casimir trick and trace theorems

Trace theorems

- General identities:

$$
\begin{aligned}
\mathrm{Tr}(A+B) &= \mathrm{Tr}(A)+\mathrm{Tr}(B) \\
\mathrm{Tr}(\alpha A) &= \alpha \mathrm{Tr}(A) \\
\mathrm{Tr}(AB) &= \mathrm{Tr}(BA) \\
\mathrm{Tr}(ABC) &= \mathrm{Tr}(CAB) = \mathrm{Tr}(BCA)
\end{aligned}
$$

- Important relations among gamma matrices:

$$
\begin{aligned}
g_{\mu\nu} g^{\mu\nu} &= 4 \\
\{\gamma^\mu, \gamma^\nu\} &= 2g^{\mu\nu}
\end{aligned}
$$

$$
\begin{aligned}
\gamma_\mu \gamma^\nu \gamma^\mu &= \gamma_\mu \left(2g^{\mu\nu} - \gamma^\mu \gamma^\nu\right) \\
&= 2\gamma^\nu - \gamma_\mu \gamma^\mu \gamma^\nu \\
&= 2\gamma^\nu - 4\gamma^\nu \\
&= -2\gamma^\nu
\end{aligned}
$$

$$\gamma_\mu \gamma^\mu = 4$$

$$\gamma_\mu \gamma^\nu \gamma^\lambda \gamma^\mu = 4g^{\nu\lambda}$$

Casimir trick and trace theorems

Trace theorems

$$
\begin{aligned}
\mathrm{Tr}(\gamma^\mu \gamma^\nu) &= \mathrm{Tr}\left(\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu\right)/2 \\
&= \mathrm{Tr}(2g^{\mu\nu})/2 \\
&= g^{\mu\nu}\,\mathrm{Tr}(1) \\
&= 4g^{\mu\nu}
\end{aligned}
$$

$$
\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma) = 4\left(g^{\mu\nu} g^{\lambda\sigma} - g^{\mu\lambda} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\lambda}\right)
$$

Note: The trace of the product of an odd number of gamma matrices is zero!

Casimir trick and trace theorems

- Trace theorems involving γ^5 matrix

$$\mathrm{Tr}(\gamma^5) = 0$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\begin{aligned} \mathrm{Tr}(\gamma^5\gamma^\mu) &= 0 \\ \mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\lambda) &= 0 \end{aligned}$$

Note: The trace of the product of an odd number of gamma matrices multiplied by a γ^5 matrix is zero!

$$\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu) = 0$$

$$\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma) = 4i\epsilon^{\mu\nu\lambda\sigma}$$

Casimir trick and trace theorems

Contraction of ε tensor

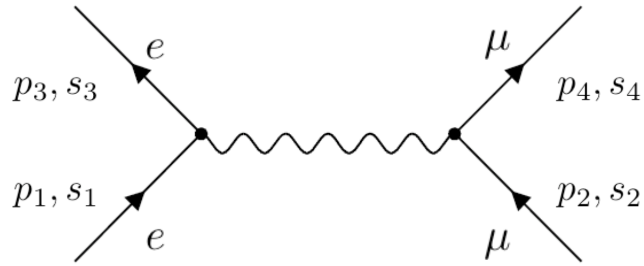
$$
\begin{aligned}
\epsilon^{\mu\nu\lambda\sigma}\epsilon_{\mu\nu\lambda\sigma} &= -24 \\
\epsilon^{\mu\nu\lambda\sigma}\epsilon_{\mu\nu\lambda\tau} &= -6\,\delta^{\sigma}_{\tau} \\
\epsilon^{\mu\nu\lambda\sigma}\epsilon_{\mu\nu\theta\tau} &= -2\left(\delta^{\lambda}_{\theta}\delta^{\sigma}_{\tau} - \delta^{\lambda}_{\tau}\delta^{\sigma}_{\theta}\right) \\
&\vdots
\end{aligned}
$$

$$\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma) = 4i\epsilon^{\mu\nu\lambda\sigma}$$

$$
\epsilon^{\mu\nu\lambda\sigma} \equiv \begin{cases} -1 & \text{for } \textit{even} \text{ permutations of } 0123 \\ +1 & \text{for } \textit{odd} \text{ permutations of } 0123 \\ 0 & \text{if any 2 indices are the same} \end{cases}
$$

Renormalization

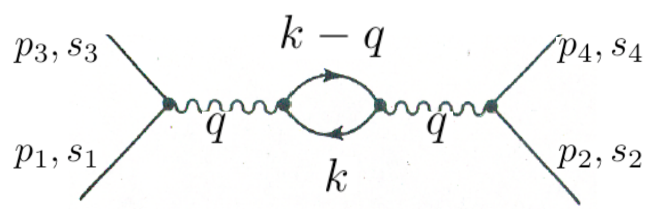
Introduction



Leading order:

$$\mathcal{M} = -\frac{g_e^2}{(p_1 - p_3)^2}[\bar{u}^{(s_3)}(p_3)\gamma^\mu u^{(s_1)}(p_1)][\bar{u}^{(s_4)}(p_4)\gamma_\mu u^{(s_2)}(p_2)]$$

Same process involving loop diagram: Vacuum polarization



4 internal lines!

$$\mathcal{M} = \frac{-g_e^4}{q^4}[\bar{u}(p_3)\gamma^\mu u(p_1)]\left\{\int \frac{d^4k}{(2\pi)^4}\frac{Tr[\gamma_\mu(\not{k} + mc)\gamma_\nu(\not{q} - \not{k} + mc)]}{(k^2 - m^2c^2)((q-k)^2 - m^2c^2)}\right\}[\bar{u}(p_4)\gamma^\nu u(p_2)]$$

Divergent integral!

Renormalization

Dealing with divergent terms (1)

- The solution to this problem contributed enormously to the development of QED (Dirac, Pauli, Kramers, Weisskopf, Bethe besides Tomonaga, Schwinger and Feynman)
- The procedure to deal with divergent loop terms is call renormalization!
- General concept: Redefinition of masses and coupling constants

$$m_{physical} = m_{bare} + \delta m$$

$$g_{physical} = g_{bare} + \delta g$$

Contains divergent terms!

What we measure!

Special case: Lowest order

$$g_{physical} = g_{bare}$$

Renormalization

Dealing with divergent terms (2)

- Technical idea:

$$\int_{m^2}^{\infty} \frac{dx}{x} \rightarrow \int_{m^2}^{M^2} \frac{dx}{x} = \ln \frac{M^2}{m^2}$$

- With this procedure M becomes:

$$\mathcal{M} = -g_e^2[\bar{u}(p_3\gamma^\mu u(p_1)] \frac{g_{\mu\nu}}{q^2} \left\{ 1 - \frac{g_e^2}{12\pi^2} \left[\ln\left(\frac{M^2}{m^2}\right) - f\left(\frac{-q^2}{m^2c^2}\right) \right] \right\}$$

$$\times [\bar{u}(p_4)\gamma^\nu u(p_s)]$$

- Introduce renormalized coupling constant:

$$g_R = g_e \sqrt{\left(1 - \frac{g_e^2}{12\pi^2} \ln\left(\frac{M^2}{m_2}\right)\right)}$$

Renormalization

Dealing with divergent terms (3)

- In terms of g_R we find then for M:

$$\mathcal{M} = -g_R^2 \left[\bar{u}(p_3)\gamma^\mu u(p_1)\right] \frac{g_{\mu\nu}}{q^2} \left\{1 + \frac{g_R^2}{12\pi^2} f\left(\frac{-q^2}{m^2c^2}\right)\right\} \left[\bar{u}(p_4)\gamma^\mu u(p_2)\right]$$

- Two terms:
  - Infinite term: absorbed in g_R
  - Finite term: depends on q^2 (f-term)

- Express end result in terms of α:

$$g_e = \sqrt{4\pi\alpha}$$

$$\alpha(q^2) = \alpha(0)\left\{1 + \frac{\alpha(0)}{3\pi} f\left(\frac{-q^2}{m^2c^2}\right)\right\}$$

Consequence of loop diagram: q^2 dependence
Effect in QED is small!

Note: If all infinities arising from higher-order diagrams can be accommodated through renormalized quantities, a theory is said to be renormalizable!

All gauge theories are renormalizable: t'Hooft/Feldman