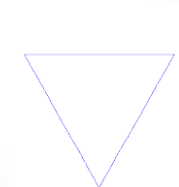


Modelagem em Engenharia C & A

Aula 15- Fractais – A revanche

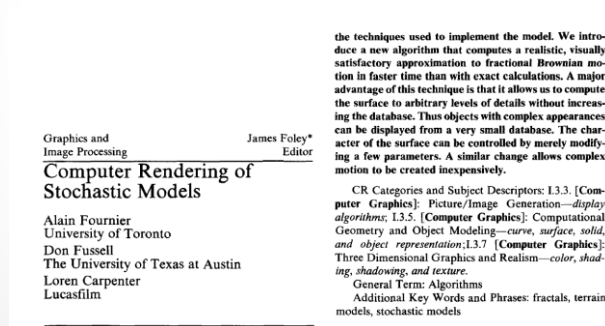
Objetivos

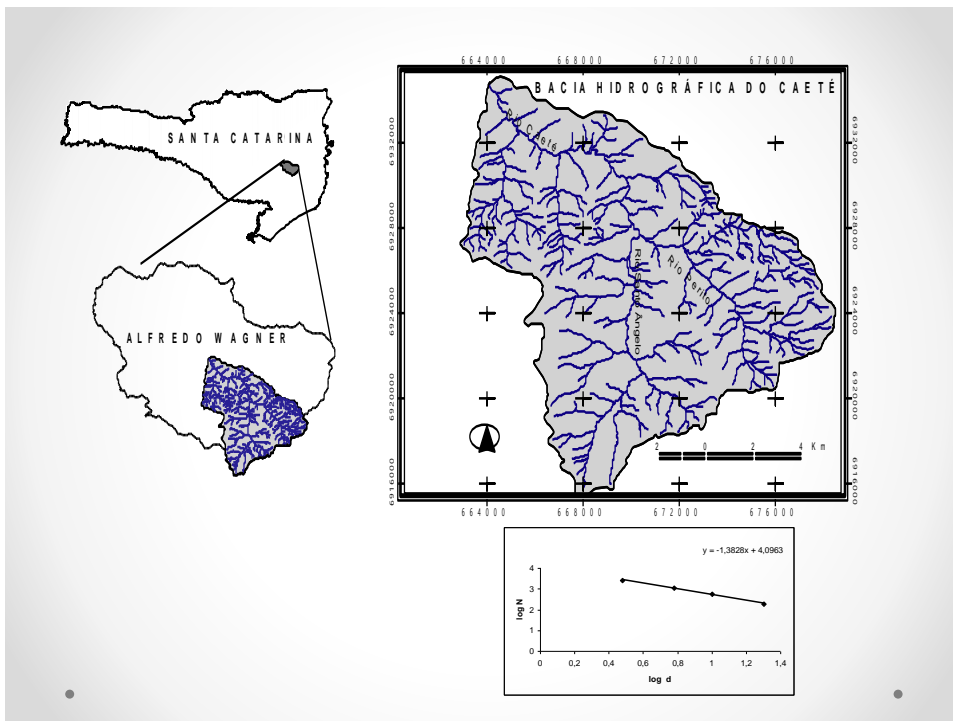
- Modelos Estocásticos
- MidPoint Displacement
- Renderização de superfícies 3D utilizando fractais



Bibliografia

- **Computer Rendering of Stochastic Models**
 - Alain Fournier, University of Toronto; Don Fussell, The University of Texas at Austin; Loren Carpenter, Lucasfilm
 - Communications the ACM Number 6 of Volume 25, June 1982,





Origem do Fractal Randômico

- Movimento Browniano Fractal (FBM)

$$B_H(0, w) = b_0$$

$$B_H(u, w) - B_H(0, w) = \left[\frac{1}{\Gamma(H + 0.5)} \int_{-\infty}^u [(u-s)^{H-0.5} - (-s)^{H-0.5}] dB(s, w) + \int_0^u (u-s)^{H-0.5} dB(s, w) \right]$$

Fig. 1. Ordinary Brownian Motion ($H = 0.5$).



Fig. 2. Fractional Brownian Motion ($H = 0.3$).

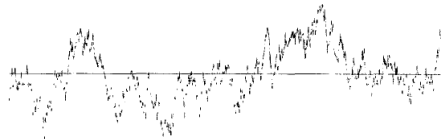
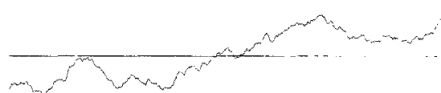
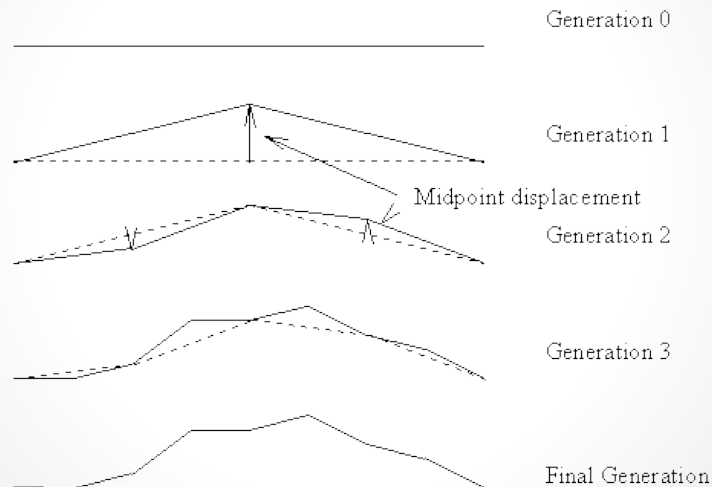


Fig. 3. Fractional Brownian Motion ($H = 0.7$).

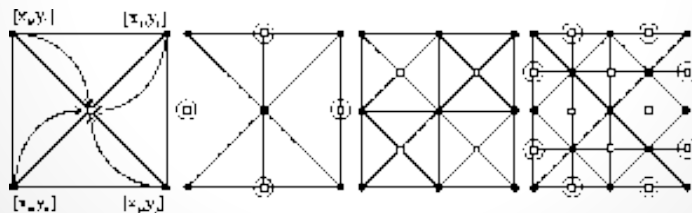


Tecnica da subdivisão recursiva randômica



Método MidPoint Displacement

- Um quadrado inicial é subdividido em sucessivos quadrados menores e a cada um é atribuído um deslocamento no eixo z em função do valor das arestas
- 1º passo: adicionar um vértice no meio
 - $[x_0, y_0, f(x_0, y_0)]$ $[x_1, y_0, f(x_1, y_0)]$ $[x_0, y_1, f(x_0, y_1)]$ $[x_1, y_1, f(x_1, y_1)]$
- 2º passo: calcular o deslocamento a partir de uma variação aleatória d com distribuição probabilística conhecida

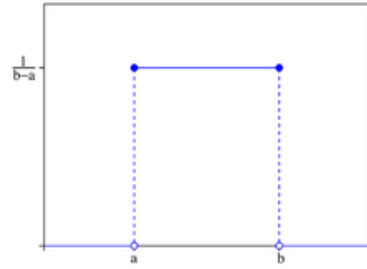


Gerador de Números aleatórios

- Distribuição Prob Uniforme

$$f(x; a, b) = \begin{cases} \frac{1}{b-a} & , a \leq x \leq b, \\ 0 & , c.c. \end{cases}$$

$$F(k; a, b) = \frac{[k] - a + 1}{b - a + 1}$$

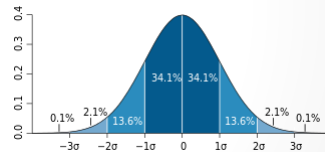


- Função no Excel: ALEATORIO() ou ALEATORIOENTRE()

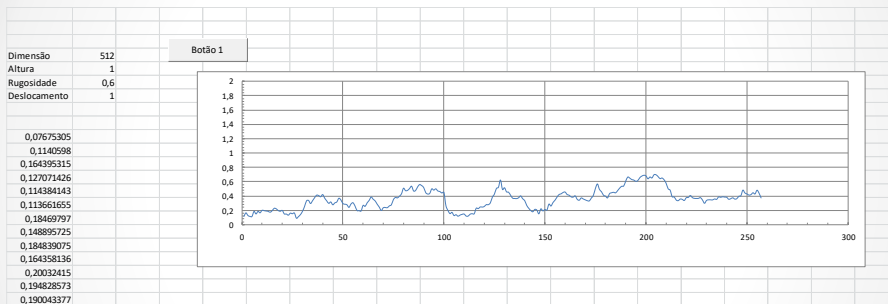
Distribuição Normal

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}, -\infty < x < \infty, \sigma > 0.$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\left(-\frac{x^2}{2}\right)}.$$



MID_1D



MID_3D

