Aula 09

13.1-1

$1/T = 10^{+6} \rightarrow \boxed{T = 10^{-6}}$

$\omega_c = 2{,}4 \cdot 10^9 \cdot 2\pi \ Hz$

a

$0{,}95\delta(t) - 0{,}3\,\delta(t - T/2)$

$$g(t) = \sum_{i=0}^{K} \alpha_i \exp(j\omega_c \tau_i)\, p(t - KT - \tau_i) =$$

$$= 0{,}95\, p(t) - 0{,}3 \cdot \exp(j\,\omega_c T/2)\, p(t - T - T/2)$$

$$= 0{,}95\, p(t) - 0{,}3 \exp(j\, 1{,}2 \cdot 10^9 \cdot 10^{-6} \cdot 2\pi)\, p(t - 3/2\, T)$$

$$\boxed{g(t) = 0{,}95\, p(t) - 0{,}3 \exp(j\, 1200 \cdot 2\pi)\, p(t - 1{,}5 \cdot 10^{-6})}$$

1

$$\boxed{g(t) = 0{,}95\, p(t) - 0{,}3\, p(t - 1{,}5 \cdot 10^{-6})}$$

b

$$y(t) = \sum_K a_K\, g(t - KT) + n_e(t)$$

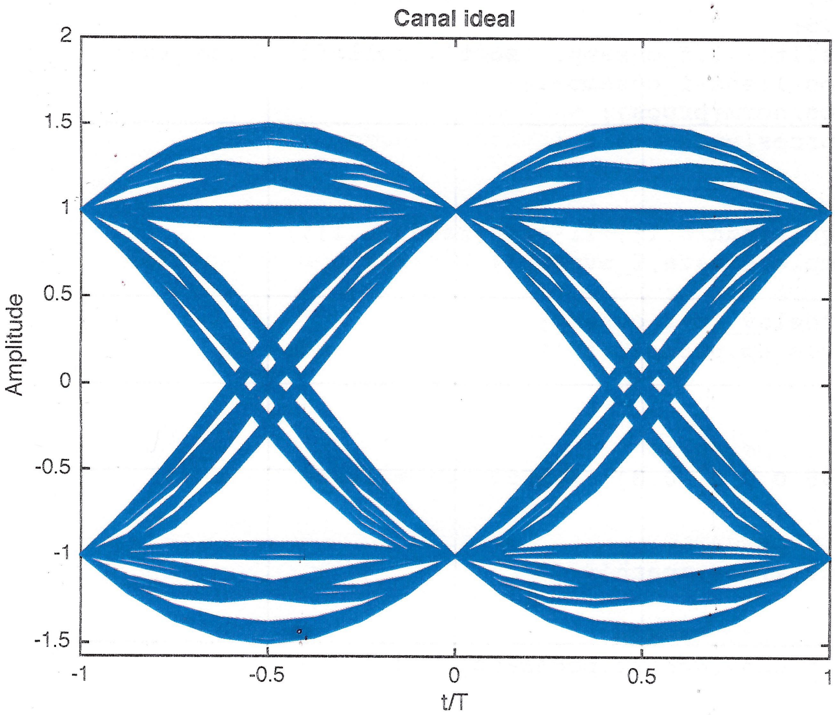
Veja as figura do Matlab Diagrama de olho

não está aberto em $t = KT$

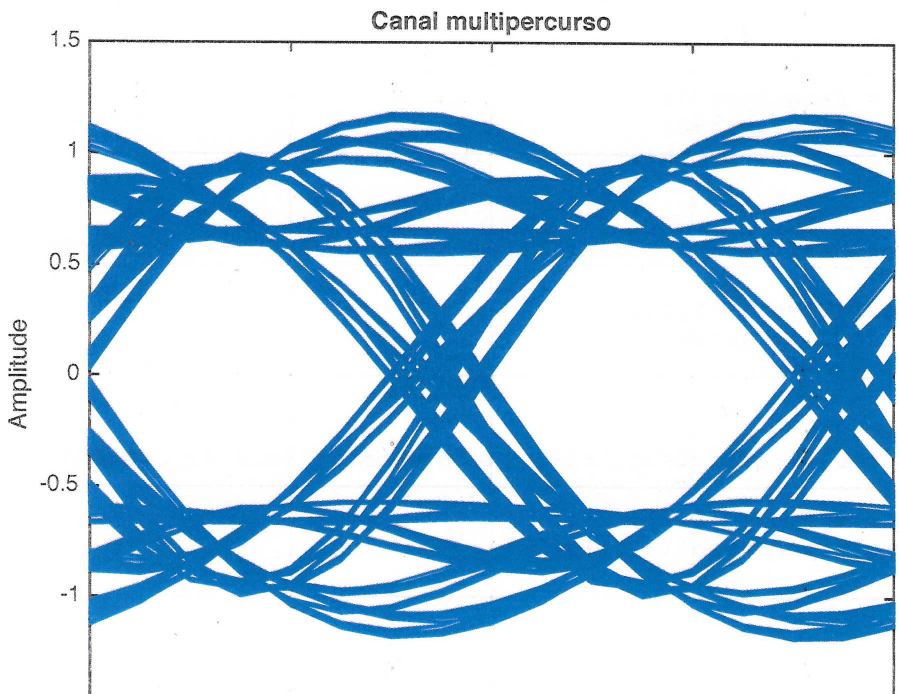
2

```
%ExercÌcio 13.1.1
close all;
L = 1e6; %N˙mero de sÌmbolos
f_ovsamp = 8; % Fator de superamostragem - fazer gr·ficos
delay_rc = 4;
rolloff = 0.5;
prcos = rcosflt(1,1,f_ovsamp, 'sqrt', rolloff, delay_rc); %Pulso
prcos = prcos(1:end-f_ovsamp+1); %remove 0s
prcos = prcos/norm(prcos); %normaliza
pccasado = prcos(end:-1:1); %Filtro casado

%SÌmbolos QAM
s_data = sign(randn(L,1))+1i*sign(randn(L,1));
s_up = upsample(s_data,f_ovsamp); %Deixar com mesmo tamanho devido a
sobreamostragem
delayrc = 2*delay_rc*f_ovsamp;
xrcos = conv(s_up,prcos); %Sinais transmitidos


%Canal de Multipercurso
mpath = [0.95 0 0 0 -0.3]; %Canal com multipercurso

%Aplicar multipercurso
xchout = conv(xrcos,mpath);

%sinal pelo filtro casado
xrxoutmulti = conv(xchout,pccasado); %com canal

xrxoutideal = conv(xrcos,pccasado); %com canal

%Diagrama de olho

delaychbmulti = 2*delayrc+4;
delaychbideal = 2*delayrc+0;
%eyevec = conv(xchout, prcos);

%CANAL IDEAL
eyevec = xrxoutideal(delaychbideal+1:(delaychbideal+800)*f_ovsamp);
%h1 = figure(1);
eyediagram(real(eyevec),16,2);
title('Canal ideal'); grid;
xlabel('t/T');

%CANAL Multipercurso
eyevec = xrxoutmulti(delaychbmulti+1:(delaychbmulti+800)*f_ovsamp);
%h2 = figure(2);
eyediagram(real(eyevec),16,2);
title('Canal multipercurso'); grid;
xlabel('t/T');
```

3



Canal ideal
Amplitude
t/T
2
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1



Canal multipercurso
Amplitude
1.5
1
0.5
0
-0.5
-1

13.2-1 4

a) Filtro casado

$$\boxed{T=10^{-6}}$$

$$g(-t)=0{,}35\,p(-t)-0{,}3\,p(-t-1{,}5\cdot10^{-6})$$

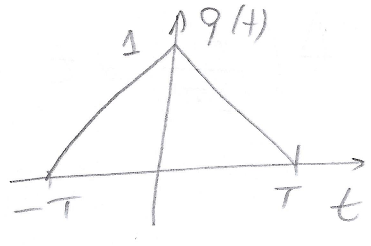
b) $h(t)=g(t)*g(-t)$

$$=(0{,}35)^2\,p(t)*p(-t)-0{,}265\,p(t)*p(-t-1{,}5\cdot10^{-6})$$

$$-0{,}265\,p(-t)*p(t-1{,}5\cdot10^{-6})+0{,}09\,p(t-1{,}5\cdot10^{-6})*p(-t-1{,}5\cdot10^{-6})$$

e $\boxed{h[n]=h(nT)}$

13.2.2 $g(t)=\Delta\left(\frac{t}{2T}\right)$



$G(f)=T\,\mathrm{sinc}^2(\pi fT)$

$G(-f)=T\,\mathrm{sinc}^2(\pi fT)$

a) $S_w(f)=|G(-f)|^2\,S_n(f)=\frac{N}{2}\,|G(-f)|^2$

$$\boxed{\bar{S}_N(f)=\frac{N}{2}\cdot T^2\,\mathrm{sinc}^4(\pi fT)}$$

(5)

(b)

Correlação

$$R_w[\ell] = \frac{N}{2} \int_{-\infty}^{\infty} |Q(f)|^2 e^{-j2\pi f \ell T} df$$

$$R_w[\ell] = \frac{N}{2} \int_{-\infty}^{\infty} T^2 \text{sinc}^4 (\pi f T) e^{-j2\pi f \ell T} df$$

$$R_w[0] = E[w^2] = \frac{N}{2} \int_{-\infty}^{\infty} T^2 \text{sinc}^4 (\pi f T) df =$$

$$= \frac{N}{2} \cdot \int_{-\infty}^{\infty} (\underbrace{T \text{ inc}^2 (\pi f T)}_{|P(f)|^2})^2 df =$$

$$= \frac{N}{2} \int_{-\infty}^{\infty} \Delta\left(\frac{t}{2T}\right) dt = \frac{N}{2} \frac{2T \cdot 1}{2} = \frac{N}{2} T$$

$$R_w[0] = \frac{N}{2} T^2$$

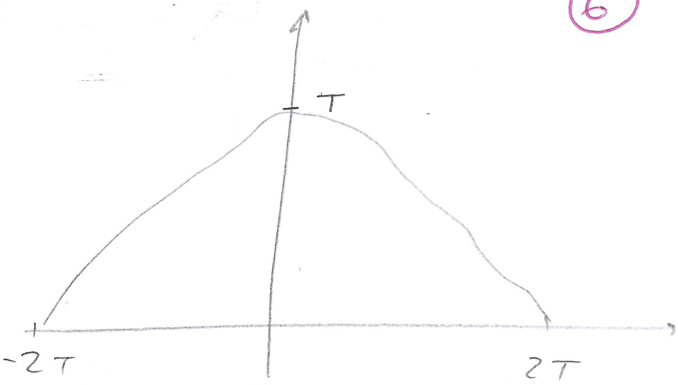
$$\boxed{\sigma_w = \frac{N}{2} T}$$

Como o ruído no canal tem média nula, $w[n]$ também tem média nula

(c) $$R_w[\ell] = \frac{N}{2} \int_{-\infty}^{\infty} |Q(f)|^2 e^{-j2\pi f \ell T} = \frac{N}{2} \int_{-\infty}^{\infty} Q(f)\, Q(-f)\, e^{-j2\pi f T} df$$

$=$

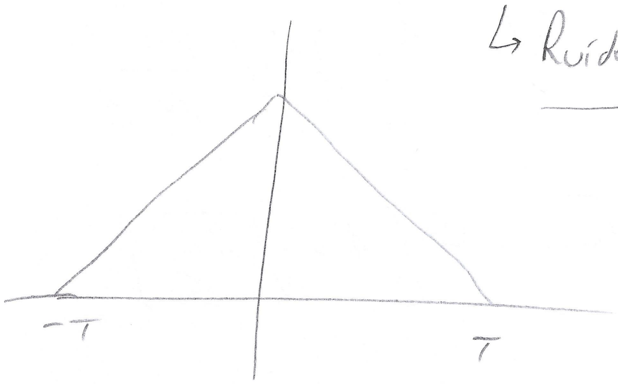
$$= \frac{N}{2}\, g(t) * g(-t) \Big|_{t=lT}$$

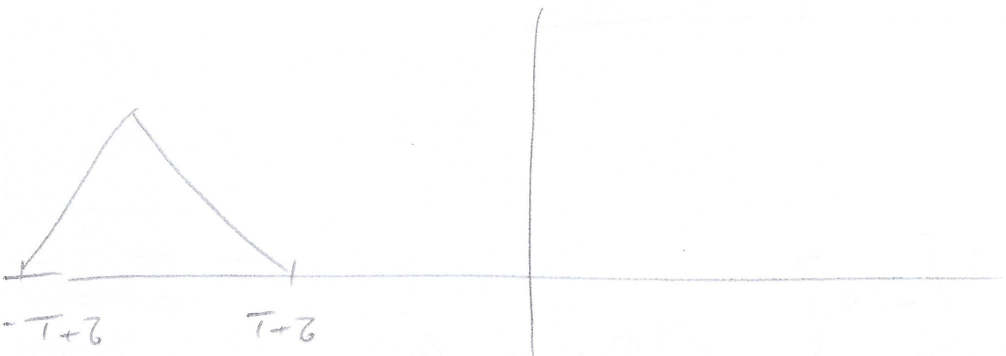


$$= \begin{cases} \frac{N}{2} \cdot T, & l = 0 \\ a, & l = \pm 1 \\ 0, & l \geq 2 \end{cases}$$

→ $\boxed{\text{As amostras tem correlação}}$

↳ Ruído não independente





13.3-1

$$H(z) = 1 + 0{,}6 z^{-1}$$

$$z[k] = H(z) a_k + w[k]$$

$$E_b/N = 18$$

$$z[k] = a_k + 0{,}6\, a_{k-1} + w[k]$$

a_k	a_{k-1}	$\bar{z}[k]$
-1	-1	$-1{,}6\sqrt{E_b}$
-1	1	$-0{,}4\sqrt{E_b}$
1	-1	$0{,}4\sqrt{E_b}$
1	1	$1{,}6\sqrt{E_b}$

$\overset{5}{\text{6}}$

$$P(e|\text{"}-1\ -1\text{"}) = P(w > 1{,}6\sqrt{E_b}) = Q\left[\frac{1{,}6\sqrt{E_b}}{\sqrt{N/2}}\right] =$$

$$= Q\left[1{,}6\sqrt{\frac{2E_b}{N}}\right]$$

$$P(e|\text{"}-1\ 1\text{"}) = P(w > 0{,}4\sqrt{E_b}) = Q\left[0{,}4\sqrt{\frac{2E_b}{N}}\right]$$

$$P(e|\text{"}1\ -1\text{"}) = P(w < 0{,}4\sqrt{E}) = Q\left[0{,}4\sqrt{\frac{2E_b}{N}}\right]$$

$$P(e|\text{"}+1\ 1\text{"}) = P(w < -1{,}6\sqrt{E_b}) = Q\left[1{,}6\sqrt{\frac{2E_b}{N}}\right]$$

$$P_e = \frac{2Q\left[1{,}6\sqrt{\frac{2E_b}{N}}\right] + 2Q\left[0{,}4\sqrt{\frac{2E_b}{N}}\right]}{4} =$$

$$= \frac{2Q[9{,}6] + 2Q[2{,}4]}{4} = \underline{\underline{0{,}0041}}$$

(sem canal) $P_e = Q(\sqrt{36}) = Q(6) = 9{,}8 \cdot 10^{-10}$

b) Com zero-forcing,

$$P_s = Q\left(\sqrt{\frac{2E_b}{N \cdot \sum_{i=0}^{\infty} |f[i]|^2}}\right)$$

⑦

$$F(z) = \frac{z^{-\mu}}{H(z)} = \frac{z^{-\mu}}{1+0{,}6z^{-1}}$$

$$f[i] = \begin{cases} 0, & i<\mu \\ (-0{,}6)^{i-\mu}, & i \geq \mu \end{cases} \quad e$$

$$\sum_{i=0}^{\infty} |f[i]|^2 = \sum_{i=\mu}^{\infty} (0{,}36)^{i-\mu} = \frac{1}{1-0{,}36} = \frac{1}{0{,}64}$$

$$P_b = Q\left(\sqrt{\frac{2E_b}{N \cdot 1/0{,}64}}\right) = Q\left(\sqrt{1{,}28\frac{E_b}{N}}\right) = Q(4{,}8)$$

$$= 7{,}93 \cdot 10^{-7} \quad \text{(Melhor)}$$

13.3-2 $\boxed{H(z) = 1+0{,}9z^{-1}}$

Nesse caso $z[k] = a_k + 0{,}9\, a_{k-1} + w[k]$

a_k	a_{k-1}	$z[k]$
-1	-1	-1,9
-1	1	-0,1
1	-1	0,1
1	1	1,9

→ VAI GERAR ERROS!

$$P(e|\text{"}-1\ -1\text{"}) = P(e|\text{"}1\ 1\text{"}) = Q\left(1{,}9\sqrt{\frac{2E_b}{N}}\right)$$

$$P(e|\text{"}1\ -1\text{"}) = P(e|\text{"}-1\ 1\text{"}) = Q\left(0{,}1\sqrt{\frac{2E_b}{N}}\right)$$

$$P_e = \frac{Q\left(1{,}9\sqrt{\frac{2E_b}{N}}\right) + Q\left(0{,}1\sqrt{\frac{2E_b}{N}}\right)}{2}$$

$$= \frac{\overbrace{\cancel{Q(11{,}4)}}^{\approx 0} + Q(0{,}6)}{2} \approx 0{,}1371$$

(b) Com zero-forcing,

$$F(z) = \frac{z^{-\mu}}{H(z)} = \frac{z^{-\mu}}{1+0{,}9z^{-1}} \xleftrightarrow{Z^{-1}} f[i] = \begin{cases} 0, & i<\mu \\ (-0{,}9)^{\mu-i} & i\geq\mu \end{cases}$$

$$\sum_{i=0}^{\infty} |f(i)|^2 = \sum_{i=\mu}^{\infty} \left[(-0{,}9)^2\right]^{\mu-i} = \frac{1}{1-0{,}81} = 5{,}263$$

$$P_e = Q\left(\sqrt{\frac{2E_b}{N\cdot \frac{1}{0{,}19}}}\right) = Q\left(\sqrt{0{,}38\cdot 18}\right) =$$

$$= Q(2{,}615) = 0{,}004461$$

13.3-3

$$H(e^{j\omega}) = 1 + a\, e^{-j\omega}$$

$$|H(e^{j\omega})| = \sqrt{(1 + a\cos\omega)^2 + a^2 \operatorname{sen}^2\omega}$$

$$= \sqrt{1 + 2a\cos\omega + a^2}$$

$$= \sqrt{1 + a^2 + 2a\cos\omega}$$

$|H(e^{j\omega})|$ é mínimo em $\boxed{\omega = \pi}$

$$|H(e^{j\omega})|_{min} = \sqrt{(1-a)^2} = |1-a|$$

Como o equalizador tem resposta em frequência

$$F(z) = \frac{z^{-\mu}}{H(z)} \Rightarrow |F(e^{j\omega})| = \frac{1}{|H(e^{j\omega})|}$$

e ele terá máximos em $\omega = \pi$ $\quad F(e^{j\omega})|_{MAX} = \dfrac{1}{|1-a|}$

Assim, em $\omega = \pi$ o equalizador do exercício 13.3-1 multiplica o ruído nessa frequência por $\dfrac{1}{|1-0{,}6|} = 2{,}5$ e o do 13.3-2 por $\dfrac{1}{|1-0{,}9|} = 10$. Daí o desempenho pior nesse 2º caso.

```
[Hf1,f1] =freqz([1 0.6],1,1024);
[Hf2,f2] =freqz([1 0.9],1,1024);

subplot(211); plot(f1/pi,abs(Hf1));
axis([0 1 0 2]);grid;
xlabel('\omega/\pi');ylabel('H_1(e^{j\omega})');
title('a=0.6');
subplot(212); plot(f2/pi,abs(Hf2));
axis([0 1 0 2]);grid;
xlabel('\omega/\pi');ylabel('H_2(e^{j\omega})');
title('a=0.9');
```