Nonlinear $\mathcal{H}_\infty$ Control

Applied to Biped Robots

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Outline

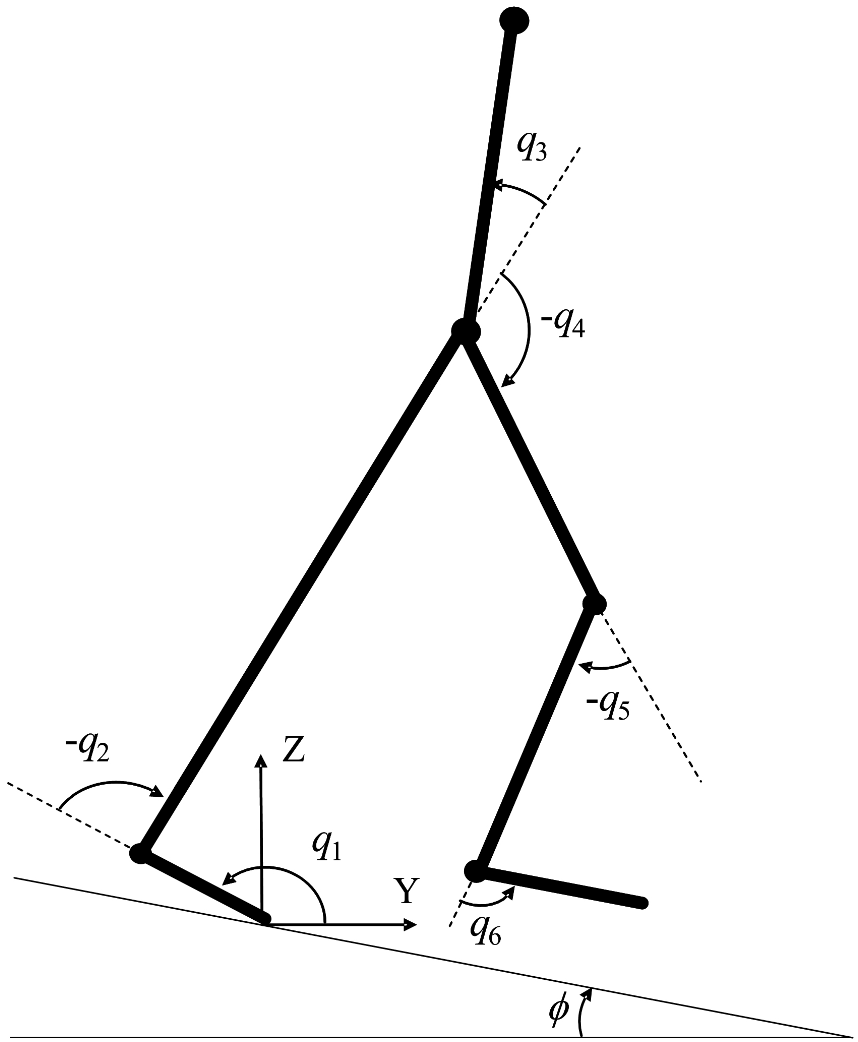
- Introduction
- Dynamic Model
- Hybrid Control Strategy
- Nonlinear $\mathcal{H}_\infty$ Control
- Results

Introduction

- Passive walking
  - Energy efficiency
  - Smooth and anthropomorphic movement
  - Limit cycles
  - Small basin of attraction
- Nonlinear $\mathcal{H}_\infty$ Control
  - Robustness against disturbances and parametric uncertainties

Dynamic Model

- Planar biped robot with torso, knees and feet



Dynamic Model

- Swing phase
  - Before foot rotation
  - Between foot rotation and knee strike
  - After knee strike
- Ground collision phase

Dynamic Model

- Swing phase - Before foot rotation
  - Foot link is parallel to the ground
  - Torsional spring at ankle joint

$$\tau_M = K_0 - K_M q_2,$$

    where $K_0 > K_M > 0$ are adjustable parameters
  - End: Foot Rotation Indicator (FRI) point outside support area [Goswani, 1999]

Dynamic Model

- Swing phase - Before knee strike
  - Dynamic equations

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

  with $\tau = [0 \;\; \tau_2 \;\; \tau_3 \;\; \tau_4 \;\; \tau_5 \;\; \tau_6]^T$

  - No actuation in the toe joint of stance leg
  - Underactuated system after foot rotation

- Swing phase - Knee strike
  - Knee joint: $q_5^+ = 0$ and $\dot{q}_5^+ = 0$
  - Velocity change

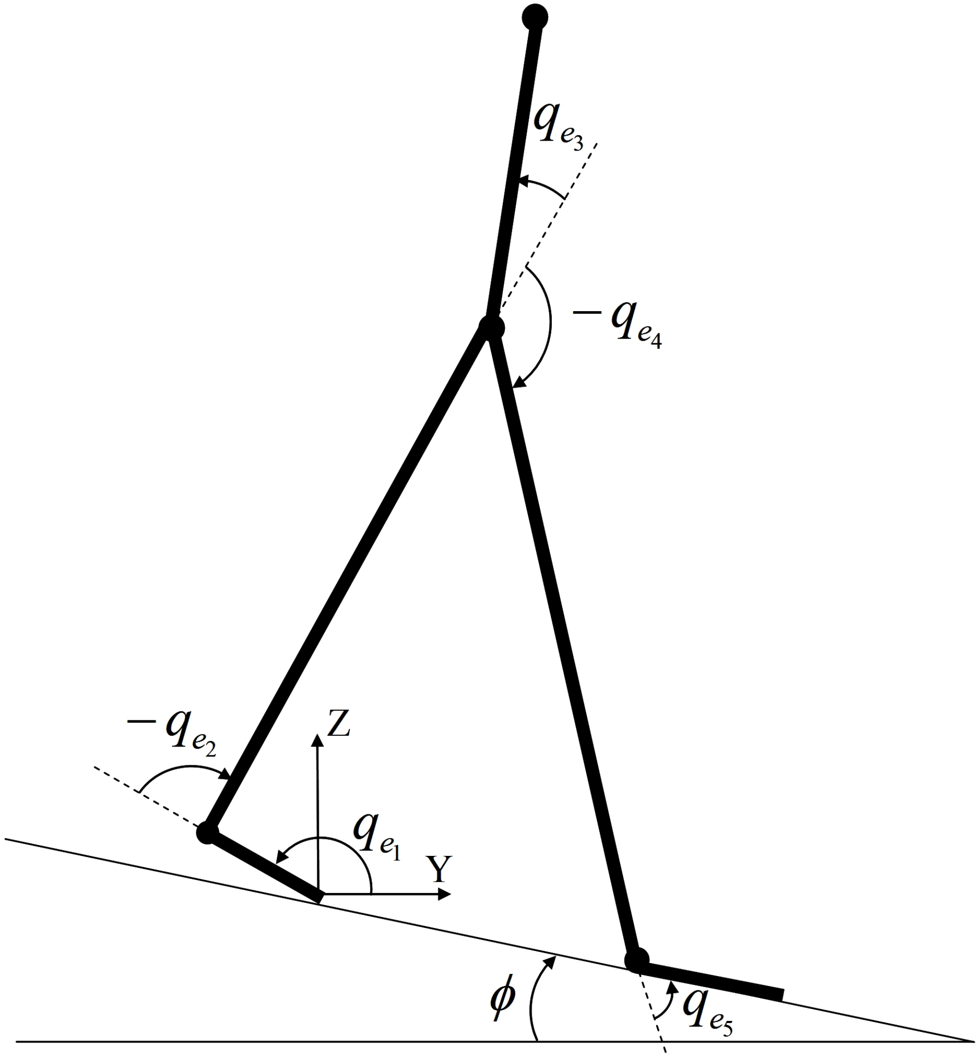
$$\dot{q}^+ = \dot{q}^- - M(q)^{-1} J_i^T \lambda_i$$

with $J_i = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 0]$, since the constrain $J_i \dot{q}^+ = 0$, and

$$\lambda_i = X_i^{-1} J_i \dot{q}^-$$

where $X_i = J_i M(q)^{-1} J_i^T$

- Ground collision model



Dynamic Model

- Ground collision model
  - Velocity change

$$\begin{bmatrix} M_e(q_e) & -E_h(q_e)^T & -E_t(q_e)^T \\ E_h(q_e) & 0 & 0 \\ E_t(q_e) & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_e^+ \\ F \end{bmatrix} = \begin{bmatrix} \dot{q}_e^- \\ 0 \end{bmatrix}$$

with $F = [F_{h_T} \;\; F_{h_N} \;\; F_{t_T} \;\; F_{t_N}]^T$, impulsive forces

Dynamic Model

- Ground collision model
  - Coordinates change

$$\dot{q}^+ = [\dot{q}_{e_1}^+ \;\; \dot{q}_{e_2}^+ \;\; \dot{q}_{e_3}^+ \;\; \dot{q}_{e_4}^+ \;\; 0 \;\; \dot{q}_{e_5}^+]^T$$

and

$$x = \Delta(x)$$

with $x = [q^T \;\; \dot{q}^T]^T$

Stable Limit Cycle

- Torso joint control $\Rightarrow$ absolute value $\theta_4 = q_1 + q_2 + q_3$

$$\tau_4 = K_{P_4}(\theta_4^d - \theta_4) - K_{D_4}\dot{\theta}_4$$

- Swing foot joint control $\Rightarrow$ absolute value $\theta_6 = q_1 + q_2 + q_4 + q_5 + q_6$

$$\tau_6 = K_{P_6}(\theta_6^d - \theta_6) - K_{D_6}\dot{\theta}_6$$

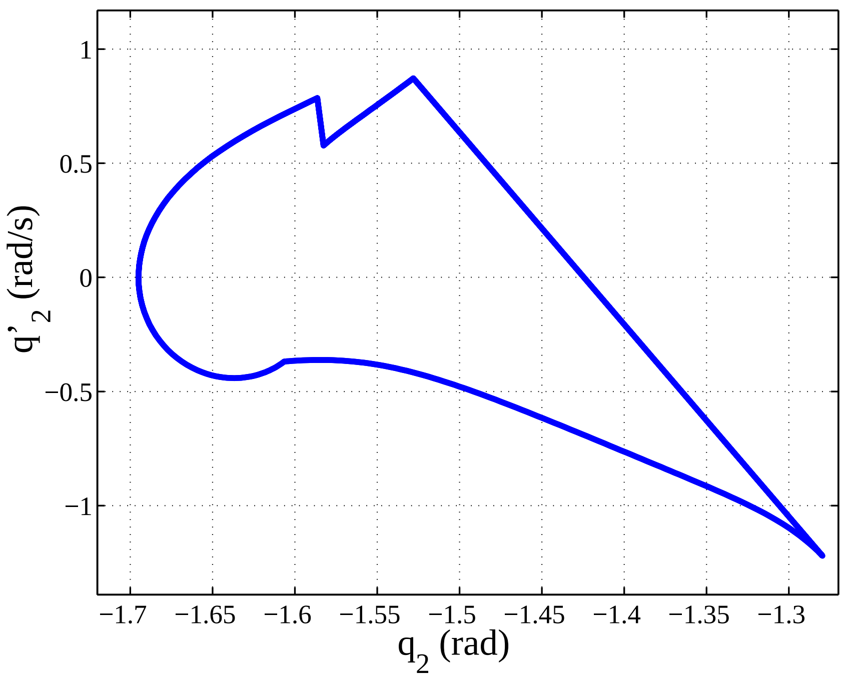
Stable Limit Cycle

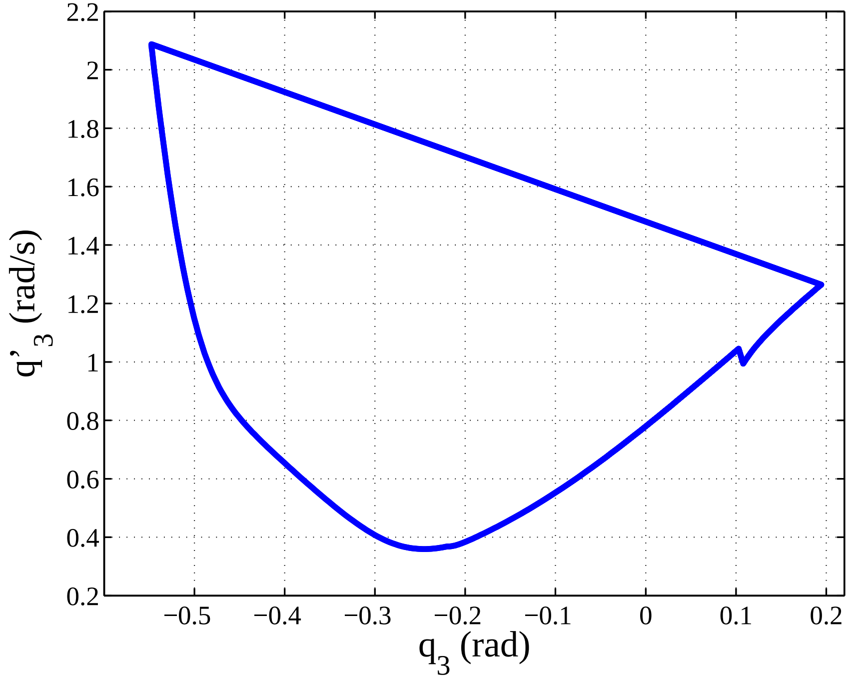
- Ground slope: 3°
- Desired absolute value: $\theta_4^d = 80^\circ$ and $\theta_6^d = 0^\circ$
- PD Gains: $K_{P_4} = 150$, $K_{D_4} = 100$, $K_{P_6} = 80$, and $K_{D_6} = 10$
- Spring parameters: $K_0 = -10$ and $K_M = 30$
- Initial conditions:

$$x_c = [3.1 \;\; -1.3 \;\; -0.5 \;\; -3.9 \;\; 0 \;\; 1.5 \;\; 0 \;\; -1.2 \;\; 2.1 \;\; 0.3 \;\; 0.3 \;\; -1.3]^T$$

Stable Limit Cycle

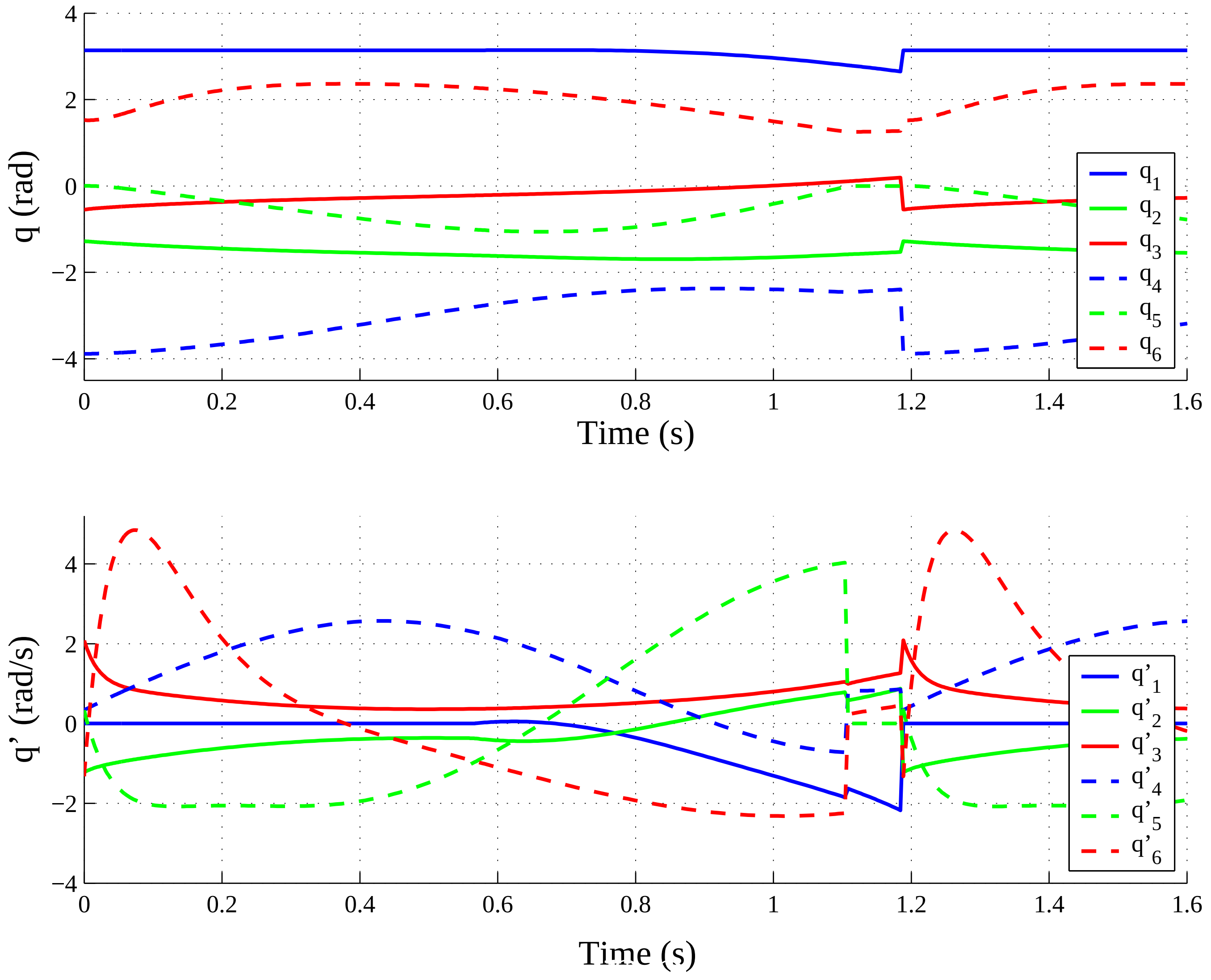
- Limit cycle trajectory for joints 2 and 3





Stable Limit Cycle

- Joint position and velocity time variations



- Hybrid Control Strategy
  - Minimum distance between the current position and the limit cycle

$$\bar{d} = min_i||q - q_r(i)||,$$

$q_r(i)$ are the discrete points on the limit cycle.

  - Estimate of the basin of attraction: C
  - If $\bar{d} \geq C \Rightarrow$ Nonlinear $\mathcal{H}_\infty$ Control

Nonlinear $\mathcal{H}_\infty$ Control

- Quasi-LPV representation of nonlinear systems
  - Before the foot rotation $\Rightarrow$ fully actuated system
  - After foot rotation $\Rightarrow$ underactuated system

$$\bar{M}(q)\ddot{q}_a + \bar{C}(q,\dot{q})\dot{q}_a + \bar{D}(q,\dot{q})\dot{q}_1 + \bar{g}(q_a) = \tau_a + \bar{d}$$

$$\begin{aligned}
\bar{M} &= M_{aa} - M_{a1}{M_{11}}^{-1}M_{1a} \\
\bar{C} &= C_{aa} - M_{a1}{M_{11}}^{-1}C_{1a} \\
\bar{D} &= C_{a1} - M_{a1}{M_{11}}^{-1}C_{11} \\
\bar{g} &= g_a - M_{a1}{M_{11}}^{-1}g_1
\end{aligned}$$

Simulation Results

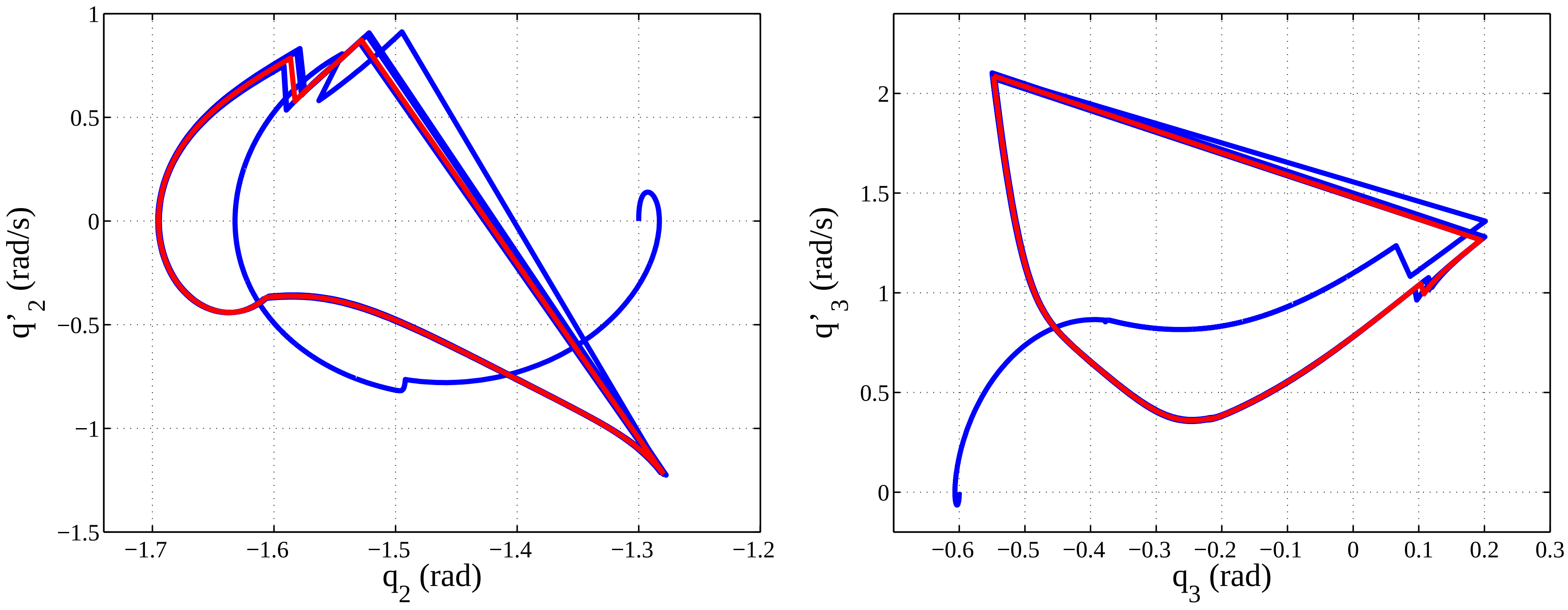
- Initial conditions with zero velocities

$$x_0 = [3.09 \quad 1.3 \quad -0.6 \quad -3.68 \quad 0 \quad 1.84 \quad 0 \quad 0 \quad 0 \quad 0]^T$$

- Nonlinear $\mathcal{H}_\infty$ Control
  - From initial conditions to knee strike
  - Estimate of the basin of attraction: $C = 0, 15$
  - Three steps to reach the basin of attraction
  - Smooth trajectory

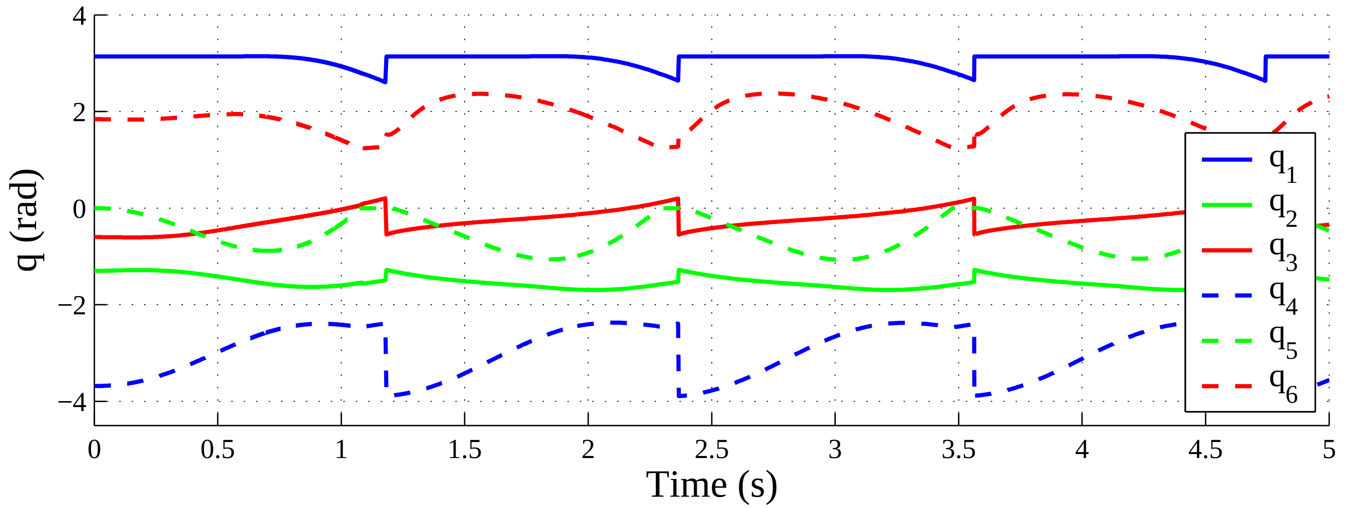
Initial conditions with zero velocities

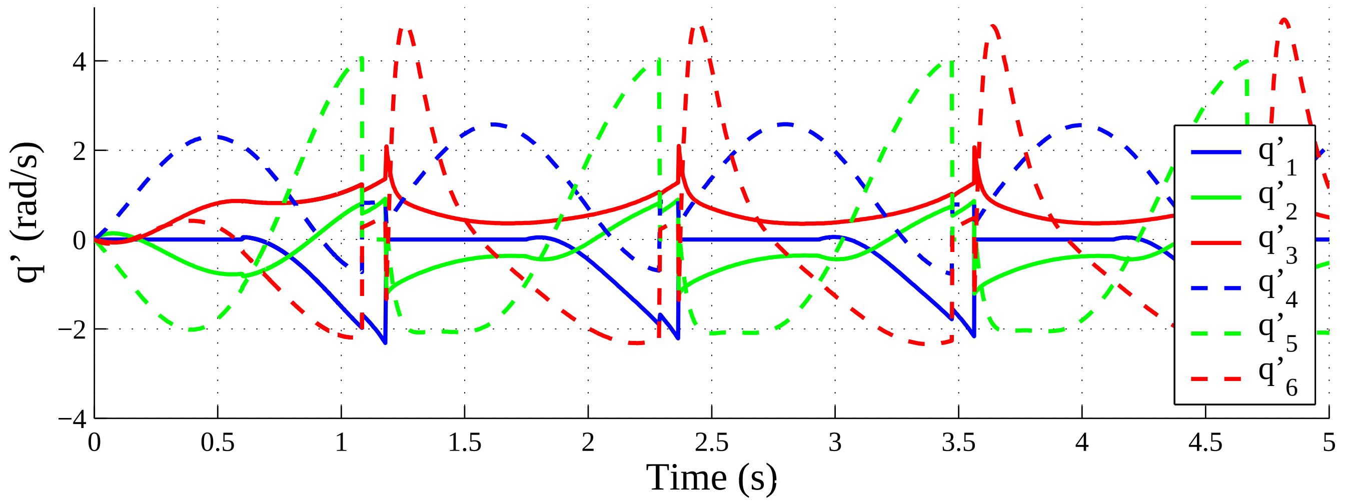
- Limit cycle trajectory for joints 2 and 3



Initial conditions with zero velocities

- Joint position and velocity time variations

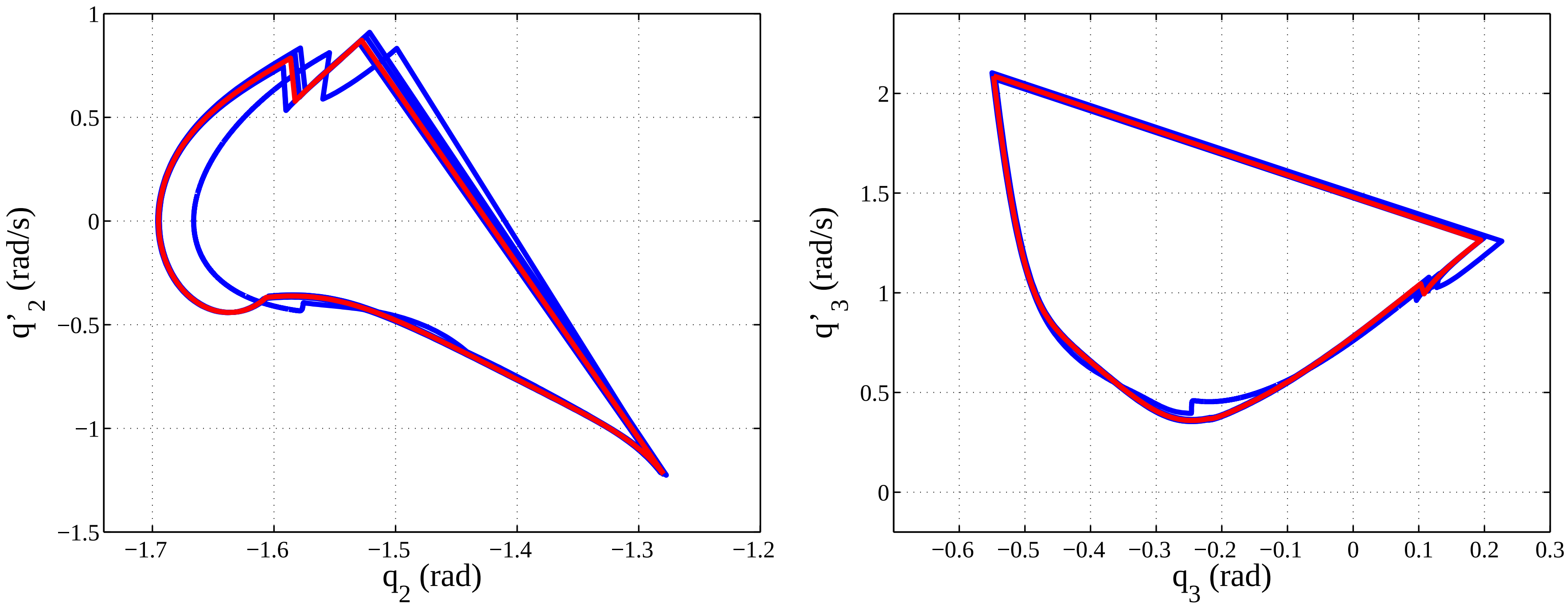




- Disturbance Rejection
  - Limit cycle initial conditions
  - External disturbances composed of normal and sine functions
  - Instant of maximum peak of disturbance: $t_f = 1$ s
  - Only one step to reach the basin of attraction

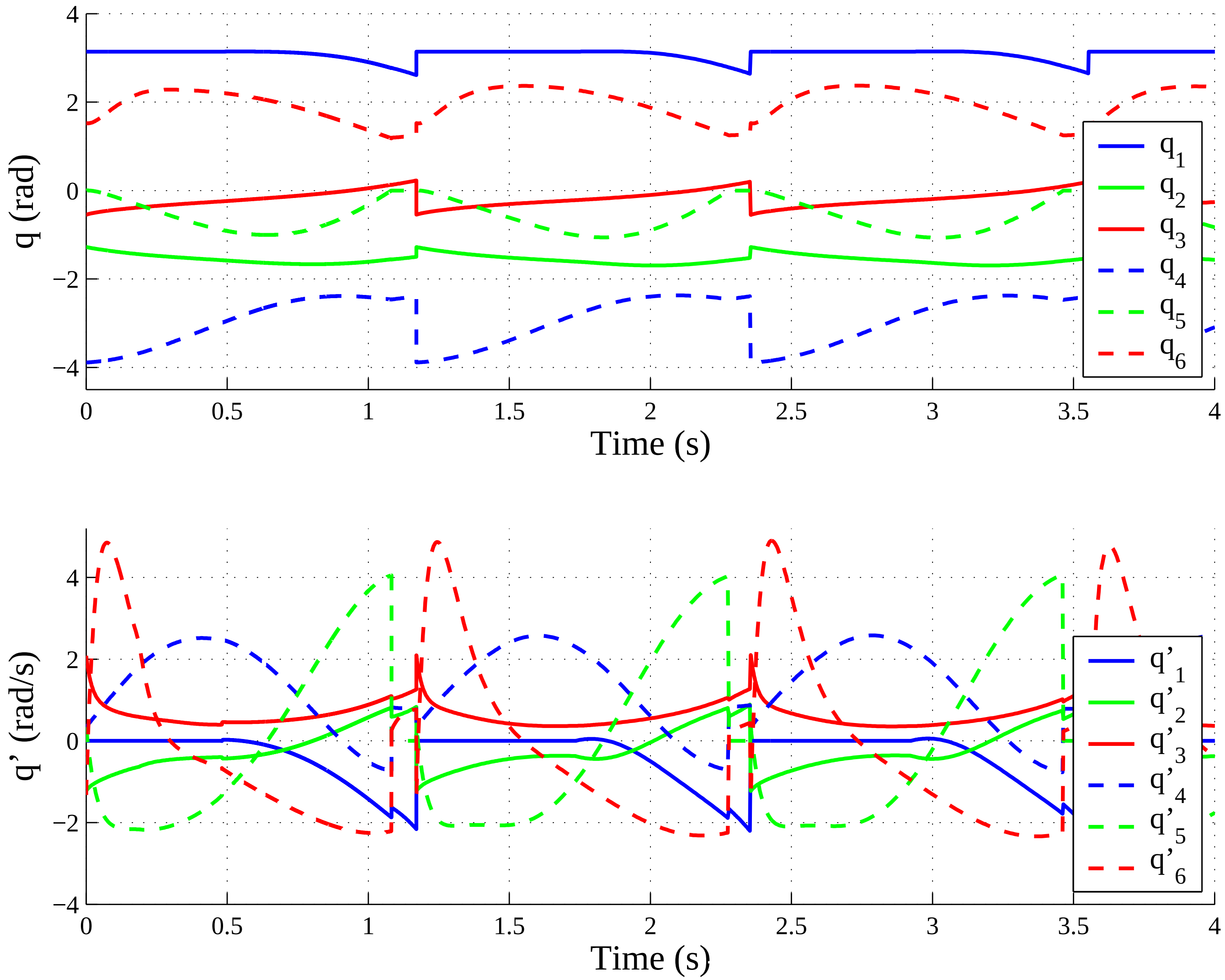
Disturbance Rejection

- Limit cycle trajectory for joints 2 and 3



Disturbance Rejection

- Joint position and velocity time variations



Conclusions

- Dynamic Model
  - Torsional spring at ankle
  - Foot Rotation Indicator (FRI) point
  - Underactuated configuration after foot rotation
  - Problem: find C
- Nonlinear $\mathcal{H}_\infty$ Control
  - Increases the basin of attraction
  - Increases the robustness