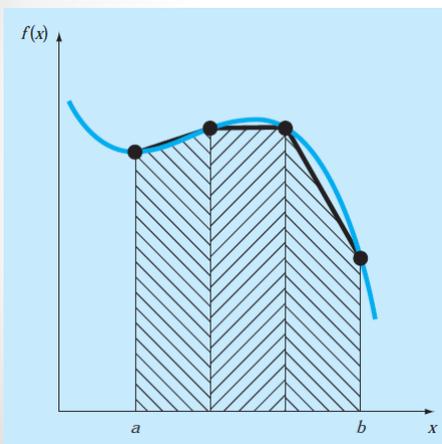


# Modelagem em Engenharia C & A

Aula 13- Integração Numérica

## Regra do Trapézio



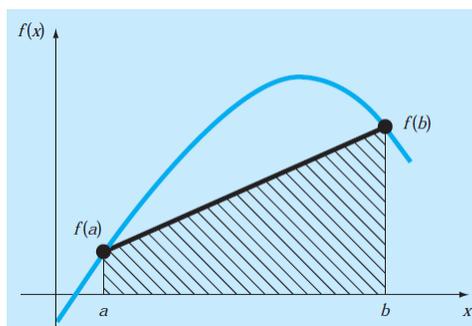
$$I = \int_a^b f(x) dx \cong \int_a^b f_1(x) dx$$

$$f_1(x) = f(a) + \frac{f(b) - f(a)}{b - a}(x - a)$$

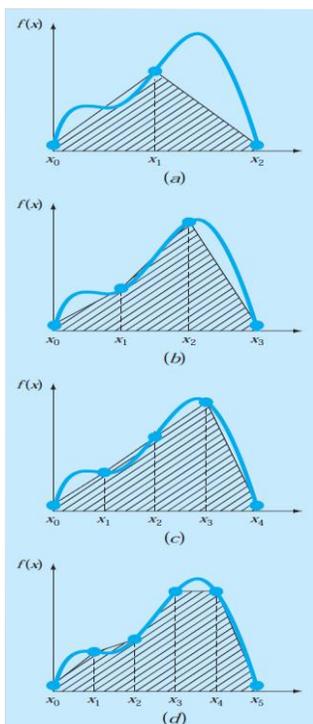
$$I = \int_a^b \left[ f(a) + \frac{f(b) - f(a)}{b - a}(x - a) \right] dx$$

$$I = (b - a) \frac{f(a) + f(b)}{2}$$

## Resultado - Erro



$$E_t = -\frac{1}{12} f''(\xi)(b-a)^3$$



## Multiplos trapézios

$$h = \frac{b-a}{n}$$

$$I = \int_{x_0}^{x_1} f(x) dx + \int_{x_1}^{x_2} f(x) dx + \dots + \int_{x_{n-1}}^{x_n} f(x) dx$$

$$I = h \frac{f(x_0) + f(x_1)}{2} + h \frac{f(x_1) + f(x_2)}{2} + \dots + h \frac{f(x_{n-1}) + f(x_n)}{2}$$

$$I = \frac{h}{2} \left[ f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right]$$

# Fórmula de recorrência

## Problema

- Calcular o volume do hidrograma sintético dado pela fórmula abaixo com duração total de 5 horas:

$$Q(t) = Q_{base} + (Q_{pico} - Q_{base}) \left[ \frac{t}{T_p} \exp \left( 1 - \frac{t}{T_p} \right) \right]^\beta$$

Qbase	1
Qpico	100
Tempo de Concentração	5
Tempo de Pico	2
Coef Beta	3

- Comparar o resultado para n=5,10 e 50

$$I = \underbrace{(b-a)}_{\text{Width}} \underbrace{\frac{f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n)}{2n}}_{\text{Average height}}$$

$$E_t = -\frac{(b-a)^3}{12n^3} \sum_{i=1}^n f''(\xi_i)$$

# Rotina VBA

Function Integral(X As Range, Y As Range) As Single

Dim i As Integer, N As Integer

N = X.Rows.count

If N < 3 Then Exit Function

Integral = (Y(1, 1) + Y(N, 1)) / 2

For i = 2 To N - 1

Integral = Integral + Y(i, 1)

Next i

Integral = Integral \* (X(N, 1) - X(1, 1)) / (N - 1)

End Function

# Regra de Simpson

- Substituir a função por polinômios de grau 2!!!

$$I \cong \underbrace{(b-a)}_{\text{Width}} \underbrace{\frac{f(x_0) + 4f(x_1) + f(x_2)}{6}}_{\text{Average height}}$$

- Regra Geral

$$I \cong \underbrace{(b-a)}_{\text{Width}} \underbrace{\frac{f(x_0) + 4 \sum_{i=1,3,5}^{n-1} f(x_i) + 2 \sum_{j=2,4,6}^{n-2} f(x_j) + f(x_n)}{3n}}_{\text{Average height}}$$