Aula 08

11.7-2 Comparando-se ao exemplo 11.4, se a_1 for transmitido, temos

$$b_1 = E + a + \sqrt{E}\, n_1 \qquad b_{-1} = -E + a - \sqrt{E}\, n_1$$

$$b_2 = a + \sqrt{E}\, n_2 \qquad b_{-2} = a - \sqrt{E}\, n_2$$

$$\vdots \qquad\qquad \vdots$$

$$b_K = a + \sqrt{E}\, n_K \qquad b_{-K} = a - \sqrt{E}\, n_K$$

e

$$P(C|m_2) = Prob\,(b_1 > b_{-1}, b_2, b_{-2}, \ldots, b_K, b_{-K})$$

Note que

$$b_1 > b_{-1} \Rightarrow E + a + \sqrt{E}\, n_1 > -E + a - \sqrt{E}\, n_1 \quad \text{ou} \quad n_1 > -\sqrt{E}$$

$$b_1 > b_2 \Rightarrow E + a + \sqrt{E}\, n_1 > a + \sqrt{E}\, n_2 \Rightarrow n_2 < \sqrt{E} + n_1$$

$$b_1 > b_{-2} \Rightarrow E + a + \sqrt{E}\, n_1 > a - \sqrt{E}\, n_2 \Rightarrow n_2 > -(\sqrt{E} + n_1)$$

Assim,

$$b_1 > b_2 \text{ e } -b_2 \Rightarrow \boxed{-(n_1 + \sqrt{E}) < n_2 < (n_1 + \sqrt{E})}$$

Similarmente,

$$b_1 > b_K \text{ e } b_{-K} \Rightarrow \boxed{-(n_1 + \sqrt{E}) < n_K < (n_1 + \sqrt{E})}$$

Assim,

$$P(C|m_1) = Prob.\,(b_1 > b_{-1},\ b_2,\ b_{-2},\ b_3,\ b_{-3},\ \dots,\ b_K,\ b_{-K})$$

$$= Prob.\,\left[n_1 > -\sqrt{E},\ |n_2| < (n_1+\sqrt{E}),\ |n_3| < (n_1+\sqrt{E}),\ \dots \right.$$
$$\left. |n_K| < (n_1+\sqrt{E})\right]$$

Como $n_1, n_2, \dots, n_K$ são todas VAs gaussianas independentes, cada uma com variância $N/2$,

$$P(C|m_1) = \left[P(n_1 > -\sqrt{E}) \cdot P(|n_2| < n_1+\sqrt{E}) \cdot P(|n_3| < n_1+\sqrt{E})\right.$$
$$\left. \dots\ P(|n_K| < n_1+\sqrt{E})\right]$$

$$= \frac{1}{\sqrt{\pi N}} \int_{-\sqrt{E}}^{+\infty} e^{-n_1^2/N} \left[\int_{-(n_1+\sqrt{E})}^{(n_1+\sqrt{E})} e^{-n}\, dn\right]^{N-1} dn_1$$

$$= \frac{1}{\sqrt{\pi N}} \int_{-\sqrt{E}}^{+\infty} e^{-n_1^2/N} \left[1 - 2Q\left(\frac{n_1+\sqrt{E}}{\sqrt{N/2}}\right)\right]^{N-1} dn_1$$

Fazendo $y = \dfrac{n_1+\sqrt{E}}{\sqrt{N/2}}$, $\quad dn_1 = \dfrac{1}{\sqrt{N/2}}\, dy$

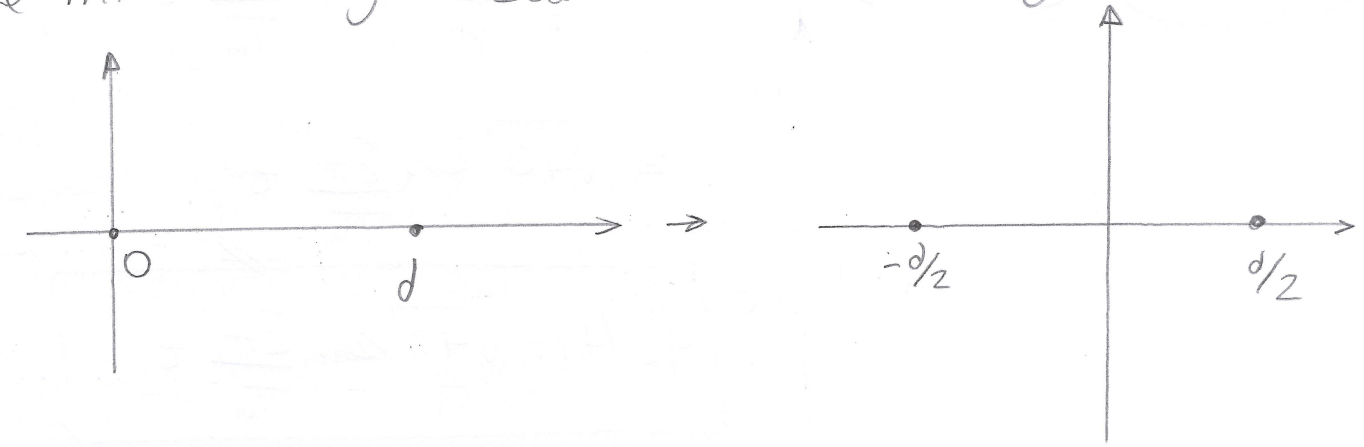
$$P_{e_M} = 1 - P(C|m_2) =$$

$$= 1 - \frac{1}{\sqrt{2\pi}} \int_{-\sqrt{2E/N}}^{+\infty} e^{-\left(y-\sqrt{2E/N}\right)^2/2} \left[1 - 2Q(y)\right]^{N-1} dy$$

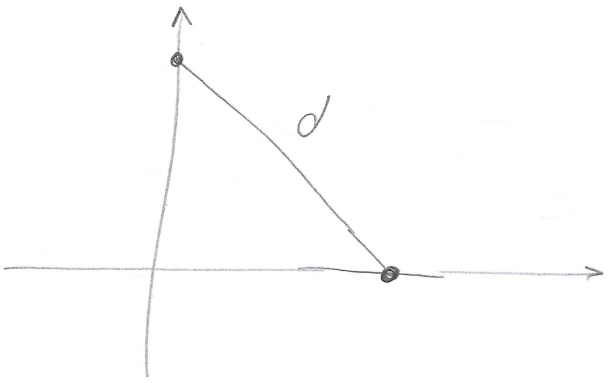
com $\boxed{E_b = \frac{E}{\log_2 2N}}$ ou $E = E_b \log_2 2N$

Como não a mudança na dimensionalidade do conjunto de sinais, concluímos que a largura de banda não se altera.

11.6-1 (a) O conjunto de sinais On-OFF e seu equivalente de mínima energia são mostrados a seguir.



(b) Para um FSK binário, com mesma BER



O conjunto equivalente de mínima energia é o mesmo (vide Exemplo 11.7).

(c) Do item (a) e (b), concluímos que uma sinalização on-off é uma sinalização ortogonal têm probabilidades de erro idênticas. O conjunto equivalente é uma sinalização polar com metade da energia por bit.

11.8.2 (a) Sejam $\varphi_1(t) = \sqrt{\frac{2}{T_M}} \cos \frac{2\pi}{T_M} t =$

$$= \sqrt{40} \cos \frac{2\pi}{T_M} t$$

e $\boxed{\varphi_2(t) = \frac{\sqrt{40}}{2\sqrt{10}} \operatorname{sen} \frac{2\pi_2}{T_M} t}$

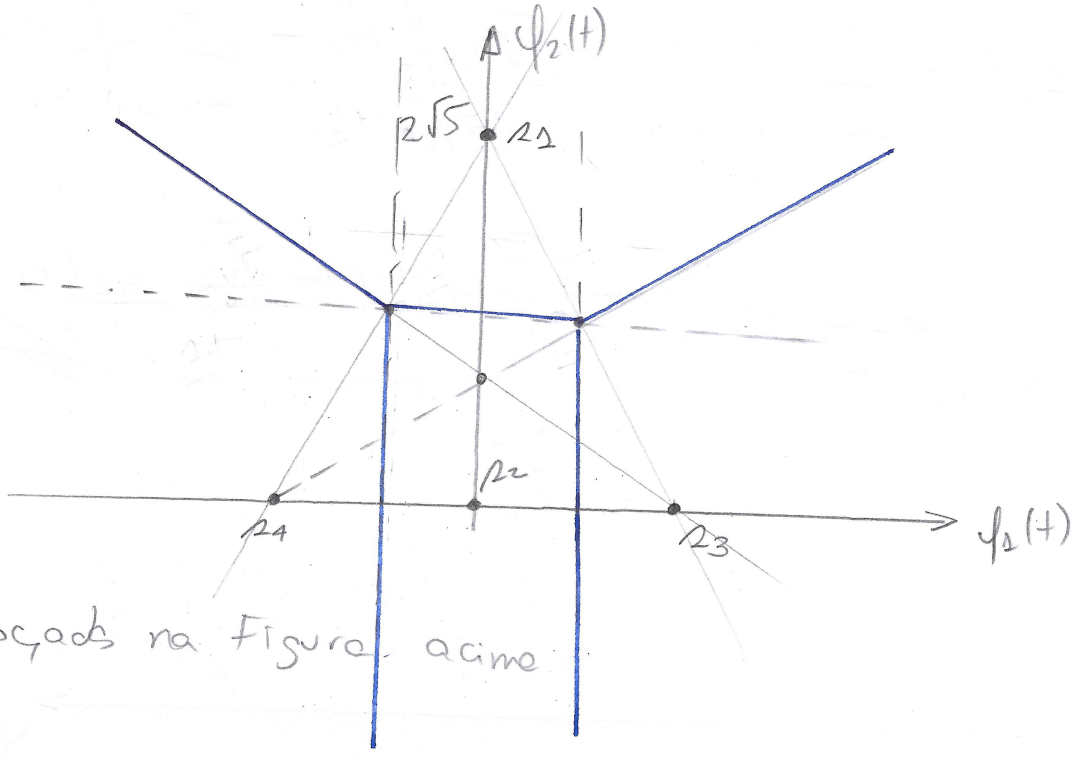
Assim,

$$z_1(t) = \frac{20\sqrt{2}}{\sqrt{40}} \cdot \varphi_2(t) = \frac{\overset{10}{\cancel{20}}\sqrt{2}}{2\sqrt{2}\cdot\sqrt{5}}\varphi_2(t) = 2\sqrt{5}\,\varphi_2(t)$$

$$z_2(t) = 0$$

$$z_3(t) = \frac{10\sqrt{2}}{\sqrt{40}}\varphi_1(t) = \frac{\overset{5}{\cancel{10}}\sqrt{2}}{2\sqrt{2}\sqrt{5}}\varphi_1(t) = \sqrt{5}\,\varphi_1(t)$$

$$z_4(t) = -\frac{10\sqrt{2}}{\sqrt{40}}\varphi_1(t) = \frac{-\overset{5}{\cancel{10}}\sqrt{2}}{2\sqrt{2}\sqrt{5}}\varphi_1(t) = -\sqrt{5}\,\varphi_1(t)$$



(b) Esboçado na Figura acima

(c) O vetor $\underline{a} = \frac{1}{4}\sum \underline{z}_i = \frac{1}{4}\left[(0, 2\sqrt{5}) + (0,0) + (\sqrt{5},0) + (-\sqrt{5},0)\right] =$

$$\underline{a} = (0,\ 2\sqrt{5}/4) \Rightarrow \boxed{a = \sqrt{5}/2\ \varphi_2(t)}$$

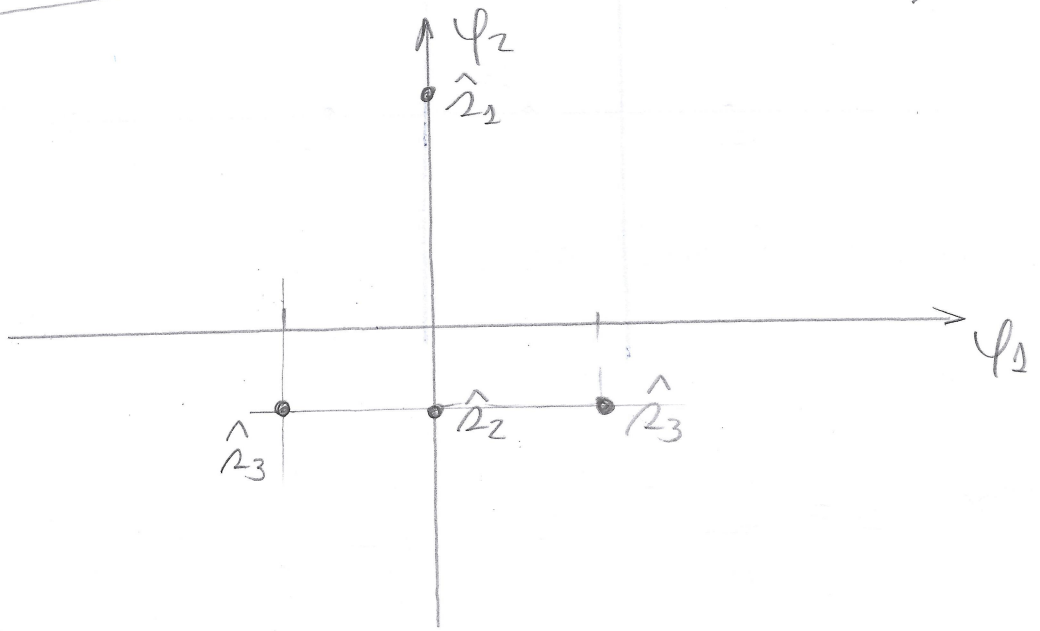
Assim, o conjunto de mínima energia é dado por

$$\hat{s}_1(t) = s_1(t) - \frac{\sqrt{5}}{2}\varphi_2(t) = \left(2\sqrt{5} - \frac{\sqrt{5}}{2}\right) = \frac{3\sqrt{5}}{2}\varphi_2(t)$$

$$\hat{s}_2(t) = s_2(t) - \frac{\sqrt{5}}{2}\varphi_2(t) = -\frac{\sqrt{5}}{2}\varphi_2(t)$$

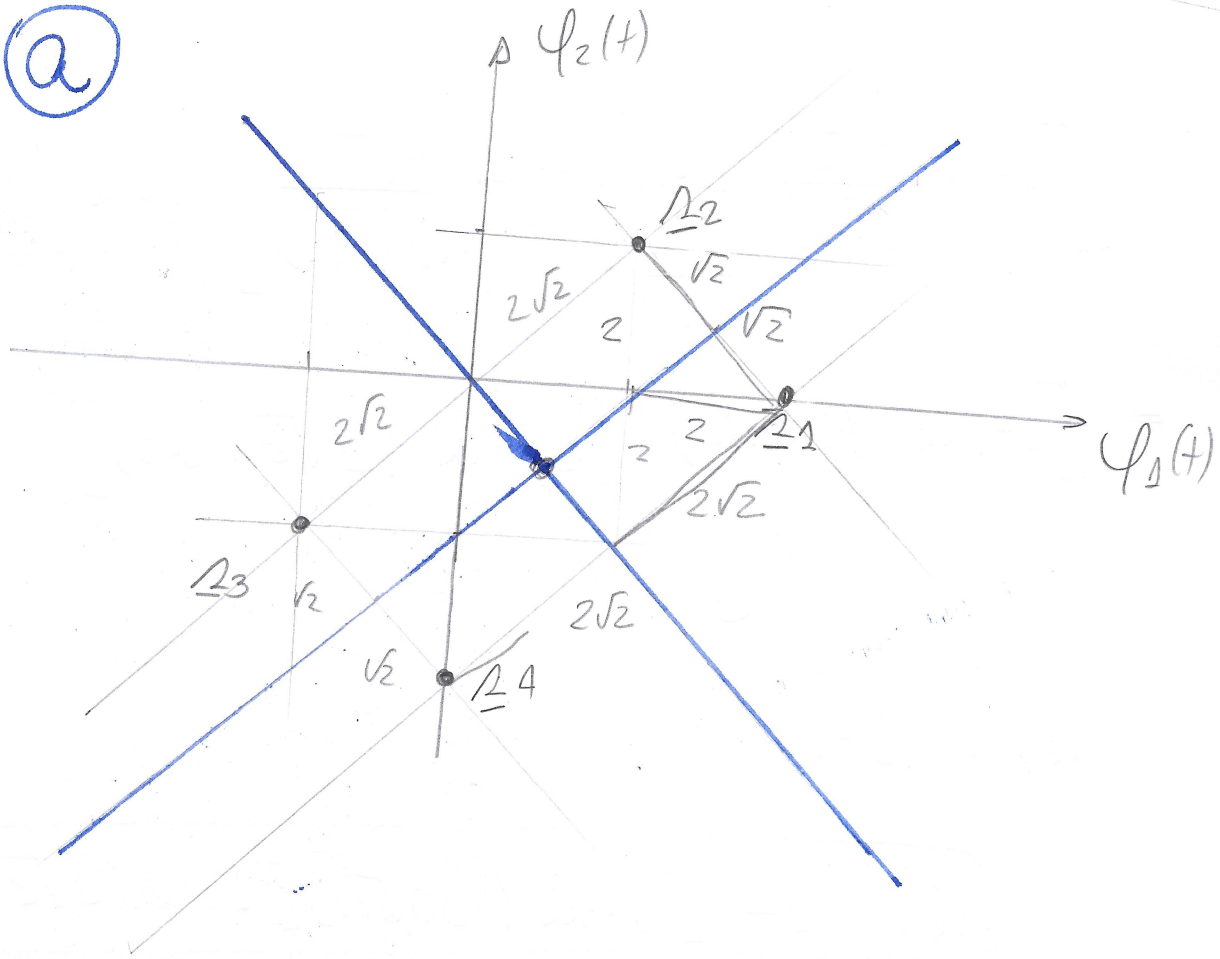
$$\hat{s}_3(t) = s_3(t) - \frac{\sqrt{5}}{2}\varphi_2(t) = \sqrt{5}\,\varphi_1(t) - \frac{\sqrt{5}}{2}\varphi_2(t)$$

$$\hat{s}_4(t) = s_4(t) - \frac{\sqrt{5}}{2}\varphi_2(t) = -\sqrt{5}\,\varphi_1(t) - \frac{\sqrt{5}}{2}\varphi_2(t)$$

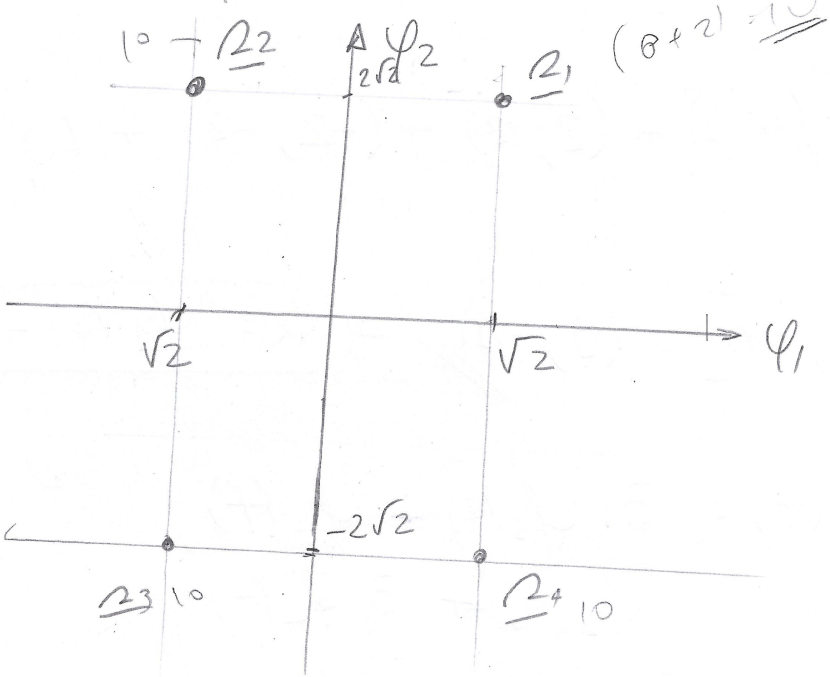


(d) O receptor ótimo é o mostrado nas figuras 11.19(a) ou (b).

11.8-3 (a)



(b) Conjunto equivalente



(e)

e $P(C) = P(C|m_1) = P(C|m_2) = P(C|m_3) = P(C|m_4)$

$= P(n_1 > -\sqrt{2} \text{ e } n_2 > -2\sqrt{2}) =$

$= \left[1 - Q\left(\frac{\sqrt{2}}{\sqrt{N_2}}\right)\right]\left[1 - Q\left(\frac{2\sqrt{2}}{\sqrt{N_2}}\right)\right]$ M. $\boxed{N_2 = 10^{-4}}$

$= \left[1 - Q(100\sqrt{2})\right]\left[1 - Q(2\sqrt{2}\,100)\right]$

e

$P_{eM} = 1 - P(C) \Rightarrow$

$\Rightarrow \boxed{P_{eM} = 1 - \left[1 - Q(100\sqrt{2})\right]\left[1 - Q(2\sqrt{2}\,100)\right]}$

(c) $a = \frac{1}{4}\left[(4,0) + (2,2) + (-2,-2) + (0,-4)\right] =$

$= \frac{1}{4}\left[(4,-4)\right] = (1,-1) \Rightarrow \boxed{a = \varphi_1(t) - \varphi_2(t)}$

e $\begin{cases} \hat{r}_1(t) = 3\varphi_1(t) + \varphi_2(t) \\ \hat{r}_2(t) = \varphi_1(t) + 3\varphi_2(t) \\ \hat{r}_3(t) = -3\varphi_1(t) - \varphi_2(t) \\ \hat{r}_4(t) = -\varphi_1(t) - 3\varphi_2(t) \end{cases}$

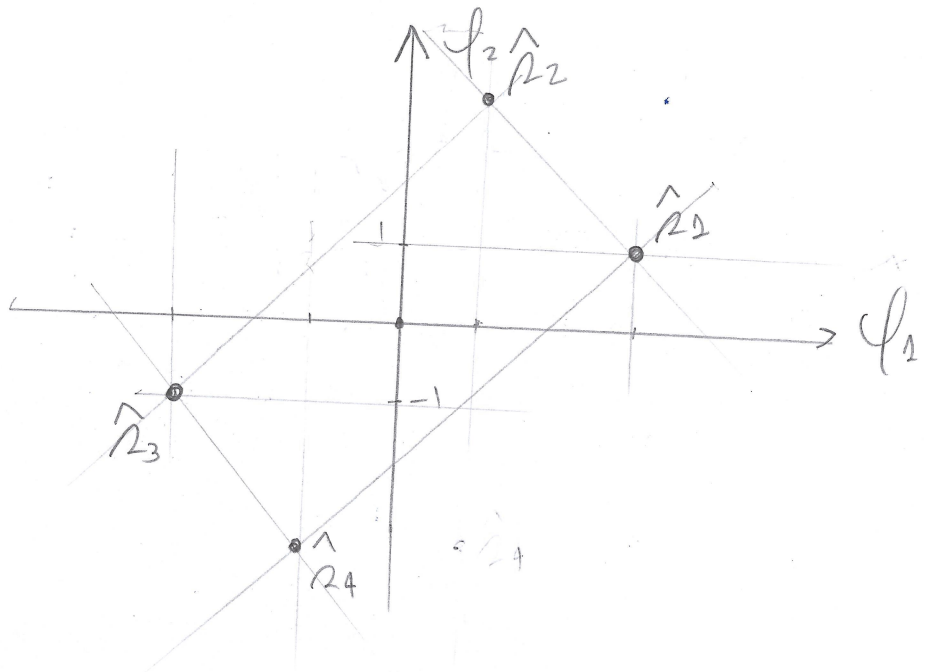
d

$$E_{m\,original} = \frac{16 + (8) + (8) + 16}{4} = 12$$

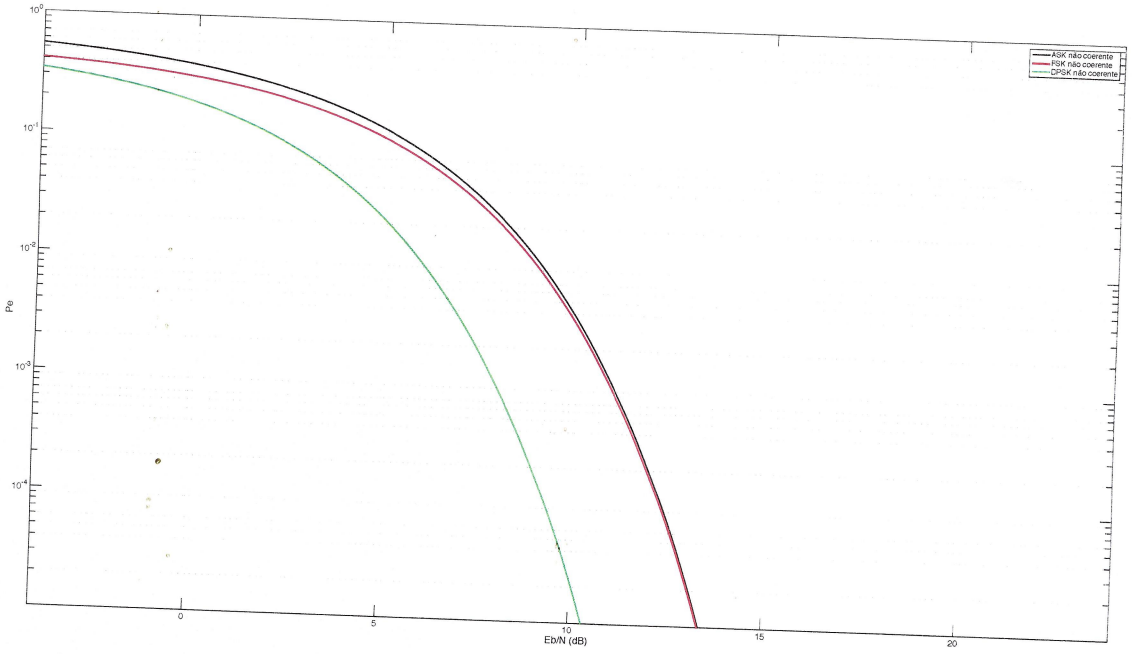
9

$$E_{M\,mínimo} = \frac{10 + 10 + 10 + 10}{4} = 10$$

Redução = 21



```
% Exercìcio 11.11.1 do Lathi

clear all; close all;
EbNdB = linspace(-4,24,1000);
EbN = 10.^(EbNdB/10);
%M = 16;

%ASK

PeASK = 1/2*(exp(-1/2*EbN)+Que(sqrt(EbN)));

%FSK

PeFSK = 1/2*(exp(-1/2*EbN));


%DPSK
PeDPSK = 1/2*(exp(-EbN));

figure(1);
semilogy(EbNdB, PeASK, 'k',EbNdB, PeFSK, 'r', EbNdB, PeDPSK, 'g');
legend('ASK n„o coerente', 'FSK n„o coerente', 'DPSK n„o coerente');
grid;
xlabel('Eb/N (dB)');
ylabel('Pe');
axis([-4 24 1e-5 1])
```



DPSK tem melhor desempenho em termos de BER, seguido pelo FSK e ASK não coerentes que têm desempenho similar.