



Specifically, Duverger argued that social forces are more likely to produce additional parties when countries employ multimember districts than when they do not. We will test Duverger's claims on the determinants of party system size with the following model:

$$\text{Legislative Parties} = \beta_0 + \beta_1 \text{Multimember District} + \beta_2 \text{Social Heterogeneity} + \beta_3 \text{Multimember District} \times \text{Social Heterogeneity} + \varepsilon$$

As Clark, Gilligan and Golder (2006) summarize, "Duverger's theory leads us to believe that both multi-member districts and social heterogeneity are necessary, but not sufficient, to produce more legislative parties." In other words, this implies that  $\beta_1 = \beta_2 = 0$  and that  $\beta_3 > 0$ .

## Part I. X and Z are dichotomous variables

**Exercise 1.** First, we are going to test the hypotheses by analyzing the results reported for the following regression:

$$\text{Legislative Parties} = \beta_0 + \beta_1 \text{Multimember District} + \beta_2 \text{Social Heterogeneity} + \beta_3 \text{Multimember District} \times \text{Social Heterogeneity} + \varepsilon$$

where

Legislative Parties = effective number of legislative parties;

Multimember District = dichotomous variable indicating whether a country has single- or multi-member districts; and,

Social Heterogeneity = dichotomous variable indicating whether a country is ethnically heterogeneous (more ethnic groups than the median country) or ethnically homogenous (less ethnic groups than the median country).

- a) Please estimate the regression model and discuss whether what the results reveal about the effect of ethnic diversity and district type on party size.

```
. regress enps heterogeneity multimember multi_heterogeneity
```

Source	SS	df	MS	Number of obs =	54
Model	33.4971669	3	11.1657223	F( 3, 50) =	7.35
Residual	75.9721185	50	1.51944237	Prob > F =	0.0004
Total	109.469285	53	2.06545822	R-squared =	0.3060
				Adj R-squared =	0.2644
				Root MSE =	1.2327

enps	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
heterogeneity	-.8336323	.554038	-1.50	0.139	-1.94645 .2791856
multimember	.5455444	.50323	1.08	0.284	-.4652227 1.556312
multi_heterogeneity	1.654754	.6973784	2.37	0.022	.254028 3.05548
_cons	2.516478	.4108855	6.12	0.000	1.69119 3.341766

"As predicted, the coefficients on Multimember District and Social Heterogeneity are indistinguishable from zero and the coefficient on the interaction term is both positive and statistically significant...In contrast, the difference in the expected number of parties

between heterogeneous societies that employ multimember districts and those that employ single-member districts is statistically significant (this difference is captured by the coefficient on the interaction term).“

“If our asymmetric claim is correct, we should also expect the marginal effect of Social Heterogeneity ( $\beta_1 + \beta_3$ ) to be positive and statistically significant when there are multi-member districts (see footnote 15). This is indeed the case. The marginal effect of Social Heterogeneity in a country with multi-member districts is 0.82. This effect is statistically significant at the 94% level.”

Students should be able to understand how 0.82 was obtained.

- b) Now, let’s examine the model’s predictions on the effective number of legislative parties depending on both social heterogeneity and district type. Please fill in the following table and include the commands in your do-file.

Table 3. The Predicted Number of Legislative Parties

Social Heterogeneity	Single-Member Districts	Multi-Member Districts
Heterogeneous		
Homogenous		

Table 3. The Predicted Number of Legislative Parties

Social Heterogeneity	Single-Member Districts	Multi-Member Districts
Heterogeneous	1.68 [0.94 – 2.43]	3.88 [3.26 – 4.50]
Homogenous	2.52 [1.69 – 3.34]	3.06 [2.48 – 3.65]

**Notes:** 95% confidence intervals in parentheses

The table shows that there is no difference in the number of parties in Single Member Districts depending on heterogeneous or homogenous societies, and that single member districts produce a low number of parties. As the authors explain, “the model predicts that there will be 2.52 legislative parties in homogenous societies employing single-member districts compared to 1.68 legislative parties in heterogeneous societies with single-member districts. Both of these predictions are consistent with Hypothesis 1—single-member districts almost never produce multiparty systems. Although the predicted number of parties differs between homogenous and heterogeneous societies employing

single-member districts, this difference is not statistically significant (this difference is captured by the coefficient on Social Heterogeneity).” This can be verified by comparing the confidence intervals which overlap.

“In contrast, the difference in the expected number of parties between heterogeneous societies that employ multimember districts and those that employ single-member districts is statistically significant (this difference is captured by the coefficient on the interaction term).” The model predicts there will be 1.68 legislative parties in heterogeneous societies with single-member districts versus 3.88 parties in heterogeneous societies with multi-member districts. The confidence intervals do not overlap.

For this reason, the authors conclude that “both multimember districts and social heterogeneity are necessary, but not sufficient, for more legislative parties.”

**X and Z are continuous variables.**

**Exercise 2.** We will now treat multi-member district magnitude and ethnic diversity as continuous variables. Our revised model is:

$$\text{Legislative Parties} = \gamma_0 + \gamma_1 \ln(\text{Median District Magnitude}) + \gamma_2 \text{Social Heterogeneity} + \gamma_3 \ln(\text{Median District Magnitude}) \times \text{Social Heterogeneity} + \varepsilon$$

- a) Please estimate the revised model and discuss whether what the results reveal about the effect of ethnic diversity and district type on party size. Do the results confirm earlier findings?

```
. regress enps eneth lnml lmleneth
```

Source	SS	df	MS	Number of obs =	54
Model	39.7248824	3	13.2416275	F( 3, 50) =	9.49
Residual	69.744403	50	1.39488806	Prob > F =	0.0000
				R-squared =	0.3629
				Adj R-squared =	0.3247
Total	109.469285	53	2.06545822	Root MSE =	1.1811

enps	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
eneth	-.3619712	.3486305	-1.04	0.304	-1.062216 .3382738
lnml	-.1911174	.2967357	-0.64	0.522	-.7871287 .4048939
lmleneth	.4833254	.1805094	2.68	0.010	.1207616 .8458893
_cons	2.671367	.6072149	4.40	0.000	1.45174 3.890994

As predicted, the coefficients on both constitutive terms are statistically indistinguishable from zero, and the coefficient on the interaction term is relatively large and positive. This suggests that social heterogeneity has no distinguishable causal effect on party system size when district magnitude is one, i.e., when  $\ln(\text{Magnitude}) = 0$ . Similarly, an increase in district magnitude also has no distinguishable causal effect on party system size when a society is entirely homogenous. However, the positive and relatively large coefficient on the interaction term means that social heterogeneity will increase party system size when the district magnitude is sufficiently large.”

- b) To determine exactly how large the district magnitude needs to be for social heterogeneity to have its hypothesized positive effect on party system size, we need to calculate marginal effect of social heterogeneity ( $\gamma_1 + \gamma_3 \ln(\text{District Magnitude})$ ) and its associated confidence intervals across the observed range of district magnitudes. To do so, let's first examine the model's predictions using the margins command in Stata, which we learned in class last week. Margins uses the delta method approximation to examine how small changes in the explanatory variable affects the dependent variable. Try to use this command to examine the effect of district magnitudes and heterogeneity on party size. What are some reasonable criteria to use in such an estimation?

It would be reasonable to examine the average marginal effects, but this is exactly what we can interpret by examining the regression output. The next type of hypotheses to examine are conditional marginal effects. Theory should help us to select critical cases to examine. To help understand the marginal effect plots in c) and d), we will try to execute similar commands.

```
margins, dydx(eneth) at(lnml=(0 1 2 3 4 5))
```

```
Average marginal effects      Number of obs =      54
Model VCE   : OLS
```

```
Expression   : Linear prediction, predict()
dy/dx w.r.t. : eneth
```

```
1._at      : lnml      =      0
```

```
2._at      : lnml      =      1
```

```
3._at      : lnml      =      2
```

```
4._at      : lnml      =      3
```

```
5._at      : lnml      =      4
```

6.\_at : lnml = 5

		Delta-method				
		dy/dx	Std. Err.	t	P> t	[95% Conf. Interval]
-----+-----						
eneth						
_at						
1		-.3619712	.3486305	-1.04	0.304	-1.062216 .3382737
2		.1213543	.2498486	0.49	0.629	-.3804813 .6231899
3		.6046798	.2616729	2.31	0.025	.0790943 1.130265
4		1.088005	.3737491	2.91	0.005	.3373081 1.838702
5		1.571331	.5254249	2.99	0.004	.5159837 2.626678
6		2.054656	.6911018	2.97	0.005	.6665375 3.442775

margins, dydx(lnml) at(eneth=(1 2 3 4 ))

Average marginal effects                      Number of obs =      54  
Model VCE : OLS

Expression : Linear prediction, predict()  
dy/dx w.r.t. : lnml

1.\_at : eneth = 1

2.\_at : eneth = 2

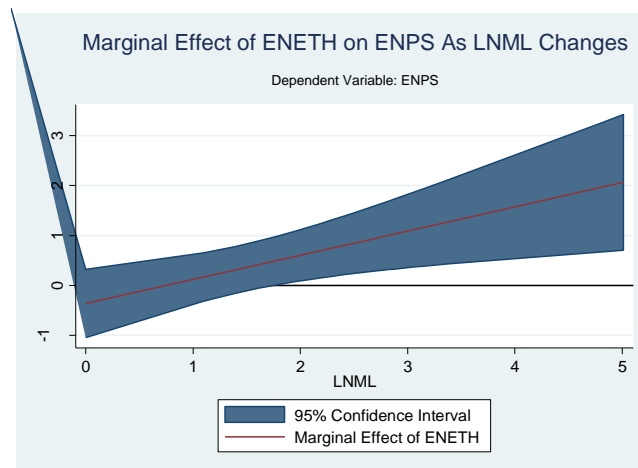
3.\_at : eneth = 3

4.\_at : eneth = 4

		Delta-method				
		dy/dx	Std. Err.	t	P> t	[95% Conf. Interval]
-----+-----						
lnml						
_at						
1		.292208	.1498859	1.95	0.057	-.0088466 .5932627
2		.7755335	.1484815	5.22	0.000	.4772997 1.073767
3		1.258859	.2946099	4.27	0.000	.6671175 1.8506
4		1.742184	.4655218	3.74	0.000	.8071564 2.677213

- c) Next, let's replicate the Clark, Gilligan and Golder (2006) marginal effect plot of the marginal effect of social heterogeneity across a range of district magnitudes on the number of parties (our dependent variable) using the code provided by Guy Whitten. What additional insights do you gain from this plot?

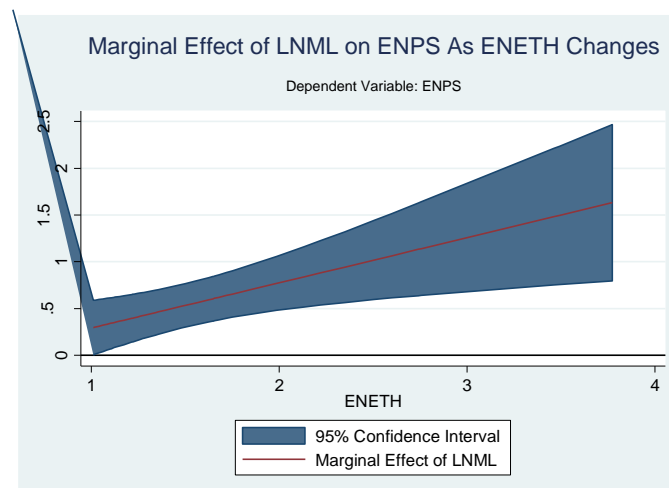
Case 1 ) The effect of social heterogeneity across a range of district magnitudes.



“Social heterogeneity has no discernible effect on the number of legislative parties when logged average district magnitude is close to zero. However, as predicted, the effect of social heterogeneity becomes both clear and pronounced once the district magnitude becomes sufficiently large. Specifically, social heterogeneity has a positive and significant effect on party system size whenever the district magnitude is greater than six (or whenever  $\ln(\text{Magnitude})$  is greater than 1.8).”

- d) Next, let's do a second marginal effect plot that is not reported by Clark, Gilligan and Golder (2006) of the marginal effect of district magnitudes across a range of levels of social heterogeneity on the number of parties (our dependent variable) using the code provided by Guy Whitten. What additional insights do you gain from this plot?

Case 2 ) The effect of district magnitudes across a range of levels of ethnic heterogeneity.



District Magnitudes has a statistically significant effect on the number of legislative parties at any level of social heterogeneity. The effect of district magnitude on the number of parties increases as social heterogeneity becomes large. Specifically, district

magnitude has a positive and significant effect on party system size at any level of social heterogeneity.

**Exercise 3.** Clark, Gilligan and Golder (2006) argue that “We should note that our interaction model allows us to talk about degrees of “necessity” or degrees of “sufficiency.” For example, the magnitude of the coefficient on the cause that is purported to be necessary, but not sufficient, is a measure of “sufficiency.” Conversely, the magnitude of the coefficient on the interaction term is a measure of the extent to which the purported cause is necessary. If  $b_1$  is large relative to  $b_3$  (and they have the same sign), the more “sufficient” is  $X_1$  and the less “necessary” is  $X_2$  for  $Y$ . Similarly, if  $b_2$  is large relative to  $b_3$ , the less “necessary” is  $X_1$  and the more “sufficient” is  $X_2$  for  $Y$ . Do you agree? Explain with figures to illustrate your reasoning (if necessary).

Clark, Gilligan e Golder’s statement (2006) concerning the relationship between the constitutive terms of the regression model  $(\beta_1, \beta_2, \beta_3)$  regarding necessity and sufficiency needs to be interpreted with caution. As this exercise has attempted to illustrate, it is much easier to make conclusions about necessity and sufficiency in the case of discrete variables as opposed to continuous variables.

This is due to two motives both explained by Brambor, Clark and Golder (2006) in their second article on interactions, “Understanding Interaction Models: Improving Empirical Analyses.” Political Analysis 14: 63-82.” As they show in this article, the marginal effect of variables cannot be interpreted directly only examining the beta parameters in interaction models. Instead, we should examine the marginal effect of  $x$  on  $y$  given all values of  $z$ :

$$\frac{\partial y}{\partial x} = \beta_1 + \beta_3 z$$

(and vice-versa, the marginal effect of  $z$  on  $y$  given values of  $x$ ).

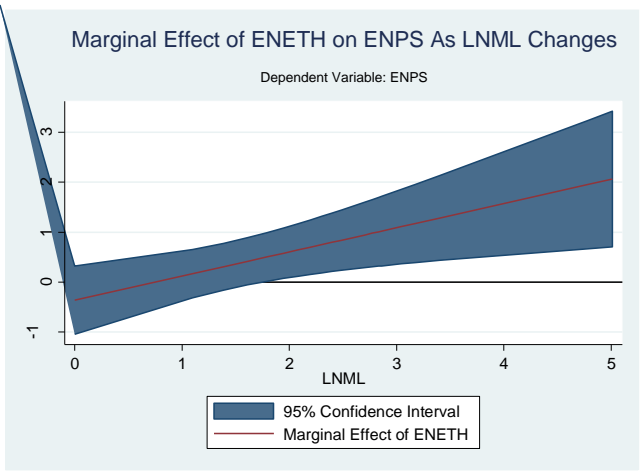
The second issue that makes this statement to be qualified further has to do with observing what they call *substantively meaningful marginal effects* (Brambor, Clark & Golder, 2006, pg 71-6). (See more on this issue in next question).

**Exercise 4.** Go back and look at the descriptive statistics for the explanatory variables. Do any of your findings change considering the distribution of cases?

If we examine the distribution of cases, we can see that the majority of values are in the lower values of  $\ln ml$  and  $eneth$ . Thus, marginal effects that depend on high values of  $\ln ml$  (log of



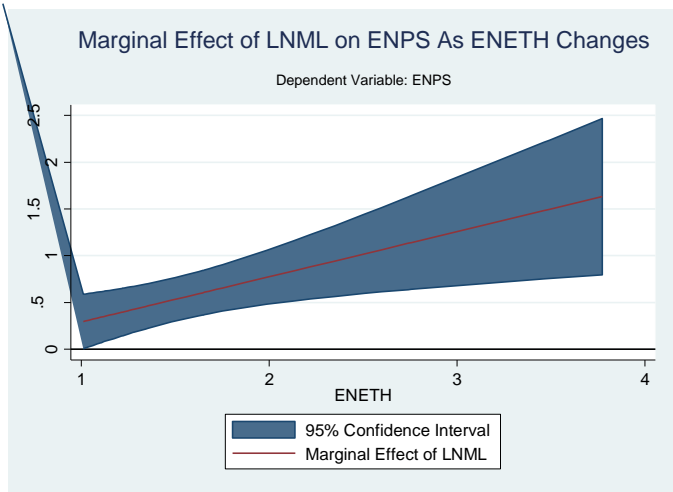
median district magnitude) or eneth (effective number of ethnic groups) may not be substantively meaningful.



In the case of lnml, 100% of the values are between zero and 5.01. Recall that the marginal effect of ENETH is only statistically significant when lnml is greater than 1.8. Per the summary statistics, there are roughly 55% of cases for which lnml is greater than 1.8. There are more than 50% of the sample for which the marginal effect of eneth, which depends on values of lnml, is not significantly different from zero.

```
. sum lnml, detail
```

Log Med Mag				
Percentiles		Smallest		
1%	0	0		
5%	0	0		
10%	0	0	Obs	54
25%	0	0	Sum of Wgt.	54
50%	1.700599		Mean	1.530797
75%	2.484907	Largest	Std. Dev.	1.37122
90%	3.178054	3.401197	Variance	1.880245
95%	3.401197	4.787492	Skewness	.3341832
99%	5.010635	5.010635	Kurtosis	2.335585



With respect to ENETH, the distribution of cases across different levels of ethnic heterogeneity does not appear to be relevant as the marginal effect of LNML is statistically significant at all values of ENETH.

