A NON-LINEAR THEORY OF CONSOLIDATION

by

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SYNOPSIS

A theory of non-linear consolidation for oedometer boundary conditions has been developed assuming the coefficient of consolidation to be constant, Darcy's Law to be valid, and the soils obey the law

$$e = e_0 - I_c \log_{10} \frac{\sigma'}{\sigma_0}.$$

The solutions show that for oedometer boundary conditions Terzaghi's theory predicts satisfactorily the rate of settlement but not the rate of dissipation of pore pressures. Terzaghi's solution for the latter case is on the unsafe side.

Experimental results are presented which show the validity, for normally consolidated clay, of the new theory.

On a développé une théorie de consolidation non-linéaire des conditions limites œdométriques en admettant que le coefficient de consolidation soit constant, que la Loi de Darcy soit valable et que le sol obéïsse à la loi

$$e = e_0 - I_c \log_{10} \frac{\sigma'}{\sigma_0}.$$

Les solutions montrent que pour des conditions limites œdométriques, la théorie de Terzaghi prédit convenablement la vitesse de tassement mais pas la vitesse de disparition de la pression interstitielle. La solution de Terzaghi pour ce dernier cas est plutôt douteuse.

On présente les résultats expérimentaux qui montrent la validité de la nouvelle théorie pour de l'argile consolidée normalement.

NOTATION

A	constant
B	$\sum_{N=1}^{N=\infty} \frac{2}{M}\left\{\sin M \frac{z}{H}\right\} \epsilon^{-M^2 T_v}$
c_v	coefficient of consolidation
e	void ratio
e_0	void ratio for effective pressure σ_0
H	distance apart of permeable and impermeable boundaries
I_c	compression index
i	hydraulic gradient
k	permeability of soil
M	$(2N+1)\frac{\pi}{2}$
m_v	coefficient of compressibility of the soil skeleton
N	integer from zero to infinity
σ	total pressure
σ'	effective pressure
σ'_f	final effective pressure
σ'_i	initial effective pressure
t	time
T_v	time factor
u	pore-water pressure
u_z	pore-water pressure at a distance z from the impermeable boundary
v	velocity of pore water
w	$\log_{10} (\sigma'/\sigma'_f)$
z	distance from permeable boundary
λ_w	density of water
S	degree of settlement

INTRODUCTION

Attention has recently been focused on the non-linear behaviour of soils during consolidation.

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[1] The references are given on p. 173.

McNabb (1960)[1] has derived the one dimensional consolidation equation in very general form. Lo (1960) has attempted to take into account the variability of the coefficient of consolidation. Richart (1957) has solved by means of finite differences the problem considering a variable void ratio, and Schiffman (1958) has considered the case of a non-linear relationship for the value of the coefficient of permeability. Hansbo (1960) has considered variable permeability for the solution to the problem of the consolidation of clays by sand drains. Numerous authors have considered the problem of combined primary and secondary consolidation. In this Paper secondary consolidation will be ignored.

In a mass of real soil the compressibility, permeability, and coefficient of consolidation vary during the consolidation process. The least variable factor for normally consolidated clays is the coefficient of consolidation. It seems reasonable, therefore, that a more accurate consolidation theory could be developed by assuming the coefficient of consolidation to be constant while the compressibility and permeability were both allowed to decrease with increasing pressure.

ASSUMPTIONS AND DEVELOPMENT OF GENERAL EQUATION

(1) The coefficient of compressibility m_v of the soil skeleton is given by:

$$m_v = -\frac{1}{1+e} \cdot \frac{\partial e}{\partial \sigma'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where e is the void ratio and σ' the effective pressure.

(2) Results of oedometer tests on normally consolidated soils have shown that the empirical law:

$$e = e_0 - I_c \log_{10} \frac{\sigma'}{\sigma'_0}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where e_0 is the void ratio of soil subjected to pressure σ'_0, σ'_0 the effective pressure at point '0' on the e–log σ' curve, and I_c the compression index of soil assumed constant, is approximately valid. Differentiating void ratio (e) with respect to effective pressure and substituting in equation (1) gives:

$$m_v = \frac{0{\cdot}434\, I_c}{(1+e)\sigma'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(3) During the consolidation process $(1+e)$ varies with time far less than the effective pressure σ' so that $(1+e)$ may be considered constant for any load increment. With this assumption equation (3) becomes:

$$m_v = \frac{A}{\sigma'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where A is a constant.

(4) Results of oedometer tests on normally consolidated soils have shown that the coefficient of consolidation c_v varies much less than the coefficient of compressibility m_v and may be taken as relatively constant. It is therefore assumed that:

$$c_v = \frac{k}{m_v \gamma_w} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where k is the permeability of the soil and γ_w the bulk density of water. This is equivalent to assuming that as the soil particles are moved closer together, the decrease in permeability is proportional to the decrease in compressibility.

(5) The soil is laterally confined.

(6) The total stresses are the same, and the effective stresses are the same for every point on a horizontal layer.

(7) Darcy's Law is applicable to the movement of water through the soil:

$$v = ki = -\frac{k}{\gamma_w}\cdot\frac{\partial u}{\partial z} \qquad (6)$$

where v is the velocity of flow, i the hydraulic gradient, k the coefficient of permeability, u excess pore-water pressure, and z the height above the impermeable boundary. The rate of water lost per unit area in a small element of soil thickness dz is:

$$\frac{\partial v}{\partial z}dz = \frac{\partial}{\partial z}\left(-\frac{k}{\gamma_w}\cdot\frac{\partial u}{\partial z}\right)dz.$$

Substituting equations (4) and (5) gives:

$$\frac{\partial v}{\partial z}dz = -\frac{\partial}{\partial z}\left(m_v c_v\frac{\partial u}{\partial z}\right)dz = -\frac{\partial}{\partial z}\left(\frac{Ac_v}{\sigma'}\cdot\frac{\partial u}{\partial z}\right)dz$$

$$= -Ac_v\left[\frac{1}{\sigma'}\cdot\frac{\partial^2 u}{\partial z^2}-\left(\frac{1}{\sigma'}\right)^2\frac{\partial u}{\partial z}\cdot\frac{\partial \sigma'}{\partial z}\right]dz \qquad (7)$$

(8) The degree of saturation of the soil is 100% and the pore water and soil particles are incompressible relative to the soil skeleton. Therefore

$$\sigma = \sigma'+u \qquad (8)$$

where σ is total pressure.

Differentiating with respect to depth gives:

$$\frac{\partial \sigma}{\partial z} = \frac{\partial \sigma'}{\partial z}+\frac{\partial u}{\partial z}. \qquad (9)$$

It should be noted that if, and only if, the total stress σ is constant with depth does:

$$\frac{\partial \sigma}{\partial z} = 0. \qquad (10)$$

(9) The soil skeleton does not creep under constant effective stress (i.e. secondary consolidation effects are ignored).

(10) The strain developed within the soil element is given by:

$$f = \frac{e_n-e}{1+e_n} = \frac{I_c}{1+e_n}\log_{10}\frac{\sigma'}{\sigma'_n} \qquad (11)$$

where f is strain and e_n the void ratio corresponding to zero strain and to the stress σ'_n. Differentiating strain with respect to time to obtain the rate of water lost per unit area:

$$\frac{\partial f}{\partial t} = \frac{I_c}{1+e_n}\cdot\frac{0{\cdot}434}{\sigma'}\cdot\frac{\partial \sigma'}{\partial t} \qquad (12)$$

where t is time. Assuming that $(1+e_n)$ is approximately equal to $(1+e)$ in equations (3) and (4):

$$\frac{\partial f}{\partial t} = \frac{A}{\sigma'}\cdot\frac{\partial \sigma'}{\partial t}. \qquad (13)$$

Equating the rate of water lost per unit area to the rate of volume decrease per unit area both within a small element of soil thickness dz (this is possible since the pore-water and soil particles are assumed incompressible relative to the soil skeleton):

$$\frac{\partial v}{\partial z}dz = \frac{\partial f}{\partial t}dz. \qquad (14)$$

Substituting equations (7) and (13) gives:

$$-c_v\left[\frac{1}{\sigma'}\cdot\frac{\partial^2 u}{\partial z^2}-\left(\frac{1}{\sigma'}\right)^2\frac{\partial u}{\partial z}\cdot\frac{\partial \sigma'}{\partial z}\right]=\frac{1}{\sigma'}\cdot\frac{\partial \sigma'}{\partial t} \quad \ldots\ldots \quad (15)$$

Equation (15) is the general equation for one-dimensional strain and drainage since no assumptions as to type of loading, rate of change of loading etc, have been made.

APPLICATION OF THEORY TO OEDOMETER TEST

In an oedometer test the applied total load is theoretically constant with depth and increments of loading are applied instantaneously on a sample which is at equilibrium.

For a thin layer of soil the weight of the soil can be ignored so that equation (10) is valid. Substituting equations (9) and (10) into equation (15) gives:

$$-c_v\left[\frac{1}{\sigma'}\cdot\frac{\partial^2 u}{\partial z^2}+\left(\frac{1}{\sigma'}\right)^2\left(\frac{\partial u}{\partial z}\right)^2\right]=\frac{1}{\sigma'}\cdot\frac{\partial \sigma'}{\partial t} \quad \ldots\ldots \quad (16)$$

Using the substitution:

$$w=\log_{10}\frac{\sigma'}{\sigma_f'}=\log_{10}\frac{\sigma_f'-u}{\sigma_f'} \quad \ldots\ldots \quad (17)$$

where σ_f' is final effective pressure.

Differentiating with respect to z:

$$\frac{\partial w}{\partial z}=-\frac{0{\cdot}434}{\sigma'}\cdot\frac{\partial u}{\partial z} \quad \ldots\ldots \quad (18)$$

$$\frac{\partial^2 w}{\partial z^2}=-\frac{0{\cdot}434}{\sigma'}\cdot\frac{\partial^2 u}{\partial z^2}-\frac{0{\cdot}434}{(\sigma')^2}\left(\frac{\partial u}{\partial z}\right)^2 \quad \ldots\ldots \quad (19)$$

Differentiating equation (17) with respect to time:

$$\frac{\partial w}{\partial t}=\frac{0{\cdot}434}{\sigma'}\cdot\frac{\partial \sigma'}{\partial t} \quad \ldots\ldots \quad (20)$$

Substituting equations (19) and (20) into equation (16) gives a simpler version of the differential equation in terms of the function w:

$$c_v\cdot\frac{\partial^2 w}{\partial z^2}=\frac{\partial w}{\partial t} \quad \ldots\ldots \quad (21)$$

This equation is identical in form to that of the ordinary Terzaghi linear theory and can be solved in the same way since the boundary conditions for the oedometer test are similar in terms of u and w. These conditions are:

$$\left.\begin{array}{llllllll} t=0 & \text{and} & 0\leqslant z\leqslant H & \text{and} & u=\sigma_f'-\sigma_i' & \text{then} & w=\log\sigma_i'/\sigma_f' \\ 0\leqslant t\leqslant\infty & ,, & z=H & ,, & \partial u/\partial z=0 & ,, & \partial w/\partial z=0 \\ 0\leqslant t\leqslant\infty & ,, & z=0 & ,, & u=0 & ,, & w=0 \\ t=\infty & ,, & 0\leqslant z\leqslant H & ,, & u=0 & ,, & w=0 \end{array}\right\} \quad (22)$$

where σ_i' is initial effective pressure at time $t=0$.

Therefore:

$$w_z=\log_{10}\frac{\sigma_f'-u}{\sigma_f'}=\left[\log_{10}\frac{\sigma_i'}{\sigma_f'}\right]\sum_{N=0}^{N=\infty}\frac{2}{M}\left(\sin M\frac{z}{H}\right)\epsilon^{-M^2T_v} \quad \ldots \quad {}^{*}(23)$$

where

$$M=(2N+1)\frac{\pi}{2} \quad \ldots\ldots \quad (24)$$

$$T_v=\frac{c_v t}{H^2}. \quad \ldots\ldots \quad (25)$$

* It appears that Gray (1936) obtained this solution. However, he does not derive the equation, nor does he obtain a solution for the degree of settlement.

Therefore
$$\frac{\sigma_f' - u}{\sigma_f'} = \left(\frac{\sigma_i'}{\sigma_f'}\right)^B \quad \ldots \ldots \ldots \quad (26)$$

where
$$B = \sum_{N=1}^{N=\infty} \frac{2}{M}\left(\sin M \frac{z}{H}\right)\epsilon^{-M^2 T_v} \quad \ldots \ldots \quad (27)$$

B is the same as Terazghi's value of:

$$\frac{u}{\sigma_f' - \sigma_i'}. \quad \ldots \ldots \ldots \quad (28)$$

Simplifying equation (26):

$$-u = \sigma_f'\left[\left(\frac{\sigma_i'}{\sigma_f'}\right)^B - 1\right] \quad \ldots \ldots \ldots \quad (29)$$

$$\frac{u}{\sigma_f' - \sigma_i'} = \frac{\sigma_f'}{\sigma_f' - \sigma_i'}\left[1 - \left(\frac{\sigma_i'}{\sigma_f'}\right)^B\right] \quad \ldots \ldots \ldots \quad (30)$$

The degree of settlement for the assumed ideal soil will be given by:

$$S = \frac{\int_0^H (e_0 - e)}{\int_0^H (e_0 - e_f)}\, dz$$

$$= \frac{\int_0^H \left(\log_{10} \frac{\sigma_f' - u}{\sigma_i}\right) dz}{\int_0^H \left(\log_{10} \frac{\sigma_f'}{\sigma_i'}\right) dz} \quad \ldots \ldots \ldots \quad (31)$$

Substituting equation (26) gives:

$$S = \frac{\int_0^H \left(\log_{10} \frac{\sigma_f'}{\sigma_i'} \cdot \left(\frac{\sigma_i'}{\sigma_f'}\right)^B\right) dz}{\int_0^H \left(\log_{10} \frac{\sigma_f'}{\sigma_i'}\right) dz} \quad \ldots \ldots \ldots \quad (32)$$

$$= \frac{1}{H}\int_0^H (1 - B)\, dz \quad \ldots \ldots \ldots \quad (33)$$

But B is the same as Terzaghi's value of $\dfrac{u}{\sigma_f' - \sigma_i'}$.

Therefore S = degree of settlement given by Terzaghi's theory

$$= 1 - \sum_{N=0}^{N=\infty} \frac{2}{M}\,\epsilon^{-M^2 T_v} \quad \ldots \ldots \ldots \quad (34)$$

Furthermore, equation (33) implies that the contribution to the total settlement of a thin layer of thickness dz at depth z for the assumed ideal soil is the same as the contribution computed by Terzaghi's theory.

CALCULATION

Calculations of the value of pore pressure u (equation (29)) were done with the aid of SILLIAC, an automatic computer.

The first twenty terms in the summation equation were used for evaluating B and thus the pore pressure.

Values of pore pressure were computed for $z/H = 1{\cdot}0$, 0·9, 0·8, 0·7, 0·6, 0·5, 0·4, 0·3, 0·2, 0·1, and for $\sigma_f'/\sigma_i' = 1$, $1\frac{1}{2}$, 2, 4, 8, 16. The results for $z/H = 1{\cdot}0$ are tabulated in Table 1 and plotted graphically in Fig. 1.

Table 1. Percentage values of centre pore-pressures u_z for different values σ'_f/σ'_i

S	T_v	$u_{z=H}$ for σ'_f/σ'_i					
		1	$1\frac{1}{2}$	2	4	8	16
16·0	0·02	100	100	100	100	100	100
22·6	0·04	99·9	99·9	99·9	100	100	100
31·9	0·08	97·5	98·0	98·3	98·8	99·2	99·5
35·7	0·10	94·9	95·9	96·4	97·6	98·4	99·0
50·4	0·20	77·2	80·7	82·9	87·6	91·4	94·1
61·3	0·30	60·7	65·4	68·7	75·8	81·9	86·8
69·8	0·40	47·5	52·5	56·1	64·3	71·7	78·1
76·4	0·50	37·1	41·9	45·3	53·6	61·4	68·5
81·6	0·60	29·0	33·3	36·4	44·1	51·7	58·9
85·6	0·70	22·6	26·3	29·0	35·9	42·9	49·7
88·7	0·80	17·7	20·8	23·1	29·0	35·2	41·3
91·2	0·90	13·8	16·4	18·3	23·3	28·5	34·0
93·1	1·0	10·8	12·9	14·4	18·5	23·0	27·6
99·4	2·0	0·92	1·11	1·27	1·68	2·16	2·67
100	3·0	0·08	0·09	0·11	0·14	0·18	0·23
100	4·0	0·01	0·01	0·01	0·01	0·02	0·02

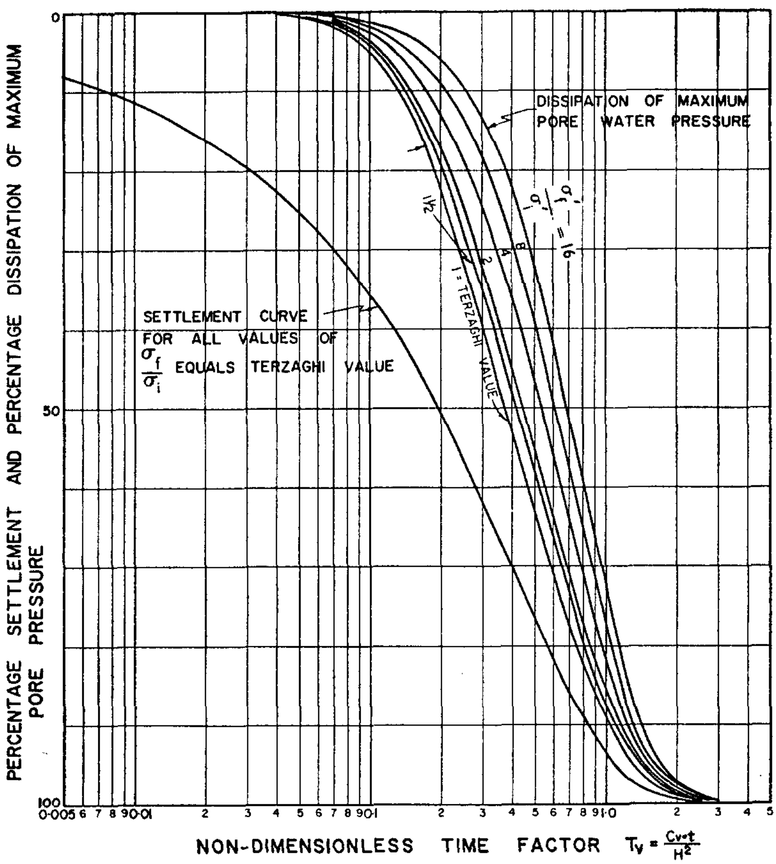

Fig. 1. Solution for percentage settlement and percentage dissipation of maximum pore pressure

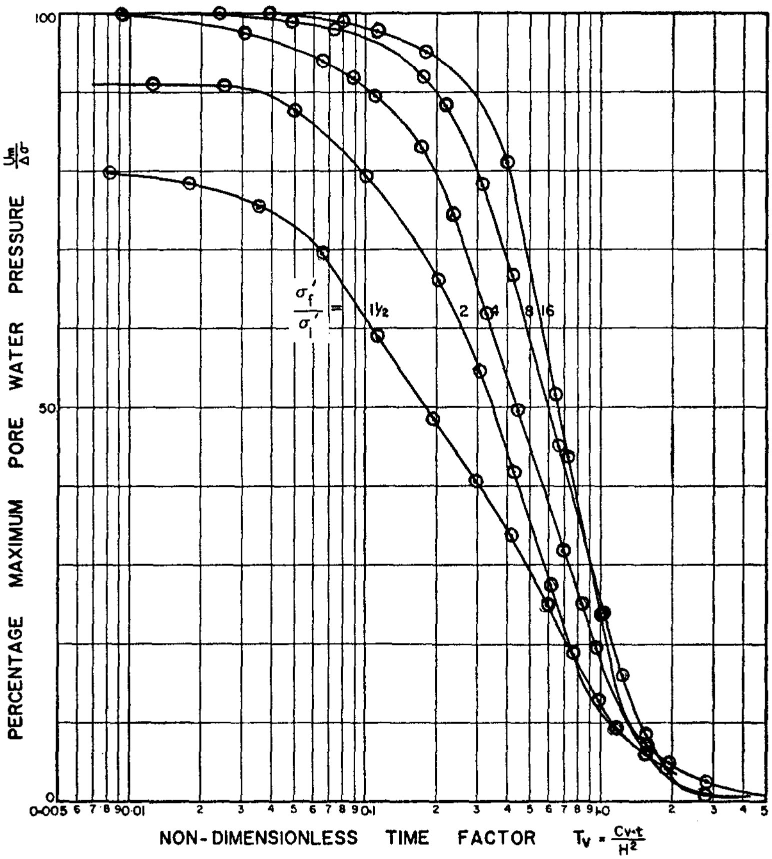

Fig. 2. Remoulded Port Kembla clay. Percentage maximum pore-water pressure *versus* log time factor

EXPERIMENTAL PROCEDURE AND APPARATUS

A Casagrande-type fixed ring oedometer suitable for permeability measurements was modified so that metal-to-metal contact of ring and base was obtained. The ring portion was then fixed to the base with a hard drying steam pipe jointing compound. (Stag brand joining compound made by Imperial Chemical Industries, England.) The ring size was 3·21 in. internal diameter.

The oedometer was then immersed in a basin full of de-aired distilled water. The inside of the ring was carefully greased with high vacuum silicone grease so that no air adhered to it. The bottom porous stone was then placed in position and one layer of Whatman's 54 filter paper was used to prevent clogging of the stone.

With the oedometer still in water a soil specimen was pushed out of a cutting ring into the oedometer. The pore-pressure gauge was next connected to the base of the oedometer which was only then removed from the basin.

As a special precaution against trapped air, a special top was placed on the oedometer and the specimen and surrounding water subjected to an overnight pressure of 100 lb/sq. in.

The oedometer pot was then placed in position in an oedometer press. Several presses were tried but the Bishop-type press was the only one which appeared to be sufficiently friction-free in its moving parts. For this reason all recorded experimental work was confined to the Bishop-type press.

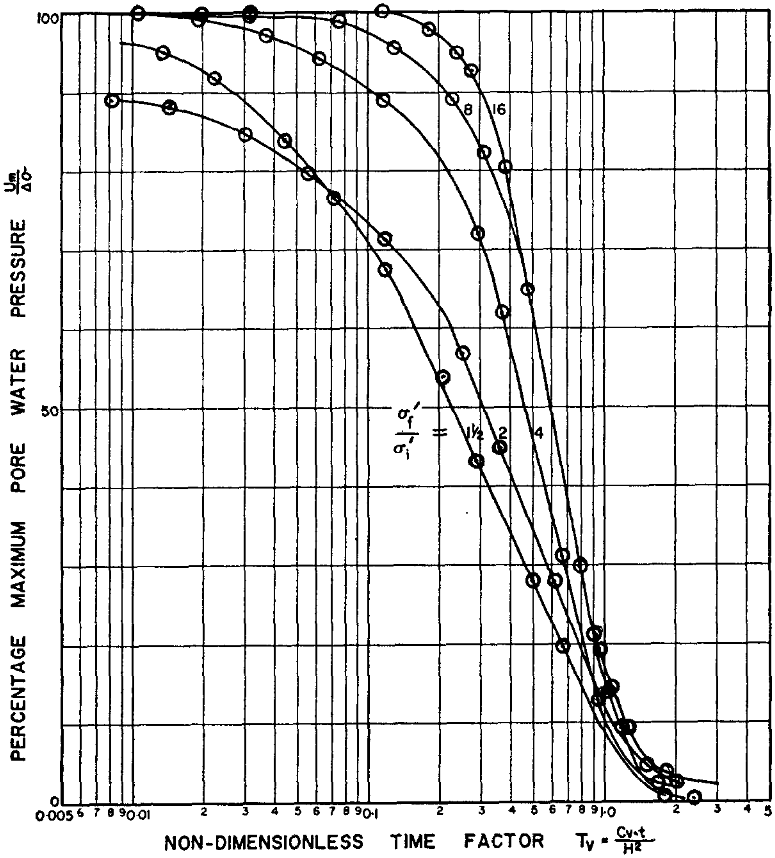

Fig. 3. Remoulded Mascot clay. Percentage maximum pore-water pressure *versus* log time factor

Owing to the large pressure increments used in some of the tests it was essential to have close fitting porous stones. These were manufactured to $\frac{10 \pm 2}{1000}$ in. in diameter less than the oedometer ring.

All tests where tilting occurred caused jamming of the stones in the ring and were rejected.

A sufficiently high pressure was applied to the soil so that the void ratio would plot on the straight portion of the $e - \log \sigma$ curve.

Tests were carried out on three remoulded New South Wales clays for a series of different ratios of final pressure to initial pressure. The results of these tests plotted graphically in Figs 2, 3, and 4 are also tabulated in Tables 2, 3, and 4, together with index and grading properties. The end of primary consolidation was determined by Casagrande's method using a log time graph.

DISCUSSION

The main factors affecting the measurements of settlement and pore-water pressure in oedometer testing are considered to be; permeability of porous stones; side friction; temperature; vibration; compressibility of pore-pressure gauge, and electro-chemical differences in pore fluid and fluid in pore-pressure gauge.

Discussion and investigation of the above factors have been carried out by Taylor (1942), Zan Zelst (1948), Newland and Alley (1960), Hansbo (1960), Leonards and Girault (1961), Whitman, Richardson and Healy (1961), Lo (1961), and many others.

The rate of settlement was found to be affected by the permeability of the porous stone. Similar results to those recorded by Newland and Alley were obtained with old stones, and for this reason new stones were used and one layer of filter paper was used between the stone and the soil.

Very little is known about the effects of side friction, particularly during the consolidation process. It is also questionable whether the type of oedometer used will affect the friction in different ways; for example, will the friction vary in the same manner for tests performed during this investigation with the drainage face moving and the impermeable boundary stationary as for tests with the impermeable boundary moving and the drainage face stationary? Similarly, are the effects of friction the same in a fixed ring consolidation test with drainage on both boundaries? Although during this investigation no measurements were made of the reduction in load at the base due to side friction, consideration of the possible effects of such friction is desirable in discussing the test results.

Table 2. Results from remoulded Port Kembla clay

σ_f'/σ^1 =	16		4		1½		8		2	
S%	T_v	u%	T_v	u%	T_v	u%	T_v	u%	T_v	u%
0	0	100	0		0		0		0	
10	0·009	100	0·008	100	0·008	79·7	0·007	100	0·007	91·0
20	0·033	100	0·0318	97·5	0·030	76·5	0·027	99·8	0·036	90·0
30	0·072	99·2	0·0705	93·5	0·064	69·7	0·064	98·5	0·069	84·2
40	0·110	98·0	0·125	88·0	0·119	58·2	0·120	96·0	0·125	76.0
50	0·196	94·7	0·196	80·5	0·196	48·2	0·196	90·5	0·196	67·4
60	0·286	90·3	0·282	68·0	0·305	40·2	0·286	82·0	0·286	57·3
70	0·399	82·0	0·411	54·0	0·440	32·7	0·400	69·0	0·400	45·0
80	0·560	61·5	0·585	39·0	0·595	24·7	0·574	53·5	0·555	31·4
90	0·770	40·5	0·862	23·5	0·805	17·5	0·811	36·0	0·785	17·7
95	0·970	27·0	1·19	12·0	0·970	12·9	1·05	23·0	1·01	10·9
97·5	1·14	15·5	1·47	7·0	1·06	11·1	1·22	16·0	1·20	8·65
100	1·52	7·0	1·91	3·0	1·15	9·55	1·48	10·0	1·44	5·46
102·5	2·12	4·0	2·68	1·0	1·32	7·6	1·96	3·5	1·80	3·64
105	4·52	0·5	5·36	0·0	1·50	6·2	3·11	0·3	2·34	1·82
110					2·10	3·2			4·60	0·91
120					4·10					
σ_i'	1·7		27·2		108·8		6·8		54·4 lb/sq. in.	
σ_f'	27·2		108·8		163·2		54·4		108·8 lb/sq. in.	
u_{max}	25·5		81·6		43·3		47·6		49·5 lb/sq. in.	
Δe_P	0·762		0·354		0·0722		0·345		0·127	
H_{50}	0·785		0·617		0·551		0·910		0·788 in.	
t_{50}	148		99		89		240		235 min	
$c_v \times 10^4$	8·08		7·56		6·70		6·78		5·18 in²/min	
H_s	0·277						0·376 inches			

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Grain size, mm:	Percentage smaller than:
0·2	100
0·08	98
0·04	96
0·01	86

2+

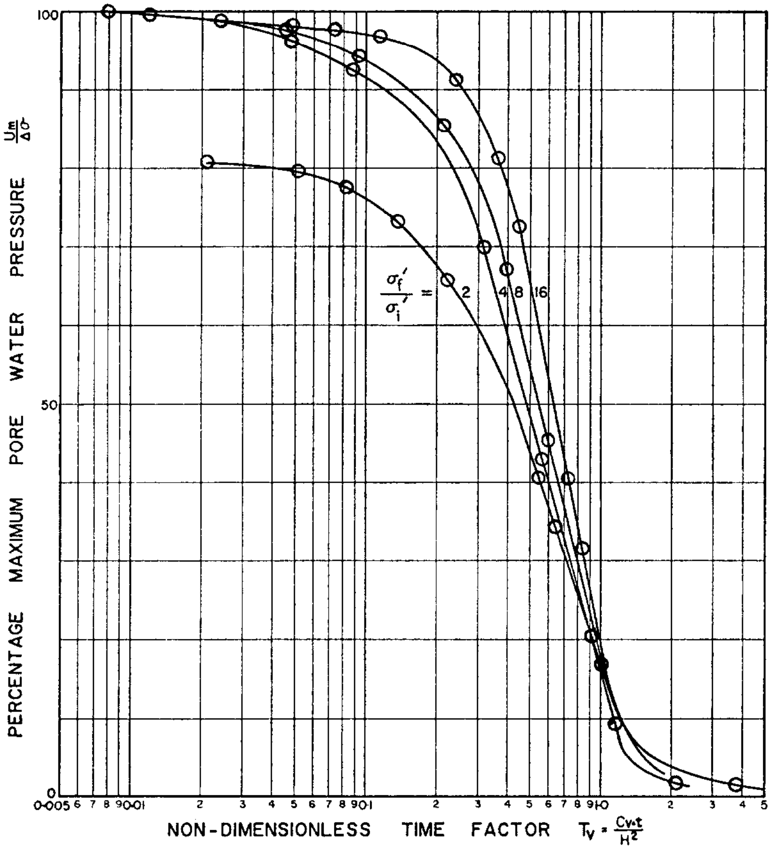

Fig. 4. Remoulded Hurstville clay. Percentage maximum pore-water pressure *versus* log time factor

Consider first that the friction increases suddenly as the load is applied and then decreases throughout the test, the final value of the frictional force being greater than that existing before the load was applied. Thus, if the pore-pressure results were plotted in terms of the total load increase on the base of the oedometer (which would be less than on the drainage face) the pore-pressure dissipation curves would yield slower rates of dissipation. A similar argument may be presented with the same conclusion for the case of friction increasing throughout the test.

A possible explanation of the deviation from Terzaghi's theory is an error in T_v. This error would be caused by an increase in friction during consolidation. Such an increase would reduce the load on the base of the oedometer and thus pore pressure by a greater amount than the percentage decrease in average void ratio which, according to Terzaghi's theory, is dependent on the integral with depth of the pore-pressure dissipation.

It may therefore be concluded that the only possible way in which friction could increase the pore pressures when plotted as a percentage of the applied load would be for the frictional force to be less under the new load than under the previous load. Making the extreme assumption that the whole of the previous load is taken by friction but that this friction reduces to and remains at zero on application of the new load, then the error involved in plotting the pore pressures as a percentage of the applied load would be:

$$\text{error} = \frac{\sigma_i'}{\sigma_f' - \sigma_i'} \times 100\% \qquad \qquad (35)$$

Even this highly improbable explanation does not account for the slow dissipation of pore-water pressure with load ratios of 8 and 16. Friction cannot explain the deviation of test results from Terzaghi's theory.

Temperature variation over the main part of the test was less than 2°F, this being recorded by a minimum and maximum thermometer. It is questionable whether the soil temperature varied by this amount. Unfortunately, little has been published on the effects of temperature, but it is believed that these low temperature variations would not explain the very much slower dissipation results recorded with large load ratios.

Table 3. Results from remoulded Mascot clay

$\sigma_f'/\sigma_i' = 8$			2		16		4		$1\frac{1}{2}$	
$S\%$	T_v	$u\%$	T_v	$u\%$	T_v	$u\%$	T_v	$u\%$	T_v	$u\%$
0	0		0		0		0		0	
10	0·009	100	0·009	89·0	0·008	100	0·0081	100	0·009	96·5
20	0·035	99·5	0·033	84·0	0·034	100	0·0317	98·0	0·037	86·5
30	0·076	99·0	0·072	77·2	0·071	100	0·0709	93·5	0·076	76·0
40	0·126	96·0	0·122	71·0	0·129	100	0·124	88·7	0·132	66·0
50	0·196	91·5	0·196	63·5	0·196	97·5	0·196	82·5	0·196	54·3
60	0·292	84·5	0·283	53·0	0·280	92·5	0·274	74·5	0·283	45·0
70	0·400	74·0	0·390	43·2	0·382	81·5	0·385	61·0	0·410	33·0
80	0·529	58·5	0·530	32·5	0·522	60·0	0·548	41·5	0·587	22·3
90	0·742	35·0	0·740	22·1	0·712	37·5	0·771	23·5	0·835	14·0
95	0·950	20·0	0·920	15·0	0·906	20·5	1·06	9·0	1·06	8·0
97·5	1·13	12·5	1·08	10·0	1·08	12·5	1·35	4·2	1·26	4·5
100	1·46	6·0	1·30	6·5	1·40	4·0	1·87	2·0	1·53	2·0
102·5	2·80	2·0	1·68	3·8	4·04	0	4·22	0·5	1·88	0·5
105			2·15	2·0					2·36	
110			4·32						4·25	
σ_i'	3·4		27·2		1·7		27·2		108·8 lb/sq. in.	
σ_f'	27·2		54·4		27·2		108·8		163·2 lb/sq. in.	
u_{max}	23·8		24·2		25·5		81·6		53·0 lb/sq. in.	
Δe_p	0·222		0·0592		0·2695		0·1272		0·0345	
H_{50}	0·8805		0·8104		0·9740		0·8723		0·8306 in.	
t_{50}	310		197		365		158		139 min	
$c_v \times 10^4$	4·92		6·56		5·11		9·59		9·75 in²/min	
H_s	0·489				0·479				inches	

LL 52 PL 21 PI 31 Activity = 0·70

Grain size, mm:	Percentage smaller than:
0·2	99
0·08	98
0·05	91
0·03	79
0·01	64
0·002	44

Care was taken to see that the oedometers were not vibrated during tests.

Whitman *et al.* (1961) have shown that the effect of a compressible pore-water gauge increases the rate of dissipation of pore-water pressure, thus eliminating this as a possible explanation.

In regard to the electro-chemical differences in the pore fluid and pore-pressure gauge, this was minimized by remoulding the soil with distilled water and by allowing the specimen to consolidate for one week before conducting a test.

Finally, in regard to the validity of the theory, it should be noted that equation (2) is an empirical law and is only valid over a certain range. For the three soils tested this range extended beyond 16 tons/sq. ft. However, soils deviate from the law at much lower values of vertical stress. The law of course cannot be valid for very high stress because, knowing the compression index, initial void ratio, and stress, it is possible to calculate the imaginary load above which the void ratio becomes negative.

CONCLUSIONS

A non-linear theory of consolidation has been developed for an ideal normally consolidated soil.

Table 4. Results from remoulded Hurstville clay

	$\sigma'_f/\sigma'_i = 8$		4		16		2	
$S\%$	T_v	$u\%$	T_v	$u\%$	T_v	$u\%$	T_v	$u\%$
0	0		0		0		0	
10	0·008	100	0·009	100	0·008	100	0·005	
20	0·032	98·5	0·033	98·0	0·031	98·5	0·021	80·5
30	0·070	96·0	0·070	94·0	0·071	97·5	0·055	79·3
40	0·125	91·7	0·126	89·5	0·125	96·5	0·120	74·5
50	0·196	86·8	0·196	84·0	0·196	94·0	0·196	68·0
60	0·270	81·0	0·278	75·0	0·287	88·0	0·300	60·0
70	0·380	69·5	0·390	60·5	0·405	78·0	0·437	49·0
80	0·530	52·0	0·540	45·5	0·545	60·5	0·633	35·2
90	0·745	33·5	0·895	21·5	0·740	39·5	0·980	20·0
95	0·970	19·0	1·22	6·5	0·880	29·0	1·34	12·0
97·5	1·23	9·5	1·47	3·5	1·01	18·0	1·61	9·0
100	2·00	3·8	2·14	1·5	1·35	7·0	2·10	6·0
102·5	13·8	0	7·35	0·3	10·8	0	4·10	2·5
105							12·0	0
σ'_i	3·4		27·2		3·4		54·4 lb/sq. in.	
σ'_f	27·3		108·8		54·4		108·8 lb/sq. in.	
u_{max}	23·8		81·6		51·0		43·8 lb/sq. in.	
Δe_p	0·2325		0·1668		0·337		0·0763	
H_{50}	0·974		0·8696		0·937		0·8283 in.	
t_{50}	128		67		146		72 min	
$c_v \times 10^4$	14·56		22·17		11·82		18·55 in²/min	
H_s	0·509				0·498 inches			

LL 60 PL 20 PI 40 Activity = 0·78

Grain size, mm:	Per cent smaller than:
0·2	99
0·1	98
0·04	89
0·01	71
0·002	51

Within experimental accuracy the results are in agreement with the theory for three remoulded normally consolidated soils of different engineering properties. For high ratios of final to initial effective pressure, the pore pressure in a normally consolidated soil can be expected to be considerably higher at any particular time than that predicted by the Terzaghi theory. On the other hand, the degree of settlement can be expected to be largely independent of the ratio of final to initial effective pressure and to be that predicted by the Terzaghi theory.

Further work is required to investigate whether the theory is valid for overconsolidated soils and for soils subjected to small load increments. However, further research on side friction is needed before these can be satisfactorily investigated.

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