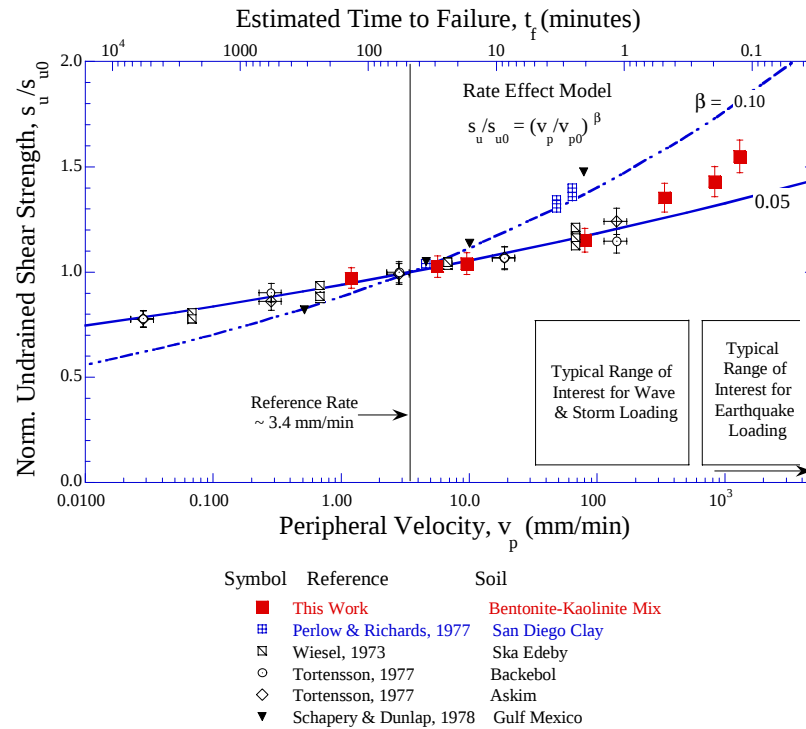




# INFLUENCE OF PERIPHERAL VELOCITY ON UNDRAINED SHEAR STRENGTH AND DEFORMABILITY CHARACTERISTICS OF A BENTONITE-KAOLINITE MIXTURE

by

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## **INFLUENCE OF PERIPHERAL VELOCITY ON MEASUREMENTS OF UNDRAINED SHEAR STRENGTH FOR AN ARTIFICIAL SOIL**

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### ***ABSTRACT***

The rate of rotation is among the most important factors affecting the measurements of undrained strength. In particular, for seismic or fast wave loading conditions, the shear rate is much higher than that used in many common laboratory tests, let alone standard procedures for field tests. The testing program described here evaluates the effect of peripheral velocity on the undrained strength and deformability characteristics inferred from the shear vane test. The peripheral velocities used in this study correspond to rotation rates ranging from approximately 2°/min to 3000 °/min. The study was conducted on a slightly cemented bentonite-kaolinite soil mixture manufactured in the laboratory that possesses many characteristics similar to those of natural materials. Results show that the shear strength increases with increasing peripheral velocity and is similar to that reported for many soils in the literature while the residual shear strength seems to be nearly independent of rotation rate. The ‘torque-rotation’ curves obtained from the shear vane test can be used to estimate qualitatively the value of the secant shear modulus as a function of the shear deformation. The stiffness at small strains is nearly constant, regardless of rotation rate, while at higher angles of rotation the stiffness increases with increasing peripheral velocity but these changes are, by and large, controlled by the increases in the undrained strength.

## INTRODUCTION

The problem of relating undrained shear strength measured in the laboratory to that measured in the field and, ultimately, to the actual resistance of the soil has been a permanent concern in geotechnical engineering. Among all the insitu techniques available today, the field vane is probably the most widely used method to estimate the undrained strength of soft clays. Its use follows the original conception and development in Sweden in late 1919, and initial intense research work by Cadling and Odenstad (1948), Carlsson (1948) and Skempton (1948). Flodin and Broms (1981) have presented a detailed history of its use since 1950.

Field vane shear testing are widely used for their simplicity, speed and relative low cost, which allow the gathering of extensive information during a site investigation program. Nevertheless, the results of these tests are affected by many factors such as rate of rotation of the vane, setup time (i.e., time elapsed between the insertion of the vane and the beginning of the test), shape and aspect ratio of the vane blade, drainage conditions, disturbance effects and non-uniform stress distribution leading to progressive failure and strength anisotropy. There have been many studies to evaluate the significance of these factors (e.g., Aas, 1965; Flaate, 1966; Wiesel, 1973; Arman et al., 1975; Menzies and Mailey, 1976; Torstensson, 1977; Menzies and Merrifield, 1980; Roy and LeBlanc, 1988). Chandler (1988) presented a comprehensive summary and discussion of these elements and their influence in the interpretation of the vane test results, which is still valid today. Some of these issues have been, to some extent, resolved by standardization of the technique. Currently, the standard field vane is rectangular with a diameter of approximately 65mm (or 55mm), an aspect ratio of height to diameter,  $H/D$ , of 2, a 1.95mm thick blade and an area ratio less than 12% in order to minimize disturbance. In addition, the rate of loading is generally chosen as  $0.1^\circ/\text{sec}$  (i.e.,  $6^\circ/\text{min}$ ) with minimal delay times after vane insertion ranging from 1 minute to less than 5 minutes. These specifications, with minor variations, are common to many standards for Field Vane Testing including the British, Swedish, Norwegian, European, Indian and ASTM among others (Lunne, 1999). Additional precautions, such as minimization- elimination or the measurement of rod friction render the new estimates of shear strength more consistent and reliable.

The rate of rotation is among the most important factors affecting the measurements of undrained strength (e.g., Leroueil and Marques, 1996) and its relevance was recognized early in the development of the vane shear tests. (i.e., Cadling and Odenstad, 1950). Historically, evaluation of rate effects for vane testing has been directed to estimate the undrained shear strength for 'static' problems where the straining rate is much slower than that of conventional testing. Based on numerous cases studies, Bjerrum proposed a reduction factor for  $s_u$  measured with the vane to account for the longer time to failure in the field, which has been typically assumed to occur within 7 days (i.e., 10,000 minutes) (e.g., Bjerrum, 1972; Torstensson, 1977; Chandler, 1988). The original correction was a function of plasticity index, recognizing that more plastic soils will exhibit a higher rate dependent behavior. More recently, additional considerations such as stress history (i.e., OCR related to  $s_u/\sigma_{vo}$ ) and ageing conditions (i.e., young vs. aged) have been incorporated in the interpretation (e.g., Aas et al., 1986).

For seismic or wave loading conditions, on the other hand, the shear rate is much faster than that in many common laboratory tests, let alone standard procedures for field tests. In particular, the evaluation of the response of laterally loaded pile foundations in soft clays under cyclic and seismic excitation requires an accurate assessment of both the stiffness and the undrained strength of the soil at higher rates of loading than that for which the traditional field vane correction was developed. The testing program described here was developed in conjunction

with a coordinated testing and analytical effort to evaluate seismic soil-pile-superstructure interaction of single piles and pile groups in soft clays on the large shaking table at Richmond Field Station (i.e., Meymand, 1998; Lok, 1999).

### ***Time Effects- Effect of Peripheral Velocity***

It is generally recognized that an increase in the rate of shear results in an increase in undrained shear strength,  $s_u$ . This observation is supported by a large database from several test types, other than the shear vane test, such as cone penetration tests (e.g., Powell and Quaternmann, 1988) and standard triaxial tests (e.g., Kulhawy and Mayne, 1990). The dependence between the two has been traditionally interpreted according to a power or a logarithmic law in terms of the angular rotation rate,  $\dot{w}$ , or time to failure,  $t_f$ :

$$s_u / s_{u0} = 1 + \alpha \cdot \log(\dot{w} / \dot{w}_0) \approx 1 + \alpha \cdot \log(t_{f0} / t_f) \quad ; \text{ semi-logarithmic law} \quad (1a)$$

$$s_u / s_{u0} = (\dot{w} / \dot{w}_0)^\beta \approx (t_{f0} / t_f)^\beta \quad ; \text{ power law} \quad (1b)$$

where  $s_{u0}$  is the undrained strength corresponding to the reference time to failure,  $t_{f0}$ , or the reference angular rotation rate,  $\dot{w}_0$ , and  $\alpha$  and  $\beta$  are soil dependent material parameters. For uncemented and lightly cemented silts and clays, several researchers report values of  $\alpha$  ranging from 1 and 2% (e.g., Aas, 1965, Roy and LeBlanc, 1988) up to a maximum of 20-30% for 1 log cycle increase in angular rotation rates ( $^\circ/\text{min}$ ) (e.g., Smith and Richards, 1975, Perlow and Richards, 1977). In general, the lower values are reported for disturbed specimens and laboratory test programs using miniature vanes. In particular, the latter may result in ambiguous interpretations due to partial drainage effects as will be discussed latter. Higher values tend to be representative of cemented or carbonatic materials (e.g., Perlow and Richards, 1977) and excellent quality "undisturbed" specimens. Although there has been a large research effort in this area over the last 40-50 years, direct comparison is difficult because of the difference in test type (i.e., field vs. laboratory), vane sizes, shapes and the setup time (ranging from a few minutes to 24 hr after insertion, e.g., Torstensson, 1977) used among different testing programs. A summary of previous investigations is shown in Table 1 along with reported or computed values of parameters  $\alpha$  and  $\beta$  for the different soils.

Perlow and Richards (1977) were the first to recognize and document the significance of both vane size (i.e., diameter,  $D$ ) and rotation rate. They introduced a new parameter referred to as angular shear velocity, which is simply the velocity at the edge of the blade (i.e.,  $v_p = \dot{w} \cdot D/2$ ). The term was later corrected to the more appropriate peripheral velocity. Their work included extensive in situ and laboratory vane shear testing at three different sites. The field vane results corresponded to shallow offshore sediments (depth < 1-2m) while laboratory testing was performed on retrieved specimens up to a depth of 10 m. They concluded that peripheral velocity uniquely characterizes viscous effects and suggested the use of a standard rotation rate resulting in a 9 mm/minute peripheral velocity for all tests, since this rate typically guarantees undrained response for most soil types. For a 65mm vane this corresponds to approximately 16 $^\circ/\text{min}$  which is slightly higher than the typical range of 6 $^\circ/\text{min}$  to 12 $^\circ/\text{min}$ .

Most of the previous investigations used rates of rotation between 6 $^\circ/\text{min}$  and 90 $^\circ/\text{min}$  which resulted in peripheral velocities lower than 1mm/second. Vane sizes and shapes varied considerably and testing procedures or equipment not always allowed a constant rate, making more difficult to estimate these quantities. It must be stressed that a significant portion of the existing database is not internally consistent since researchers have used different measures, such

as time to failure (e.g., Torstensson, 1977) or reported results for the same rotation rate but used different vane diameters. In addition some of these results include the use of miniature vanes, which in some cases lead to partially drainage conditions, making the conclusions somewhat unclear (e.g., Blight, 1968, 1977). This situation has also been observed in cone penetration testing at low insertion rates (e.g., Campanella and Robertson, 1981).

The following sections describe a laboratory testing program at the University of California at Berkeley directed at assessing the stress-deformation response of a soft cohesive "model" soil from shear vane testing at angular rotation rates ranging from  $0.04^\circ/\text{sec}$  to  $48^\circ/\text{sec}$  ( $\sim 2$  to  $3000^\circ/\text{min}$ ). The rates of rotation were chosen to obtain the widest range of peripheral velocities ( $0.02$  and  $20 \text{ mm/sec}$ ) within the limitations of the testing setup.

## EXPERIMENTAL PROGRAM

### *Material*

"Model" (i.e., artificial) soils have been used extensively at the University of California at Berkeley since the late 60's to investigate soil behavior in combination to "static and dynamic" 1g scale modeling in the shaking table (e.g., Seed and Clough, 1963). Most recently, a large coordinated research program to evaluate the seismic soil-pile-superstructure interaction (Meymand, 1998; Lok, 1999) and seismic slope stability (Wartman, 1999) has been conducted using two small shaking tables in Davis Hall and the large shaking table at the Richmond Field Station. In these studies, a suitable mix was developed to simultaneously match the scaled undrained strength,  $s_u$ , and small strain shear stiffness,  $G_{\max}$ , for a representative profile of soft San Francisco Bay Mud. After numerous tests, the following mix was selected: 72% kaolinite, 24% bentonite and 4% type C fly ash. The mixture has a liquid limit of 115, plastic limit 40 and plasticity index of 75. The soil was mixed at a target water content of 130% with a corresponding unit weight is about  $14.8 \text{ kN/m}^3$  (94 pcf). The addition of fly ash to the mixture provided a slight cementation which increased the shear wave velocity (and thus the small strain shear stiffness  $G_{\max}$ ) from 16-18 m/sec to approximately 30 m/sec after a curing period of 5 days. There was no significant increase in the shear strength and increases in shear velocity were minimal after a period of 5-7 days. The target undrained shear strength was approximately 4.3 kPa ( $90 \pm 10$  psf) and a shear wave velocity of approximately 30-32 m/sec ( $\sim 100$ -105 ft/sec) after a curing period of 5 days, which was the typical time interval between model soil placement and testing or between successive tests (Wartman, 1996; Meymand, 1998).

### *Preparation and Testing*

The testing program in the laboratory was conducted parallel to the shaking table tests to minimize small differences in composition of the mix bound to vary from batch to batch, and in the curing period, both of which influence the properties of the model soil. The soil was carefully mixed and pumped to a flexible-walled container of 2.3m in diameter and approximately 2m in height placed on large shaking table as shown in figure 1a. During filling of the container, four 10 gallons plastic buckets were filled with model soil directly from the mixing batch. Three batches at nominal depths of 0.25m, 0.70m and 1.45m were used and a total of 12 containers were stored in the wet room for the curing period to ensure that the water content would be preserved. Water content determination of the soil placed was conducted as part of the characterization program and is shown in figure 1b. The measured water content was within 5-10% of the target for most of the vertical soil profile. Water content determination

performed for each batch yielded results which are representative of those in the container. The testing program had to be completed in less than a week following the 5 days of curing time. First a pot test was performed in the middle of each bucket (Lok, 1999) and then four vane shear tests were carried out in the remaining soil.

Since the scope of this study was to characterize the influence of “shearing rate” on  $s_u$ , careful attention was given to eliminate all possible causes for discrepancies among different tests. The soil was obviously as standardized and homogenous as it is realistically possible to obtain from such a large scale mixing procedure ( $\sim 6 \text{ m}^3$ ). In order to achieve the large range of peripheral velocities required for this study, the selection of the vane was a key process. A single standard field vane was used throughout the program, with 55mm in diameter, aspect ratio H/D of 2, blade thickness less than 2mm, with an area ratio less than 12%. The size of the bucket was also chosen to insure that the vane could penetrate for twice its height ( $\sim 4D$ ), still leaving suitable distance between the tip of the vane and the bottom of the bucket. During vane shear laboratory testing, consolidation after the initial insertion is often allowed to be completed before testing begins. This is not standard practice in the field and would prolong the testing program unnecessarily, thus only a couple of minutes were allowed between insertion and testing. The device consisted of a small variable speed electrical motor connected to a set of gears, which would further widen the range of rotation rates attainable. The torque was measured by strain gages attached to a thin bar of metal on the vane rod (cf., figure 2). The vane was inserted by slowly and carefully rising the bucket with a jack to minimize swinging and disturbance. Full detail of these tests are provided by Biscontin and Pestana (1999).

### **Results and Analysis**

The undrained shear strength can be determined from measurement of torque and assuming a prescribed shear stress profile along the potential cylindrical failure surface. The shear mechanism is relatively complex and the distribution of shear stresses in the periphery of the potential shear surface has been investigated theoretically through 3D finite element studies using an elastic (Donald et al., 1977) as well as elastoplastic with strain softening constitutive models (De Alencar et al., 1988) in order to capture the progressive failure mechanism. Menzies and Merrifield (1980) have found these analyses in good agreement with experimental measurements on a carefully instrumented vane. The most general expression for the undrained strength in the vertical plane is given by:

$$s_{uv} = \frac{2T}{x\pi D^3 \left( \frac{H}{D} + \frac{1}{(n+3)} \frac{s_{uh}}{s_{uv}} \right)} \quad (2)$$

where T is the maximum torque measured, H/D is the aspect ratio of the vane, D is the diameter of the vane,  $s_{uh}/s_{uv}$  is the ratio of the undrained strength in both vertical and horizontal planes describing anisotropy, x is a factor describing the location of the failure surface with respect to the diameter of the vane ( $\sim 1.05$ , Skempton 1948) and is typically considered as 1, and n is the power law describing the shear stress distribution on the horizontal planes (e.g., Donald et al., 1977). The later is taken as zero in many cases (e.g., full mobilization, ASTM standards) while careful measurement indicate n is approximately 4-5 (e.g., Menzies and Merrifield, 1980). This difference is expected to be small and will only introduce a consistent bias in the results but will not affect the conclusions regarding the rate of increase in shear strength at faster rates. In our case, we use a standard aspect ratio of 2 and the soil was placed (and not deposited) and thus

anisotropic conditions are expected to be negligible ( $s_{uh}/s_{uv} \sim 1$ ). The undrained strength can be estimated as:

$$s_u = \frac{6T}{7\pi.D^3} \sim 0.857 \frac{T}{\pi.D^3} \quad (3)$$

Maximum torque was determined as the highest value recorded while the “residual” strength was determined based on the lowest value in the record. ASTM Standard D4648 recommends five to ten fast rotations before repeating the test. The peripheral velocity achievable with our testing setup ranges from a low value of approximately 1 mm/min to an upper bound of 1500 mm/min. The standard peripheral velocity,  $v_{p0}$ , was chosen as approximately 3.4 mm/min corresponding to an angular rotation rate of  $6^\circ/\text{min}$  for a field vane of 65 mm. Equations 1a and 1b can be then written in terms of peripheral velocity:

$$s_u/s_{u0} = 1 + \alpha \cdot \log(v_p/v_{p0}) \quad ; \text{ semi-logarithmic law} \quad (4a)$$

$$s_u/s_{u0} = (v_p/v_{p0})^\beta \quad ; \text{ power law} \quad (4b)$$

Figure 3 shows measurements of peak and residual shear strength for the three batches of model soil. It can be clearly seen that the undrained shear strength increases significantly with increases in peripheral velocity and this effect becomes more pronounced as the velocity becomes closer to expected earthquake values. For the highest peripheral velocity ( $\sim 1500$  mm/min) the measured shear strength was in the order of 45 to 65% higher than that determined at the standard shearing rate. These results are in excellent agreement with Seed and Clough recommendation of an increase of 60% in undrained strength for a similar soil mixture for earthquake engineering analyses (Seed and Clough, 1963). The graphs also show, for reference, estimated changes in strength based on the power law expression with  $\beta$  of 0.05 which has been observed by other researchers (e.g., Wiesel, 1973; Torstensson, 1977). On the other hand, the residual strength has no statistical dependence on peripheral velocity for all three batches and can be considered, for most practical purposes, independent of peripheral velocity (or rotation rate). In contrast, Skempton (1948) reports increases in the remoulded strength of about 1.5-2% for approximate rotation rates of  $0.1^\circ/\text{sec}$  to  $1^\circ/\text{sec}$ .

Batch I shows the highest scatter in the measured values of strength which has been associated with initial operation of the mixing equipment and small differences in the fraction of the constituents (Meymand, 1998; Lok, 1999). The vertical profile of water content (cf., figure 1) also shows the highest variation in the initial state in the first 40-50 cm of placed soil. This is corroborated further by the fact that batch I having a higher initial water content than batches II and III, has an undrained strength about 10% higher than the others. In contrast, Wartman (1996) shows that for a given soil mixture, the undrained shear strength and shear wave velocity decreases with increasing water content. The remainder of the paper will concentrate primarily on the results of batches II and III and treat them as the same soil.

Figure 4 shows a summary of normalized undrained strengths as a function of peripheral velocity for bentonite-kaolinite mixture used in this study. The figure also shows the predicted changes in undrained strength by the two laws described earlier. The data used for the regression analysis only included data for equivalent rotation rates less than  $700^\circ/\text{min}$ , corresponding to the fastest rate reported in the literature to date. This was done to evaluate the predictive capabilities for rotation rates corresponding to those estimated using numerical analysis for earthquake type loading. The "rounded-off" values for parameters  $\alpha$  and  $\beta$  corresponded to 0.10 and 0.055 respectively. It can be seen that both the power law and the semi-logarithmic law give practically identical predictions with excellent description of the measured "average" strength in

the range of 1 to 100 mm/min, but underpredict the increase in strength for higher values of peripheral velocity. In particular, the power law gives a closer description over the complete range of peripheral velocity underestimating the shear strength at 1400 mm/min by only 15%. The figure shows, for reference, the estimated range in shearing rate for wave/storm loading (used in offshore applications) and earthquake loading based on extensive numerical simulations of seismic pile foundation performance.

Figure 5 shows a summary of existing data documenting changes in undrained shear strength for soft soils as a function of peripheral velocity. A significant effort was dedicated to correctly summarize previous results in terms of peripheral velocity, as appropriate. For the sake of clarity, experimental data that may have included partial drainage effects, excessive disturbance or very limited range in shear rate were omitted. It can be seen that most existing data fall within the limits established by a power law with  $\beta = 0.05$  and  $0.10$  over a wide range of shearing rates ( $\sim 5$  orders of magnitude). In particular, the power law with  $\beta = 0.05$  predicts a decrease in undrained strength of 25 to 30% ( $s_u/s_{u0} = 0.70-0.75$ ) for the lowest rates of shearing corresponding (approximately) to a time to failure of 10,000 minutes which is in good agreement with correction values by Bjerrum (1972).

### ***Partial Drainage Effects***

Figure 6 shows the values of angular velocity as a function of vane diameter required to achieve a standard peripheral velocity of 3.4 mm/min ( $\sim 0.057$  mm/sec). The figure also shows the recommended relationship by Perlow and Richards (1977) for a standard peripheral velocity of 9 mm/min (0.15 mm/sec). The faster rate was proposed with the objective of achieving and maintaining undrained response during testing for most soil types. Nevertheless, most of the correlations, including Bjerrum's correction for field vane results are based on a large database obtained at the conventional rate of 6°/min. Blight (1968) proposed a practical criterion to verify that the degree of drainage is less than 10%:

$$T_f = c_v \cdot t_f / D^2 < 0.02 - 0.04 \quad \text{for essentially undrained conditions} \quad (5)$$

where  $T_f$  is the time factor at failure,  $t_f$  is the time to failure and  $c_v$  is the coefficient of consolidation. The problem of partial drainage arises especially in miniature vane tests, where  $D$  is small and the drainage path becomes short, allowing consolidation to occur. For field vanes the diameter is such that for the typical ranges of coefficient of consolidation and rotation rates the tests are always undrained. For the model soil used in this work, the  $c_v$  was estimated to be approximately  $1.8 \times 10^{-5}$  m<sup>2</sup>/day (10 in<sup>2</sup>/yr) which ensures that tests are undrained for a time to failure lower than 80 hours (Wartman, 1996). This criterion is illustrated in figure 7, where the rate of peripheral velocity required to achieve a given time factor is related to the coefficient of consolidation of the soil, and is referred to as critical peripheral velocity. The figure contains two sets of curves for a typical field vane (i.e.,  $D = 65$ mm) and the laboratory miniature vane (i.e.,  $D = 12.7$ mm) and time factors of 0.02 and 0.04. These curves were obtained assuming an average angle of rotation at failure of 5° which has been reported by many authors and seems to be conservative in this particular case. When the critical peripheral velocity exceeds the standard velocity of 3.4 mm, partial drainage will affect the results of shear strength obtained with the vane apparatus. This will occur for coefficients of consolidation higher than  $7 \times 10^{-4}$  cm<sup>2</sup>/sec for miniature vanes and  $3 \times 10^{-3}$  cm<sup>2</sup>/sec for field vane tests. These limits will correspond (approximately) to liquid limits of 70 and 35-40% respectively for normally consolidated soils based on accepted correlations between this index property and  $c_v$ . As can be seen from the

figure, increases in peripheral velocity to 9 mm/min, as suggested by Perlow and Richards (1977), will only produce marginal benefits and it is not warranted.

### ***Deformability of Soil- Estimation of Shear Modulus.***

Examples of the measured torque,  $T$ , versus angle of rotation,  $w$ , for the full range of peripheral velocities are shown in figure 8. In general, peak condition of shear stress was reached for all tests between  $3.5^\circ$  to  $5^\circ$  with a very slight increase in the angle of rotation at failure with increases in peripheral velocity. Time to failure was proportional to these values, approximately between 0.1 seconds and 2 minutes. After peak the "stress-strain" curve shows an increase in strain softening response since the residual shear stress was approximately constant in all cases.

Several researchers have been advocating the use of torque vs. angle of rotation curves for estimating the shear modulus of the soil (Cadling and Odenstad, 1950; Madhav and Krishna, 1977; Selvadurai, 1979; Pamukcu and Suhayda, 1988). A detailed discussion of the different methods used is presented in appendix A. In general, the equivalent "secant" shear modulus,  $G_w$ , can be estimated as:

$$G = \frac{T / (\pi \cdot D \cdot H)}{m \cdot w \cdot D} \quad (6)$$

where  $w$  is the angle of rotation in radians and  $m$  is a coefficient function of the vane shape. Figure 9 compares predicted values of  $m$  using rectangular vanes (i.e., Madhav and Krishna, 1977) and elliptical vanes (i.e., Selvadurai, 1979). As can be seen,  $m$  decreases from a value of 0.95-1 for  $H/D=1$  to a value of 0.80-85 for the common aspect ratio of  $H/D=2$ . The figure also shows the empirically estimated value originally proposed by Cadling and Odenstad (1950) for the field vane and the typical range of interest for vanes used both in the lab and in the field. Based on these results a value of 0.80 was used in our analyses.

Figure 10a shows the equivalent "secant" shear modulus computed for batch II as a function of the angular rotation rate,  $w$ . Similar results were obtained for batch III. At small angles of rotation, the estimated shear modulus was approximately constant with values ranging from 160 to 170 kPa, which is much lower than the values estimated for the small strain shear modulus. Measurements of shear wave velocity using bender elements and estimated from the vertical array in the soil profile in the shaking table (cf., figure 1) gave very consistent results ranging from 28-30 m/sec ( $\sim 95$ -100 ft/sec) for batches II and III. These values of  $v_s$  correspond to values of the small strain shear modulus,  $G_{max}$ , in the order of 1400-1500 kPa. While the soil used in this study is primarily a mixture of bentonite and kaolinite, a small fraction of "reactive" fly ash was added in order to increase the shear wave velocity (with minimal increases in shear strength) and thus achieve the correct 1-g scaling of relevant soil properties for the shaking table (cf., Meymand, 1998). The addition of fly ash results in a slight cementation in the soil which increases the shear wave velocity from about 16.5-18 m/sec (i.e.,  $G_{max} \sim 500$  kPa, no fly ash added) to approximately 30 m/sec achieved after 5 days of curing time. The low value of shear modulus obtained can be partially explained by the disturbance (i.e., complete destruction of the cementation in the vicinity of the blades) caused by the insertion of the vane, corresponding to the lowest value of  $G_{max}$  of approximately 500 kPa, and by the inherent difficulty of measuring small angles of rotation (system compliance), particularly at the higher rates of rotation. Pamukcu and Suhayda (1988) used a similar expression (cf., equation 6,  $m=1$ ) for a miniature laboratory shear vane with  $H/D=1$  and report good correlation of their measurements with estimation of the small strain shear modulus,  $G_{max}$ . They used a laser to obtain very precise

rotation measurements but their technique required small rotation rates and therefore it was not applicable for the range of rates used in this study.

For larger angles of rotation ( $w > 1^\circ$ ), on the other hand, it can be clearly seen that the equivalent shear modulus increases with increases in the values of peripheral velocity. Figure 10b shows the equivalent secant shear modulus normalized by the undrained shear strength corresponding to the respective value of  $v_p$ . It is observed that for larger angles of rotation, the normalized modulus,  $G_w/s_u$ , is independent of the peripheral velocity and increases in secant stiffness are, by and large, accounted for by increases in the undrained strength. Since the secant stiffness at small strains is nearly constant, values of  $G_w/s_u$  decrease monotonically with increases in peripheral velocity.

## CONCLUSIONS

The peripheral velocity in the vane test has a significant influence on measurements of undrained strength,  $s_u$ , especially at very high shearing rates representative to fast storm-wave loading and earthquake loading. For peripheral velocities less than 500-700 mm/min, the increase in  $s_u$  can be described by either a power law or semi-logarithmic law in terms on the peripheral velocity. For the artificial clay used in this study, the increase in undrained strength per log cycle is approximately 10% ( $\alpha \sim 0.10$ ,  $\beta \sim 0.055$ ) and is similar to that reported for many soils in the literature. For values of peripheral velocity higher than 600 mm/min, the undrained strength increases faster than that predicted by either model. The power law underestimates the change by 10-20% for peripheral velocities of 1400 mm/minute, but gives a slightly better fit over the entire range. In contrast, the residual shear strength appears to be nearly independent of shearing rate. More testing in the same range of rotation rates is required for natural materials in order to extend the validity of these observations.

The 'torque-rotation' curves obtained from the shear vane test can be used to estimate qualitatively the value of the secant shear modulus as a function of the shear deformation. The stiffness at small strains is nearly constant, regardless of rotation rate, and it appears to represent the remoulded state of the soil after the insertion of the shear vane. As a result, the corresponding shear stiffness at small strains can be significantly lower than that estimated from shear wave velocity measurements, especially for cemented materials. At higher angles of rotation the stiffness increases with increasing peripheral velocity but the normalized modulus,  $G_w/s_u$ , is nearly independent of the peripheral velocity. This observation suggests that changes in the secant shear stiffness are, by and large, controlled by the increases in the undrained strength (i.e., peak shear conditions).

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## NOTATION

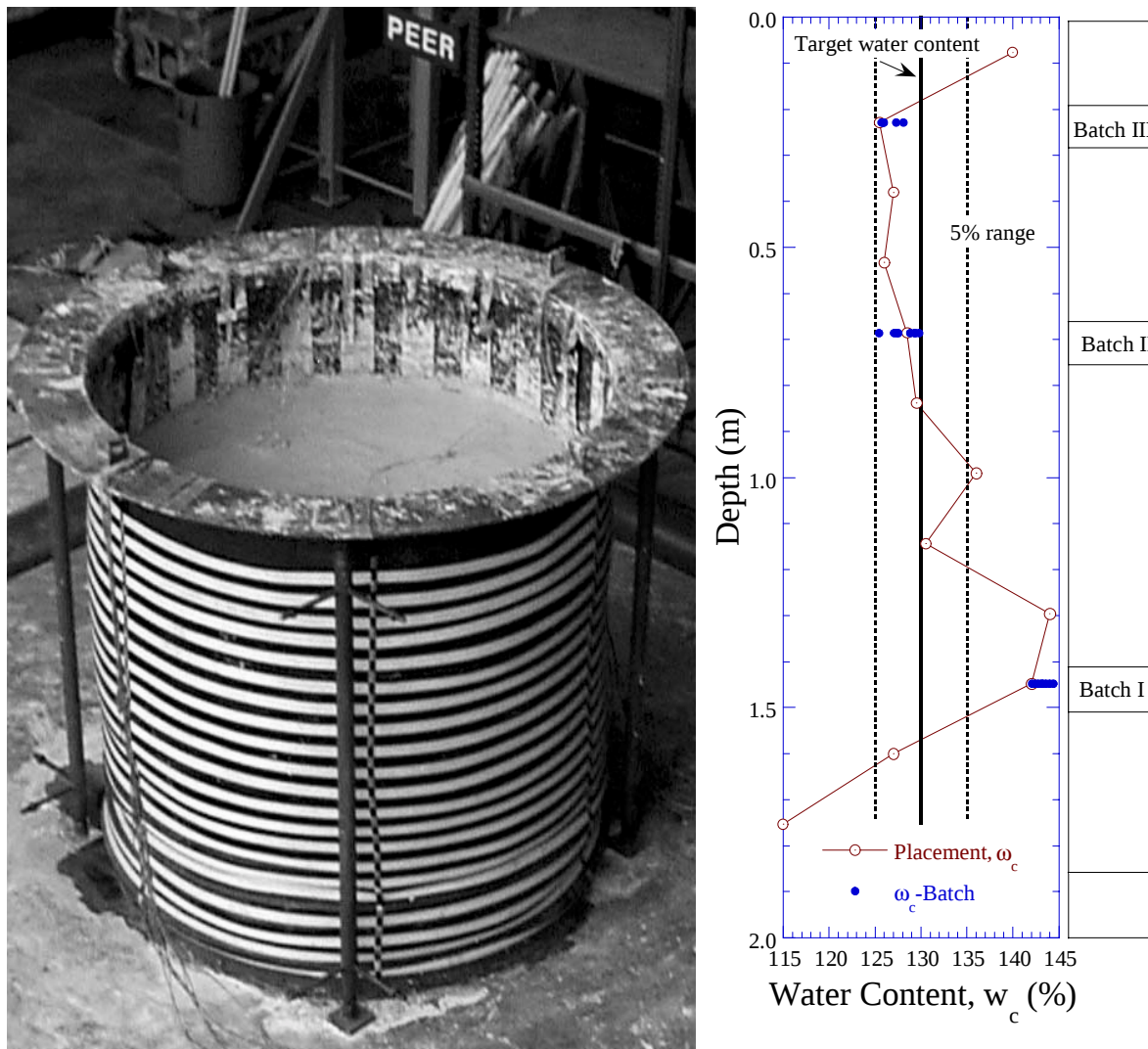
|                  |                                                                          |
|------------------|--------------------------------------------------------------------------|
| $c_v$            | coefficient of consolidation;                                            |
| $D$              | diameter of the vane;                                                    |
| $H$              | height of the vane;                                                      |
| $G_{\max}$       | small strain shear modulus;                                              |
| $G_w$            | equivalent shear modulus based on torque-rotation measurements;          |
| $s_u$            | undrained strength;                                                      |
| $s_{uv}, s_{uh}$ | undrained strength in vertical and horizontal planes, respectively;      |
| $T$              | measured torque;                                                         |
| $t_f, T_f$       | time to failure and Time factor at failure, respectively;                |
| $v_p, v_{p0}$    | peripheral velocity and reference peripheral velocity, respectively;     |
| $v_s$            | shear wave velocity                                                      |
| $w, w_0$         | angular rotation rate and reference angular rotation rate, respectively; |
| $\alpha, \beta$  | constants for rate dependent models;                                     |
| $\rho$           | soil density.                                                            |

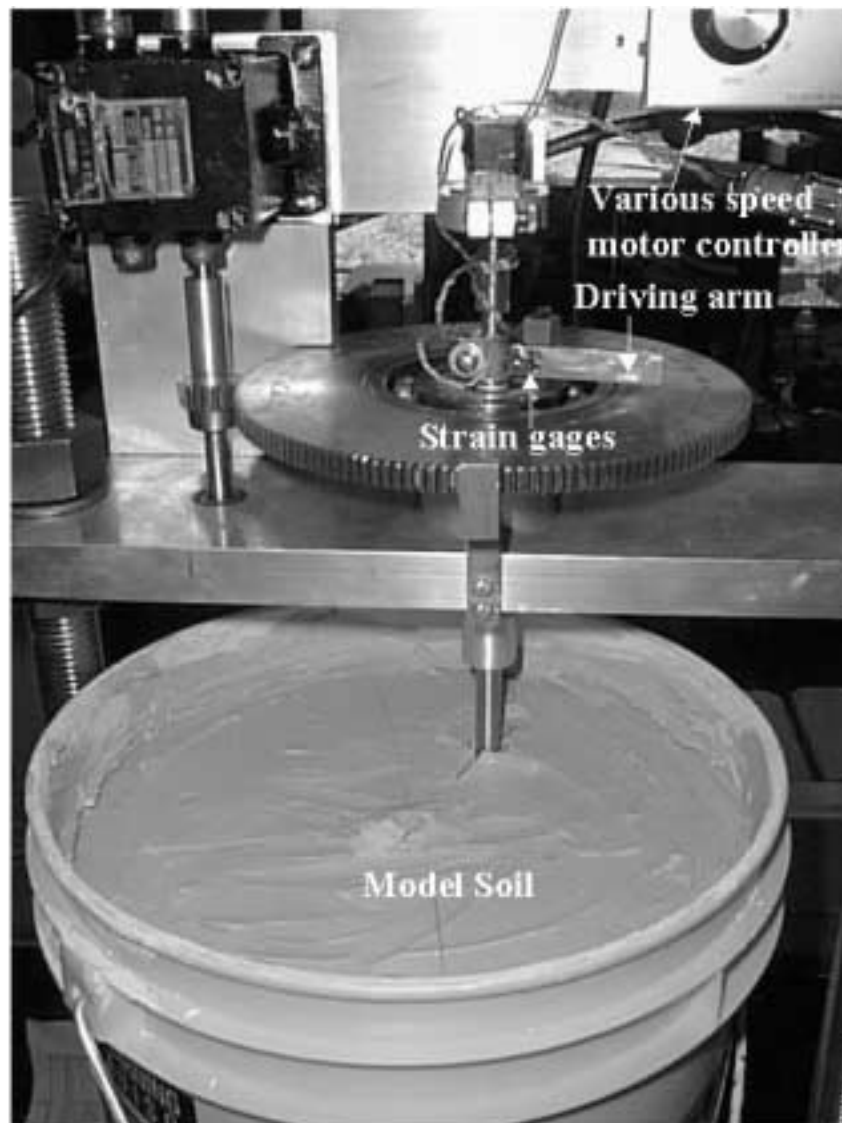
| Clay                                                        | $I_p$<br>(%)   | Shear Vane Test Details |                               |                              | Time Effect <sup>1</sup> |                      | Reference                              |
|-------------------------------------------------------------|----------------|-------------------------|-------------------------------|------------------------------|--------------------------|----------------------|----------------------------------------|
|                                                             |                | H/D                     | D<br>(mm)                     | Rate, $\dot{w}$<br>(°/min)   | $\alpha$<br>(%)          | $\beta$              |                                        |
| Grangemouth                                                 | ~22            | 1.5                     | 50, 75                        | 6- 300                       | 5-6                      | 0.025                | Skempton, 1948                         |
| Bromma                                                      | ~31            | 2.5                     | 80                            | 6 - 600                      | ~20                      | 0.086                | Cadling & Odenstad, 1950               |
| Åserum, Drammen,<br>Lierstranda, Manglerud                  | 8-9            | ¼ - 4                   | 65-130                        | 6 - 60                       | ~1-2                     | 0.006                | Aas, 1965                              |
| Silts and clays<br>(disturbed & remoulded)                  | -              | 2                       | 12.7                          | 6- 672<br>6 – 720            | ~16<br>±5                | 0.05<br>±0.01        | Migliori & Lee, 1971<br>Halwachs, 1972 |
| Skå Edeby                                                   | 50-100         | ¼ , ½,<br>1, 2          | 16.2, 65,<br>130              | 0.06-600                     | 3-6                      | 0.02                 | Wiesel, 1973                           |
| Bäckebo<br>Askim                                            | 50-65<br>80-90 | 2                       | 65                            | ~0.006 <sup>2</sup><br>- 300 | 12                       | 0.05<br>±0.01        | Torstensson, 1977                      |
| Gulf of Maine & Mexico<br>Exuma Sound, Bahamas              | -              | 2                       | 12.7                          | 21-79                        | 13<br>27                 | ~0.05<br>~0.10       | Smith & Richards, 1975                 |
| San Diego silt I<br>San Diego silt II<br>Gulf of Maine clay | 64<br>78       | 1,2<br>1,2              | 12.7,<br>76,101<br>12.7,51,76 | 72- 79<br>21-79              | 36<br>21<br>60           | 0.13<br>0.08<br>0.20 | Perlow & Richards, 1977                |
| Gulf of Mexico clay                                         | -              | 2                       | 12.7                          | 4.8-708                      | 33                       | 0.107                |                                        |
| Pierre shale                                                | 103            | 1,1.5, 2                | 12.7                          | 4.8- 107                     | 12                       | 0.05                 | Sharifounnasab & Ullrich, 1985         |
| Saint-Louis de Beaucours<br>Saint-Alban clay                | 13-19<br>6-18  | 2                       | 65                            | 6 - 120                      | 2-3<br>1                 | 0.01<br>0.004        | Roy & LeBlanc, 1988                    |
| Bentonite-Kaolinite mix                                     | 75             | 2                       | 55                            | 2-3000                       | 15±                      | 0.055                |                                        |
|                                                             |                |                         |                               |                              |                          |                      | This work                              |

Note: Time elapsed after insertion of vane and performance of shear test is typically less than a minute to a few (<5) minutes. Wiesel and Torstensson performed tests with periods after insertion of vane ranging from 15 to 24 hrs.

- Parameters  $\alpha$  and  $\beta$  refer to the time effect models described earlier. (i.e.,  $s_u/s_{u0} = 1 + \alpha \log(v/v_0)$ ;  $s_u/s_{u0} = (v/v_0)^\beta$ )
- Approximate conversion (actual rate reported in terms of time to failure)

Table 1: Previous Studies of Rate Effect on Undrained Shear Strength from Field Vane Tests





**Figure 2: Schematic of Shear Vane Testing**

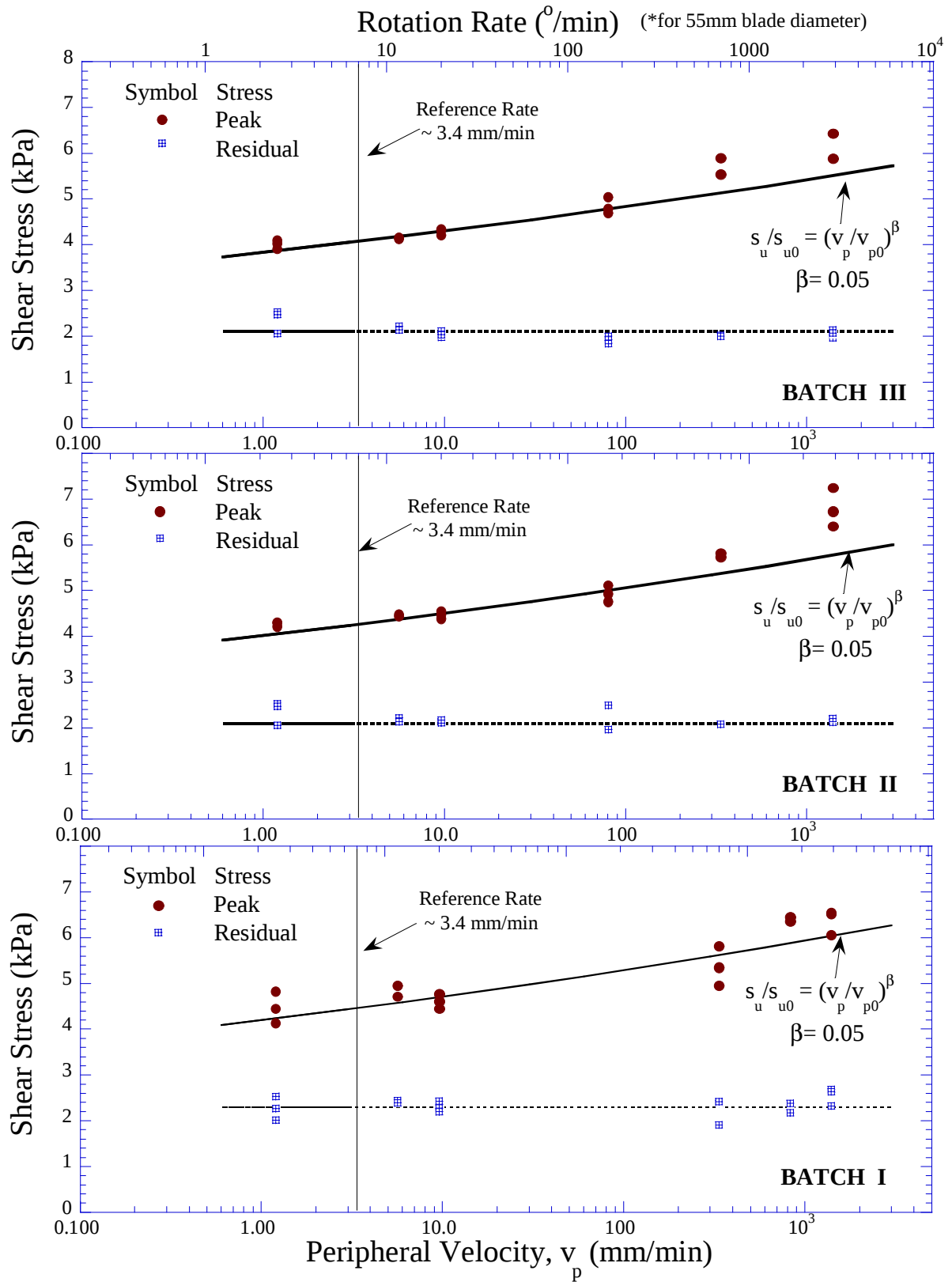
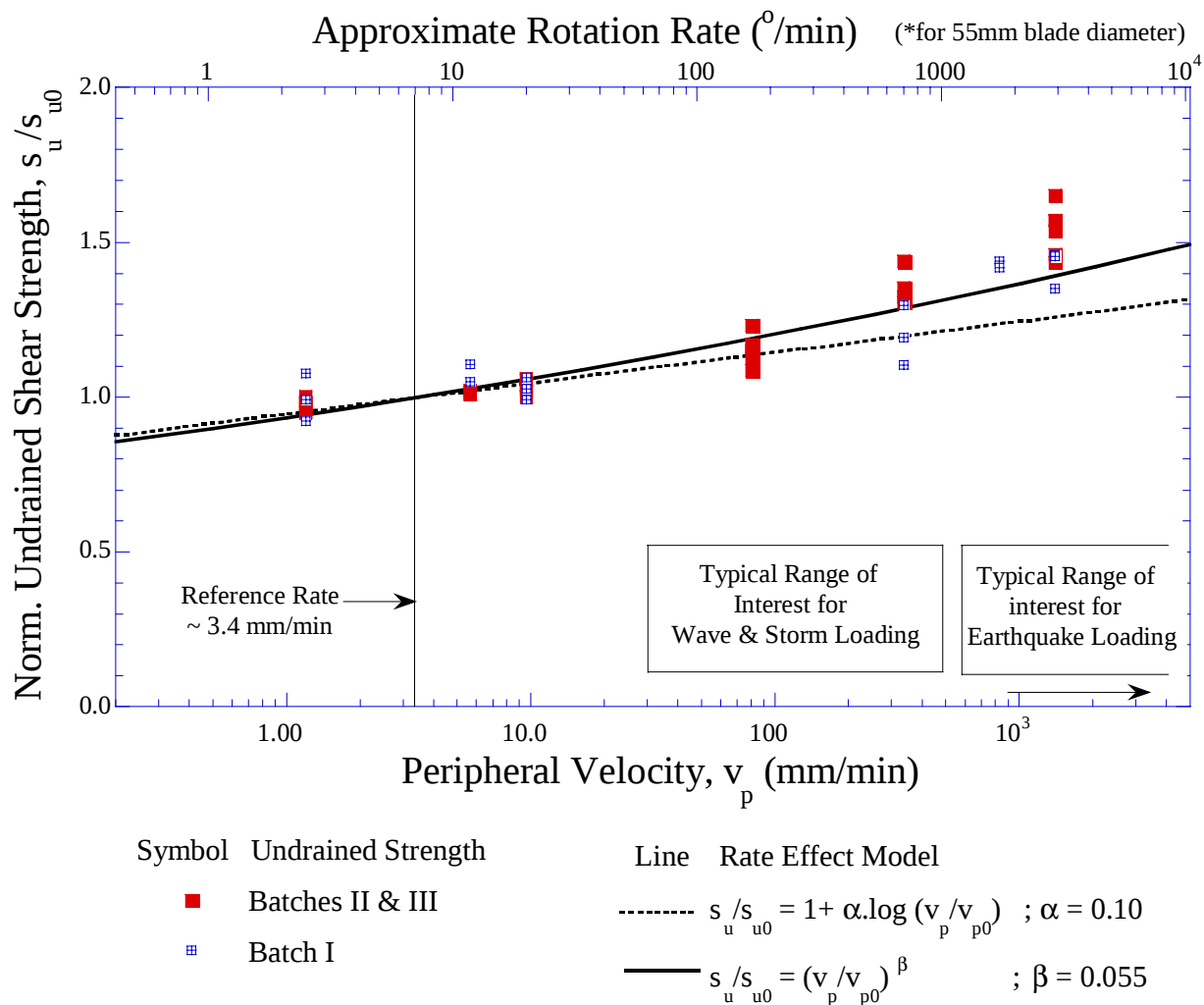
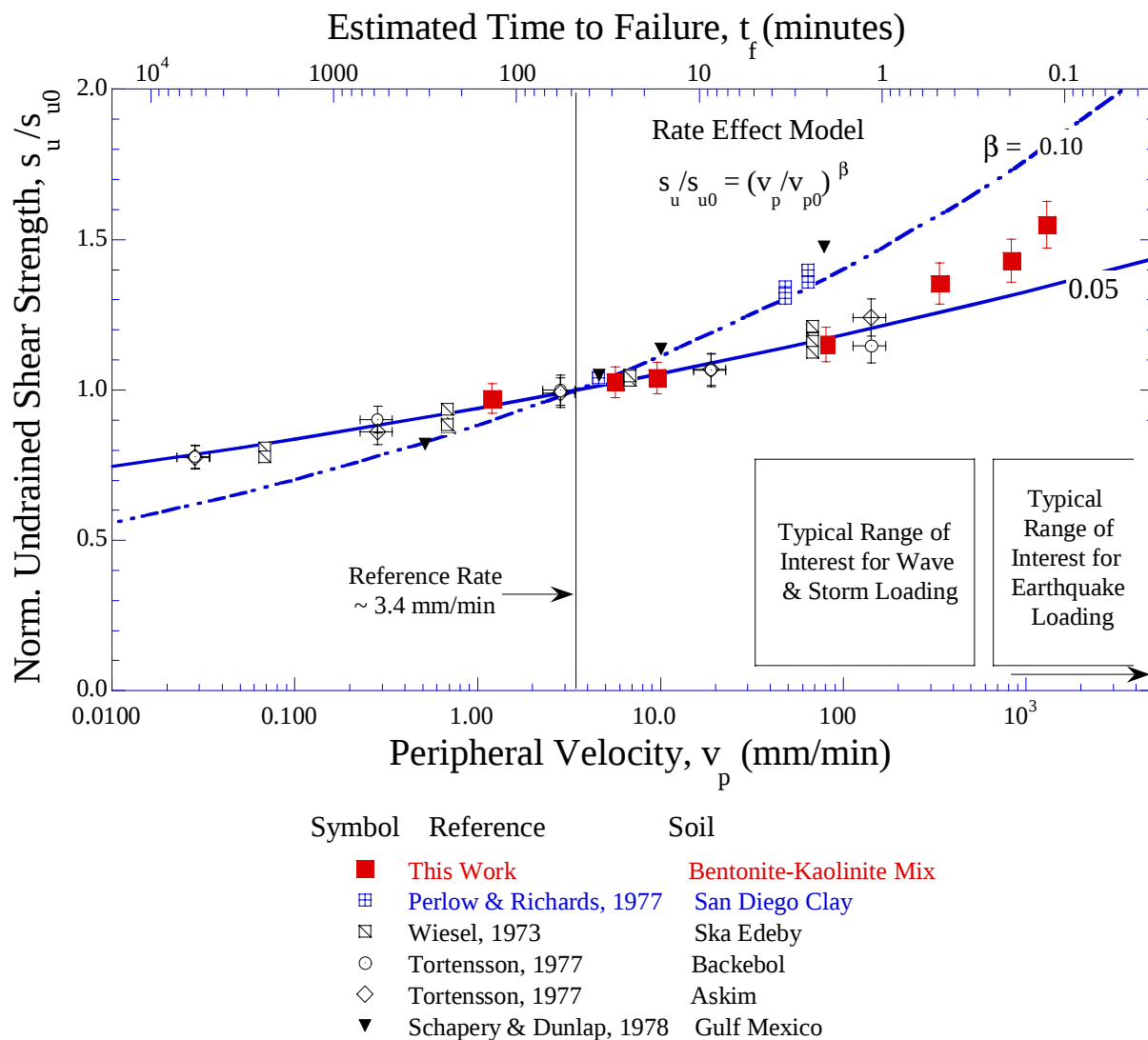


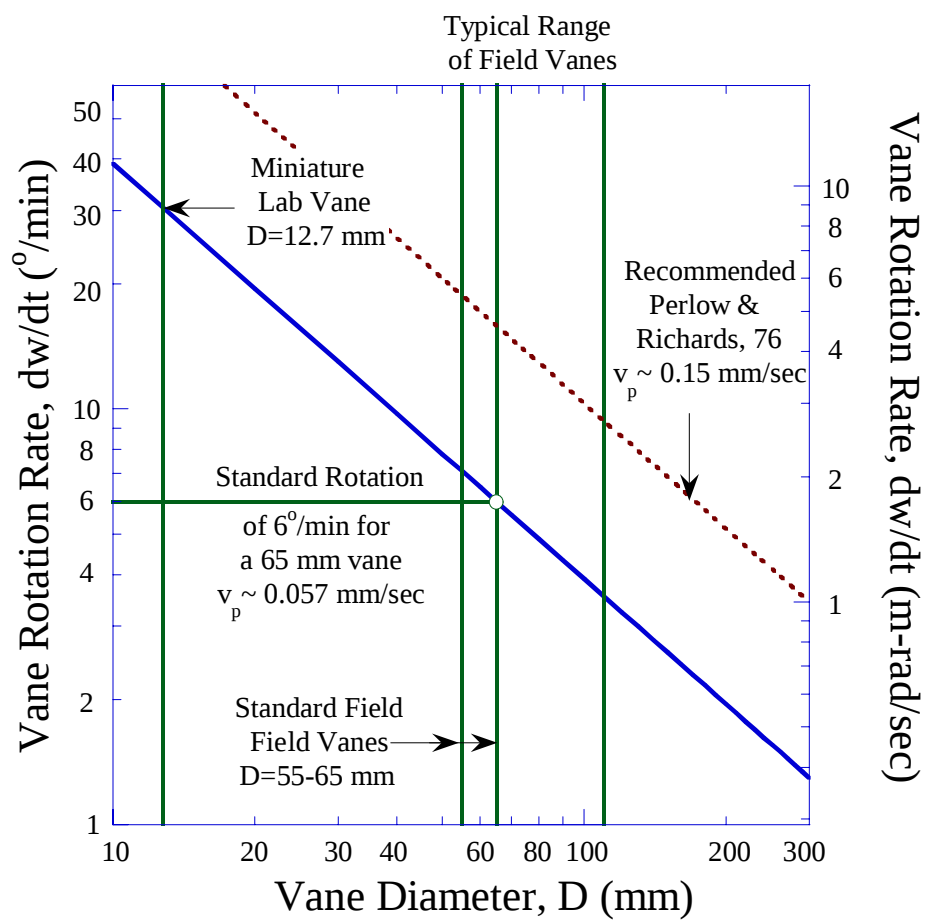
Figure 3: Peak and Undrained Residual Strength Measured from Vane Tests.



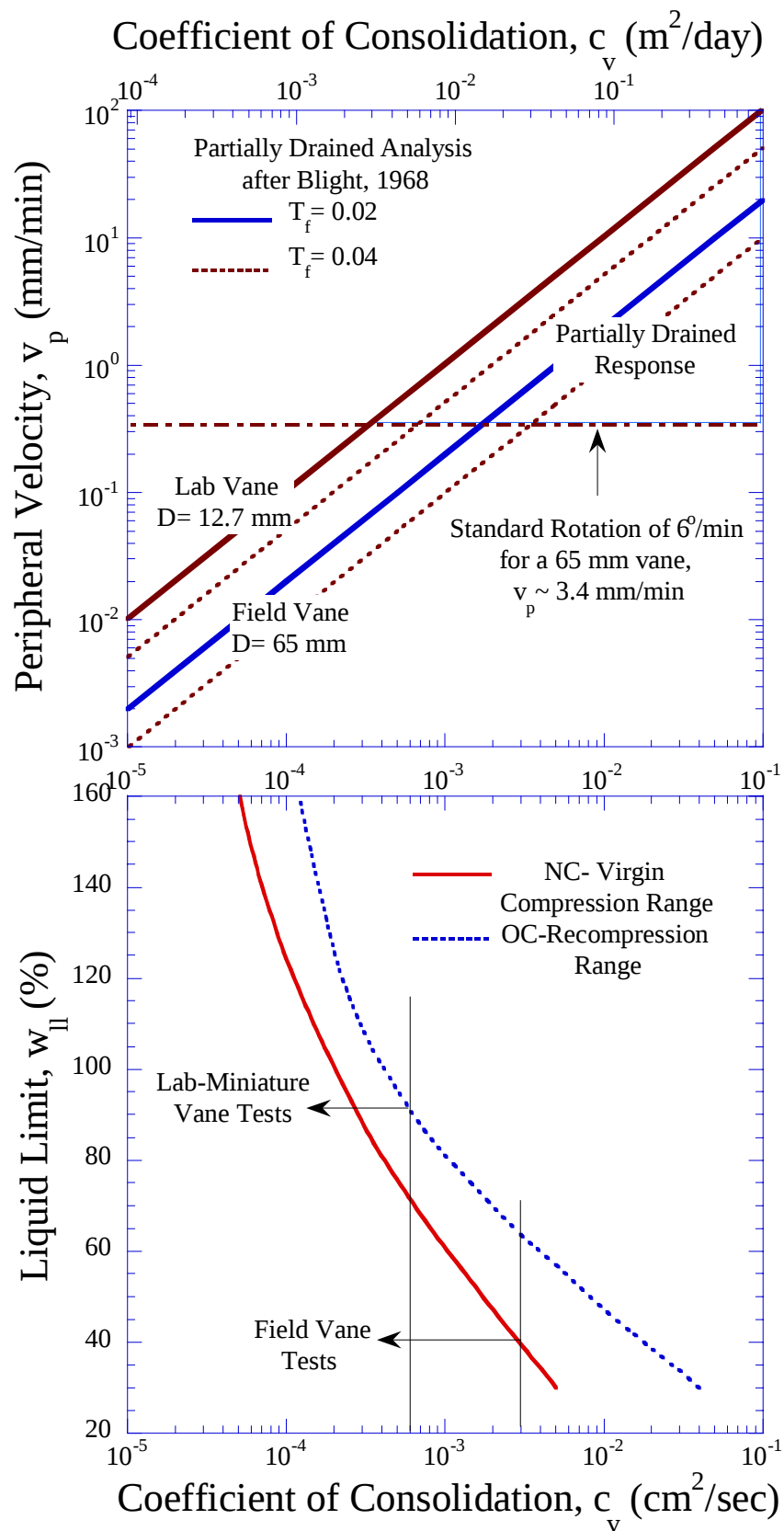
**Figure 4: Summary of Peak Conditions for Shear Vane Tests.**



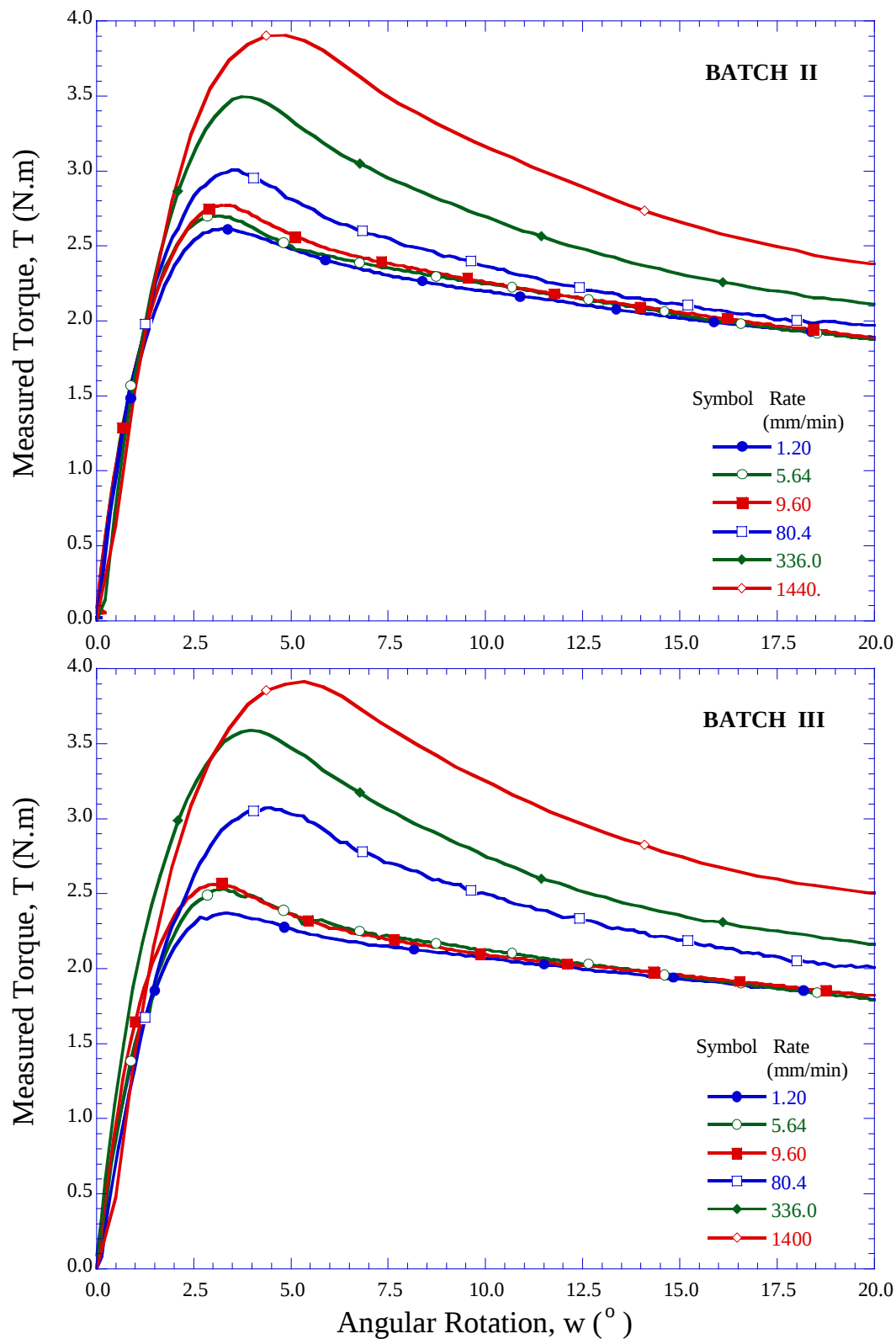
**Figure 5: Normalized Undrained Shear Strength as a function of Peripheral Velocity.**



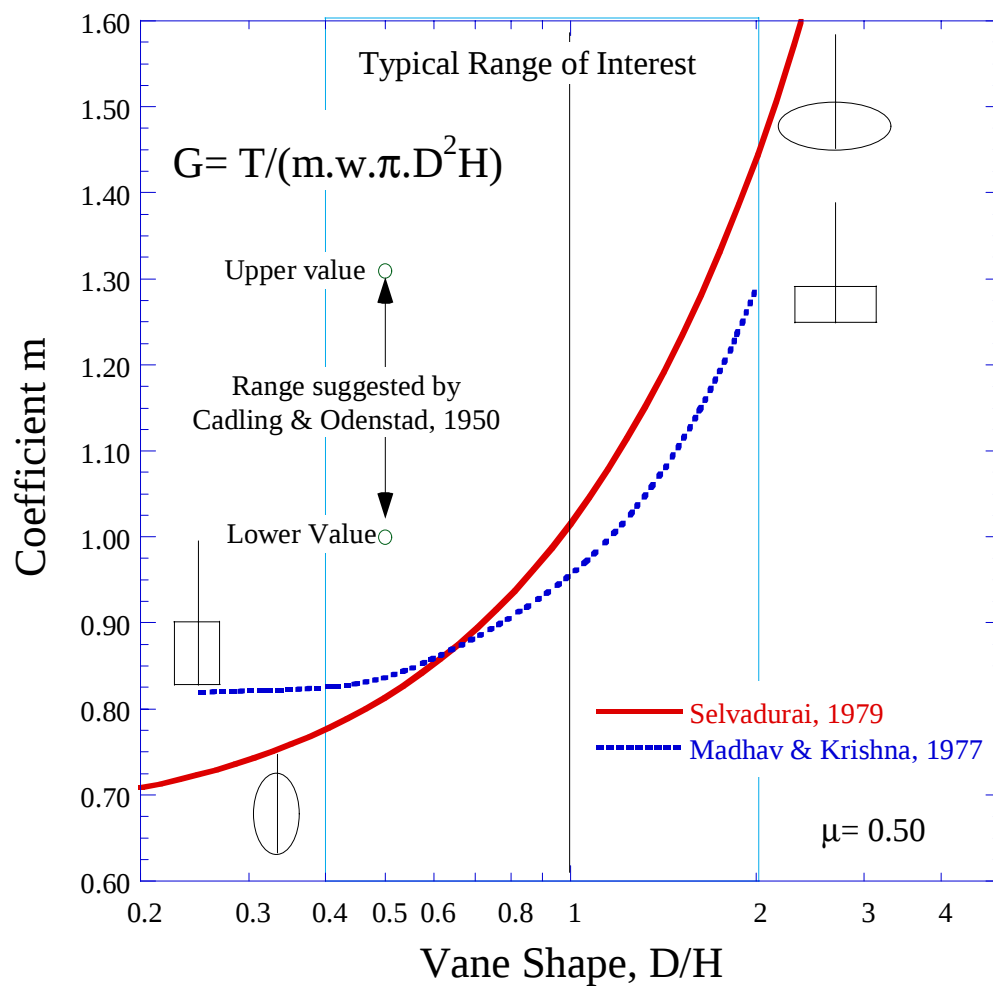
**Figure 6: Recommended Vane Rotation Rates for a Standard Peripheral Velocity of 0.057 mm/min (modified from Perlow & Richards, 1976)**



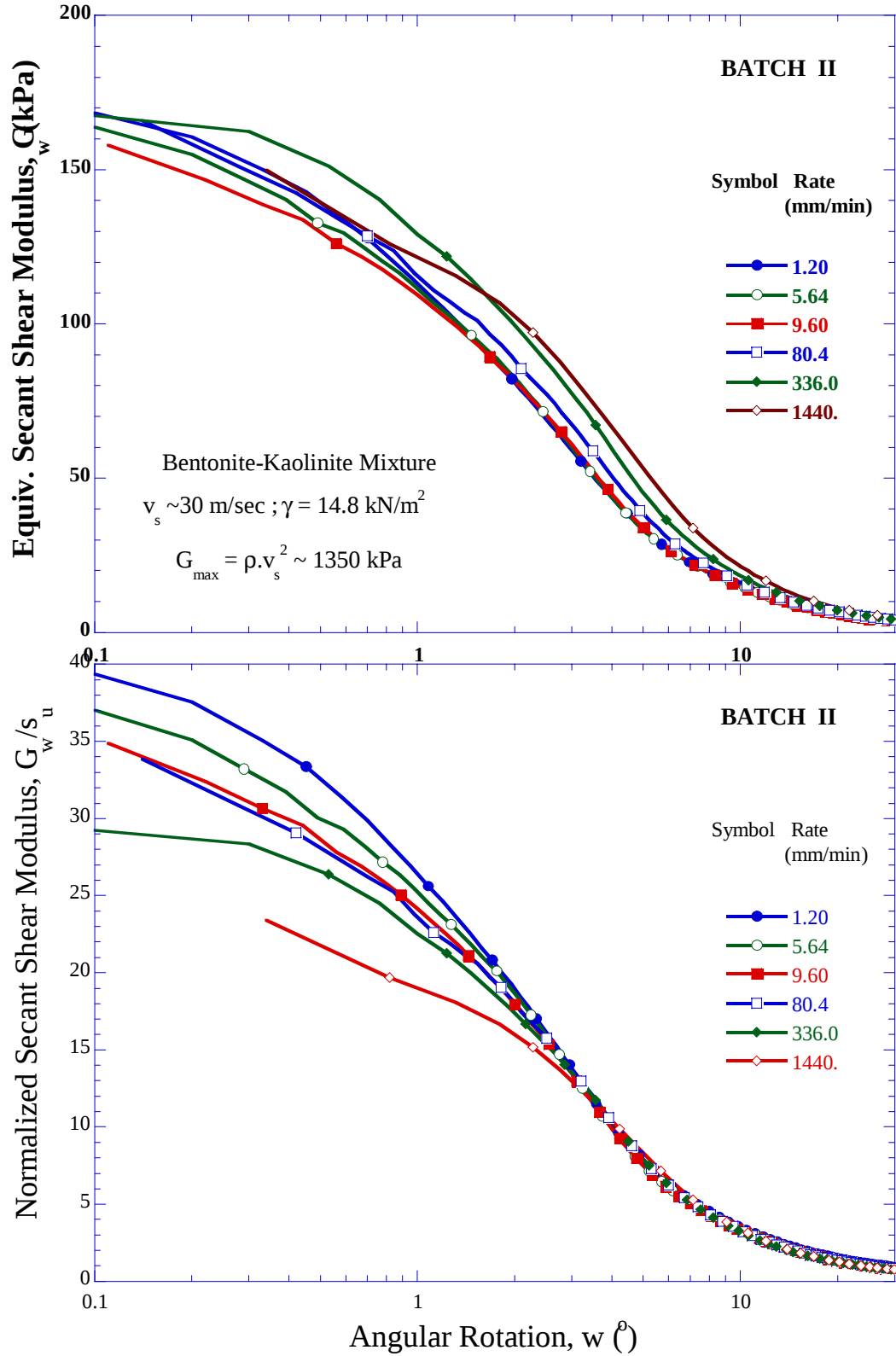
**Figure 7: Verification of Undrained Behavior Condition.**



**Figure 8: Typical Shear Stress-Deformation Behavior for Vane Tests on Batches II & III**



**Figure 9: Determination of Shear Modulus Based on Shear Vane Measurements**



**Figure 10: Equivalent Shear Modulus as a function of Peripheral Velocity.**

## APPENDIX A: SHEAR MODULUS ESTIMATION.

Many researchers have advocated the use of torque vs. angle of rotation curves for estimating the shear modulus of the soil. Selvadurai (1979) analyzed the case of an elliptical vane with different shapes (i.e., prolate vane,  $D/H < 1$ , oblate vane,  $D/H > 1$ ) embedded in an elastic medium. He assumed that the ellipsoid circumscribing the vane was solid (i.e., no volume change) and there was full contact over the entire surface. As a result, close-form analytical solutions were obtained for the torque,  $T$ , as a function of the angle of rotation,  $w$  (in radians):

$$T = \frac{4\pi.D^2.H.G.w.(1-\lambda^2)^{3/2}}{3 \left[ 2(1-\lambda^2)^{3/2} - \lambda^2 \cdot \ln \left( \frac{1+\sqrt{1-\lambda^2}}{1-\sqrt{1-\lambda^2}} \right) \right]}; \quad \text{Prolate vane, } \lambda < 1 \quad (\text{A.1a})$$

$$T = \frac{2\pi.D^2.H.G.w.(\lambda^2-1)^{3/2}}{3 \left[ \lambda^2 \cdot \tan^{-1}(\sqrt{\lambda^2-1}) - \sqrt{\lambda^2-1} \right]}; \quad \text{Oblate vane, } \lambda > 1 \quad (\text{A.1b})$$

where  $\lambda$  ( $=D/H$ ) defines the shape of the vane. From these expressions the equivalent shear modulus,  $G$ , can be determined:

$$G = \frac{T/(\pi.D.H)}{m.w.D} \quad (\text{A.2})$$

where  $m$  is a function of the shape of the vane, given by:

$$m = \frac{4.(1-\lambda^2)^{3/2}}{3 \left[ 2(1-\lambda^2)^{3/2} - \lambda^2 \cdot \ln \left( \frac{1+\sqrt{1-\lambda^2}}{1-\sqrt{1-\lambda^2}} \right) \right]}; \quad \text{prolate vane, } \lambda < 1 \quad (\text{A.3a})$$

$$m = \frac{2.(\lambda^2-1)^{3/2}}{3 \left[ \lambda^2 \cdot \tan^{-1}(\sqrt{\lambda^2-1}) - \sqrt{\lambda^2-1} \right]}; \quad \text{oblate vane, } \lambda > 1 \quad (\text{A.3b})$$

Madhav and Krishna (1977) analyzed the case of rectangular vane using elastic (i.e., Mindlin's) theory and determined the value of elastic Young Modulus,  $E$  by numerical integration. Their expression can be rewritten as:

$$G = \frac{T/(\pi.D.H)}{m.w.D}; \quad \text{where } m = (2(1+\mu).I_0.\lambda/\pi) \quad (\text{A.4})$$

where  $I_0$  is a factor dependent on the vane shape,  $\lambda$ , and the Poisson's ratio of the soil,  $\mu$ . Based on some heuristical arguments, Cadling and Odenstad (1950) proposed similar expressions based on the assumption of a cylindrical vane ( $D/H=0$ ) and a equivalent sphere, giving values of  $m$  between 1 and 1.31 for the typical field vane ( $D/H=2$ ):

$$G = \frac{T/(\pi.D.H)}{w.D} \quad (\text{cylindrical assumption}) \quad (\text{A.5a})$$

$$G = \frac{T/(\pi.D.H)}{1.31.w.D} \quad (\text{equivalent sphere for } D/H=2) \quad (\text{A.5b})$$

**APPENDIX B: SUMMARY OF RESULTS**

| Test    | Peripheral<br>velocity<br>(in/sec) | $s_u$ (peak)<br>(psf) | $s_u$ (residual)<br>(psf) | Peripheral<br>velocity<br>(mm/sec) | $s_u$ (peak)<br>(Pa) | $s_u$ (residual)<br>(Pa) |
|---------|------------------------------------|-----------------------|---------------------------|------------------------------------|----------------------|--------------------------|
| b1d1 vf | 0.918038                           | 126.427               | 48.527                    | 23.32                              | 6053.7               | 2323.6                   |
| b1d1 if | 0.544213                           | 134.815               | 45.300                    | 13.82                              | 6455.0               | 2169.0                   |
| b1d1 s  | 0.006299                           | 96.209                | 50.606                    | 0.160                              | 4606.6               | 2423.0                   |
| b1d1 vs | 0.000787                           | 100.747               | 41.993                    | 0.020                              | 4823.8               | 2010.6                   |
| b1d2 vf | 0.918038                           | 136.661               | 56.066                    | 23.32                              | 6543.4               | 2684.5                   |
| b1d2 f  | 0.220508                           | 111.744               | 49.760                    | 5.600                              | 5350.4               | 2382.5                   |
| b1d2 is | 0.003705                           | 103.511               | 50.911                    | 0.094                              | 4956.4               | 2437.7                   |
| b1d2 vs | 0.000787                           | 92.822                | 52.75                     | 0.020                              | 4444.6               | 2526.1                   |
| b1d3 vf | 0.918038                           | 136.2                 | 55.066                    | 23.32                              | 6521.3               | 2636.6                   |
| b1d3 f  | 0.220508                           | 121.511               | 50.375                    | 5.600                              | 5818.0               | 2412.0                   |
| b1d3 s  | 0.006299                           | 99.593                | 47.453                    | 0.160                              | 4768.6               | 2272.1                   |
| b1d3 is | 0.003705                           | 98.363                | 50.068                    | 0.094                              | 4709.7               | 2397.3                   |
| b1d4 if | 0.544213                           | 132.739               | 49.683                    | 13.82                              | 6355.6               | 2378.8                   |
| b1d4 f  | 0.220508                           | 103.28                | 39.916                    | 5.600                              | 4945.3               | 1911.2                   |
| b1d4 s  | 0.006299                           | 92.903                | 45.992                    | 0.160                              | 4448.2               | 2202.1                   |
| b1d4 vs | 0.000787                           | 86.289                | 47.222                    | 0.020                              | 4131.6               | 2261.1                   |

vs- very slow, is-intermediate slow, s- slow, f-fast, if-intermediate fast, vf- very fast

First two symbols denote batch # while following two denote bucket number

**Table B.1 Summary of Test Results for Batch I**

| Test    | Peripheral<br>velocity<br>(in/sec) | $s_u$ (peak)<br>(psf) | $s_u$ (residual)<br>(psf) | Peripheral<br>velocity<br>(mm/sec) | $s_u$ (peak)<br>(Pa) | $s_u$ (residual)<br>(Pa) |
|---------|------------------------------------|-----------------------|---------------------------|------------------------------------|----------------------|--------------------------|
| b2d1 vf | 0.918038                           | 151.343               | 45.605                    | 23.32                              | 7246.7               | 2183.7                   |
| b2d1 if | 0.220508                           | 119.742               | 43.531                    | 5.600                              | 5733.4               | 2084.3                   |
| b2d1 s  | 0.006299                           | 93.518                | 44.684                    | 0.160                              | 4477.7               | 2139.5                   |
| b2d1 vs | 0.000787                           | 89.904                | 42.91                     | 0.020                              | 4304.7               | 2054.8                   |
| b2d2 vf | 0.918038                           | 140.66                | 44.300                    | 23.32                              | 6734.9               | 2121.1                   |
| b2d2 f  | 0.052734                           | 106.67                | 51.990                    | 1.340                              | 5107.4               | 2489.3                   |
| b2d2 is | 0.003705                           | 93.591                | 46.297                    | 0.094                              | 4481.4               | 2216.8                   |
| b2d2 vs | 0.000787                           | 89.592                | 52.757                    | 0.020                              | 4289.9               | 2526.1                   |
| b2d3 vf | 0.918038                           | 133.816               | 46.068                    | 23.32                              | 6407.2               | 2205.8                   |
| b2d3 f  | 0.052734                           | 103.054               | 40.916                    | 1.340                              | 4934.3               | 1959.1                   |
| b2d3 s  | 0.006299                           | 91.750                | 44.070                    | 0.160                              | 4393.0               | 2110.0                   |
| b2d3 is | 0.003705                           | 92.519                | 44.685                    | 0.094                              | 4429.8               | 2139.5                   |
| b2d4 if | 0.220508                           | 121.280               | 43.531                    | 5.600                              | 5807.0               | 2084.3                   |
| b2d4 f  | 0.052734                           | 99.517                | 40.994                    | 1.340                              | 4764.9               | 1962.8                   |
| b2d4 s  | 0.006299                           | 94.903                | 45.377                    | 0.160                              | 4544.0               | 2172.6                   |
| b2d4 vs | 0.000787                           | 88.135                | 51.606                    | 0.020                              | 4219.2               | 2470.9                   |

vs- very slow, is-intermediate slow, s- slow, f-fast, if-intermediate fast, vf- very fast

First two symbols denote batch # while following two denote bucket number

**Table B.2 Summary of Test Results for Batch II.**

| Test    | Peripheral<br>velocity<br>(in/sec) | $s_u$ (peak)<br>(psf) | $s_u$ (residual)<br>(psf) | Peripheral<br>velocity<br>(mm/sec) | $s_u$ (peak)<br>(Pa) | $s_u$ (residual)<br>(Pa) |
|---------|------------------------------------|-----------------------|---------------------------|------------------------------------|----------------------|--------------------------|
| b3d1 vf | 0.918038                           | 122.8133              | 40.99174                  | 23.32                              | 5880.663             | 1962.805                 |
| b3d1 if | 0.220508                           | 123.0492              | 42.30078                  | 5.600                              | 5891.695             | 2025.395                 |
| b3d1 s  | 0.006299                           | 90.5193               | 41.22431                  | 0.160                              | 4334.136             | 1973.853                 |
| b3d1 vs | 0.000787                           | 85.67434              | 42.9161                   | 0.020                              | 4102.156             | 2054.857                 |
| b3d2 vf | 0.918038                           | 134.1236              | 44.53098                  | 23.32                              | 6421.945             | 2132.179                 |
| b3d2 f  | 0.052734                           | 105.3615              | 41.83929                  | 1.340                              | 5044.793             | 2003.299                 |
| b3d2 is | 0.003705                           | 86.28582              | 46.29762                  | 0.094                              | 4131.618             | 2216.866                 |
| b3d2 vs | 0.000787                           | 81.36417              | 52.7575                   | 0.020                              | 3895.954             | 2526.184                 |
| b3d3 vf | 0.895365                           | 134.2771              | 43.14685                  | 23.32                              | 6429.294             | 2065.905                 |
| b3d3 f  | 0.220508                           | 115.6667              | 41.68546                  | 5.600                              | 5538.216             | 1995.933                 |
| b3d3 s  | 0.006299                           | 88.05837              | 44.06949                  | 0.160                              | 4216.305             | 2110.082                 |
| b3d3 is | 0.003524                           | 86.82773              | 44.68481                  | 0.094                              | 4157.381             | 2139.544                 |
| b3d4 if | 0.052734                           | 99.97848              | 40.1475                   | 1.340                              | 4787.049             | 1922.294                 |
| b3d4 f  | 0.052734                           | 98.13252              | 38.3788                   | 1.340                              | 4698.663             | 1837.608                 |
| b3d4 s  | 0.005984                           | 90.13473              | 42.53153                  | 0.160                              | 4315.723             | 2036.443                 |
| b3d4 vs | 0.000787                           | 84.05947              | 51.60613                  | 0.020                              | 4024.834             | 2470.943                 |

vs- very slow, is-intermediate slow, s- slow, f-fast, if-intermediate fast, vf- very fast

First two symbols denote batch # while following two denote bucket number

**Table B.3 Summary of Test Results for Batch III.**