PEA5009 Aula 7

Modelagem e simulação numérica de componentes de sistemas elétricos de potência

Modelo Detalhado de Transformadores

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$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{bmatrix} L_{11} & L_{21} \\ L_{12} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

$$\frac{d}{dt} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \frac{1}{L_{11}L_{22} - L_{12}L_{21}} \begin{bmatrix} L_{22} & -L_{21} \\ -L_{12} & L_{11} \end{bmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\frac{di_1}{dt} = \frac{L_{22}}{L_{11}L_{22} - L_{12}L_{21}} v_1 - \frac{L_{21}}{L_{11}L_{22} - L_{12}L_{21}} v_2$$

$$\frac{di_2}{dt} = \frac{-L_{12}}{L_{11}L_{22} - L_{12}L_{21}} v_1 + \frac{L_{11}}{L_{11}L_{22} - L_{12}L_{21}} v_2$$

Substituição numérica:

$$
\begin{aligned}
i_1(t) &= \frac{L_{22}}{L_{11}L_{22} - L_{12}L_{21}} \int_0^t v_1\,dt - \frac{L_{21}}{L_{11}L_{22} - L_{12}L_{21}} \int_0^t v_2\,dt \\
&= i_1(t-\Delta t) + \frac{L_{22}}{L_{11}L_{22} - L_{12}L_{21}} \int_{t-\Delta t}^t v_1\,dt \\
&\quad - \frac{L_{21}}{L_{11}L_{22} - L_{12}L_{21}} \int_{t-\Delta t}^t v_2\,dt \\
&= i_1(t-\Delta t) + \frac{L_{22}\Delta t}{2(L_{11}L_{22} - L_{12}L_{21})}(v_1(t-\Delta t) + v_1(t)) \\
&\quad - \frac{L_{21}\Delta t}{2(L_{11}L_{22} - L_{12}L_{21})}(v_2(t-\Delta t) + v_2(t))
\end{aligned}
$$

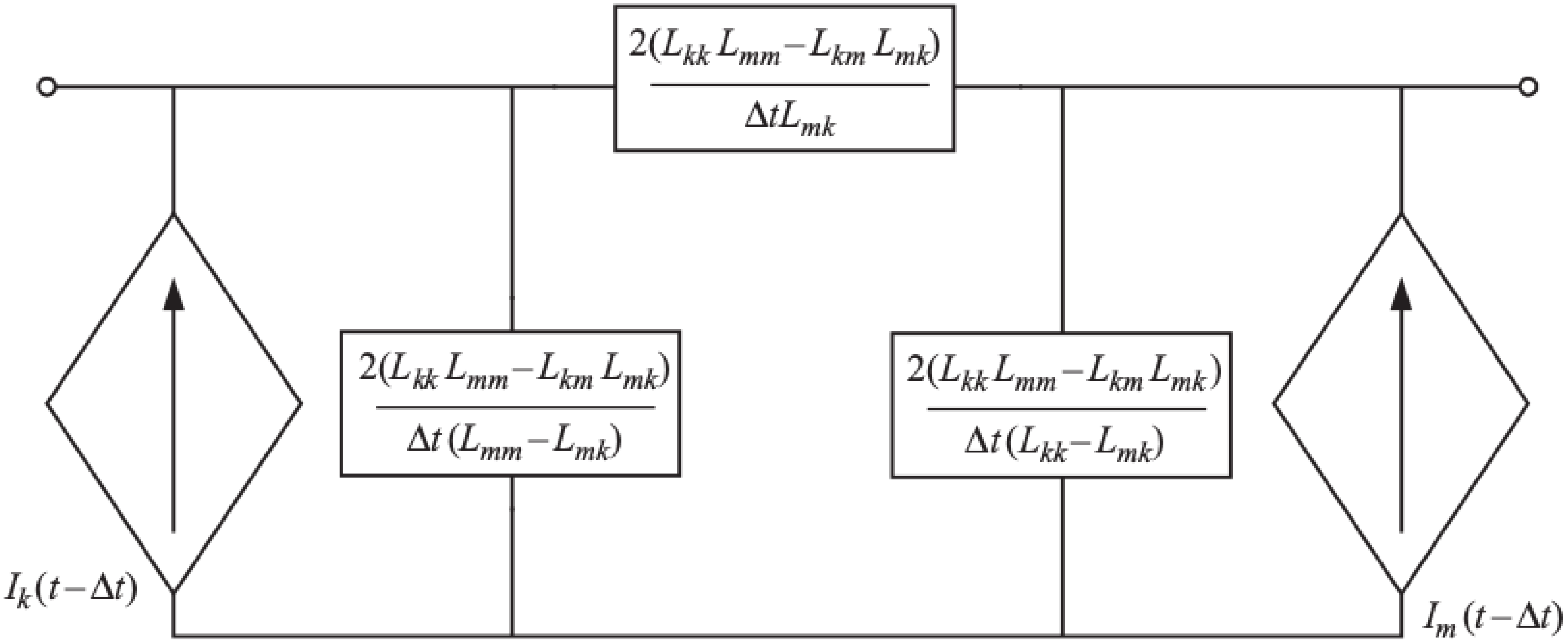
Fonte: [1]

Substituição numérica:

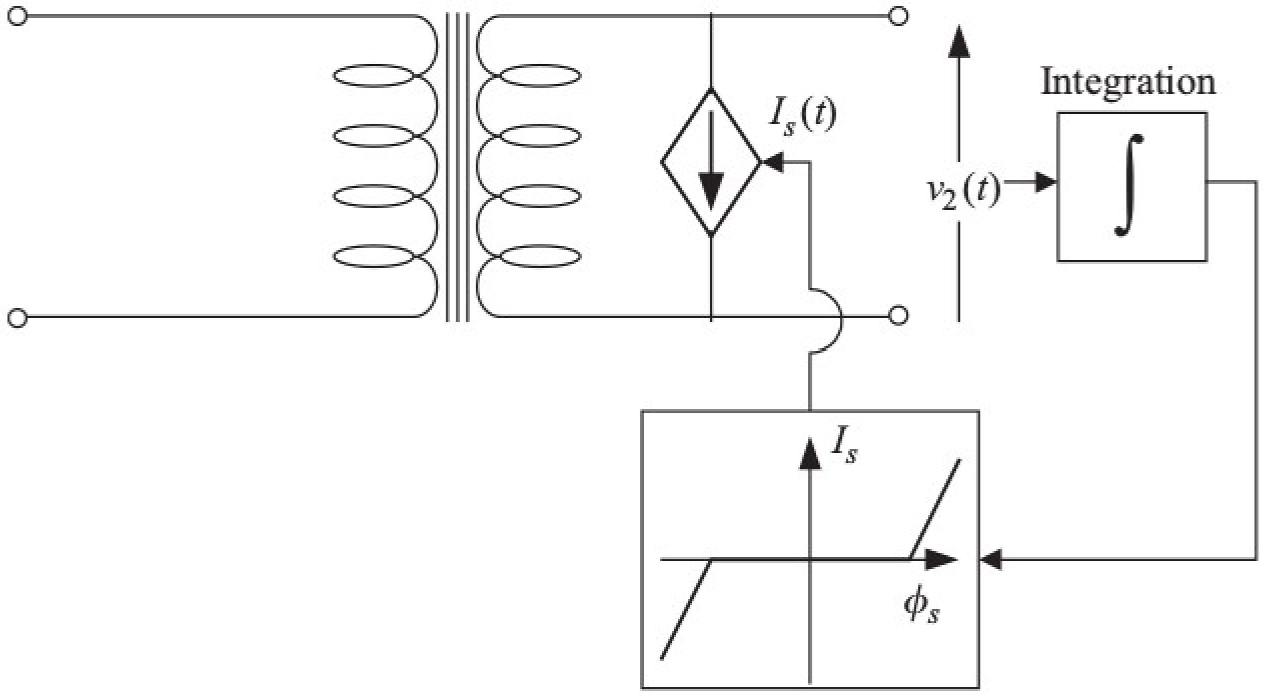
$$i_1(t) = I_h(t-\Delta t) + \left(\frac{L_{22}\Delta t}{2(L_{11}L_{22}-L_{12}L_{21})} - \frac{L_{21}\Delta t}{2(L_{11}L_{22}-L_{12}L_{21})}\right) v_1(t)$$
$$+ \frac{L_{21}\Delta t}{2(L_{11}L_{22}-L_{12}L_{21})}(v_1(t)-v_2(t)) \qquad (7.1$$

$$I_h(t-\Delta t) = i_1(t-\Delta t)$$
$$+ \left(\frac{L_{22}\Delta t}{2(L_{11}L_{22}-L_{12}L_{21})} - \frac{L_{21}\Delta t}{2(L_{11}L_{22}-L_{12}L_{21})}\right) v_1(t-\Delta t)$$
$$+ \frac{L_{21}\Delta t}{2(L_{11}L_{22}-L_{12}L_{21})}(v_1(t-\Delta t)-v_2(t-\Delta t)) \qquad (7.$$

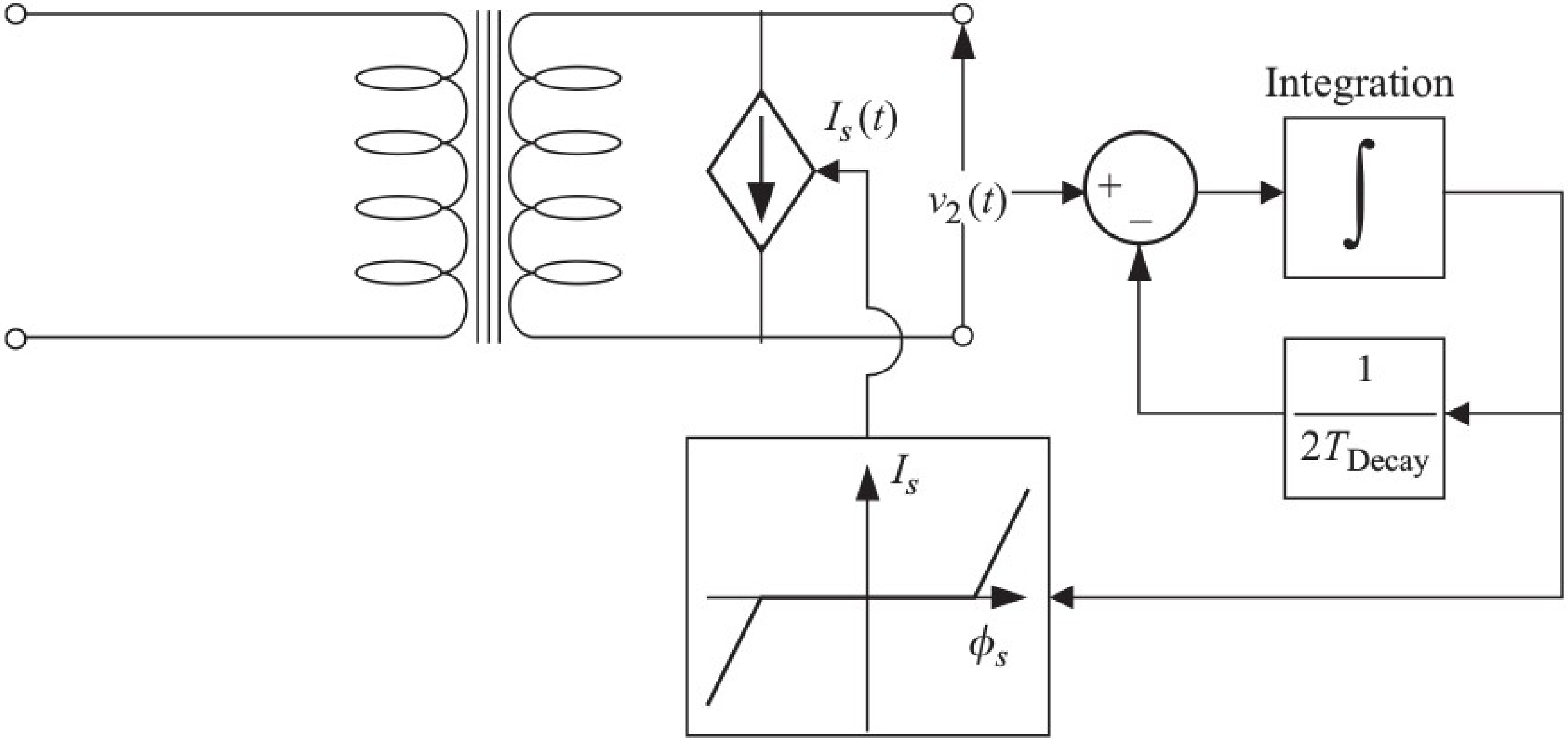
Substituição numérica:



Consideração do efeito da Saturação

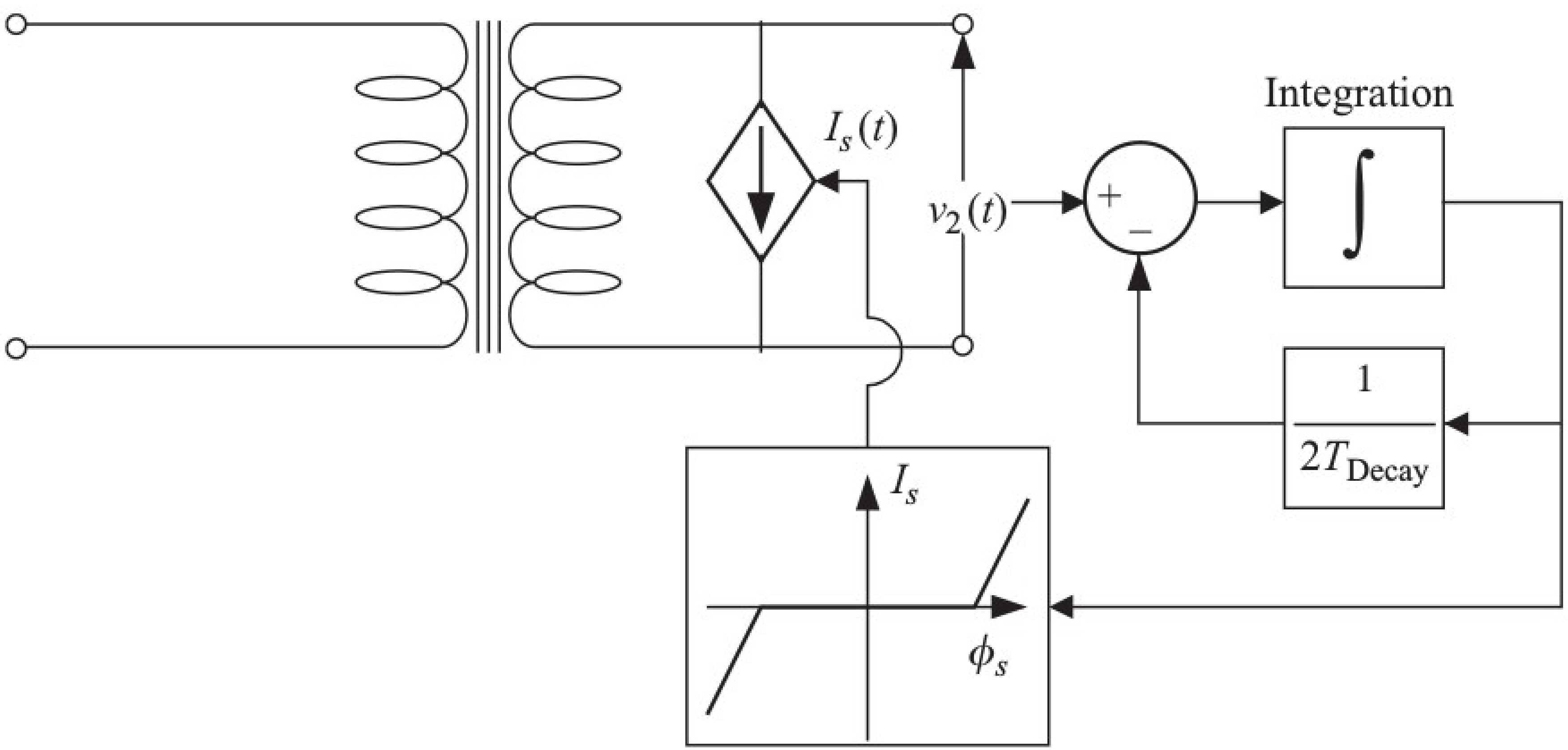


Consideração do efeito Inrush



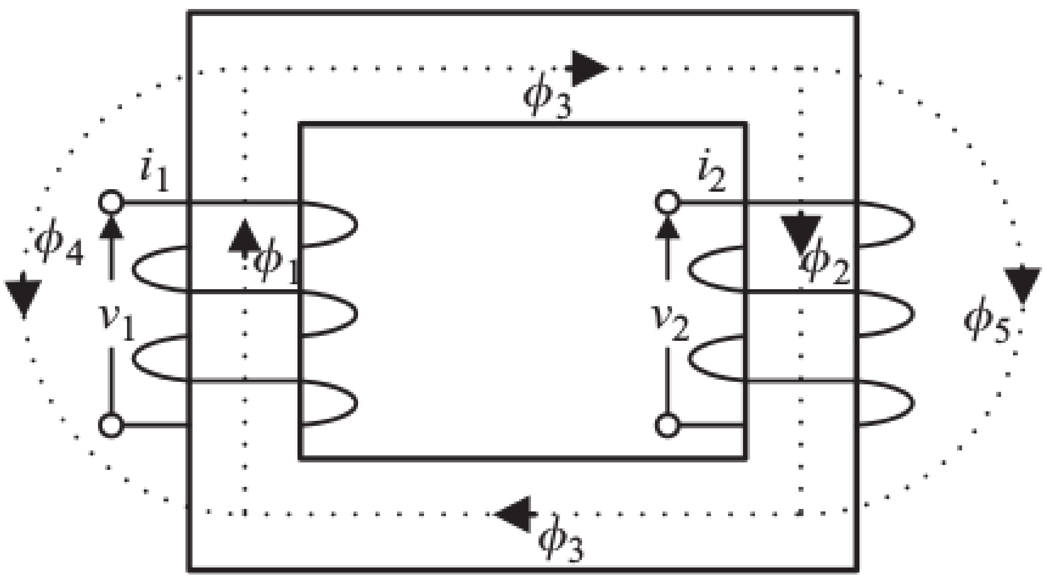
Fonte: [1]

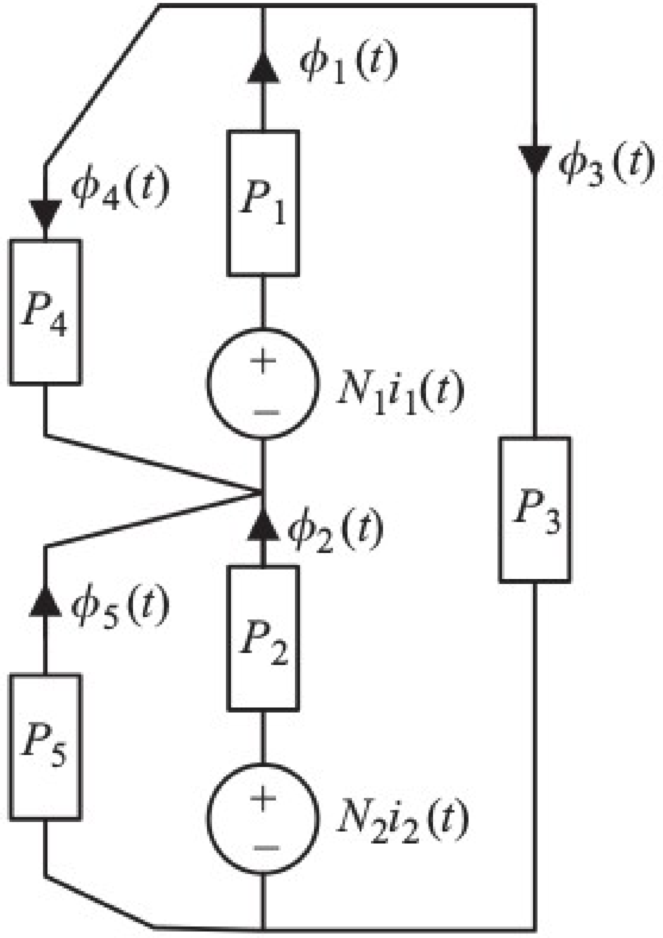
Consideração do efeito Inrush



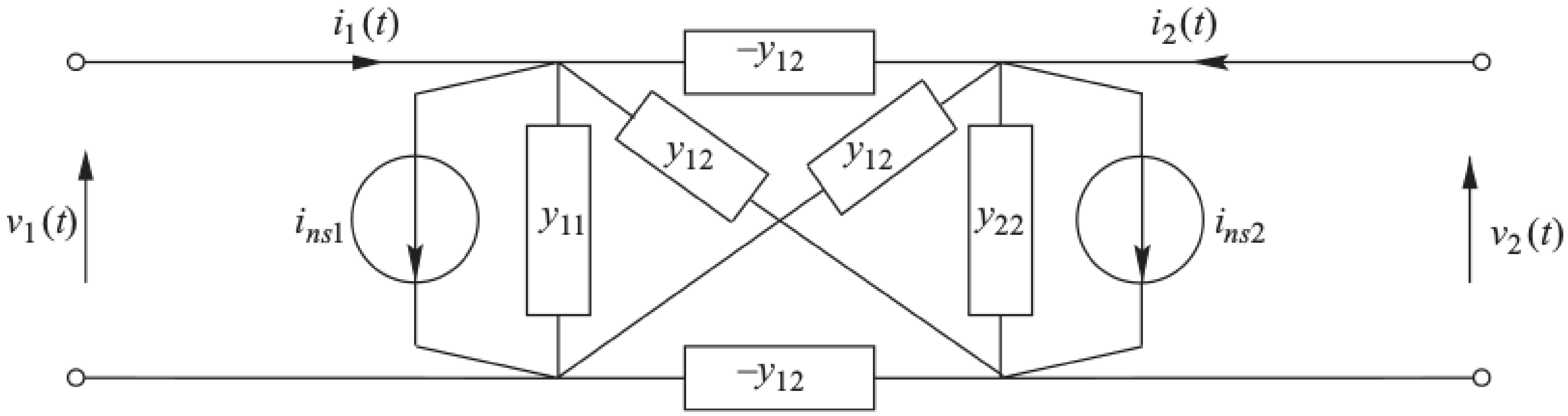
Fonte: [1]

Unified Magnetic Equivalent Circuit



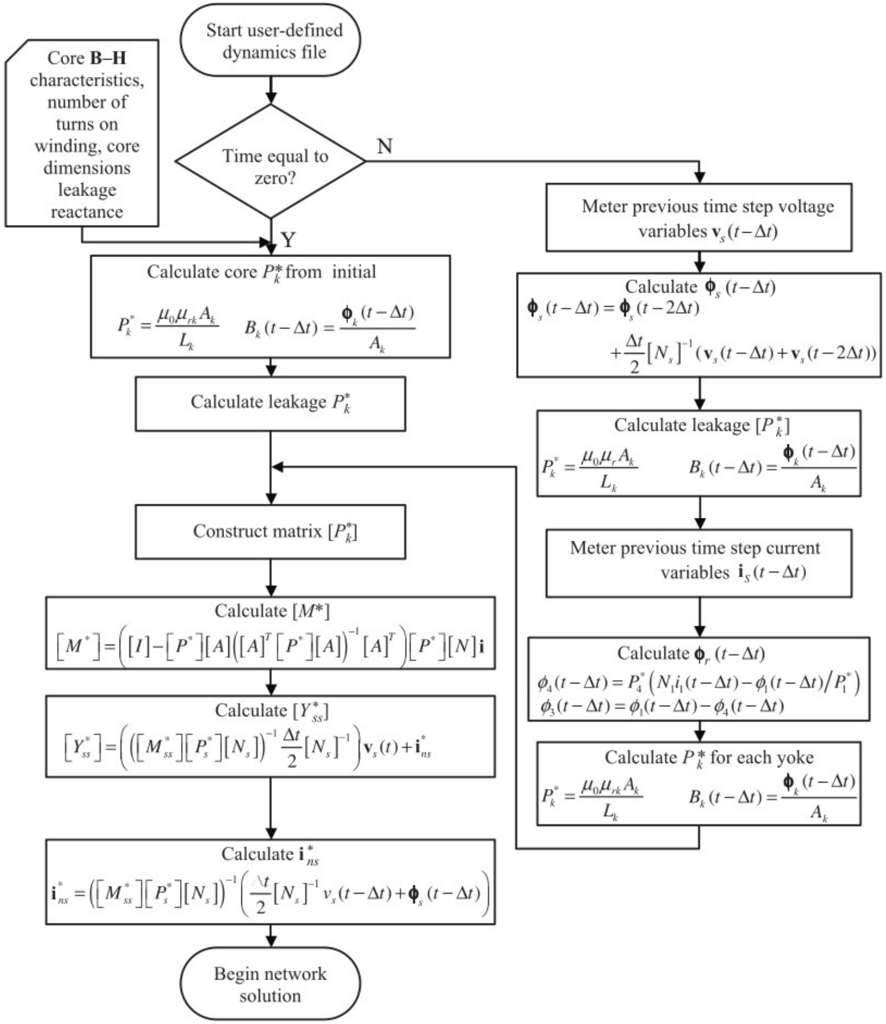


Unified Magnetic Equivalent Circuit

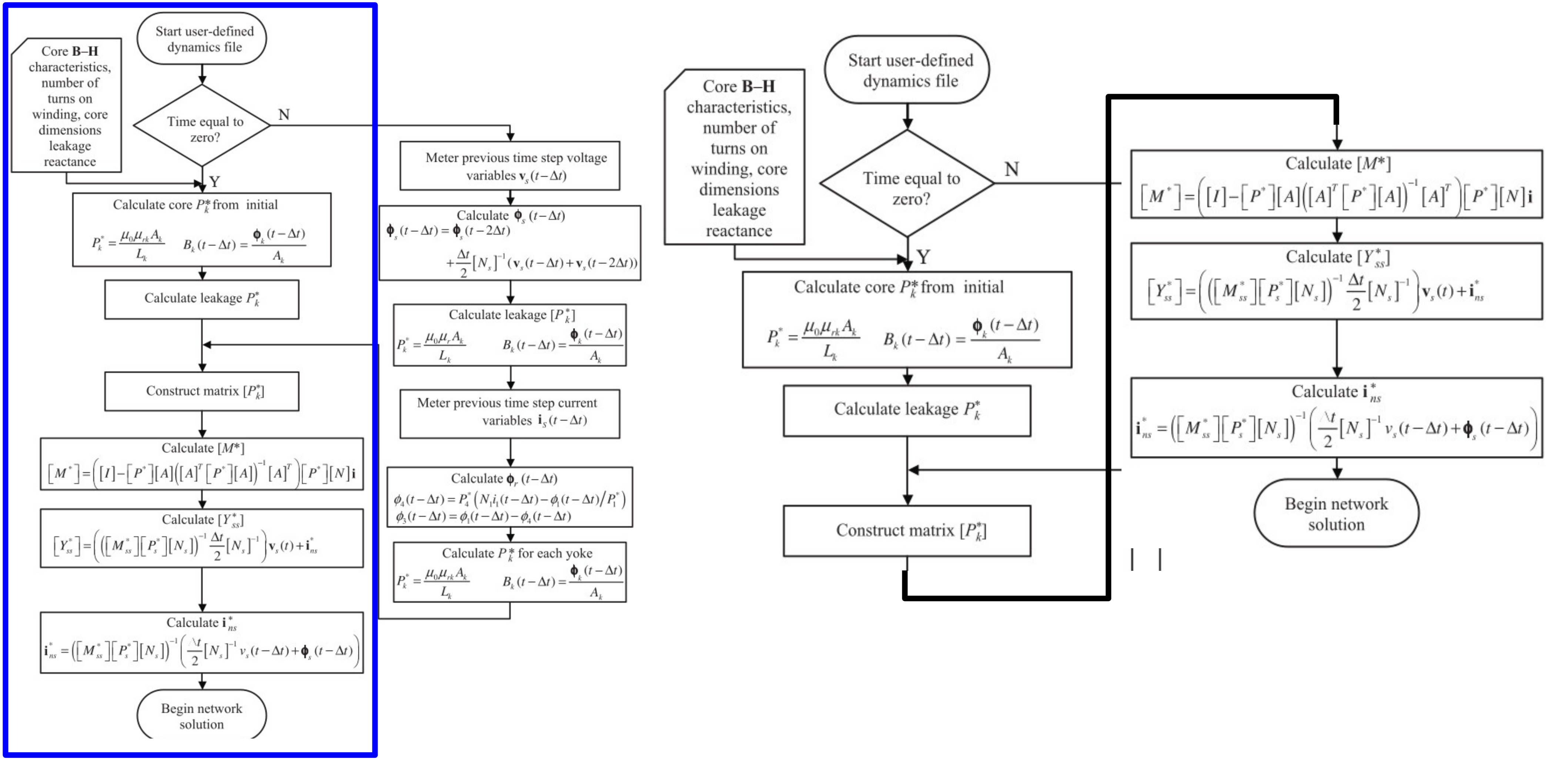


$$\begin{pmatrix} \mathbf{i}_1(t) \\ \mathbf{i}_2(t) \end{pmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{12} & y_{22} \end{bmatrix} \begin{pmatrix} \mathbf{v}_1(t) \\ \mathbf{v}_2(t) \end{pmatrix} + \begin{pmatrix} \mathbf{i}_{ns1}(t) \\ \mathbf{i}_{ns2}(t) \end{pmatrix}$$

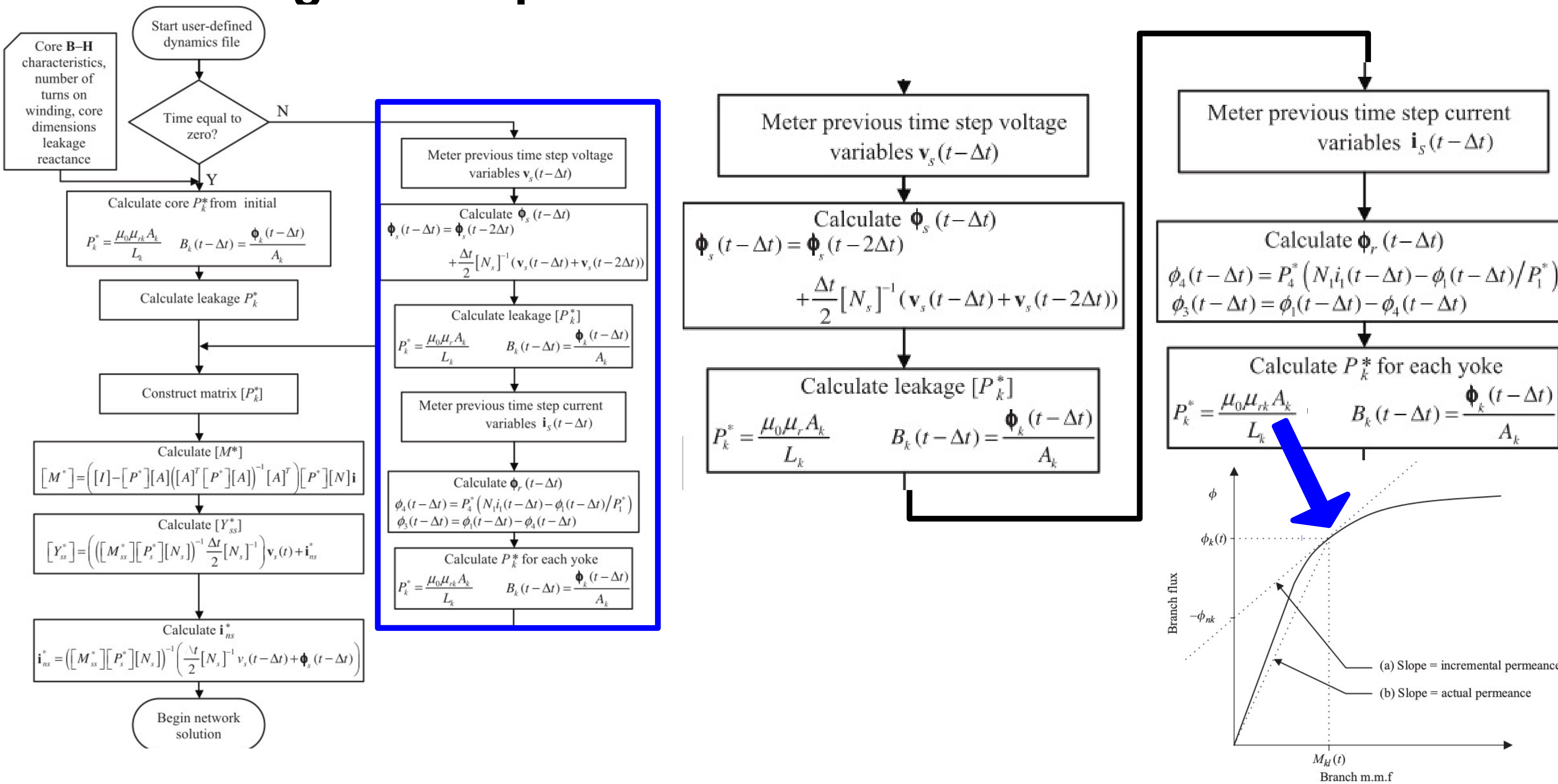
Unified Magnetic Equivalent Circuit



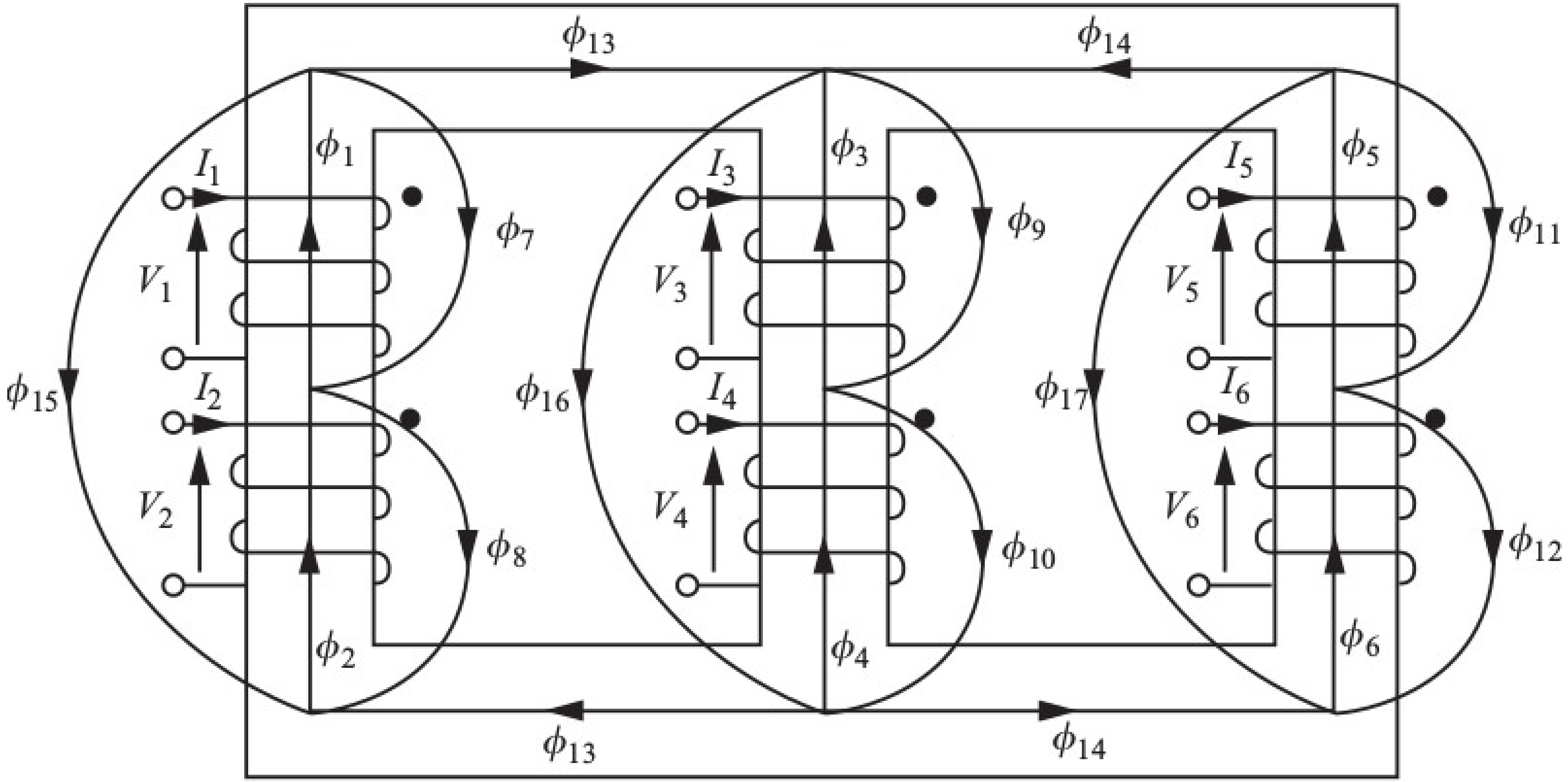
Unified Magnetic Equivalent Circuit



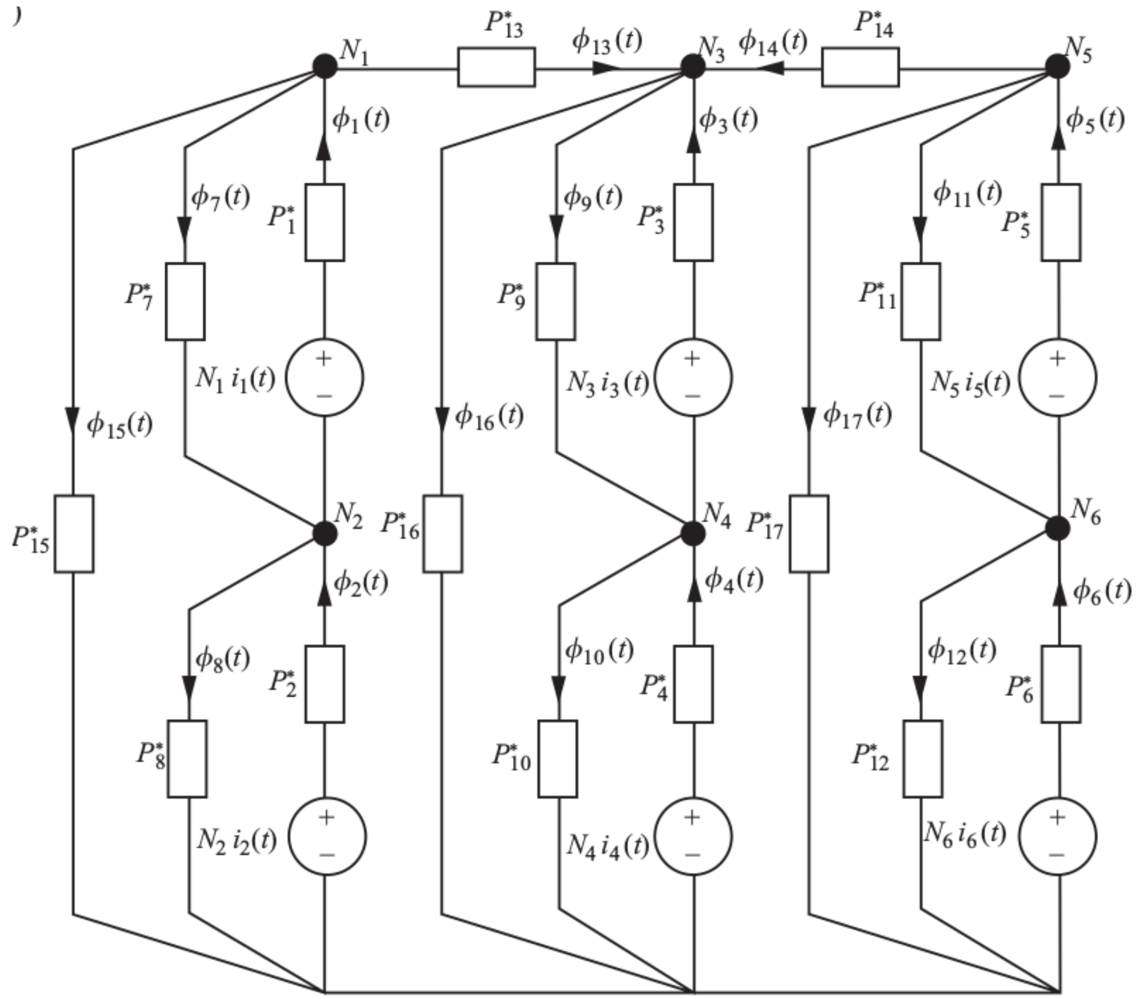
Unified Magnetic Equivalent Circuit



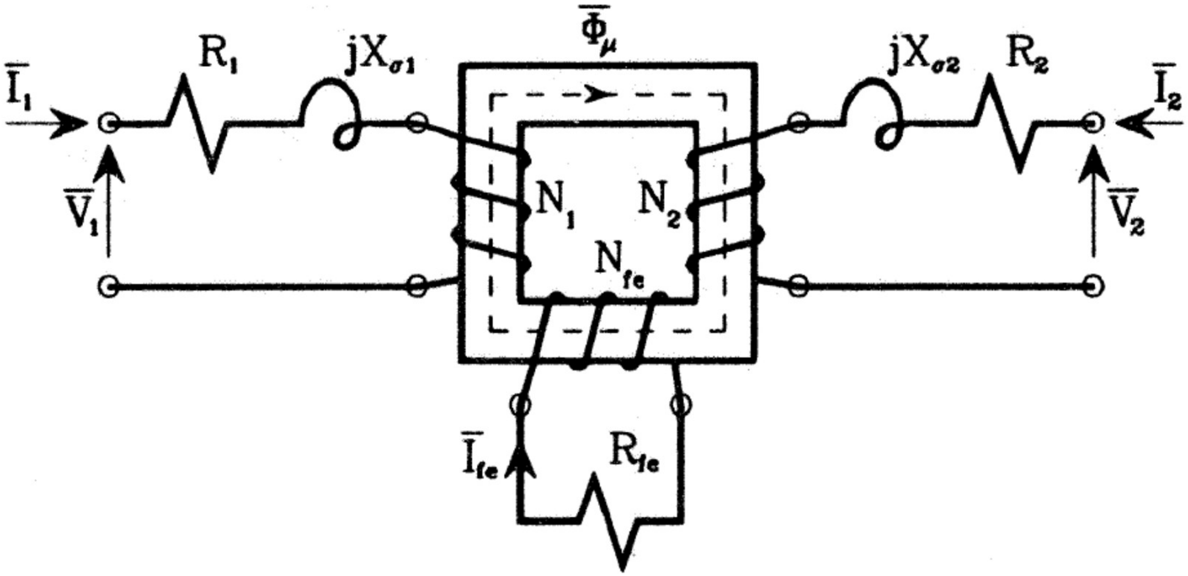
Unified Magnetic Equivalent Circuit



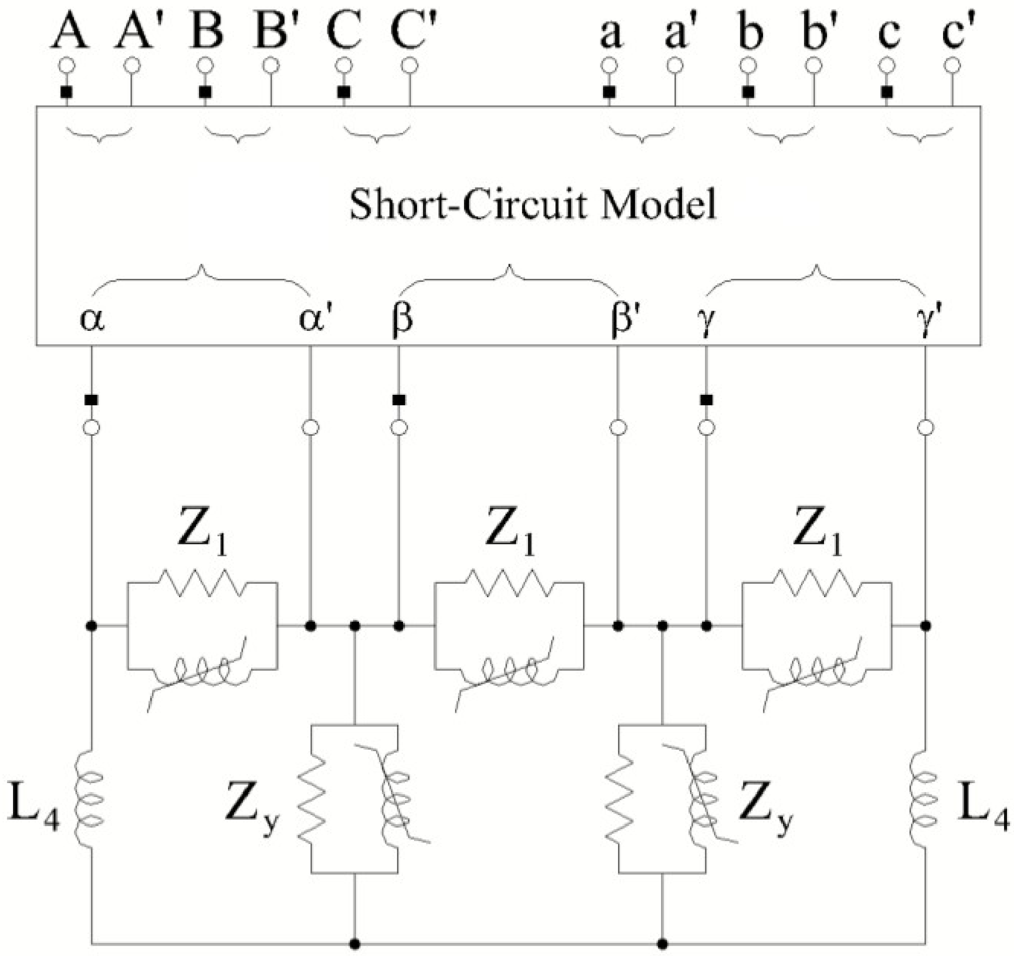
Unified Magnetic Equivalent Circuit



Perdas no Núcleo (Enrolamento Fictício)

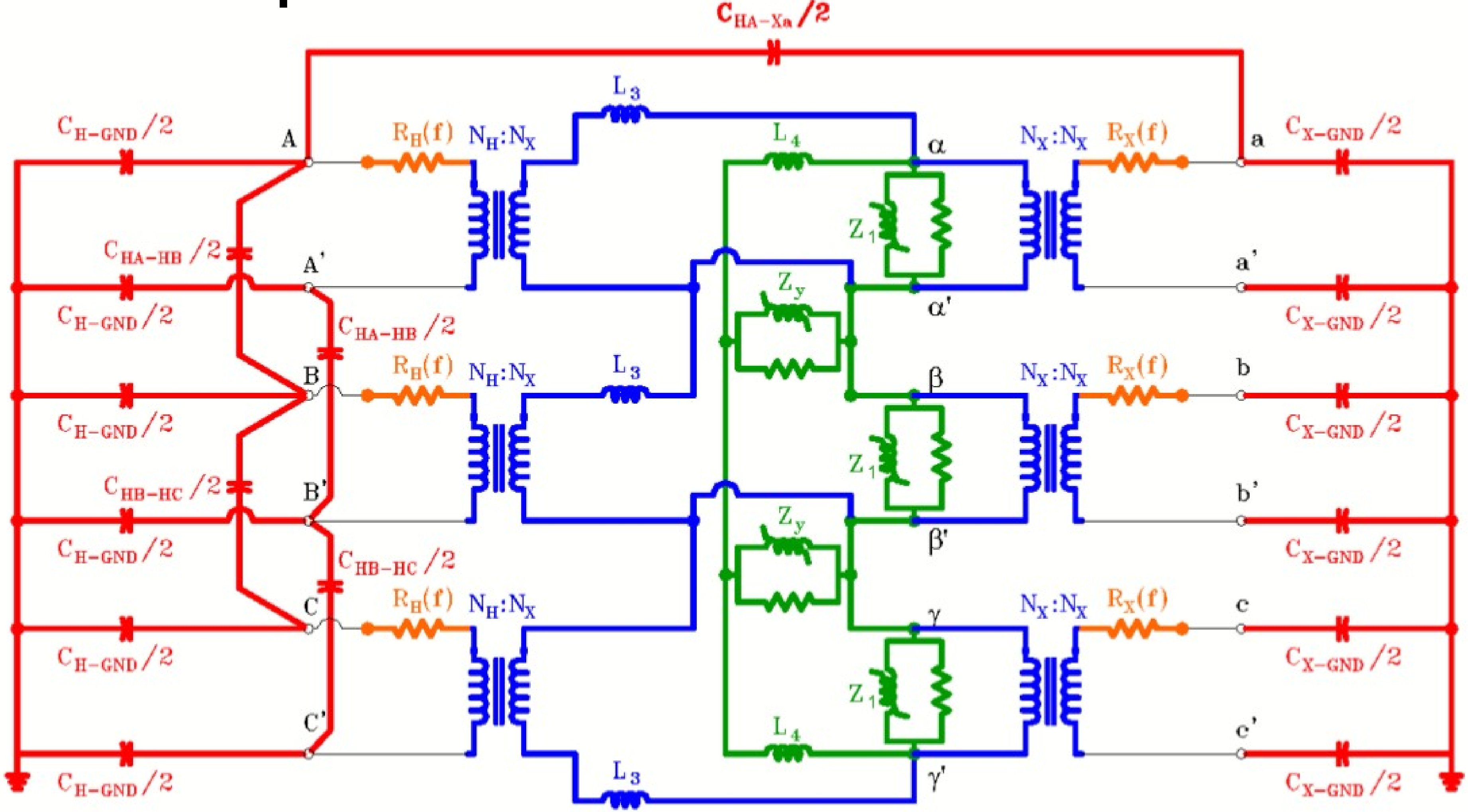


Perdas no Núcleo (Enrolamento Fictício)



Transformador 3 pernas

Efeito da Frequência



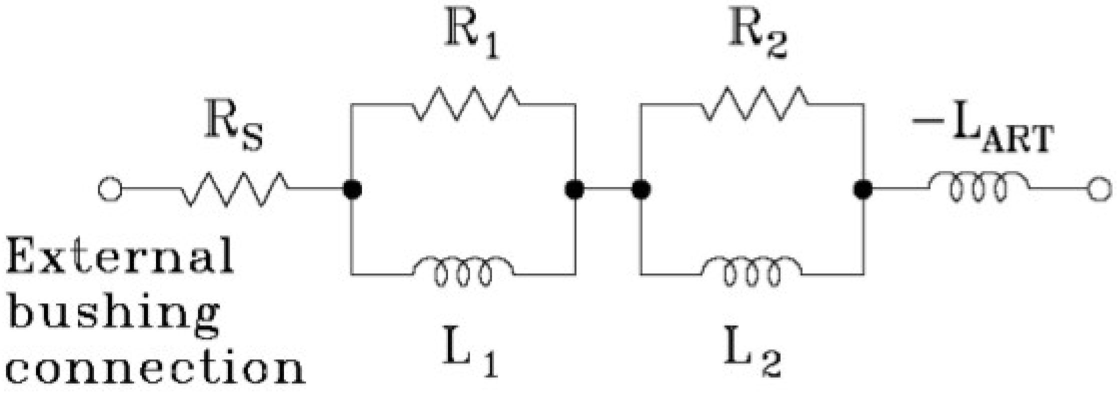
Fonte: [2]

Efeito da Frequência

Resistência

Efeito Pelicular

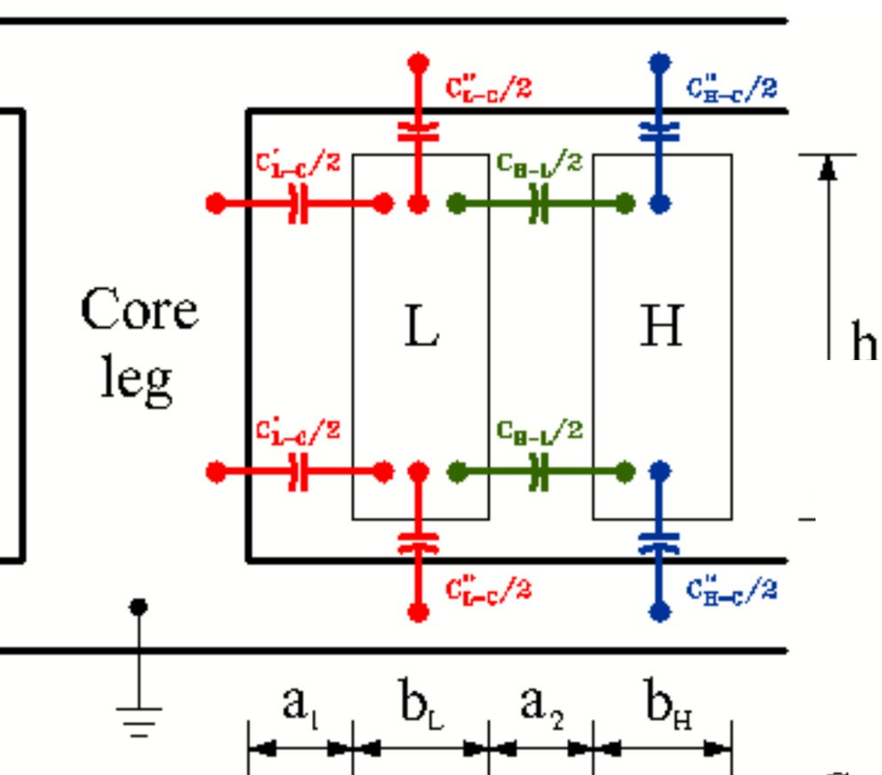
Efeito Proximidade

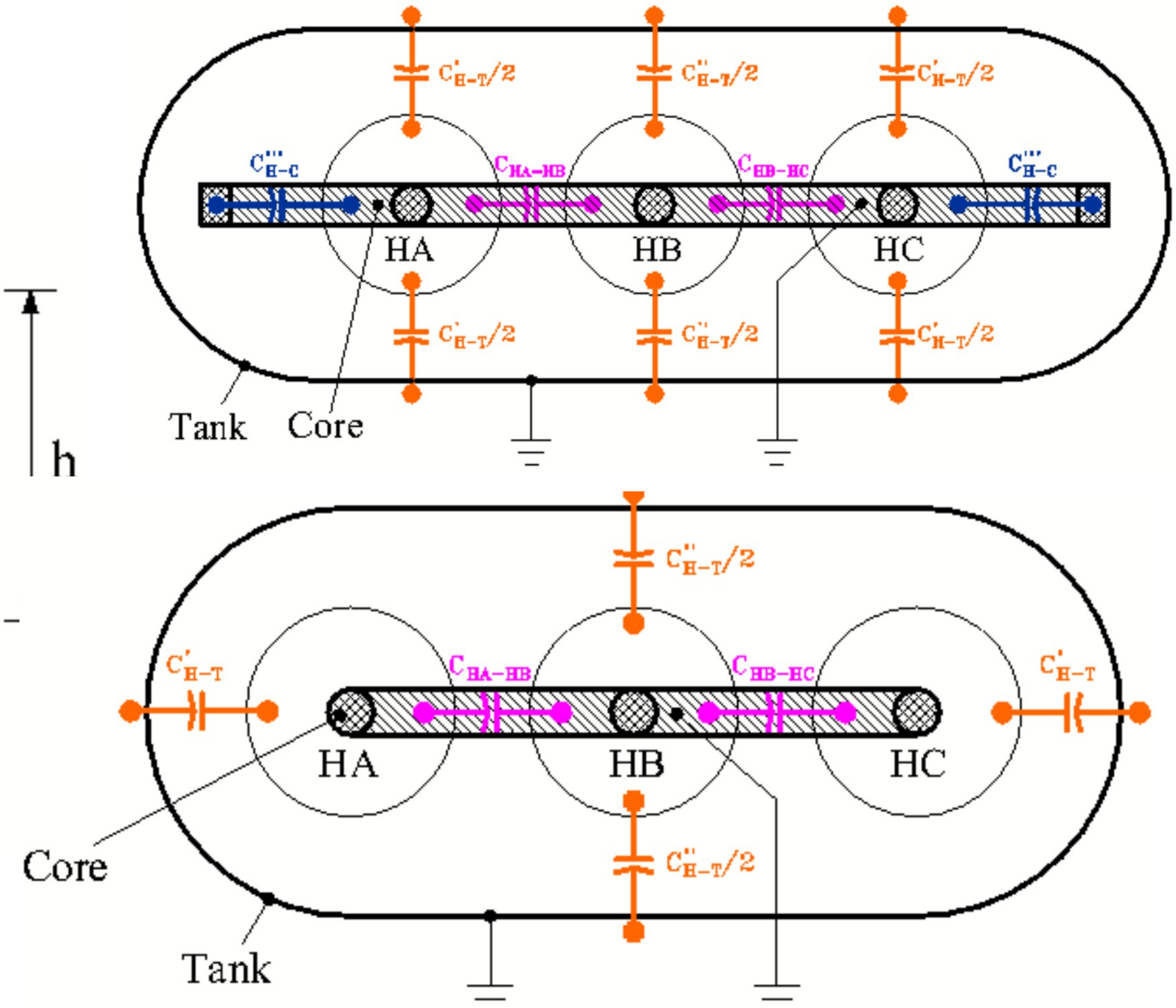


$$R_{eff} = R_S + \frac{R_1 \cdot (\omega L_1)^2}{R_1^2 + (\omega L_1)^2} + \frac{R_2 \cdot (\omega L_2)^2}{R_2^2 + (\omega L_2)^2}$$

$$F(R_1, L_1, R_2, L_2) = \sum_{i=1}^{N} [R_{given_i} - R_{eff}]^2$$

Efeito da Frequência





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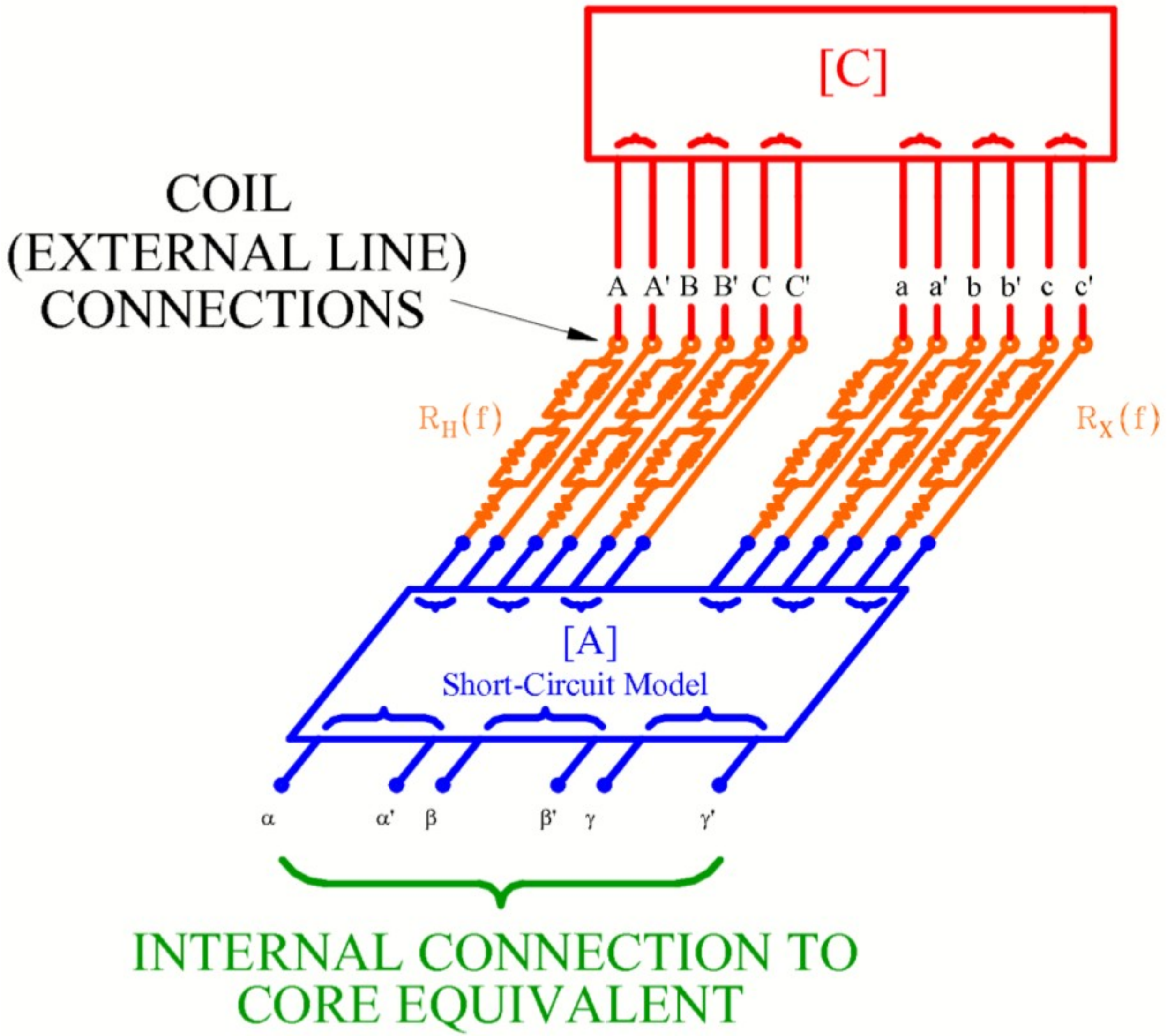

Fonte: [2]

Referências

[1] WATSON, Neville; ARRILLAGA, Jos. Power systems electromagnetic transients simulation. Iet, 2003.

[2] Chiesa, Nicola. Power transformer modelling: advanced core model. Diss. Master Thesis. Politecnico Di Milano, 2005.

[3] KRAUSE, Paul C. et al. Analysis of electric machinery and drive systems. John Wiley & Sons, 2013.