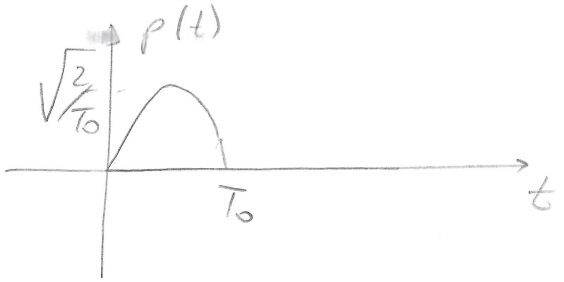
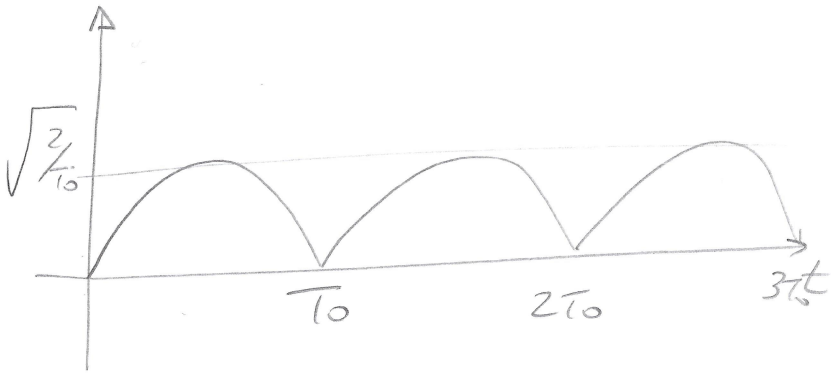
Aula 07

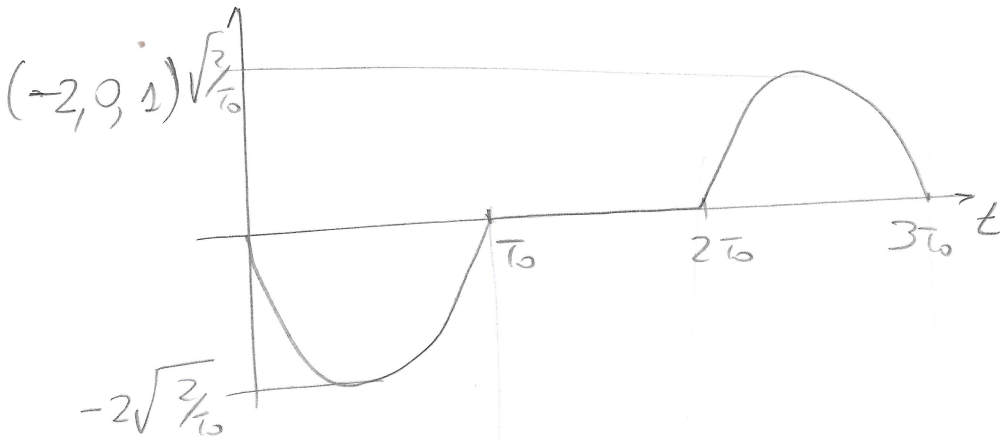
11.4-2

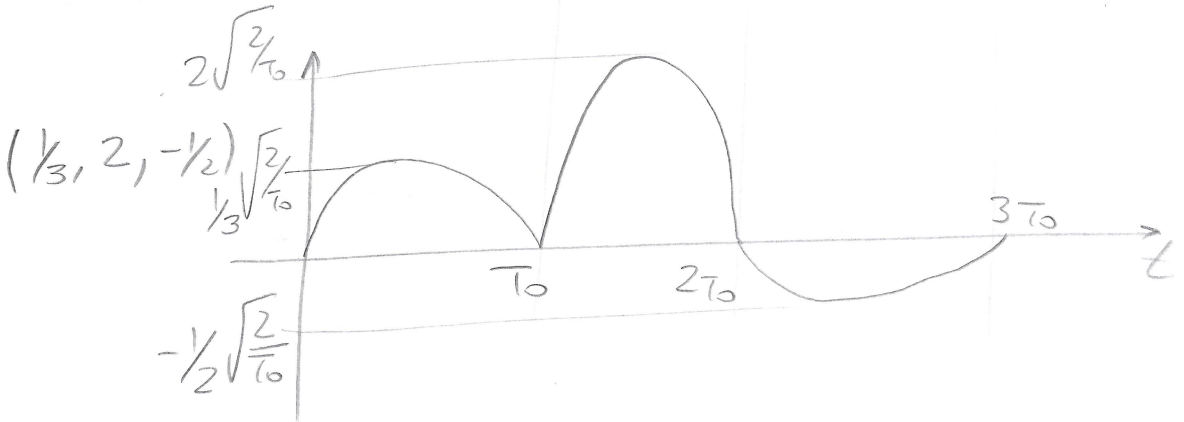
(a) $T = \frac{2\pi}{\frac{\pi}{T_0}} = 2T_0$

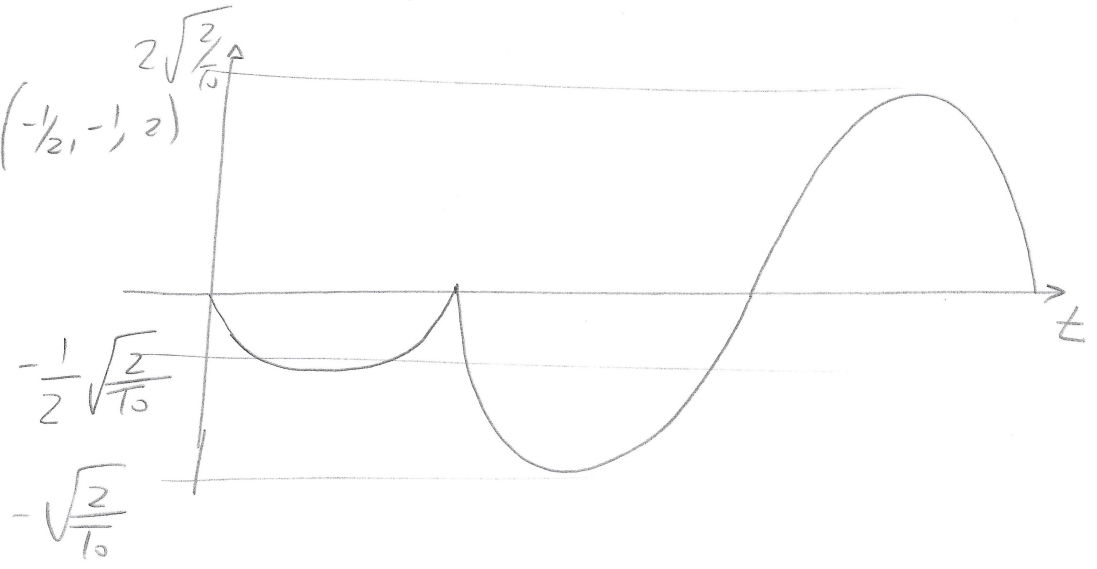


$(1, 1, 1) \rightarrow \varphi_1(t) + \varphi_2(t) + \varphi_3(t)$









(b) Sinais ortogonais

$$E_p = \left(\sqrt{\frac{2}{T_0}}\right)^2 \int_0^{T_0} \operatorname{sen}^2\left(\frac{\pi t}{T_0}\right) dt = \frac{2}{T_0} \cdot \int_0^{T_0} \left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi t}{T_0}\right)\right] dt$$

$$= \frac{2}{T_0}\left[\frac{t}{2} + \frac{1}{2}\operatorname{sen}\left(\frac{2\pi t}{T_0}\right) \cdot \frac{T_0}{2\pi}\right]_0^{T_0} =$$

$$= \frac{2}{T_0}\left[\frac{T_0}{2}\right] = 1 //$$

$E_1 = 1 + 1 + 1 = 3$

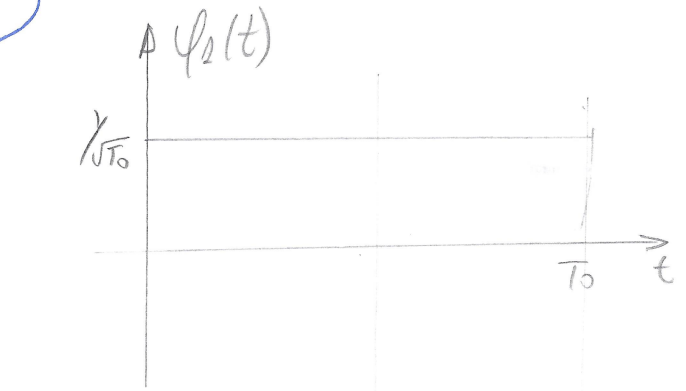
$E_2 = 4 + 0 + 1 = 5$

$E_3 = \frac{1}{9} + 4 + \frac{1}{4} \cdot \frac{4 + 144 + 9}{36} = \frac{157}{36} //$

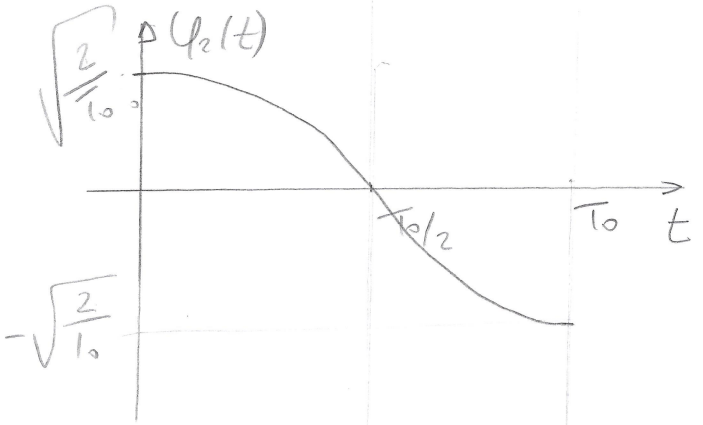
$E_4 = \frac{1}{4} + 1 + 4 = \frac{17+4+1}{4}$

$= \frac{22}{4} = \frac{11}{2} //$

11.4-3

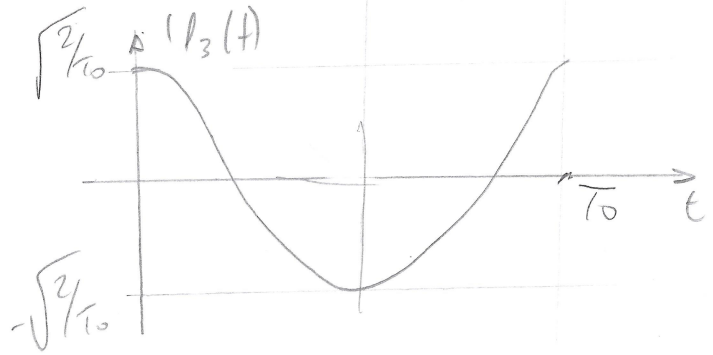


$T = \frac{2\pi}{\omega_0} = 2T_0$



Note: ortogonais entre si

Energia = 1



(a) Veja anexo - Matlab

(b) Mesmos resultados do Exercício 11.4.2:

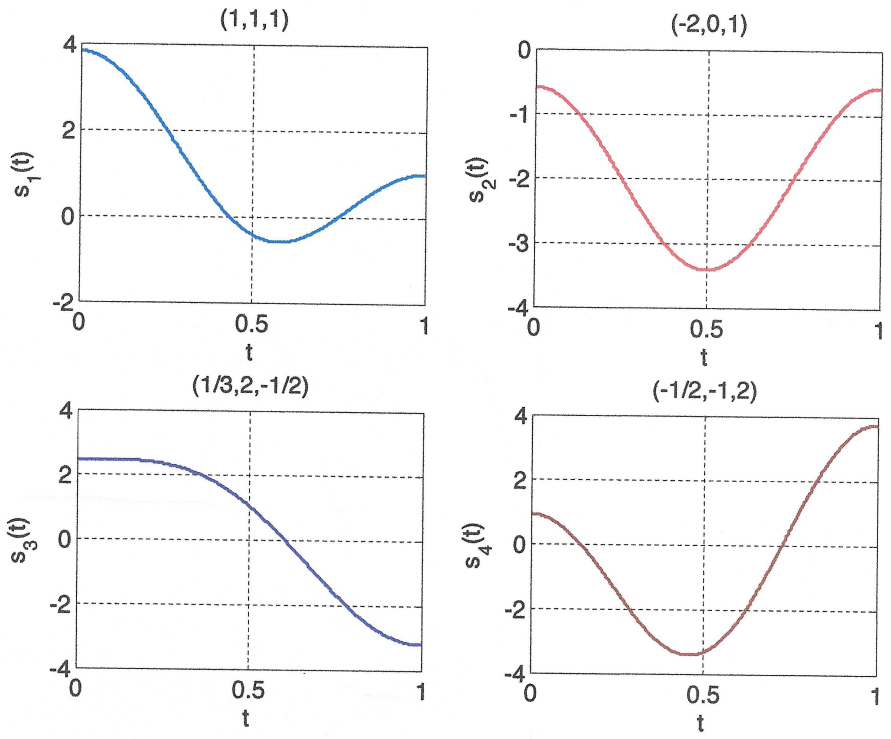
$E_1 = 3$

$E_2 = 5$

$E_3 = \frac{157}{36}$

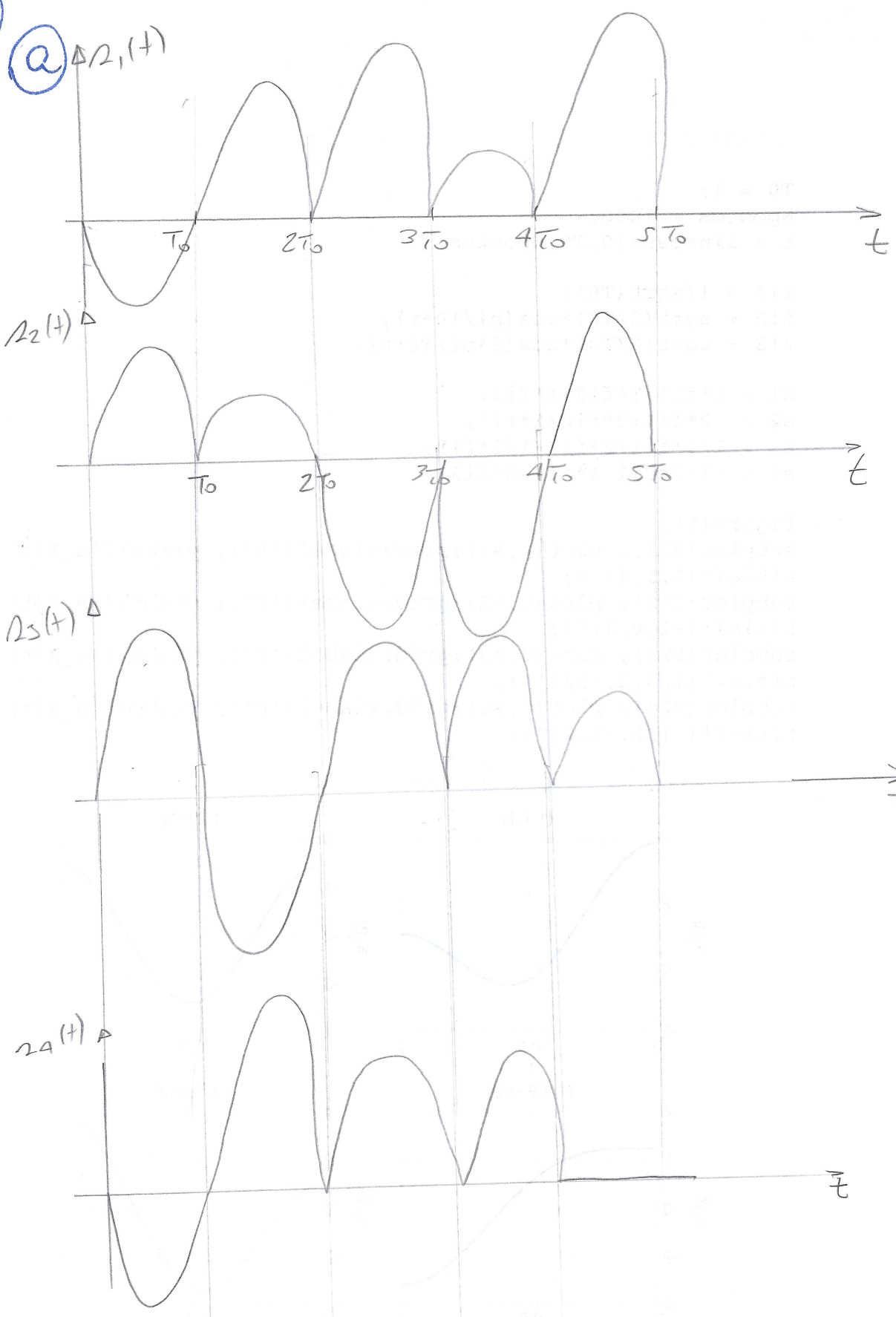
$E_4 = 11/2$

```
%%EXERCÍCIO 11.4.3

T0 = 1;
Npontos = 1000;
t = linspace(0,T0,Npontos);

fi1 = 1/sqrt(T0);
fi2 = sqrt(2/T0)*cos(pi/T0*t);
fi3 = sqrt(2/T0)*cos(2*pi/T0*t);

s1 = 1*fi1+1*fi2+1*fi3;
s2 = -2*fi1+0*fi2+1*fi3;
s3 = 1/3*fi1+2*fi2-1/2*fi3;
s4 = -1/2*fi1-1*fi2+2*fi3;

figure(1);
subplot(221); plot(t,s1);grid;xlabel('t'); ylabel('s_1(t)');
title('(1,1,1)');
subplot(222); plot(t,s2);grid;xlabel('t'); ylabel('s_2(t)');
title('(-2,0,1)');
subplot(223); plot(t,s3);grid;xlabel('t'); ylabel('s_3(t)');
title('(1/3,2,-1/2)');
subplot(224); plot(t,s4);grid;xlabel('t'); ylabel('s_4(t)');
title('(-1/2,-1,2)');
```



11.4.5

(a)



(b)

(b) Como os sinais são ortonormais,

$E_1 = (-1)^2 + (2)^2 + (3)^2 + 1^2 + 4^2 = 31$

$E_2 = 2^2 + 1^2 + (-4)^2 + (-4)^2 + \frac{2}{3}^2 = 41$

$E_3 = 3^2 + (-2)^2 + 3^2 + 4^2 + 1^2 = 39$

$E_4 = (-2)^2 + 4^2 + 2^2 + 2^2 = 28$

(c) $\cos\vartheta_{12} = \dfrac{\langle \underline{r}_1, \underline{r}_2 \rangle}{\|\underline{r}_1\| \, \|\underline{r}_2\|} = \dfrac{-2+2-12-4+8}{\sqrt{31}\sqrt{38}} = \dfrac{-8}{\sqrt{31}\sqrt{38}} \Rightarrow$

41 41

$\boxed{\vartheta_{12} = 103,46°}$

$\cos\vartheta_{13} = \dfrac{\langle r_1, r_3 \rangle}{\|r_1\| \, \|r_3\|} = \dfrac{-3-4+9+4+4}{\sqrt{31}\sqrt{39}} = \dfrac{10}{\sqrt{31}\sqrt{39}}$

$\Rightarrow \boxed{\vartheta_{13} = 73,28°}$

$\cos\vartheta_{14} = \dfrac{\langle r_1, r_4 \rangle}{\|r_1\| \, \|r_4\|} = \dfrac{2+8+6+2+0}{\sqrt{31}\sqrt{28}} = \dfrac{18}{\sqrt{31}\sqrt{28}}$

$\Rightarrow \boxed{\vartheta_{14} = 52,34°}$

$\cos\vartheta_{23} = \dfrac{\langle r_2, r_3 \rangle}{\|r_2\| \, \|r_3\|} = \dfrac{6-2-12-16+2}{\sqrt{38}\sqrt{39}} = \dfrac{-22}{\sqrt{38}\sqrt{39}}$

41 39

$\boxed{\vartheta_{23} = 123,37°}$

$$\cos\theta_{24} = \frac{\langle \underline{z}_2, \underline{z}_4 \rangle}{\|\underline{z}_2\| \|\underline{z}_4\|} = \frac{-4+4-8-8+0}{\sqrt{38}\sqrt{28}} = \frac{-16}{\sqrt{38}\sqrt{28}}$$

41

$$\boxed{\theta_{24} = 126{,}18^\circ}$$

$$\cos\theta_{34} = \frac{\langle \underline{z}_3, \underline{z}_4 \rangle}{\|\underline{z}_3\| \|\underline{z}_4\|} = \frac{-6-8+6+8+0}{\sqrt{39}\sqrt{28}} = 0$$

$$\boxed{\theta_{34} = 90^\circ}$$

11.6-3 Para $M=2$, $N = 2.10^{-8}$

Para 400 000 bits/s a transmissão banda base requer no mínimo $\frac{400\,000}{2} = 200\,\text{kHz}$

$$\boxed{B_T = 200\,\text{kHz}}$$

$$10^{-6} = Q\left(\sqrt{\frac{2E_b}{N}}\right) \Rightarrow \sqrt{\frac{2E_b}{N}} \simeq 4{,}75 \Rightarrow \frac{2E_b}{N} = 22{,}56$$

$$\Rightarrow \boxed{E_b = 22{,}56 \cdot 10^{-8}} \quad S_I = E_b R_b = 22{,}56 \cdot 10^{-8} \cdot 400\,000 = 0{,}09024\ V^2/\text{Hz}$$

Para $M = 16$

$$B_T = \frac{4}{\cancel{800}} = \frac{10}{\cancel{50}}\ kHz$$

$$P_{em} = 2 \cdot \frac{15}{16} \cdot Q\left(\sqrt{\frac{6.4}{255}\left(\frac{E_b}{N}\right)}\right) =$$

$$= \frac{15}{8} Q\left(\sqrt{\frac{24}{255}\left(\frac{E_b}{N}\right)}\right)$$

$$P_e = \frac{P_{em}}{\log_2 M} \text{ (código de Gray)} \Rightarrow P_e = \frac{15}{32} Q\left(\sqrt{\frac{24}{255}\left(\frac{E_b}{N}\right)}\right) = 10^{\cancel{-5}\ -6}$$

$$\Rightarrow \frac{\cancel{15}}{\cancel{32}} Q\left(\sqrt{\frac{24}{255}\left(\frac{E_b}{N}\right)}\right) = \frac{32}{15} \cdot 10^{-\cancel{5}\ 6} \Rightarrow$$

$$\Rightarrow \frac{24}{255} \frac{E_b}{N} = \frac{\cancel{21,16}}{\cancel{2,33}} \Rightarrow \frac{E_b}{N} = \frac{\cancel{134,68}}{\cancel{58,225}} \Rightarrow 224,82$$

$$\Rightarrow E_b = \frac{\cancel{4,496}}{\cancel{7633}} \cdot 10^{-6}$$

$$S_I = E_b \cdot R_b = \frac{\cancel{4,496}}{\cancel{7633}} 10^{-6} \cdot 400\,000 = \frac{\cancel{1,7986}}{\cancel{0,4588}}\ V^2/2$$

Para $M=32$

$$B_T = \frac{4}{5}\cancel{200} = \cancel{8}0\,KHz$$

$$P_{eM} = 2\cdot\frac{31}{32}\,Q\left(\sqrt{\frac{6\cdot 5}{1023}\,\frac{E_b}{N}}\right)$$

$$P_b = \frac{P_{em}}{log_2^M} = 2\cdot\frac{31}{32\cdot 5}\,Q\left(\sqrt{\frac{30}{1023}\,\frac{E_b}{N}}\right) = 10^{-\cancel{6}\,6}$$

$$\Rightarrow Q\left(\sqrt{\frac{30}{1023}\cdot\frac{E_b}{N}}\right) = 2{,}58\cdot 10^{-\cancel{6}\,6}$$

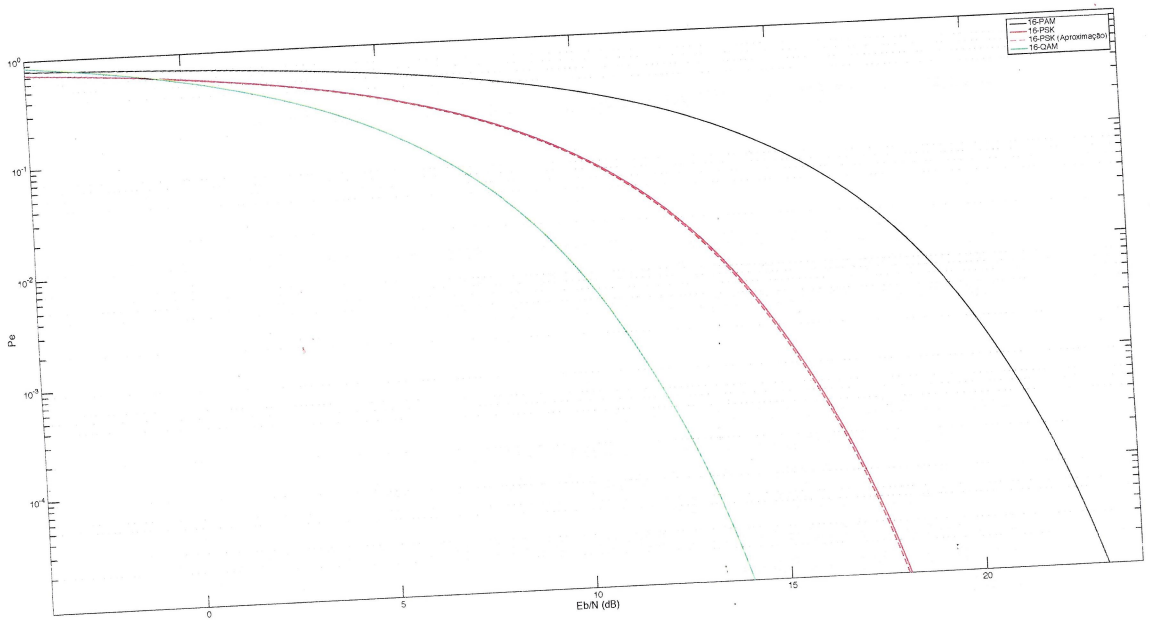
$$\sqrt{\frac{30}{1023}\,\frac{E_b}{N}} \approx \cancel{5{,}1}\,4{,}56 \Rightarrow$$

$$\frac{E_b}{N} \approx \cancel{1012{,}85}\,709{,}061 \Rightarrow E_b \approx \cancel{1025}\,1{,}118\cdot 10^{-5}$$

$$\Rightarrow S_I = E_b\cdot R_b = \cancel{8{,}1038}\,5{,}672\ W$$

11.6.9

```
% Exercício 11.6.9 do Lathi

clear all; close all;
EbNdB = linspace(-4,24,1000);
EbN = 10.^(EbNdB/10);
M = 16;

%16-PAM

PePAM = 2*(M-1)/M*Que(sqrt(6*log2(M)*EbN/(M^2-1)));

%16-PSK

fun = @(theta,EbN,M,raiz,expo) expo*(1+raiz*cos(theta).*...
exp(EbN*cos(theta).^2*log2(M)).*(1-
Que(sqrt(2*EbN*log2(M))*cos(theta))));


for indEb = 1:length(EbN),
    expo = exp(-(EbN(indEb)*log2(M)));
    raiz = sqrt(4*pi*log2(M)*EbN(indEb));
    PePSK(indEb) = 1-
integral(@(theta)fun(theta,EbN(indEb),M,raiz,expo), -
pi/M,pi/M)/(2*pi);
end

PePSKapprox  = 2*Que(sqrt(2*EbN*log2(M))*(pi/M));

%16-QAM
PeQAM = 3*Que(sqrt(4/5*EbN));

figure(1);
semilogy(EbNdB, PePAM, 'k',EbNdB, PePSK, 'r', EbNdB, PePSKapprox, 'r--
', EbNdB, PeQAM, 'g');
legend('16-PAM', '16-PSK', '16-PSK (AproximaÁ„o)', '16-QAM');
grid;
xlabel('Eb/N (dB)');
ylabel('Pe');
axis([-4 24 1e-5 1])
```



16-QAM tem desempenho superior, em termos de BER, seguido pelo 16-PSK e 16-PAM.