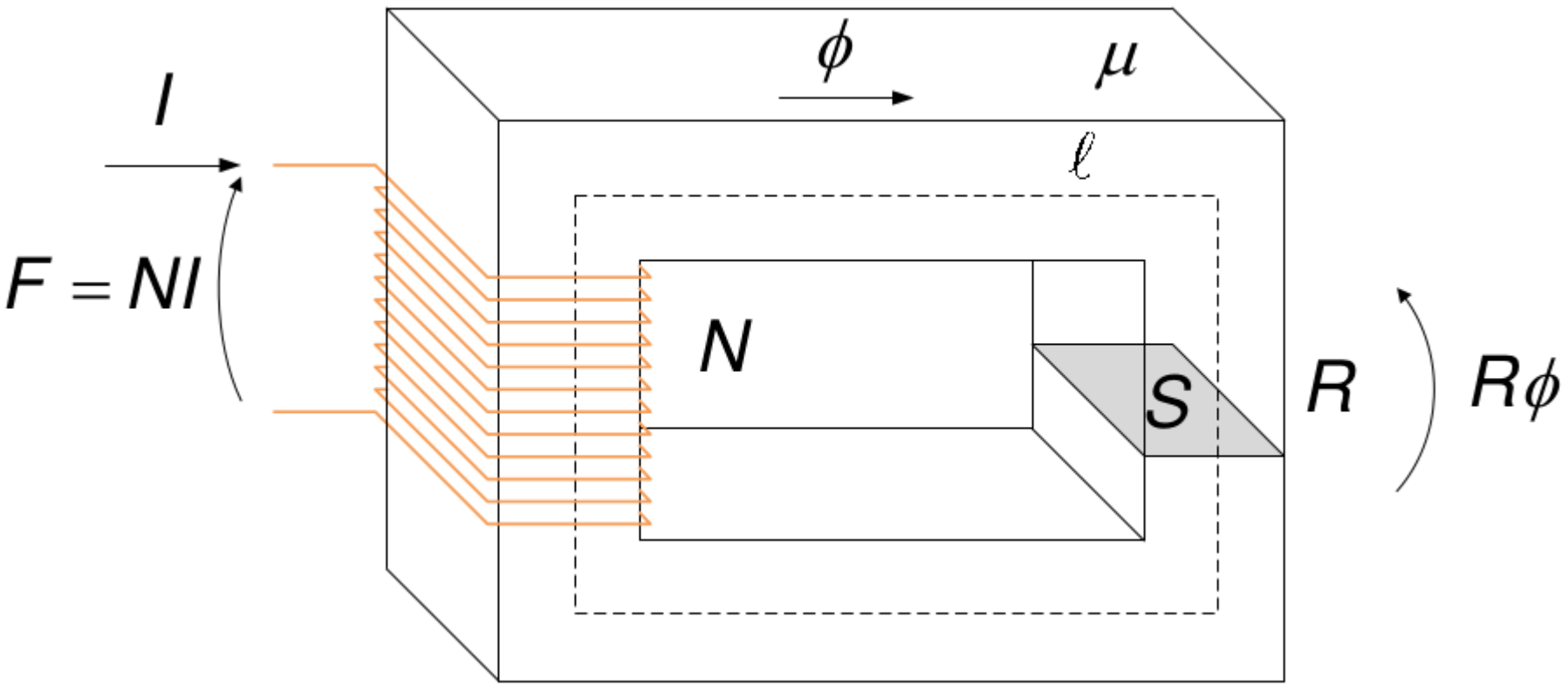
PEA5009 **Aula 6**

Modelagem e simulação numérica de componentes de sistemas elétricos de potência

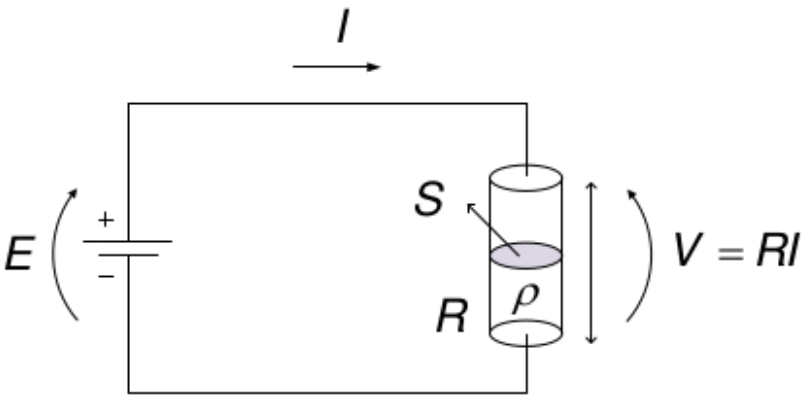
Introdução Modelos Elétricos para Transformadores

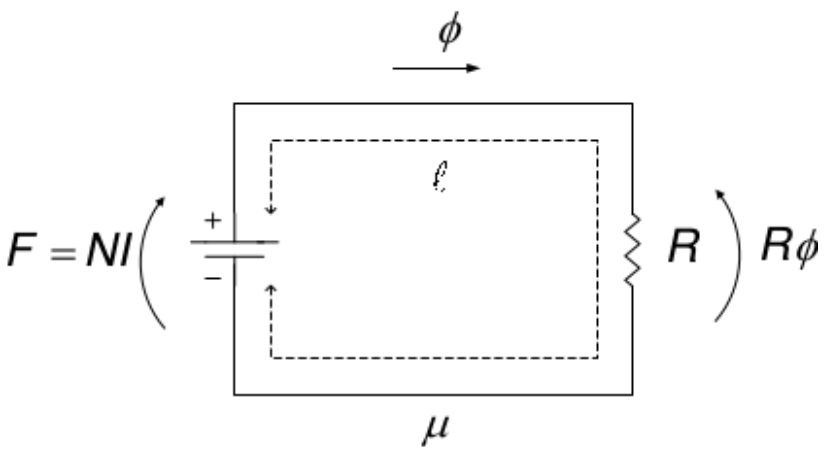
Prof. Dr. Eduardo Coelho Marques da Costa
Prof. Dr. Renato Machado Monaro



Fonte: [2]

Comparação circuito magnético x circuito elétrico





E: Força eletromotriz ou tensão interna (V)

I: corrente (A)

R: resistência $R = \dfrac{\rho}{\ell S}$ (Ω)

ρ: resistividade (Ωm)

S: seção transversal (m^2)

ℓ: comprimento (m)

$F = NI$: Força magnetomotriz (Aesp)

ϕ: fluxo magnético (Wb)

R: relutância $R = \dfrac{\ell}{\mu S}$ (Aesp/Wb)

μ: permeabilidade magnética (H/m)

S: seção transversal (m^2)

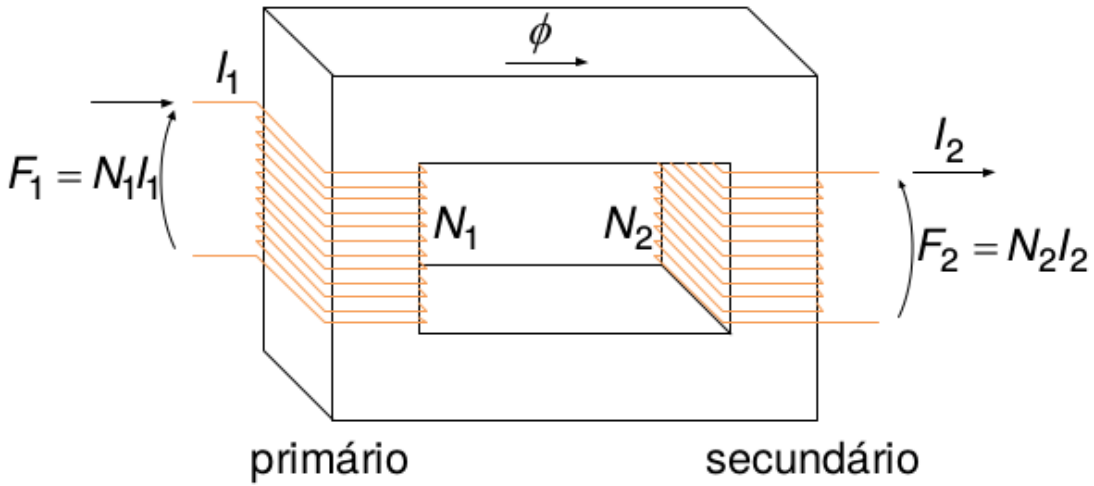
ℓ: comprimento do circuito (m)

Fonte: [2]

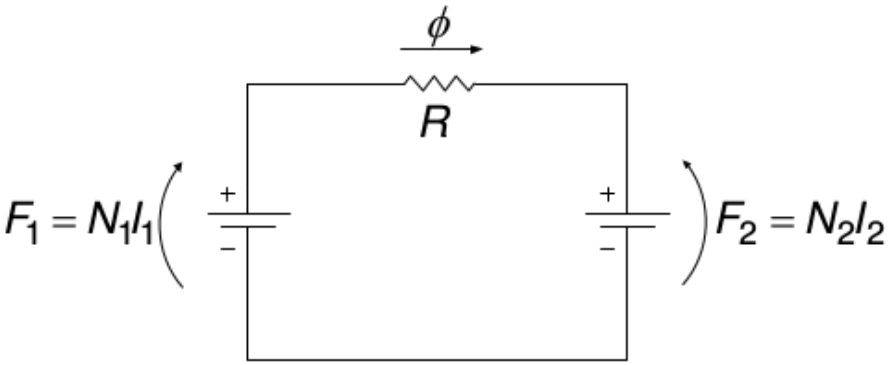
PEA5009 Aula 6

Introdução Modelos Elétricos para Transformadores

Construção básica:



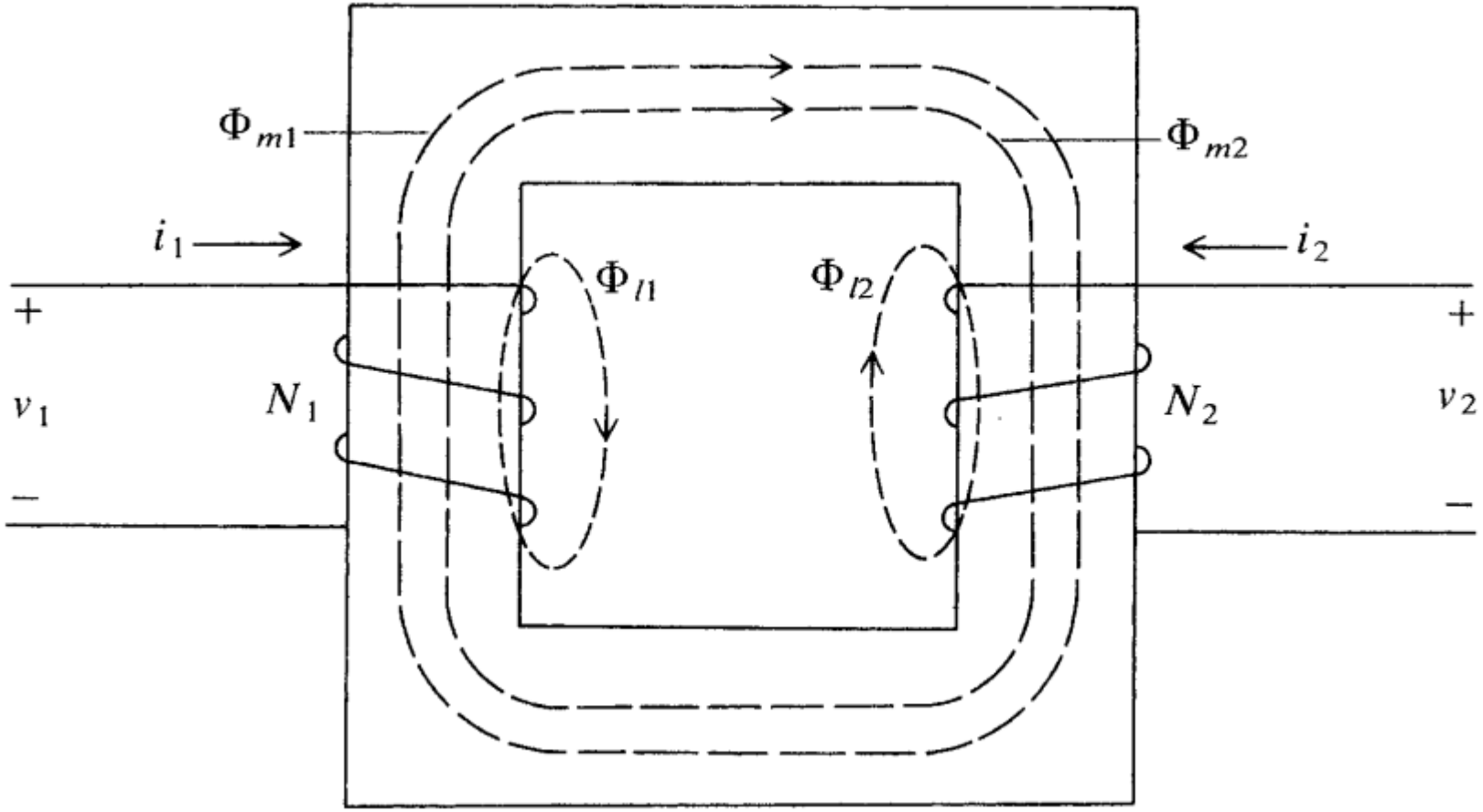
Circuito magnético:



Equação:

$$N_1 I_1 - N_2 I_2 - R\phi = 0$$

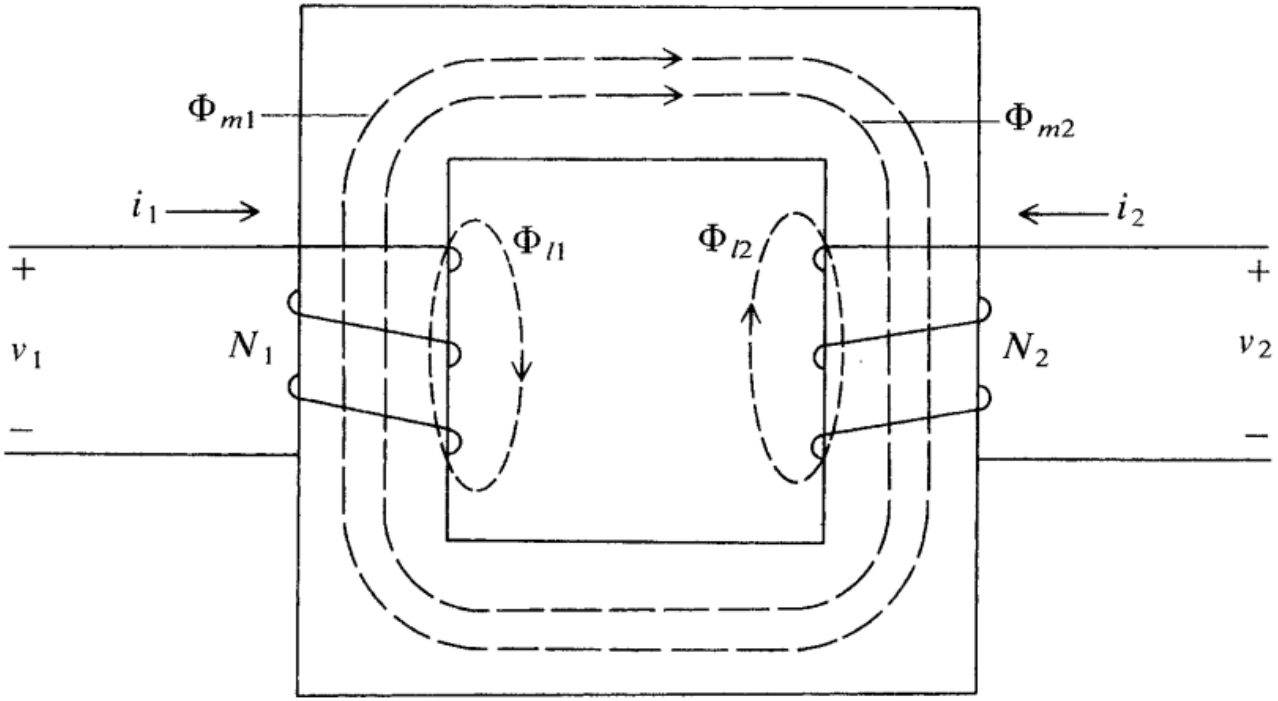
Fonte: [2]



$$\Phi_1 = \Phi_{l1} + \Phi_{m1} + \Phi_{m2}$$

$$\Phi_2 = \Phi_{l2} + \Phi_{m2} + \Phi_{m1}$$

Fonte: [1]



$$\mathbf{v} = \mathbf{ri} + \frac{d\boldsymbol{\lambda}}{dt}$$

$$\lambda_1 = N_1\Phi_1$$

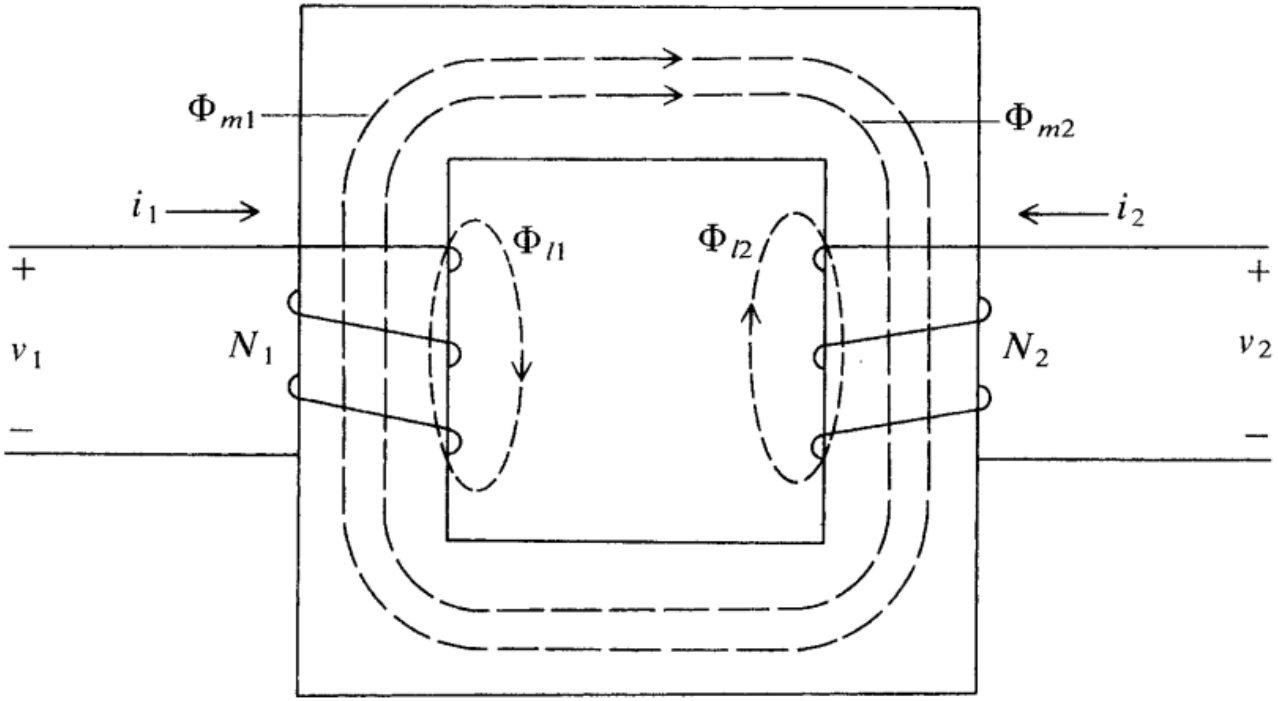
$$\lambda_2 = N_2\Phi_2$$

$$\Phi_{l1} = \frac{N_1 i_1}{\mathcal{R}_{l1}} \qquad \Phi_{m1} = \frac{N_1 i_1}{\mathcal{R}_m}$$

$$\Phi_{l2} = \frac{N_2 i_2}{\mathcal{R}_{l2}} \qquad \Phi_{m2} = \frac{N_2 i_2}{\mathcal{R}_m}$$

Fonte: [1]

Introdução Modelos Elétricos para Transformadores



$$\mathscr{R} = \frac{l}{\mu A}$$

$$\Phi_1 = \frac{N_1 i_1}{\mathscr{R}_{l1}} + \frac{N_1 i_1}{\mathscr{R}_m} + \frac{N_2 i_2}{\mathscr{R}_m} \qquad \Phi_2 = \frac{N_2 i_2}{\mathscr{R}_{l2}} + \frac{N_2 i_2}{\mathscr{R}_m} + \frac{N_1 i_1}{\mathscr{R}_m}$$

Fonte: [1]

$$\Phi_1 = \frac{N_1 i_1}{\mathscr{R}_{l1}} + \frac{N_1 i_1}{\mathscr{R}_m} + \frac{N_2 i_2}{\mathscr{R}_m}$$

$$\lambda_1 = \frac{N_1^2}{\mathscr{R}_{l1}} i_1 + \frac{N_1^2}{\mathscr{R}_m} i_1 + \frac{N_1 N_2}{\mathscr{R}_m} i_2$$

$$\Phi_2 = \frac{N_2 i_2}{\mathscr{R}_{l2}} + \frac{N_2 i_2}{\mathscr{R}_m} + \frac{N_1 i_1}{\mathscr{R}_m}$$

$$\lambda_2 = \frac{N_2^2}{\mathscr{R}_{l2}} i_2 + \frac{N_2^2}{\mathscr{R}_m} i_2 + \frac{N_2 N_1}{\mathscr{R}_m} i_1$$

Fonte: [1]

Considerando o circuito magnético linear

$$L_{11} = \frac{N_1^2}{\mathcal{R}_{l1}} + \frac{N_1^2}{\mathcal{R}_m} = L_{l1} + L_{m1} \qquad L_{22} = \frac{N_2^2}{\mathcal{R}_{l2}} + \frac{N_2^2}{\mathcal{R}_m} = L_{l2} + L_{m2}$$

$$\frac{L_{m2}}{N_2^2} = \frac{L_{m1}}{N_1^2}$$

$$L_{12} = \frac{N_1 N_2}{\mathcal{R}_m} \qquad L_{21} = \frac{N_2 N_1}{\mathcal{R}_m}$$

$$L_{12} = \frac{N_2}{N_1} L_{m1} = \frac{N_1}{N_2} L_{m2}$$

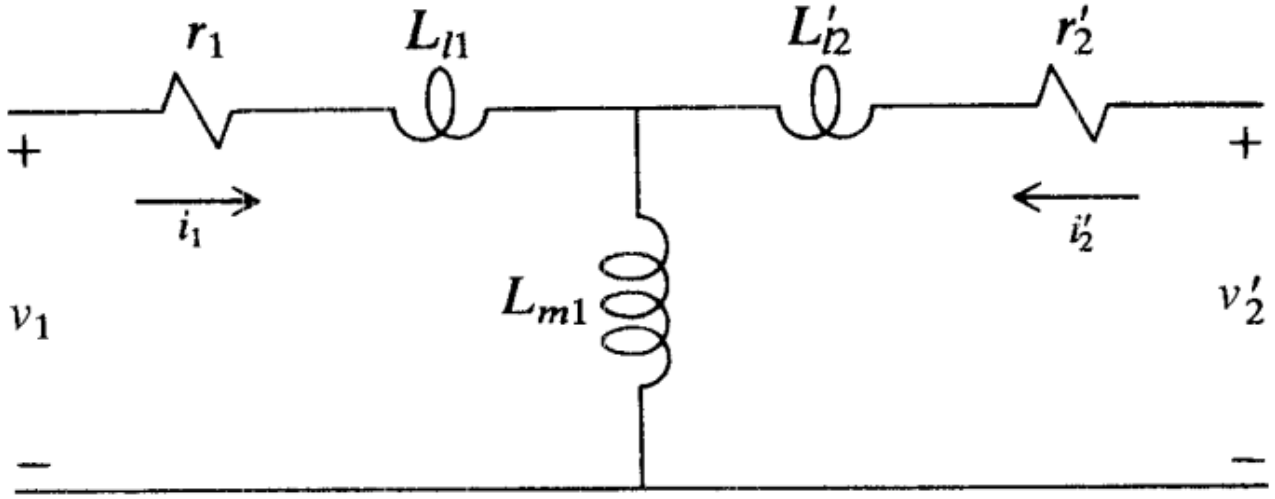
$$\boldsymbol{\lambda} = \mathbf{L}\mathbf{i}$$

$$\mathbf{L} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} = \begin{bmatrix} L_{l1} + L_{m1} & \frac{N_2}{N_1} L_{m1} \\ \frac{N_1}{N_2} L_{m2} & L_{l2} + L_{m2} \end{bmatrix}$$

$$\lambda_1 = L_{l1} i_1 + L_{m1} \left(i_1 + \frac{N_2}{N_1} i_2 \right)$$

$$\lambda_2 = L_{l2} i_2 + L_{m2} \left(\frac{N_1}{N_2} i_1 + i_2 \right)$$

Levando ao circuito T



$$i'_2 = \frac{N_2}{N_1} i_2 \qquad v'_2 = \frac{N_1}{N_2} v_2 \qquad \lambda'_2 = \frac{N_1}{N_2} \lambda_2$$

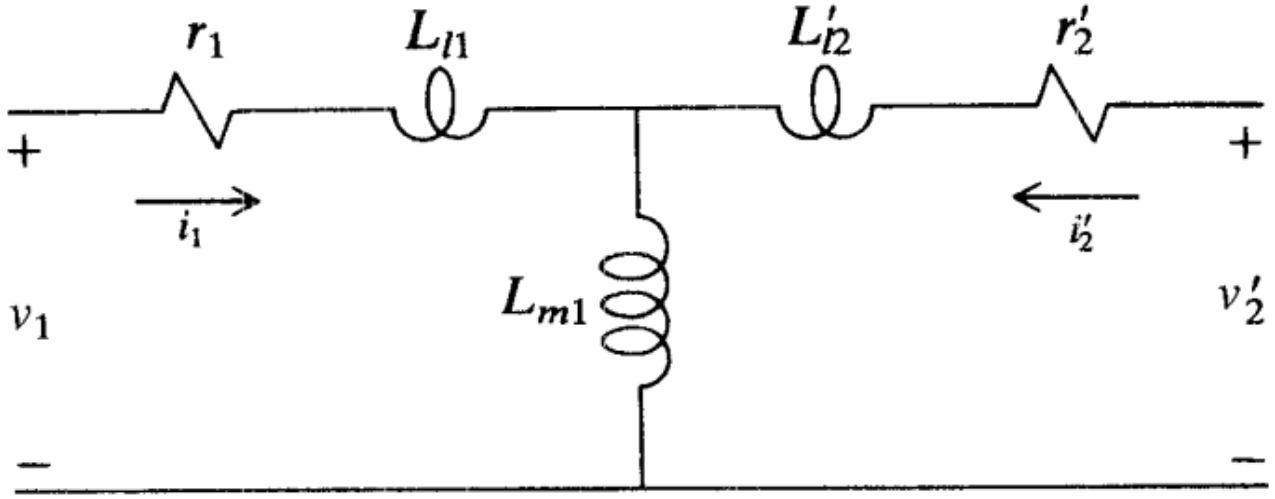
$$\lambda_1 = L_{l1} i_1 + L_{m1}(i_1 + i'_2)$$

$$\lambda'_2 = L'_{l2} i'_2 + L_{m1}(i_1 + i'_2)$$

$$L'_{l2} = \left(\frac{N_1}{N_2}\right)^2 L_{l2}$$

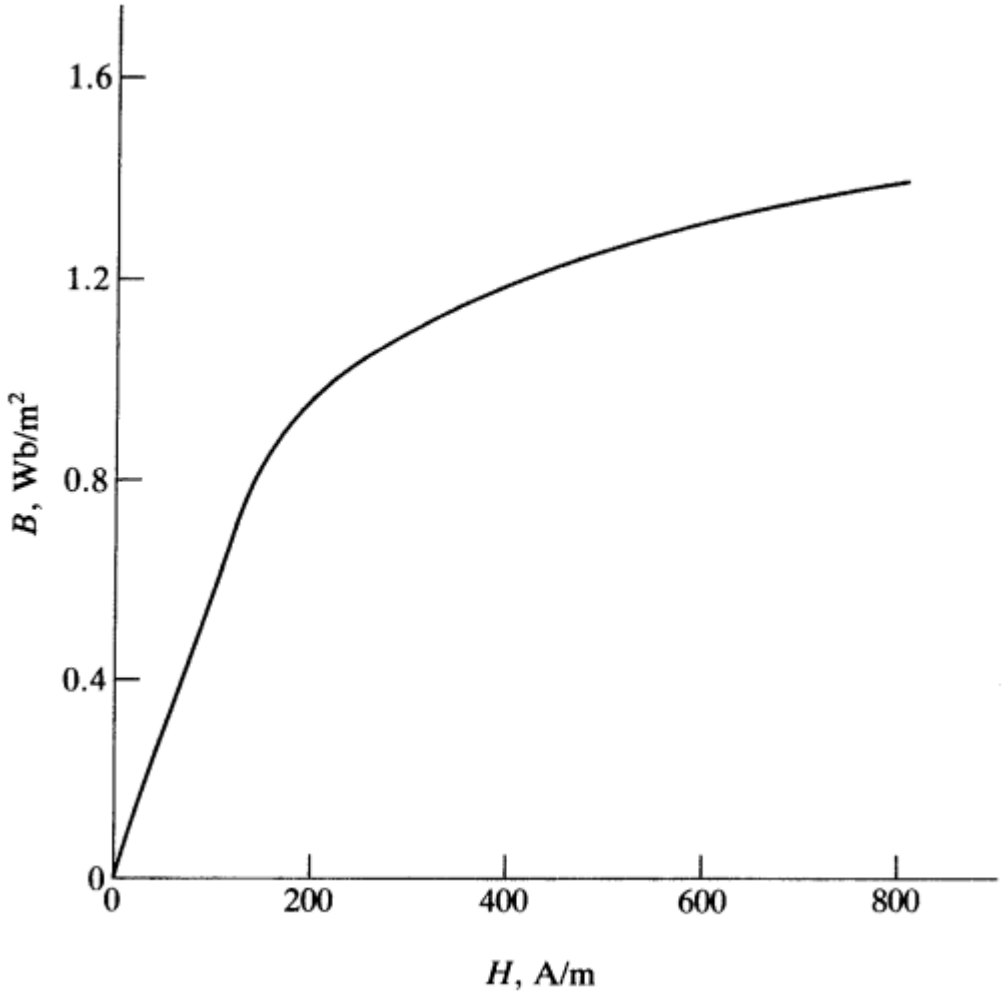
Fonte: [1]

Levando ao circuito T



$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt} \qquad v'_2 = r'_2 i'_2 + \frac{d\lambda'_2}{dt} \qquad r'_2 = \left(\frac{N_1}{N_2}\right)^2 r_2$$

Circuitos Magnéticos não Lineares



$$\lambda_1 = L_{l1} i_1 + \lambda_m$$

$$\lambda'_2 = L'_{l2} i'_2 + \lambda_m$$

$$\lambda_m = L_{m1}(i_1 + i'_2)$$

$$i_1 = \frac{1}{L_{l1}}(\lambda_1 - \lambda_m)$$

$$i'_2 = \frac{1}{L'_{l2}}(\lambda'_2 - \lambda_m)$$

Fonte: [1]

Circuitos Magnéticos não Lineares

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt} \qquad v'_2 = r'_2 i'_2 + \frac{d\lambda'_2}{dt}$$

$$\lambda_1 = \int \left[v_1 + \frac{r_1}{L_{l1}} (\lambda_m - \lambda_1) \right] dt \qquad \lambda_m = L_a \left(\frac{\lambda_1}{L_{l1}} + \frac{\lambda'_2}{L'_{l2}} \right)$$

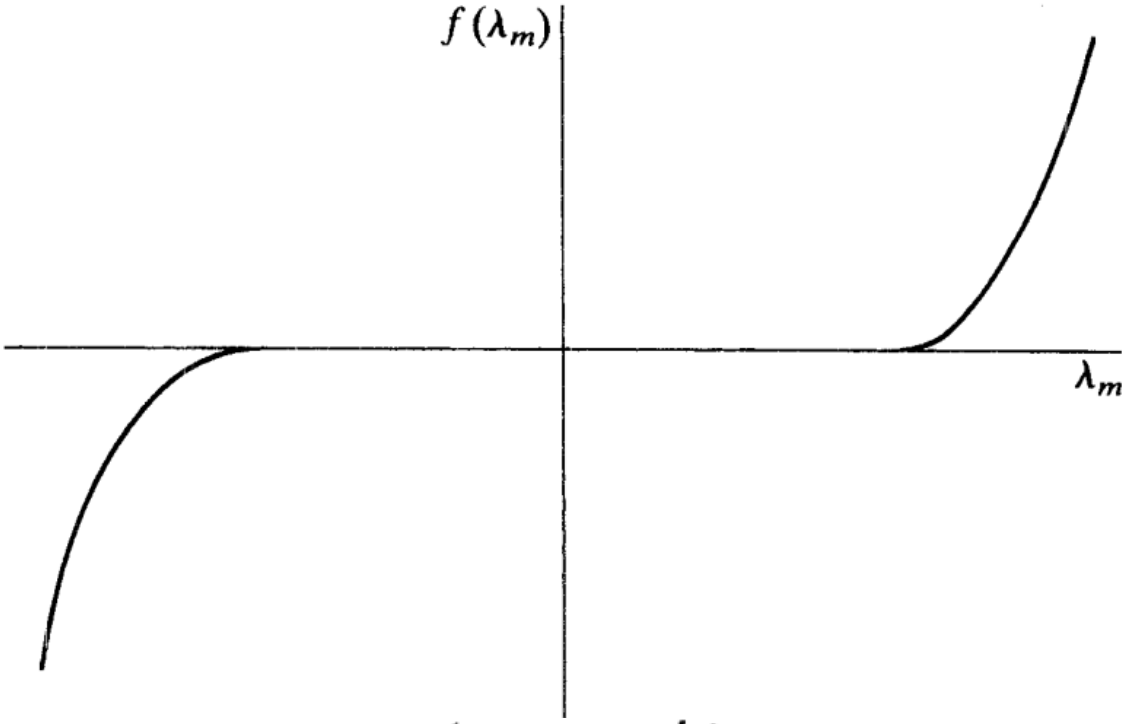
$$\lambda'_2 = \int \left[v'_2 + \frac{r'_2}{L'_{l2}} (\lambda_m - \lambda'_2) \right] dt \qquad L_a = \left(\frac{1}{L_{m1}} + \frac{1}{L_{l1}} + \frac{1}{L_{l2}} \right)^{-1}$$

Fonte: [1]

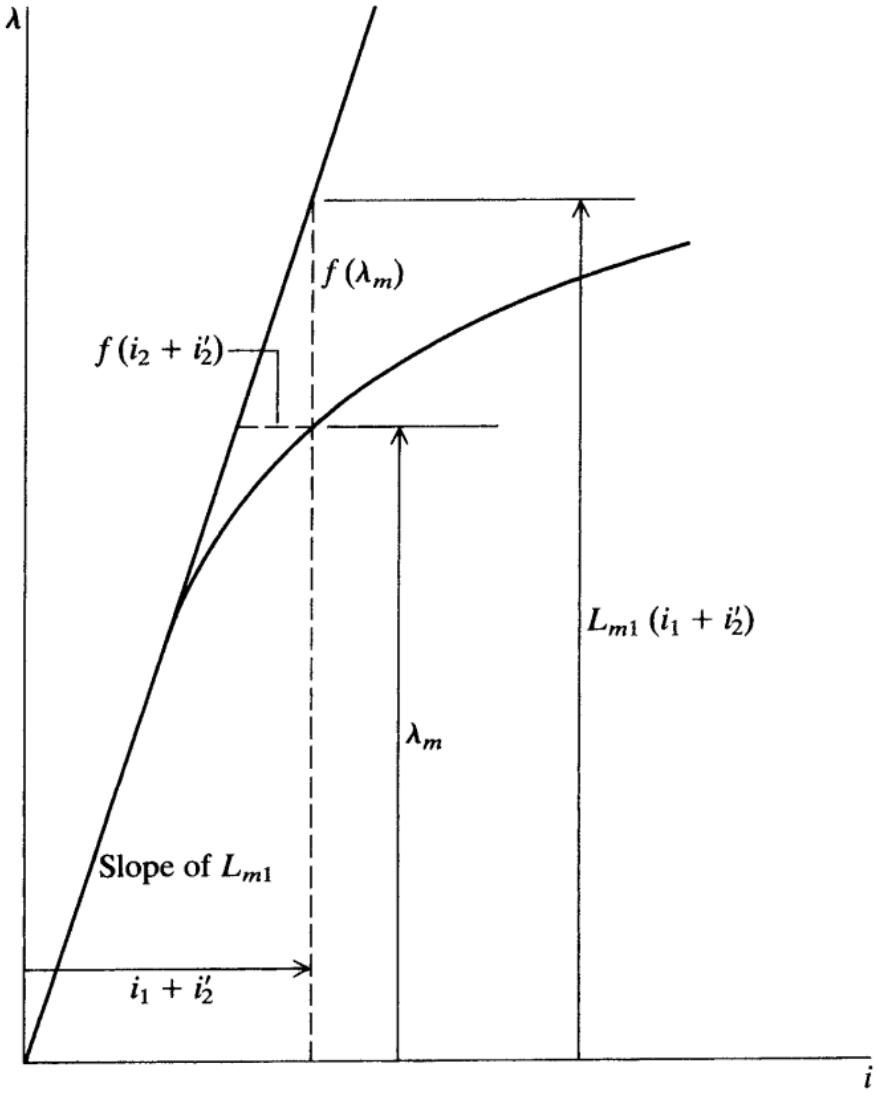
Espaço de Estado

Circuitos Magnéticos não Lineares

$$\lambda_m = L_{m1}(i_1 + i'_2) - f(\lambda_m)$$



$$\lambda_m = L_a\left(\frac{\lambda_1}{L_{l1}} + \frac{\lambda'_2}{L'_{l2}}\right) - \frac{L_a}{L_{m1}} f(\lambda_m)$$



Fonte: [1]

Referências

[1] KRAUSE, Paul C. et al. Analysis of electric machinery and drive systems. John Wiley & Sons, 2013.

[2] PEA5732- Modelagem de Componentes de Sistemas Elétricos de Potência para Cálculos Elétricos. Carlos Eduardo de Morais Pereira e Silvio Giuseppe Di Santo