SIMPLIFIED METHOD FOR CONSOLIDATION RATE OF STONE COLUMN REINFORCED FOUNDATIONS

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ABSTRACT: Field observations and numerical studies demonstrated that stone columns could accelerate the rate of consolidation of soft clays. A simplified method for computing the rate of consolidation is presented in this paper by assuming that stone columns; (1) are free draining; (2) have higher drained elastic modulus than soft clay; and (3) are deformed 1D. The formats of the final solutions in vertical and radial flows are similar to those of the Terzaghi 1D solution and the Barron solution for drain wells in fine-grained soils, respectively. Modified coefficients of consolidation are introduced to account for effects of the stone column-soil modular ratio. The new solutions demonstrate stress transfer from the soil to stone columns and dissipation of excess pore water pressures due to drainage and vertical stress reduction during the consolidation. Comparisons between the results from this simplified method and the numerical study by Balaam and Booker in 1981 exhibit reasonable agreement, when the stress concentration ratio is in the practical range (2–6). The discrepancies in the results from these two methods are discussed. This paper also includes design charts and a design example.

INTRODUCTION

Stone columns, one of the most commonly used soil improvement techniques, have been utilized worldwide to increase bearing capacity and reduce total and differential settlements of superstructures constructed on soft clays. A number of publications have been written on the development of theoretical solutions for estimating bearing capacity and settlement of reinforced foundations by stone columns (Aboshi et al. 1979; Barksdale and Bachus 1983; Priebe 1995). Therefore, these topics will not be explored in this paper. Field observations showed that stone columns could also accelerate the rate of consolidation of soft clays (Munfakh et al. 1983; Han and Ye 1992). Field pore water pressure measurement under an embankment indicated that a homogenous clay stratum outside a stone column treated area only completed 25% primary consolidation when the stone column area had reached 100% primary consolidation (Munfakh et al. 1983). Han and Ye (1992) also reported that the rates of settlement of two similar buildings, one on an unreinforced foundation and the other on a stone column reinforced foundation on the same site, reached 66 and 95%, respectively over the same time period (480 days). The acceleration of the consolidation rate was accredited to stone columns for providing a drainage path and relieving excess pore water pressures by transferring the load from the soil. A numerical study demonstrated that an increase of stone column-soil modular ratio can increase the rate of consolidation of soft clays under a rigid raft but not under a flexible raft (Balaam and Booker 1981). The numerical solutions are excellent but need high computational efforts, which may not be convenient for practitioners. The objective of this paper is to develop simplified and closed-form solutions for estimating the rate of consolidation of reinforced foundations by stone columns with reasonable accuracy. In this study, a stone column is treated as a free-draining path with a higher drained modulus than the surrounding soil. Stone columns and the surrounding soil deform equally, as was the case of equal vertical strain investigated by Barron (1947). Some studies also found that the effectiveness of a stone column as a drainage path might be degraded because installation of the stone column could disturb the surrounding soil, and fine-grained soils could be mixed into the column (Barksdale and Bachus 1983). The study of these adverse effects on the rate of consolidation is beyond the scope of this paper, and they are not included in the theoretical development herein.

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Note. Discussion open until December 1, 2001. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on October 5, 1999; revised January 5, 2001. This paper is part of the ***Journal of Geotechnical and Geoenvironmental Engineering***, Vol. 127, No. 7, July, 2001. ©ASCE, ISSN 1090-0241/01/0007-0597–0603/$8.00 + $.50 per page. Paper No. 22060.

REVIEW OF THE BARRON SOLUTION

Before jumping to the proposed simplified solutions, it is helpful to review the Barron (1947) solution, which dealt with consolidation of fine-grained soils by drain wells. Stone columns and drained wells have two major differences. First, stone columns have a larger drained elastic modulus than the surrounding soft clay. The typical elastic modulus ratios of stone column to soft clay range from 10 to 20 (Barksdale and Bachus 1983). As pointed out by Lane (1948), Barron's solution ignored the effect of the stiffness difference between the sand well and the surrounding soil on the consolidation rate. Second, stone columns have a smaller diameter ratio (influence diameter/column diameter) than drain wells. Typical diameter ratios for stone columns range from 1.5 to 5. However, the values for well diameter ratios used by Barron (1947) ranged from 5 to 100. Based on the objectives of this paper, the Barron solution for the case of equal vertical strain is reviewed below.

In his derivation, Barron (1947) assumed that: (1) water in a saturated soil is incompressible, and at the moment of loading, excess pore water pressure carries all the vertical loads; (2) soil mass only deforms vertically; (3) each drain well has a circular influence zone; and (4) loads distribute uniformly over the compressible soil zone. Considering that the reduction of soil volume is equal to the discharge of water from the soil, a partial differential equation for axisymmetric flow yields

$$c_r\left(\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}\right)+c_v\frac{\partial^2 u}{\partial z^2}=\frac{\partial \bar{u}}{\partial t} \quad (1)$$

where c_r = coefficient of consolidation in the radial direction; c_v = coefficient of consolidation in the vertical direction; u = excess pore water pressure at a certain location (r, z) in soil; $\bar{u}$ = average excess pore water pressure at a depth z in soil; r, z = cylindrical coordinates as defined in Fig. 1; and t = time.

By decomposing the total flow into radial and vertical flows, (1) can be expressed in two equations as

$$\frac{\partial u_z}{\partial t}=c_v\frac{\partial^2 u_z}{\partial z^2} \quad (2)$$

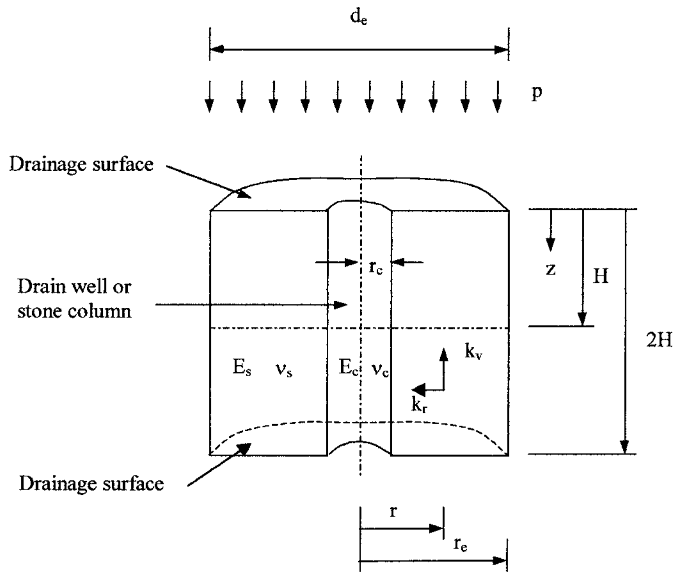

FIG. 1. Definition of Terms for Modeling

$$\frac{\partial \bar{u}}{\partial t} = c_r \left(\frac{1}{r} \frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial r^2} \right) \tag{3}$$

where u_z = excess pore water pressures due to vertical flow only; and u_r = excess pore water pressures due to radial flow only.

Eq. (2) is a Terzaghi 1D consolidation problem, and its solution is available in many soil mechanics books. Utilizing the initial and boundary conditions, a solution for (3) can also be obtained as

$$u_r = \frac{4\bar{u}}{d_e^2 F(N)} \left[r_e^2 \ln\left(\frac{r}{r_c}\right) - \frac{r^2 - r_c^2}{2} \right] \tag{4}$$

where $\bar{u} = u_0 e^{\lambda}$; $\lambda = -8T_r/F(N)$; u_0 = initial excess pore water pressure; $F(N) = [N^2/(N^2 - 1)]\ln(N) - (3N^2 - 1)/(4N^2)$; $N = d_e/d_c$ diameter ratio; $T_r = c_r t/d_e^2$ time factor in a radial flow; r_c and r_e = radii of a drain well and its influence zone, respectively, as defined in Fig. 1; and d_c and d_e = diameters of a drain well and its influence zone, respectively.

The average rate (or degree) of consolidation in the radial direction is

$$U_r = 1 - \exp^{-[8/F(N)]T_r} \tag{5}$$

SIMPLIFIED AND CLOSED-FORM SOLUTIONS FOR STONE COLUMNS

Considering that the consolidation characteristics of a stone column reinforced foundation are different from those of fine-grained soils with drain wells, the following assumptions are made:

1. Stone columns are free-draining at any time. Each stone column has a circular influence zone.
2. The surrounding soil is fully saturated, and water is incompressible.
3. Stone columns and the surrounding soil only deform vertically and have the equal strain at any depth.
4. The load is applied instantly through a rigid foundation and maintained constant during the consolidation period. At the moment of the load being applied, uniform excess pore water pressures within the surrounding soil carry all the loads. Note that the modular (stiffness) ratio between stone columns and the soil discussed later in the paper refers to the effective (drained) modulus. At the moment of loading, however, the saturated soil is under an undrained condition. The undrained elastic modulus of the saturated soil is theoretically infinite under a condition with full confinement, which results from the preceding assumption of 1D deformation. Due to the significant difference of the moduli between the surrounding soil and the stone column at the moment of loading, it is reasonable to assume that excess pore water pressures in the surrounding soil carry all the loads. This assumption is consistent with that in Barron (1947) for dealing with drain wells in fine-grained soils.
5. Total vertical stresses with stone columns and the surrounding soil, respectively, are averaged and uniform.

At any time, both the stone column and the surrounding soil carry the applied loads, i.e.,

$$\bar{\sigma}_s A_s + \bar{\sigma}_c A_c = pA \tag{6}$$

where $\bar{\sigma}_c$ and $\bar{\sigma}_s$ = average total stresses within the column and the surrounding soil, respectively; p = average applied pressure on the whole area; A_c and A_s = cross-section areas of the column and the surrounding soil, respectively; and $A = A_c + A_s$.

As shown in Fig. 1, a stone column is considered for this case instead of a drain well used in the Barron study. The similar initial and boundary conditions for the drain well problem can be used in this analysis as follows: (1) $u_0 = u|_{t=0} = (A/A_s)p$, within the surrounding soil; (2) $u|_{r=r_c} = 0(t > 0)$; (3) $(\partial u/\partial r)|_{r=r_e} = 0$; (4) $u|_{z=0} = 0(t > 0)$; and (5) $(\partial u/\partial z)|_{z=H} = 0$

The assumption of equal strain between the column and the surrounding soil yields

$$\frac{\partial e_s}{1 + e_s} = \frac{\partial e_c}{1 + e_c} \tag{7}$$

where e_c and e_s = void ratios of the stone column and the surrounding soil, respectively.

By the definition of coefficient of compressibility, we have

$$\frac{\partial e_s}{\partial \bar{\sigma}'_s} = -\alpha_{v,s}; \ \frac{\partial e_c}{\partial \bar{\sigma}'_c} = -\alpha_{v,c} \tag{8}$$

where $\bar{\sigma}'_c$ and $\bar{\sigma}'_s$ = average effective stresses within the column and the surrounding soil, respectively; and $\alpha_{v,c}$ and $\alpha_{v,s}$ = coefficients of compressibility of the column and the surrounding soil, respectively. Combining (7) and (8) yields

$$\partial \bar{\sigma}'_s = \frac{m_{v,c}}{m_{v,s}} \partial \bar{\sigma}'_c \tag{9}$$

where $m_{v,s} = (\alpha_{v,s})/(1 + e_s)$; and $m_{v,c} = (\alpha_{v,c})/(1 + e_c)$.

Considering that the excess pore water pressure within the stone column is equal to zero, i.e., $\bar{\sigma}'_c = \bar{\sigma}_c$, and the relationship in (6), (9) can be rewritten as

$$\partial \bar{\sigma}'_s = \frac{m_{v,c}}{m_{v,s}} \partial \left(\frac{pA - \bar{\sigma}_s A_s}{A_c} \right) \tag{10}$$

Due to the load being maintained constant during the consolidation, $\partial p/\partial t = 0$. Therefore

$$\frac{\partial \bar{\sigma}'_s}{\partial t} = -\frac{m_{v,c}}{m_{v,s}} \frac{A_s}{A_c} \frac{\partial \bar{\sigma}_s}{\partial t} \tag{11}$$

Using the basic soil mechanics principle $\bar{\sigma}_s = \bar{\sigma}'_s + \bar{u}$, (11) becomes

$$\frac{\partial \bar{\sigma}'_s}{\partial t} = -\frac{m_{v,c} A_s}{m_{v,c} A_s + m_{v,s} A_c} \frac{\partial \bar{u}}{\partial t} \tag{12}$$

Combining (8) and (12) yields

$$\frac{\partial e_s}{\partial t} = \alpha_{v,s} \frac{m_{v,c} A_s}{m_{v,c} A_s + m_{v,s} A_c} \frac{\partial \bar{u}}{\partial t} \tag{13}$$

The reduction rate of cylindrical soil volume due to discharge of water can be expressed as

$$-\frac{\partial V}{\partial t} = -\frac{\partial e_s}{1 + e_s}\left(\frac{2\pi rdrdz}{\partial t}\right) = -\frac{m_{v,s}m_{v,c}A_s}{m_{v,s}A_s + m_{v,s}A_c}\frac{\partial \bar{u}}{\partial t}(2\pi rdrdz) \tag{14}$$

The discharge rate of water from the cylindrical unit can be computed by

$$\frac{\partial Q}{\partial t} = -\left[\frac{k_r}{\gamma_w}\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + \frac{k_v}{\gamma_w}\frac{\partial^2 u}{\partial z^2}\right]2\pi rdrdz \tag{15}$$

where k_r, k_v = coefficients of soil permeability in radial and vertical directions, respectively; and γ_w = unit weight of water. Equalizing the volume change of the surrounding soil and the water discharge from the surrounding soil

$$\frac{k_r}{\gamma_w}\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + \frac{k_v}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{m_{v,s}m_{v,c}(1 - a_s)}{m_{v,c}(1 - a_s) + m_{v,s}a_s}\frac{\partial \bar{u}}{\partial t} \tag{16}$$

where a_s = replacement ratio of stone column over the total influence area, $a_s = A_c/A$. Eq. (16) can be further simplified as

$$c'_r\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + c'_v\frac{\partial^2 u}{\partial z^2} = \frac{\partial \bar{u}}{\partial t} \tag{17}$$

where $c'_r = (k_r/\gamma_w)[m_{v,c}(1 - a_s) + m_{v,s}a_s]/[m_{v,s}m_{v,c}(1 - a_s)]$, a modified coefficient of consolidation in the radial direction; and $c'_v = (k_v/\gamma_w)[m_{v,c}(1 - a_s) + m_{v,s}a_s]/[m_{v,s}m_{v,c}(1 - a_s)]$, a modified coefficient of consolidation in the vertical direction.

Clearly, (17) is identical to (1) in formats, except for the modified coefficients of consolidation used in (17). Like the solutions for drain wells, (17) can be decomposed into two equations with corresponding solutions. The solution for the vertical flow follows the Terzaghi 1D consolidation solution, while the solution for the radial flow follows the Barron drain-well solution. In both solutions, modified coefficients of consolidation should be used instead for stone column reinforced foundations.

Considering a combining effect of radial and vertical flows, the overall rate of consolidation can be expressed as (Carillo 1942)

$$U_{rv} = 1 - (1 - U_r)(1 - U_v) \tag{18}$$

An approximate solution can be obtained as follows if U_{rv} is greater than 30% by

$$U_{rv} = 1 - \frac{8}{\pi^2}\exp^{-[8/F(N)]T'_r - [\pi^2/4]T'_v} \tag{19}$$

where $T'_r = c'_r t/d_e^2$, a modified time factor in the radial flow; $T'_v = c'_v t/H^2$, a modified time factor in the vertical flow; and H = thickness of soil from a free-draining horizontal surface to an impervious one.

It is known that the coefficient of compressibility of an elastic body can also be expressed as

$$m_v = \frac{(1 + \nu)(1 - 2\nu)}{E(1 - \nu)} \tag{20}$$

where E = elastic modulus; and ν = Poisson ratio. Therefore:

$$\frac{m_{v,s}}{m_{v,c}} = \frac{E_c}{E_s}\frac{(1 + \nu_s)(1 - 2\nu_s)(1 - \nu_c)}{(1 + \nu_c)(1 - 2\nu_c)(1 - \nu_s)} = \xi\frac{E_c}{E_s} \tag{21}$$

where E_c and E_s = elastic moduli of the stone column and the surrounding soil, respectively; ν_c and ν_s = Poisson ratios of the stone column and the surrounding soil, respectively; and ξ = a Poisson ratio factor,

$$\xi = \frac{(1 + \nu_s)(1 - 2\nu_s)(1 - \nu_c)}{(1 + \nu_c)(1 - 2\nu_c)(1 - \nu_s)}$$

Integrating (9) yields

$$\bar{\sigma}'_s = \frac{m_{v,c}}{m_{v,s}}\bar{\sigma}'_c + C \tag{22}$$

where C = a constant.

Based on the assumption that all the applied loads at the time $t = 0$ are carried by the excess pore water pressures within the surrounding soil, then $\bar{\sigma}'_c = \bar{\sigma}'_s = 0$. Therefore, the constant, C, in (22) must be equal to 0. When consolidation of the surrounding soil is complete, the effective stresses within the stone column and the surrounding soil finally become steady and equal to the total stresses. Say the steady vertical stresses within the stone column and the surrounding soil σ_{cs} and σ_{ss}, respectively, (22) can be rewritten as

$$n_s = \frac{\sigma_{cs}}{\sigma_{ss}} = \frac{m_{v,s}}{m_{v,c}} = \xi\frac{E_c}{E_s} \tag{23}$$

in which n_s = steady-stress concentration ratio as the consolidation is complete. In the literature, the reported stress concentration ratio values mostly refer to the steady-stress concentration ratio. However, the stress concentration ratio can also be defined as the ratio of the total vertical stress on the columns to that on the soil at certain time t. Further discussion on the stress concentration ratio is presented in the Analyses and Discussions section.

The modified coefficients of consolidation in (17) can also be expressed using the stress concentration ratio

$$c'_r = c_r\left(1 + n_s\frac{1}{N^2 - 1}\right);\quad c'_v = c_v\left(1 + n_s\frac{1}{N^2 - 1}\right) \tag{24}$$

where N = a diameter ratio, defined previously in (4).

ANALYSES AND DISCUSSIONS

Stress Transfer and Stress Concentration Ratio

Eq. (23) exhibits a relationship between modular ratio, E_c/E_s, and steady-stress concentration ratio n_s by a Poisson ratio factor ξ shown in (21). This relationship is plotted with the results from a theoretical and experimental study conducted by Barksdale and Bachus (1983) in Fig. 2. The comparison shows that the calculated n_s values using parameters $\nu_c = 0.15$ and $\nu_s = 0.45$ in (23) are close to those from Barksdale and Bachus (1983) within a typical modular ratio of 10 to 20. The Barksdale and Bachus (1983) study was based on

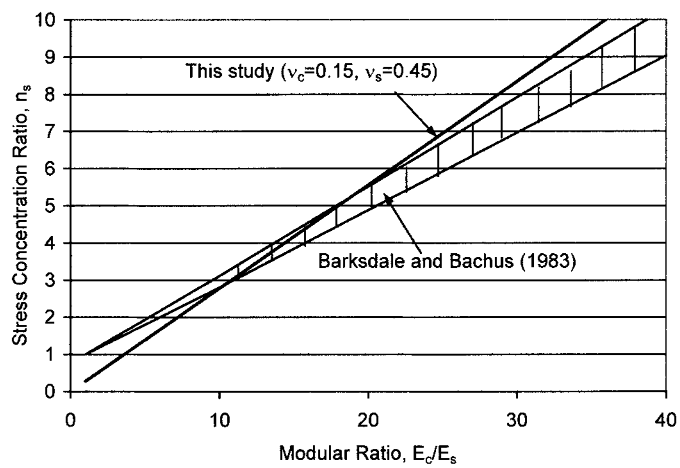

FIG. 2. Relationship between Stress Concentration Ratio and Modular Ratio

the following conditions: (1) modular ratio E_c/E_s = 0 to 40; (2) area replacement ratio a_s = 0.10 to 0.25; and (3) length-diameter ratio of columns L/d_c = 4.5 to 19.5. However, many field studies have shown that steady-stress concentration ratios for stone column reinforced foundations are in the range of 2 to 6 (Mitchell 1981).

An increase of the effective stress in the surrounding soil over a certain time period can be computed by re-arranging (12)

$$\Delta\bar{\sigma}_s' = \Delta\bar{u}\frac{1 - a_s}{1 + a_s(n_s - 1)} = u_0 U_{rv}\frac{1 - a_s}{1 + a_s(n_s - 1)} \quad (25)$$

Ignoring the consolidation due to a vertical flow, the calculated average total stresses on the soil and columns for the case N = 3 and n_s = 5 are plotted in Fig. 3. In this figure, the average total stress σ_s and σ_c are normalized by the average applied pressure p. The results demonstrate that the stress on columns increases with the time, while the stress on the soil decreases. This stress transfer process from the soil to columns is so-called "stress concentration." Due to the assumption of all the loads carried by water in the surrounding soil at the moment of loading, the stress on columns starts from zero. With the same reasoning, the average stress on the soil is slightly higher than the average applied pressure p over the influence area. The stress transfer or concentration process can also be presented in terms of stress concentration ratio, as shown in Fig. 4. It is shown that the stress concentration ratio increases with time and approaches the steady-stress concentration ratio (n_s = 5 for this case), which is in agreement with the findings from several laboratory and field studies (Juran and Guermazi 1988; Han and Ye 1991; Lawton 1999).

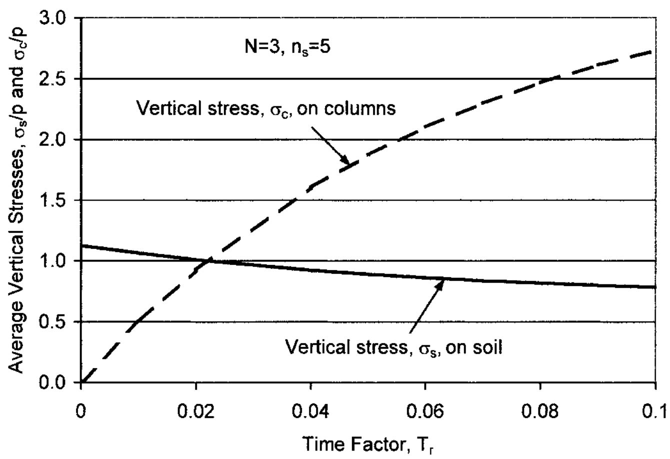

FIG. 3. Vertical Stresses on Soil and Columns with Time

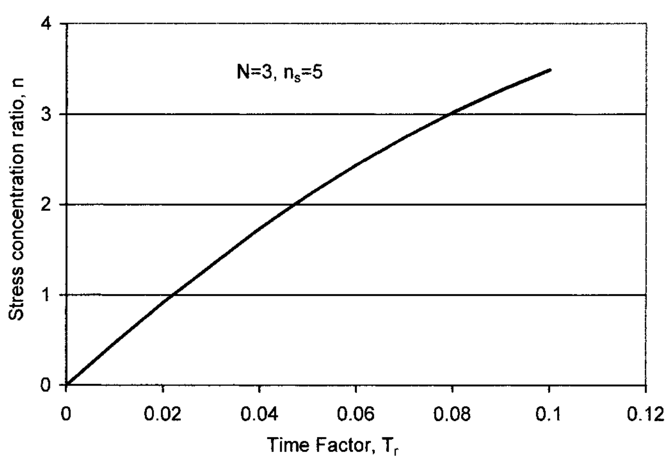

FIG. 4. Stress Concentration Ratio with Time

The experimental studies (Juran and Guermazi 1988; Han and Ye 1991; Lawton 1999) also showed that the stress concentration ratio depended on the level of loads in addition to the modular ratio and time. The general trend was observed that the steady-stress concentration ratio increased with the applied load and then started to decrease after the load reaching the yield load of stone columns. Although stone columns and the surrounding soil are assumed linearly elastic in this study, in reality, they have nonlinear behavior, and the coefficients of compressibility $m_{v,c}$ and $m_{v,s}$ should be determined as the slopes of the vertical strain $[\partial e/(1 + e)]$ versus effective vertical stress plots in terms of the range of the applied loads. For example, the coefficient of compressibility m_v of a soil is commonly determined in the range of load from 100 to 200 kPa for shallow foundation applications. Therefore, the coefficients of compressibility of stone columns and the surrounding soil are stress dependent. For the same reason, the modular ratio in terms of the coefficient of compressibility or the steady-stress concentration ratio is also stress dependent. Therefore, a steady-stress concentration ratio should be determined in terms of the level of applied loads. A full-scale plate load test can be used to determine the steady-stress concentration ratio. With lack of experimental data, the steady-stress concentration ratio is suggested to be in the range of 3.0 to 4.0, under a working load close to the allowable bearing capacity of the stone column reinforced foundation.

Excess Pore Water Pressure

It is well-known that an increase of total stress in soil can generate an excess pore water pressure. On the other hand, the excess pore water pressure would dissipate when the water drains out from the soil and/or the total stress in the soil is reduced. During the consolidation of the stone column reinforced foundation, the variation of the excess pore water pressure in the surrounding soil can be considered as a combination of different factors

$$u_t = u_0 - \Delta u_{\sigma_v} + \Delta u_{\sigma_r} - \Delta u_d \quad (26)$$

where u_0 and u_t = excess pore water pressure at t = 0 and t > 0, respectively; Δu_{σ_v} = excess pore water pressure reduced by a reduction of a vertical stress; Δu_{σ_r} = excess pore water pressure increased by an increase of a lateral stress from the column; and Δu_d = excess pore water pressure reduced by drainage of water from the soil.

The consolidation process in the soil commences right after the moment of the load applied. Under instant loading, the saturated soft clay behaves incompressible, and the stress in the stone column is relatively small. Therefore, the soft clay tends to move laterally toward the stone column. The tendency of this lateral movement acts as a "relief" of excess pore water pressures in the surrounding soil as compared with a completely confined situation, i.e., 1D deformation. Since the commencement of the consolidation, the vertical stress in the soil starts to transfer onto stone columns as shown in Fig. 3. In other words, a stress concentration onto the column along with a vertical stress reduction in the soil happens. This stress transfer or concentration induces a reduction of excess pore water pressure Δu_{σ_v} in the soil. At the same time the load is transferred onto the column, the lateral stress from the column is increased. This increase of the lateral stress increases the excess pore water pressure in the soil Δu_{σ_r}. Therefore, the overall effect of this stress transfer on the excess pore water pressure is a function of the difference $\Delta u_{\sigma_r} - \Delta u_{\sigma_v}$. Because the stress concentration at the beginning is not significant, and the soft clay tends to move laterally toward the stone column, the lateral stress plays a role in reducing the excess pore water pressures. When the stress concentration becomes significant with time, however, the lateral stress starts to play a role in increas-

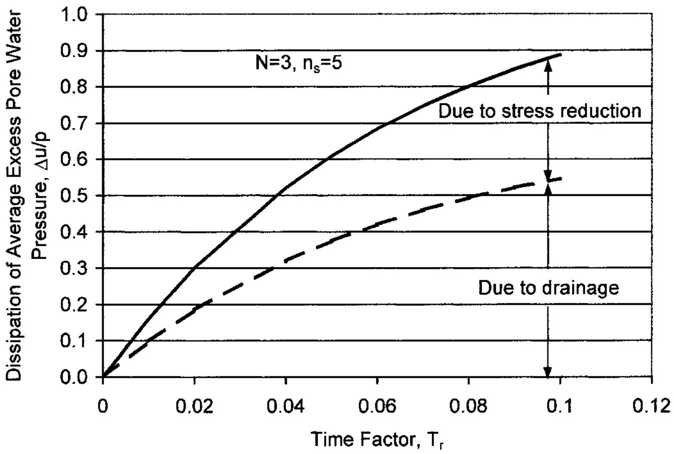

FIG. 5. Dissipation of Excess Pore Water Pressure

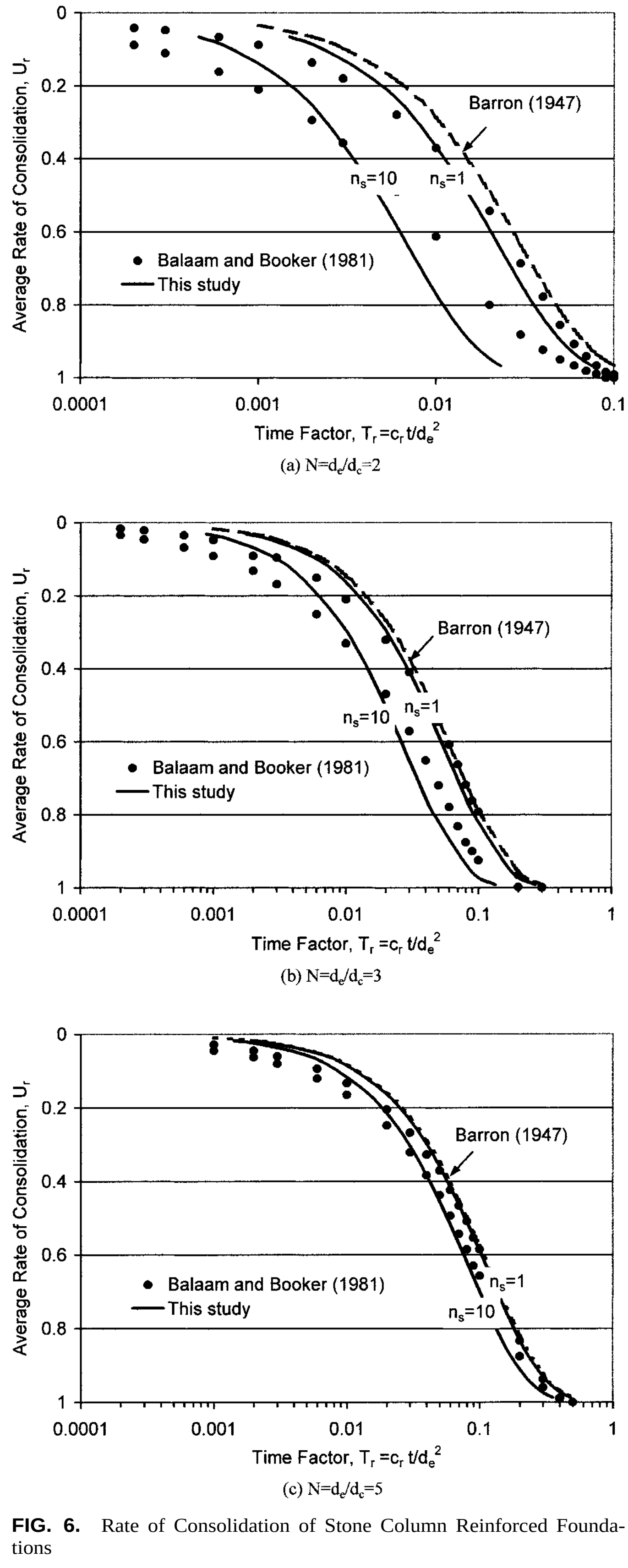

FIG. 6. Rate of Consolidation of Stone Column Reinforced Foundations

ing the excess pore water pressure. In this paper, however, no lateral movement has been assumed in the theoretical development. Therefore, the dissipation of excess pore water pressures depends on two factors, drainage and reduction of vertical stress, as shown in Fig. 5. The dissipation of excess pore water pressures, due to vertical stress reduction, is about 40% of the total dissipation for this special case. Obviously, the contribution of vertical stress reduction to the dissipation of excess pore water pressures does not exist in the foundation with drain wells. This extra contribution explains why stone columns are more effective than drain wells in accelerating the rate of consolidation of soft clays. It is expected that the portion contributed by stone columns depends on the value of stress concentration ratio. The higher the stress concentration ratio, the more dissipation of excess pore water pressures will occur by vertical stress reduction.

Consolidation Rate

Fig. 6 exhibits an increase of steady-stress concentration ratio can accelerate the rate of consolidation. As the stress concentration ratio varies from 1.0 to 10, the difference in the rate of consolidation can be as much as 40% at the diameter ratio N of 2.0. This difference becomes less as a diameter ratio N increases. This phenomenon can be easily explained using (24). It can be found that the Barron solution is a special case of (24), i.e., when $n_s = 0$. In other words, drain wells were assumed to carry no load in the Barron solution. As a result, the Barron solution underestimates the rate of consolidation of the stone column reinforced foundation.

The comparison of the results on the rate of consolidation from the numerical analysis (Balaam and Booker 1981) and the simplified method developed in this paper is shown in Fig. 6. In Balaam and Booker's (1981) study, a modular ratio was used instead of stress concentration ratio. Considering the fact that $\nu_c = \nu_s = 0.3$ was assumed in their study, the modular ratio is equivalent to the steady-stress concentration ratio. The comparison indicates the computed rates of consolidation, from the numerical and simplified methods, are in the reasonable agreement. The differences become less significant when the stress concentration ratio n_s decreases and/or the diameter ratio N increases. Within the range of typical stress concentration ratios (2 to 6), the difference in the rate of consolidation between the numerical and the simplified methods is expected to be less than 10%. For all cases, the computed rate of consolidation by the numerical method is greater than that by the simplified method at the beginning of the consolidation by the numerical method is greater than that by the simplified method at the beginning of the consolidation. However, it is reversed when the rate of consolidation is greater than approximately 40%. These discrepancies can result from the different assumptions used in the numerical and simplified methods.

In the Balaam and Booker (1981) study, the lateral movement, from the soft soil to the stone column or from the stone column to the soft soil, is permitted. However, the lateral movement is not allowed in the development of this simplified method. As discussed early, the lateral movement in the numerical study tends to reduce the excess pore water pressures at the beginning of loading, so that it accelerates the rate of consolidation. This effect may be the reason why the rate of consolidation at the beginning of loading is higher from the numerical study by Balaam and Booker (1981) than from the simplified method. When more stresses are transferred onto the stone column with time, however, the lateral movement from the stone column to the soft soil in the numerical study tends to increase the excess pore water pressures so that it slows down the process of consolidation. This ''slow down'' effect is more significant as the stress concentration becomes larger with time. Taking no account of the excess pore water pressure induced by the lateral stress, the simplified method computes a higher rate of consolidation than the numerical method does, as time increases to a certain level. As shown in Fig. 6, the difference of the rate of consolidation from the numerical and simplified methods is diminished with an increase of the diameter ratio. This is because the relative contribution to the increase of excess pore water pressures by the lateral stress is reduced when the volume of the soil involved in the consolidation increases with an increase of the diameter ratio.

DESIGN CHARTS

To assist geotechnical engineers in better utilizing the solutions developed in this paper, design charts are provided in Figs. 7 and 8 with a wide range of N values. Basically, the curve in Fig. 7 and other curves in Fig. 8 are reproductions of the Terzaghi 1D consolidation solution and the Barron (1947) consolidation solution for drain wells, respectively. The significant difference from the classical (Terzaghi and Barron's) solutions is that modified time factors with modified coefficients of consolidation presented in (19) are used in Figs. 7 and 8. When the steady-state concentration ratio n_s equals 0, all the curves in Figs. 7 and 8 become the results of the classical solutions. The stone column reinforced foundation has a typical range of N values from 1.5 to 5.0.

EXAMPLE

An example has been selected to show how to use the solutions developed in this paper or the curves in Figs. 7 and 8 for actual design. Consider a project in which stone columns with a diameter of 0.85 m and a spacing of 1.5 m (square pattern) are used for treating 10 m soft clay underlaid by a dense and permeable sand layer. The soft clay has an equal coefficient of consolidation in vertical and radial flow of 5.0 $\times$ 10^{-4} cm^2/s. The design is required to calculate the average rate of consolidation of the soft clay after 100 kPa instant loading for 15 days.

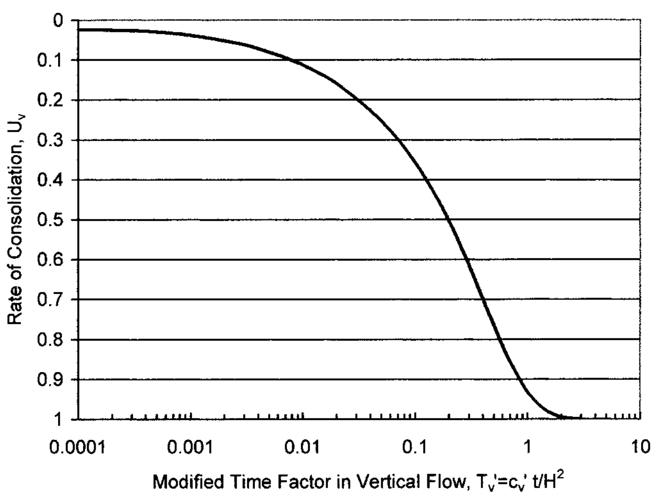

FIG. 7. Rate of Consolidation in Vertical Flow

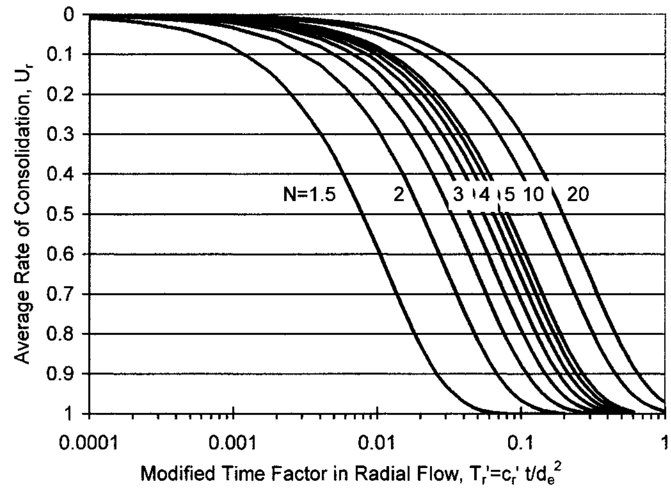

FIG. 8. Rate of Consolidation in Radial Flow

Considering the square pattern of stone columns, the equivalent influence diameter d_e = 1.13 $\times$ 1.5 m = 1.70 m and the diameter ratio N = 1.70 m/0.85 m = 2.0. With the typical modular ratio of 10 to 20 as suggested by Barksdale and Bachus (1983), the steady-stress concentration ratio can be determined in the range of 3.0 to 5.0 in Fig. 2. The steady-stress concentration ratio of 4.0 is selected in this design. The modified coefficients of consolidation in vertical and radial flows are calculated as 1.17 $\times$ 10^{-3} cm^2/s using (24). Due to the existence of top and bottom drainage surfaces between the soft clay, half the thickness of the soft clay is used for computing the modified time factor in a vertical flow, i.e., T_v' = 0.006, and the modified time factor in a radial flow T_r' = 0.053. From Figs. 7 and 8, it can be established that the average rate of consolidation is in a vertical flow, U_v = 0.09, and in a radial flow U_r = 0.83. Considering a combined effect of radial and vertical flows, the overall rate of consolidation from (18) is 0.85 or 85%. Obviously, the contribution of vertical flow to the average rate of consolidation is minimal. If the Terzaghi and Barron solutions are used instead (assuming n_s = 0), however, the time factors in a vertical flow and in a radial flow are T_v = 0.025 and T_r = 0.023, respectively. The estimated average rates of consolidation U_v = 0.06 and U_r = 0.54, the overall rate is 0.57 or 57%. Therefore, the difference of the rate of consolidation between the solutions developed in this paper and the classical (Terzaghi and Barron's) is 28%. To reach the same degree of consolidation (85%), however, the classical solutions require 35 days–2.3 times the number of days predicted by the new method developed in this paper.

SUMMARY

A simplified method for computing the rate of consolidation is developed in this paper to account for a drained modular ratio between the stone column and the soil or a stress concentration ratio. The solution supports earlier findings by a numerical study (Balaam and Booker 1981) that found the rate of consolidation can be accelerated by increasing the modular ratio and reducing the diameter ratio (influence diameter/column diameter). The Terzaghi 1D consolidation solution and the Barron (1947) solution for drain wells in fine-grained soil are special cases of the simplified solutions developed in this paper that underestimate the rate of consolidation of stone column reinforced foundations. The new solutions demonstrated the stress transfer and the dissipation of excess pore water pressures due to drainage and vertical stress reduction in the process of consolidation. The comparison of the results from the simplified method and a numerical study shows reasonable agreement, especially when the steady-stress concentration ratio is within a typical range (2–6). The discrepancies in results for these two methods are discussed, mainly resulting from different assumptions adopted in one or 3D deformation. A design example, which used the design charts developed in this paper, demonstrates the difference in prediction for the rate of consolidation from the classical solutions. The classical solutions yield a requirement for much longer time to achieve the same rate of consolidated rather than the new method.

ACKNOWLEDGMENTS

The authors express their appreciation to the reviewers and the Editorial Board Member for their review of this paper, valuable comments, and suggestions that helped the authors refine and improve the quality of the paper.

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NOTATION

The following symbols are used in this paper:

A = unit influence area;
A_c = area of column portion;
A_s = area of soil portion;
a_s = area replacement ratio;
c_r = coefficient of consolidation in radial direction;
c_r' = modified coefficient of consolidation in radial direction;
c_v = modified coefficient of consolidation in vertical direction;
c_v' = modified coefficient of consolidation in vertical direction;
d_c = diameter of stone column;
d_e = diameter of influence area;
e = void ratio of soil;
H = thickness of soil;
k_r = coefficient of soil permeability in radial direction;
k_v = coefficient of soil permeability in vertical direction;
m_v = coefficient of compressibility;
N = diameter ratio;
n = stress concentration ratio;
n_s = steady stress concentration ratio;
p = average applied pressure;
Q = discharge of water from soil;
r = radius;
r_c = radius of stone column;
r_e = radius of influence area;
T_r = time factor in radial direction;
T_r' = modified time factor in radial direction;
T_v = time factor in vertical direction;
T_v' = modified time factor in vertical direction;
t = time;
u = excess pore water pressure;
u_r = excess pore water pressure in radial direction;
u_z = excess pore water pressure in vertical direction;
u_0 = initial pore water pressure;
U_r = average rate of consolidation in radial direction;
U_{rv} = average rate of consolidation;
U_v = rate of consolidation in vertical direction;
$\bar{u}$ = average excess pore water pressure in soil;
V = volume of soil;
z = depth;
α_v = coefficient of compressibility of soil;
γ_w = unit weight of water;
Δu_{σ_v} = excess pore water pressure reduced by reduction of vertical stress;
Δu_{σ_r} = excess pore water pressure induced by lateral stress;
Δu_d = excess pore water pressure reduced by drainage;
ξ = Poisson ratio factor;
$\bar{\sigma}_c'$ = average effective stress in columns;
$\bar{\sigma}_s'$ = average effective stress in soil; and
$\bar{\sigma}'$ = average effective stress.