Consolidation of Clay with a System of Vertical and Horizontal Drains

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Abstract: Consolidation of inhomogeneous clay with a system of horizontal and vertical drains is considered. Horizontal drains in the system are made of multiple thin pervious layers such as sand layers and geotextile sheets, while vertical drains are vertical cylindrical drains. Consolidation behavior with the drain system is formulated using the transfer matrix method. Special care is given to formulation of thin pervious layers for efficient computation. The developed formulation is verified using available numerical and field information. Parametric studies are conducted to study the consolidation characteristics of clay with the drain system. Based on the findings, a design method for an optimum system of horizontal and vertical drains is proposed and design charts are presented for such a design.

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Introduction

Vertical drains such as sand drains discharge the pore water in clay out of the ground. They are widely used to accelerate the consolidation process for the improvement of soft ground: e.g., Manila Bay Reclamation Area (Tominaga et al. 1979), Changi Airport, Singapore (Choa et al. 1979) and Kansai International Airport (Takai et al. 1989; Suzuki and Yamada 1990). Horizontal drains discharge the pore water in clay, also. With sufficient horizontal drainage rate in the drains, they modify the hydraulic gradient of vertical flow of pore water in clay to accelerate the consolidation. Thin sand layers have been used for horizontal drains in reclaimed lands with clay fill (Watari 1984; Lee et al. 1987; Karunarathe et al. 1990) and in clay fill embankments (Gibson and Shefford 1968). Geotextiles have an excellent performance in discharging pore water to dissipate the pore water pressure much faster than the original soil mass (Rowe 1992; Loh 1998). Thus, they have also been used for horizontal drains in backfills of earth-retaining walls and in embankments for drainage and filtration purpose (Tatsuoka et al. 1986; Itoh et al. 1994). Horizontal drains in ground generally require some means to take the collected water out. A combination of vertical and horizontal drains is, therefore, an advantage for accelerating consolidation in soft clay.

Proper evaluation of consolidation behavior of clay with vertical and horizontal drains can be made only by rational analysis. When soil is a layered inhomogeneous medium, it is tedious and considerably difficult to develop analytical expressions for its consolidation behavior, and thus available expressions are generally limited to very simple cases (Gray 1945; Schiffman and Stein 1970). Horne (1964) developed a solution for consolidation behavior of soil stratified only in a very special manner, in which clay and thin sand are layered alternatively and the thickness and properties are identical for all clay layers and for all sand layers. Abid and Pyrah (1991) formulated a special finite element technique to handle highly permeable thin layers by using special line elements and disk ring elements.

In this paper, a powerful transfer matrix method (Nogami and Paulson 1984, 1985) is used to formulate the consolidation behavior of clay soil with a system of horizontal thin drains and vertical cylindrical drains. The consolidation behavior of clay with such a drain system is investigated, and a design method for an optimum drain system is presented.

Formulations

General Expressions for Excess Pore Water Pressure

Clay ground with a combination of vertical and horizontal drains is considered. The vertical drains are cylindrical drains installed at an equal spacing distance. The horizontal drains are a number of horizontal, thin, pervious layers such as thin layers of sand or sheets of geotextile. When the horizontal drain layers are thin, the excess pore water pressure, and thus the horizontal flow, are reasonably assumed to be uniform along the thickness in the drain layer. Load is applied uniformly over the entire horizontal ground surface to produce the one-dimensional loading condition.

The system of clay and drains are divided into I horizontal layers, in such a way that the clay can be considered homogeneous within a layer, and a thin sand layer (or geotextile) is located only at the layer–layer interface, as shown in Fig. 1. The origin of the cylindrical coordinates is set at the center of a cylindrical drain at the top of an individual clay layer, in which z is positive in the downward direction. The governing equations for the excess pore water pressures in an individual clay layer and thin sand layer are stated, respectively, with the cylindrical coordinate system as (Appendix I)

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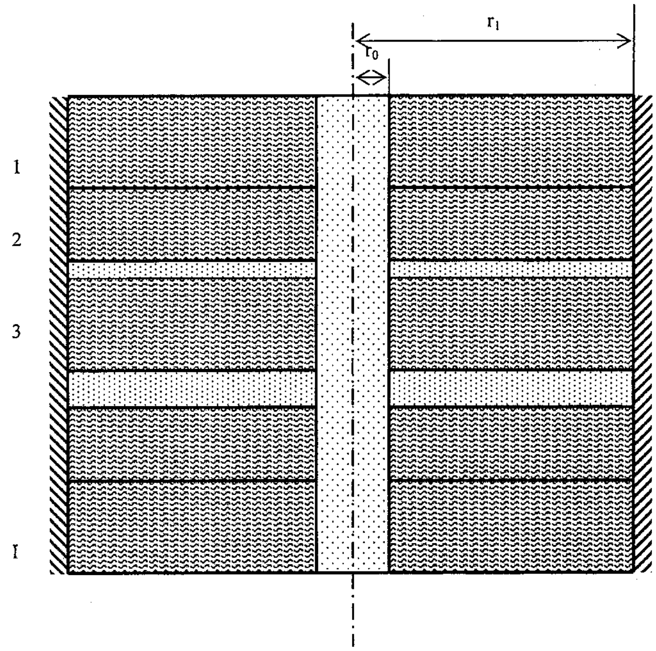

Fig. 1. Layered clay containing thin pervious layers

$$\frac{\partial u(r,z,t)}{\partial t} = c_z \frac{\partial^2 u(r,z,t)}{\partial z^2} + c_r \left(\frac{\partial^2 u(r,z,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,z,t)}{\partial r} \right) + \frac{d\sigma(r,z,t)}{dt} \tag{1}$$

$$\frac{\partial u_s(r,t)}{\partial t} = c_s \left(\frac{\partial^2 u_s(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u_s(r,t)}{\partial r} \right) + \frac{\gamma_w c_s}{k_s h_s} \Delta q(r,t) + \frac{d\sigma_s(r,t)}{dt} \tag{2}$$

where t = time; r and z = coordinates in the radial and vertical directions of the cylindrical coordinate system, respectively; u and σ = excess pore water pressure and total stress in clay, respectively; c_z and c_r = coefficients of consolidation of clay in the vertical and horizontal directions, respectively; u_s and σ_s = excess pore water pressure and total stress in sand, respectively; Δq = amount of flow per a unit area discharged into a pervious layer; h_s = thickness of pervious layer; and c_s and k_s = coefficient of consolidation and permeability of sand, respectively. The above coefficients of consolidation c_z, c_r, and c_s are written, respectively, as $k_z/(m_c\gamma_w)$, $k_r/(m_c\gamma_w)$, and $k_s/(m_s\gamma_w)$, where m_c and m_s = volume compressibility of clay and sand, respectively; k_z and k_r = coefficients of permeability of clay in the vertical and radial directions, respectively; k_s = coefficient of permeability of sand; and γ_w = unit weight of water.

With separation of variables, $u(x,y,t)$ is expressed in the form of

$$u(r,z,t) = u(r)\phi(z)T(t) \tag{3}$$

Substitution of Eq. (3) into Eq. (1*a*) results in three ordinary differential equations with respect to $u(r)$, $\phi(z)$, and $T(t)$. Their solutions are expressed as (Gibson and Schefford 1968)

$$u(r) = A_3 J_0(sr) + B_3 Y_0(sr) \tag{4a}$$

$$\phi(z) = A_2 \cos\beta z + B_2 \sin\beta z \tag{4b}$$

$$T(t) = A_1 e^{-\alpha^2 t} + \int_0^t \frac{d\sigma(\tau)}{d\tau} e^{-\alpha^2(t-\tau)} d\tau \tag{4c}$$

where $A_1 \sim A_3$, B_2, and B_3 = unknown constants; J_0 and Y_0 = first and second kinds Bessel functions of the zeroth order, respectively; s = parameter later defined by the boundary conditions in the radial direction; and

$$\alpha^2 = c_z\beta^2 + c_r s^2 \tag{5}$$

Solution forms of the expressions related to $u_s(r,t)$ in Eq. (1*b*) are the same as those above given for $u(r,z,t)$ with $\beta = 0$. Thus, substituting Eq. (3) with Eqs. (4*a*)–(4*c*) into Eq. (1*b*) and manipulating it result in the following expression for a thin pervious layer:

$$\Delta q(r,t) = \Delta Q u_s(r) T_s(t) \tag{6}$$

where the notations with subscript s correspond to those above defined without subscription; and, replacing β and c_r, respectively, with 0 and c_s,

$$\Delta Q = \frac{k_s h_s}{\gamma_w}\left(s^2 - \frac{\alpha^2}{c_s} \right)\phi_s \tag{7}$$

with ϕ_s = unknown constant defined later.

Boundary Conditions

It is reasonably assumed that an individual vertical drain and its influence zone in the clay are uncoupled with others in a group of vertical drains (Barron 1948). By denoting r_0 = radius of the vertical drain, r_1 = radial distance of its influence zone, and h = the thickness of a clay layer, the boundary conditions of the ith clay layer are given as

1. At $r = r_0$ and r_1:

$$u(r_0,z,t)_i = 0 \tag{8a}$$

$$\left.\frac{\partial u(r,z,t)_i}{\partial r}\right|_{r=r_1} = 0 \tag{8b}$$

2. At the interface between the ith and $i+1$th layers:

$$u(r,h,t)_i = u(r,0,t)_{i+1} = u_s(r,t)_{i,i+1} \tag{9a}$$

$$\frac{(k_y)_i}{\gamma_w}\left.\frac{\partial u(r,z,t)_i}{\partial z}\right|_{z=h_i} = \frac{(k_y)_{i+1}}{\gamma_w}\left.\frac{\partial u(r,z,t)_{i+1}}{\partial z}\right|_{z=h_{i+1}} - \Delta q(r,t)_{i,i+1} \tag{9b}$$

3. At the top surface of the first clay layer ($i = 1$):

$$u(r,0,t)_1 = 0 \quad \text{pervious surface} \tag{10a}$$

$$\left.\frac{\partial u(r,z,t)_1}{\partial r}\right|_{z=0} = 0 \quad \text{impervious surface} \tag{10b}$$

4. At the bottom surface of the bottom clay layer ($i = 1$):

$$u(r,h,t)_1 = 0 \quad \text{pervious surface} \tag{11a}$$

$$\left.\frac{\partial u(r,z,t)_1}{\partial r}\right|_{z=h_i} = 0 \quad \text{impervious surface} \tag{11b}$$

Substitutions of Eqs. (3) and (4*a*) into Eqs. (8*a*) and (8*b*) result in homogeneous equations to define an infinite number of eigenvalues, s_n, and their associated eigenvectors, $u_n(r)$. The nth eigenvector normalized at $r = r_1$ is expressed as

$$u_n(r) = J_0(s_n r) Y_0(s_n r_0) - J_0(s_n r_0) Y_0(s_n r) \tag{12}$$

with s_n = nth eigenvalue that is the nth root of the characteristic equation

$$J_1(sr_1) Y_0(sr_0) - Y_1(sr_1) J_0(sr_0) = 0 \tag{13}$$

Using Eq. (4*b*), the following expression is obtained for the ith layer:

$$\begin{Bmatrix} \phi(z) \\ \dot{\phi}(z) \end{Bmatrix}_i = \begin{bmatrix} \cos(\beta z)_i & \sin(\beta z)_i \\ -(\beta)_i \sin(\beta z)_i & (\beta)_i \cos(\beta z)_i \end{bmatrix} \begin{Bmatrix} A_2 \\ B_2 \end{Bmatrix}_i \quad (14)$$

where $\dot{\phi}(z) = d\phi(z)/dz$. Given $[\phi(0)\dot{\phi}(0)] = (\phi^a \dot{\phi}^a)$, Eq. (14) is solved for A_2 and B_2 and then rewritten as

$$\begin{Bmatrix} \phi(z) \\ \dot{\phi}(z) \end{Bmatrix}_i = \begin{bmatrix} \cos(\beta z)_i & \dfrac{\sin(\beta z)_i}{(\beta)_i} \\ -(\beta)_i \sin(\beta z)_i & \cos(\beta z)_i \end{bmatrix} \begin{Bmatrix} \phi^a \\ \dot{\phi}^a \end{Bmatrix}_i \quad (15)$$

Therefore, $[\phi(h)\dot{\phi}(h)] = (\phi^b \dot{\phi}^b)$ yields

$$\begin{Bmatrix} \phi^b \\ \dot{\phi}^b \end{Bmatrix}_i = \begin{bmatrix} \cos(\beta h)_i & \dfrac{\sin(\beta h)_i}{(\beta)_i} \\ -(\beta)_i \sin(\beta h)_i & \cos(\beta h)_i \end{bmatrix} \begin{Bmatrix} \phi^a \\ \dot{\phi}^a \end{Bmatrix}_i \quad (16)$$

Eq. (9*a*) leads to $u_s(r) = u(r)$, $\phi_s = \phi(h)_i$, and $T_s(t) = T(t)$. Substituting Eq. (6) with these relationships into Eqs. (9*a*) and (9*b*), the boundary conditions stated in Eqs. (9*a*) and (9*b*) result in

$$\begin{Bmatrix} \phi^a \\ \dot{\phi}^a \end{Bmatrix}_{i+1} = \begin{bmatrix} 1 & 0 \\ \dfrac{(h_s k_s)_{i,i+1}}{(k_z)_{i+1}} \left(s^2 - \dfrac{\alpha^2}{(c_s)_{i,i+1}} \right) & \dfrac{(k_z)_i}{(k_z)_{i+1}} \end{bmatrix} \begin{Bmatrix} \phi^b \\ \dot{\phi}^b \end{Bmatrix}_i \quad (17)$$

where $h_s = 0$ for no pervious layer at the interface between the ith and $i+1$th layers. Eqs. (16) and (17) transfer the quantities, respectively, from the top to bottom of the layer and from one layer to another layer by simply multiplying the matrices. Therefore, this method is often called a transfer matrix method (Nogami and Paulson 1984, 1985). Given $(\phi^a \dot{\phi}^a)_i$, $(\phi^a \dot{\phi}^a)_i$, and $(\phi^b \dot{\phi}^b)_i$ can be defined, successively, one by one from $i = 2$ through I in order by using Eqs. (16) and (17).

The value α in Eq. (4*c*) is defined to satisfy the boundary conditions at the top of the first layer [Eqs. (10*a*) or (10*b*)] and the bottom of the last layer [Eqs. (11*a*) or (11*b*)]: this process can be carried out conveniently by utilizing the transfer matrix method as explained at the later section. There is an infinite number of values α that satisfy these boundary conditions. Thus, Eq. (15) leads to the expression with mth α and nth s as

$$\begin{Bmatrix} \phi_{nm}(z) \\ \dot{\phi}_{nm}(z) \end{Bmatrix}_i = \begin{bmatrix} \cos(\beta_{nm} z)_i & \dfrac{\sin(\beta_{nm} z)_i}{(\beta_{nm})_i} \\ -(\beta_{nm} z)_i \sin(\beta_{nm} z)_i & \cos(\beta_{nm} z)_i \end{bmatrix} \begin{Bmatrix} \phi^a_{nm} \\ \dot{\phi}^a_{nm} \end{Bmatrix}_i \quad (18)$$

where

$$(\beta^2_{nm})_i = \left(\frac{1}{c_z} \right)_i \alpha^2_{nm} - \left(\frac{c_r}{c_z} \right)_i s_n^2 \quad (19)$$

With Eqs. (12) and (18), $u(r,z,t)$ in Eq. (3) can be rewritten as

$$[u(r,z,t)]_i = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} C_{nm} u_n(r) [\phi_{nm}(z)]_i T_{nm}(t) \quad (20)$$

where

$$[\phi_{nm}(z)]_i = \left(\cos(\beta_{nm} z)_i \ \frac{\sin(\beta_{nm} z)_i}{(\beta_{nm})_i} \right) \begin{Bmatrix} \phi^a_{nm} \\ \dot{\phi}^a_{nm} \end{Bmatrix}_i \quad (21a)$$

$$T_{nm}(t) = A_1 e^{-\alpha^2_{nm} t} + \int_0^t \frac{d\sigma(\tau)_i}{d\tau} e^{-\alpha^2_{nm}(t-\tau)} d\tau \quad (21b)$$

Loading and Initial Conditions

It is assumed that the load $p(t)$ is applied uniformly over the surface of the system of clay and thin pervious layers. According to this one-dimensional loading condition, the total stress in the soil is equal to $p(t)$ throughout the depth. The functions $\phi_{nm}(z)$ and $u_n(r)$ are eigenvectors obtained for homogeneous boundary conditions. It is well known that the eigenvectors are orthogonal, which is explained in Appendix II for $u_n(r)$ and by Nogami and Li (2002) for $\phi_{nm}(z)$. Utilizing it, the distribution of uniform total stress is expanded by $u_n(r)$ in the r direction and $\phi_{nm}(z)$ in the z direction to write

$$[\sigma(r,z,t)]_i = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} C'_{nm} u_n(r) [\phi_{nm}(z)]_i p(t) \quad (22)$$

where $C'_{nm} = C'_n C'_m$ with

$$C'_n = \frac{1}{\Delta_n} \frac{2 \dfrac{r_0}{s_n} u_1(s_n r_0)}{r_1^2 u_0^2(s_n r_1) - r_0^2 u_1^2(s_n r_0)} \quad (23a)$$

$$C'_m = \frac{\Sigma_i (m_c)_i (A_{nm})_i + \Sigma_i (m_s)_{i,i+1} (h_s)_{i,i+1} [\phi_{nm}(h)]_i}{\Sigma_i (m_c)_i (B_{nm})_i + \Sigma_i (m_s)_{i,i+1} (h_s)_{i,i+1} [\phi_{nm}(h)]_i^2} \quad (23b)$$

in which Eq. (23*a*) is obtained from Eq. (46) in Appendix III and Eq. (23*b*) is by Nogami and Li (2002); and

$$(A_{nm})_i = \int_0^{h_i} [\phi_{nm}(z)]_i dz = \sin(\beta_{nm})_i \left(\frac{\phi^a_{nm}}{\beta_{nm}} \right)_i + [1 - \cos(\beta_{nm})_i] \times \left(\frac{\dot{\phi}^a_{nm}}{\beta_{nm}} \right)_i \quad (24a)$$

$$(B_{nm})_i = \int_0^{h_t} [\phi_{nm}(z)]_i^2 dz = \left(\frac{h_i}{2} + \frac{\sin(\beta_m h)_i}{4(\beta_{nm})_i} \right) (\phi^a_{nm})_i^2 + \sin^2(\beta_{nm} h)_i \left(\frac{\phi^a_{nm} \dot{\phi}^a_{nm}}{\beta_{nm}} \right)_i + \left(\frac{h_i}{2} - \frac{\sin(\beta_{nm} h)_i}{4(\beta_{nm})_i} \right) \left(\frac{\dot{\phi}^a_{nm}}{\beta_{nm}} \right)_i^2 \quad (24b)$$

Under the condition $\sigma(r,z,t)_i = u(r,z,t)_i$ at $t = 0$, comparison of Eqs. (20) and (22) at $t = 0$ leads to $A_1 = p(0)$ in $T_{nm}(t)$ and $C_{nm} = C'_{nm}$ in Eq. (20). Therefore, $u(r,z,t)$ and $\Delta q(r,z,t)$ are finally written, respectively, from Eqs. (20) and (6) as

$$[u(r,z,t)]_i = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} C'_{nm} u_n(r) [\phi_{nm}(z)]_i T_{nm}(t) \quad (25a)$$

$$[\Delta q(r,t)]_{i,i+1} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} C'_{nm} (\Delta Q_{nm})_{i,i+1} u_n(r) T_{nm}(t) \quad (25b)$$

where $u_n(r)$ and $\phi_{nm}(z)$ are given, respectively, by Eqs. (12) and (21*a*); C'_{nm} is given by Eqs. (23*a*) and (23*b*); and the rest are

$$(\Delta Q_{nm})_{i,i+1} = (k_s h_s)_{i,i+1} \left(s_n^2 - \frac{\alpha^2_{nm}}{(c_s)_{i,i+1}} \right) [\phi_{nm}(h)]_i \quad (26a)$$

$$T_{nm}(t) = p(0) e^{-\alpha^2_{nm} t} + \int_0^t \frac{dp(\tau)}{d\tau} e^{-\alpha^2_{nm}(t-\tau)} d\tau \quad (26b)$$

It is noted that, if the pervious layer is a geotextile sheet, $k_s h_s$ is replaced with k_g (=permeability of a sheet of geotextile per a unit length for flow normal to its vertical cross section) and c_s is set infinity.

Average Degree of Consolidation

The degree of consolidation varies in both the vertical and radial directions. The average degree of consolidation is defined herein as the average over clay layers at a given r and is written in the following manner:

$$U(r,t)=1-\frac{\Sigma_i\int_0^{h_i}(u(r,z,t))_i dz}{\Sigma_i\int_0^{h_i}p(t)dz}=1-\frac{u^t(r,t)}{u^0(t)} \tag{27}$$

where

$$u^t(r,t)=\sum_i\sum_n\sum_m C'_{nm}u_n(r)\left(\frac{\sin(\beta_{nm}h)_i}{(\beta_{nm})_i}\quad\frac{1-\cos(\beta_{nm}h)_i}{(\beta_{nm})_i^2}\right)\times\begin{Bmatrix}\phi_{nm}^a\\ \dot{\phi}_{nm}^a\end{Bmatrix}_i T_{nm}(t) \tag{28a}$$

$$u^0(t)=p(t)\sum_l h_i \tag{28b}$$

Computational Procedure

The excess pore water pressure and average degree of consolidation in clay can be computed, respectively, from Eqs. (25) and (27) by using the transfer matrix method in the following manner:

1. Define $u_n(r)$ common for all layers:
   a. Compute s_n from Eq. (13).
   b. Define $u_n(r)$ from Eq. (12) with the above computed s_n.
2. Compute α_{nm} common for all layers and define $[\phi_{nm}(z)]_i$ for each layer:
   a. Assume α_{nm} common for all layers and compute $(\beta_{nm})_i$ for each layer from Eq. (19).
   b. Set $(\phi^a\ \dot{\phi}^a)_1=(0\ 1)$, for the pervious top surface or $(\phi^a\ \dot{\phi}^a)_1=(1\ 0)$ for the impervious top surface.
   c. Establish the square matrix on the right-hand side of Eq. (16) and compute $(\phi^b\ \dot{\phi}^b)_1$ from Eq. (16) with the above defined $(\phi^a\ \dot{\phi}^a)_1$.
   d. Establish the square matrix on the right-hand side of Eq. (17) and compute $(\phi^a\ \dot{\phi}^a)_2$ from Eq. (17) with the above computed $(\phi^b\ \dot{\phi}^b)_1$.
   e. Repeat steps c and d for $i=2$ through $I-1$ to compute $(\phi^a\ \dot{\phi}^a)_1$.
   f. Do step d with $i=I$ to compute $(\phi^b\ \dot{\phi}^b)_1$ with the above-computed $(\phi^a\ \dot{\phi}^a)_1$.
   g. Check if ϕ^b is nearly equal to 0 for the pervious bottom surface or $\dot{\phi}^b$ is nearly equal to 0 for the impervious bottom surface at the Ith layer.
   h. If the difference is larger than the set tolerance, repeat steps a–g with the revised new α_{nm} until the difference becomes within the tolerance. If the difference is less than the tolerance, define $[\phi_{nm}(z)]_i$ from Eq. (21a) with $(\phi_{nm}^a\ \dot{\phi}_{nm}^a)_i$ computed in the last iteration cycle.
3. Compute C'_{nm} from Eqs. (23a) and (23b).

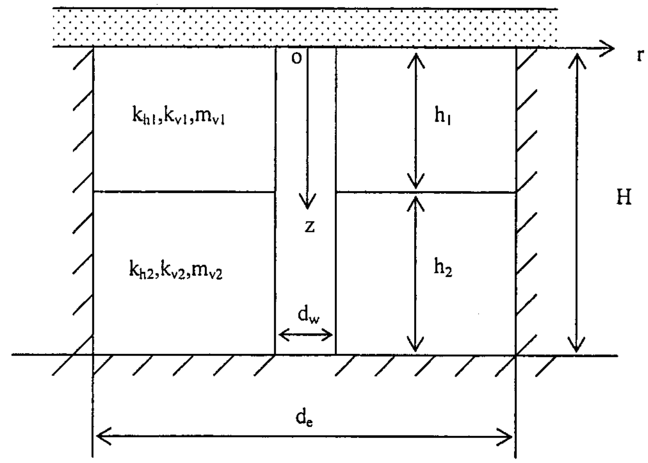

Fig. 2. Two-layer system with vertical drain

4. Define $u(r,z,t)_i$ and compute $U(r,t)$ from Eqs. (25a) and (27), respectively.

In the above process, α values are iteratively determined so that $(\phi^b)_1$ is nearly equal to 0 for the pervious bottom surface or $(\dot{\phi}^b)_1$ is nearly equal to 0 for the impervious bottom surface. This can be conveniently performed after computing the function $F(\alpha)$ $[=(\phi^b)_1$ or $(\dot{\phi}^b)_1]$ with steps 2a–2f and plotting it to observe the roots of $F(\alpha)=0$ visually.

Verifications

Comparison with Another Solution for Inhomogeneous Clay without Thin Sand Layer

First, the formulation based on the transfer matrix method is examined for a two-layer clay without thin pervious layers. The conditions considered are the same as those previously computed by the direct analytical solutions (Xie et al. 1994), which are shown in Fig. 2 and Table 1, where $c_{v,h}=c_{z,r}$ and $k_{v,h}=k_{z,r}$. The direct analytical solution adopted the superposition method proposed by Rendulic (1935) and has a very lengthy expression already even for a two-layer system. The distributions of excess pore water pressures computed by the two approaches are shown in Fig. 3 for T $(=c_{vl}t/H^2)=0.0002$ and 0.002. Good agreement between the two computed results can be seen except in the early stage of consolidation. The difference in the early stage is due to an insufficient number of terms in the lateral expansion in the direct analytical solution. It should be noted that the superposition method cannot be used when the horizontal drains are involved. This is because the lateral flow in the horizontal drain produces the flow in the clay both in the vertical and horizontal directions in general, which cannot be decoupled to apply the superposition principle.

Table 1. Geotechnical and Geometrical Parameters of Three Cases

Case No.	n	H/d_w	h_2/h_1	kv_2/kv_1	c_{v2}/c_{v1}	c_{h2}/c_{h1}
1	10	100	1	2	1	5
2	10	100	1	2	1	1
3	10	100	1	2	1	1/5

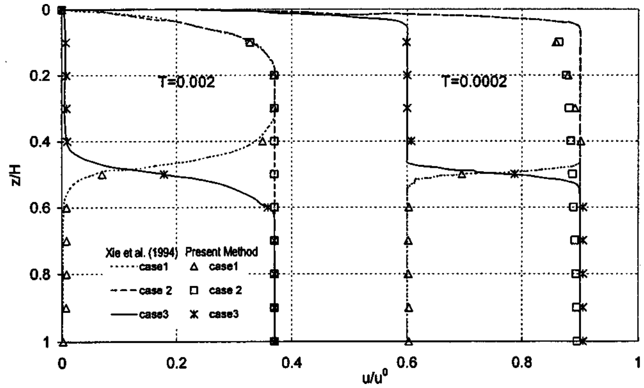

Fig. 3. Excess pore water pressure isochrone computed by Xie et al. (1994) and present formulation

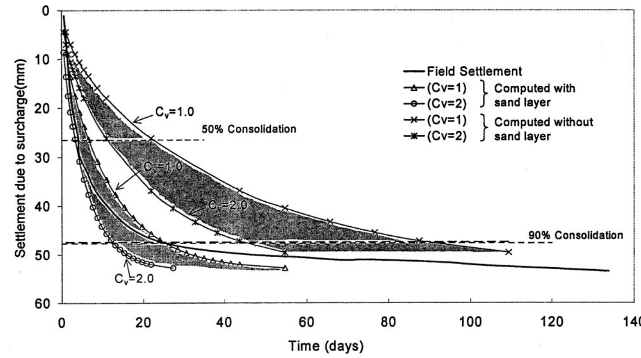

Fig. 4. Computed and observed settlement time histories

Comparison with Field Observation for Clay with Thin Sand Layer

Second, the field test result at Pulau Tekong Besar (Lee et al. 1987) is used for verification. A test pond, in a rectangular shape with a large aspect ratio, was dug in the existing land reclaimed with sand at Pulau Tekong Besar. It was then filled with clay and sand to form a test clay reclaimed land of depth 1.35 m with a horizontal sand layer of 0.55 m thickness at the middle depth of the test reclaimed land. The shorter width of the rectangular model was 28 m. The coefficient of consolidation of clay varies with the effective pressure. According to laboratory tests, the representative coefficients of consolidation were c_c $(=c_z=c_r)$ $=2.0\ \mathrm{m^2/year}$ through a 50% degree of consolidation and c_c $=1.0\ \mathrm{m^2/year}$ through 90% consolidation (Tan et al. 1992). The coefficients of permeability of clay were in the ranges of 4×10^{-9}–4×10^{-8} m/s for clay and 5×10^{-4}–1×10^{-3} m/s for sand. Accordingly, the representative permeability ratio k_s/k_c $(k_r=k_z=k_c)$ was assumed to be in the range of 1.1×10^4–5.6×10^4 (Tan et al. 1992). The previous study with the finite element method indicated that the settlement was mostly due to the primary consolidation in clay fill except in the late stage of consolidation (Tan et al. 1992).

The present approach is also applicable for the system of clay and thin sand layer tried at the Pulau Tekong Besar site, after rewriting the formulations with the Cartesian coordinates. Using the above field information and modified formulations, 50 and 90% consolidation times (t_{50} and t_{90}, respectively) were computed as shown in Table 2. The average values over those computed for various permeability ratios are $t_{50}=3.1$ days and t_{90} $=26.3$ days, while those computed without a sand layer are t_{50} $=9.84$ days and $t_{90}=90.59$ days. The estimated values by using the hyperbolic method from the field-measured settlement record are $t_{50}=3.0$ days and $t_{90}=26$ days (Lee et al. 1987).

Fig. 4 shows the observed settlement curve load and those computed with $c_c=1.0$ and $2.0\ \mathrm{m^2/year}$. It is noted that a surcharge load was applied in both field cases and the curves in Fig. 4 are for the period after the application of the surcharge load. The settlement curves for the condition without the sand layer are also potted in Fig. 4. Reasonably good agreement between the computed and observed settlements can be seen once the sand layer is taken into account in the analysis.

Table 2. Computed Results for Pulau Tegong Besar Site

k_s/k_c	t_{50} (days)	t_{90} (days)
1.1×10^4	3.5	29.4
2.8×10^4	3.0	25.6
5.6×10^4	2.7	24.1
Average	3.1	26.3

Parametric Studies

Nondimensional Forms

The expression of $u(r,z,t)$ is a product of $u(r)$, $\phi(z)$, and $T(t)$. The functions $u(r)$ and $T(t)$ are rewritten with the nondimensional parameters as

$$u(r)=J_0(\overline{sr})Y_0(\overline{s})-J_0(\overline{s})Y_0(\overline{sr}) \tag{29a}$$

$$T(t)=p(0)\left(e^{-\overline{\alpha}_n^2 T}+\int_0^T \frac{d\overline{p}(\overline{\tau})}{d\overline{\tau}}e^{-\overline{\alpha}^2\overline{\tau}}d\overline{\tau}\right) \tag{29b}$$

where $\overline{s}=sr_0$; $\overline{\alpha}=\alpha h/\sqrt{c_z}$; $\overline{p}=p/p(0)$; and

$$T=\frac{c_z t}{h^2} \tag{30}$$

with $n=r_1/r_0$. The function $\phi(r)$ is governed by the following two transfer matrices in nondimensional form:

$$\begin{Bmatrix}\phi^b\\ \dot{\phi}^b h\end{Bmatrix}_i=\begin{bmatrix}\cos(\overline{\beta})_i & \dfrac{\sin(\overline{\beta})_i}{(\overline{\beta})_i}\\ -(\overline{\beta})_i\sin(\overline{\beta})_i & \cos(\overline{\beta})_i\end{bmatrix}\begin{Bmatrix}\phi^a\\ \dot{\phi}^a h\end{Bmatrix}_i \tag{31a}$$

$$\begin{Bmatrix}\phi^a\\ \dot{\phi}^a h\end{Bmatrix}_{i+1}=\begin{bmatrix}1 & 0\\ (\lambda)_{i,i+1}\overline{s}^2-(\delta)_{i,i+1}(\overline{\beta})_{i+1}^2 & \dfrac{(k_z)_i}{(k_z)_{i+1}}\end{bmatrix}\begin{Bmatrix}\phi^b\\ \dot{\phi}^b h\end{Bmatrix}_i \tag{31b}$$

where $\overline{\beta}=\beta h$; and

$$(\lambda)_{i,i+1}=\frac{(k_s h_s)_{i,i+1}(h)_i}{r_1^2(k_z)_{i+1}} \tag{32a}$$

$$(\delta)_{i,i+1}=\frac{(m_s)_{i,i+1}}{(m_c)_i}\frac{(h_s)_{i,i+1}}{(h)_i} \tag{32b}$$

It is noted that $k_s h_s=k_g$ and $\delta=0$ for geotextile. The relationship among the parameters α, β, and s, given in Eq. (5), is rewritten in the following nondimensional form:

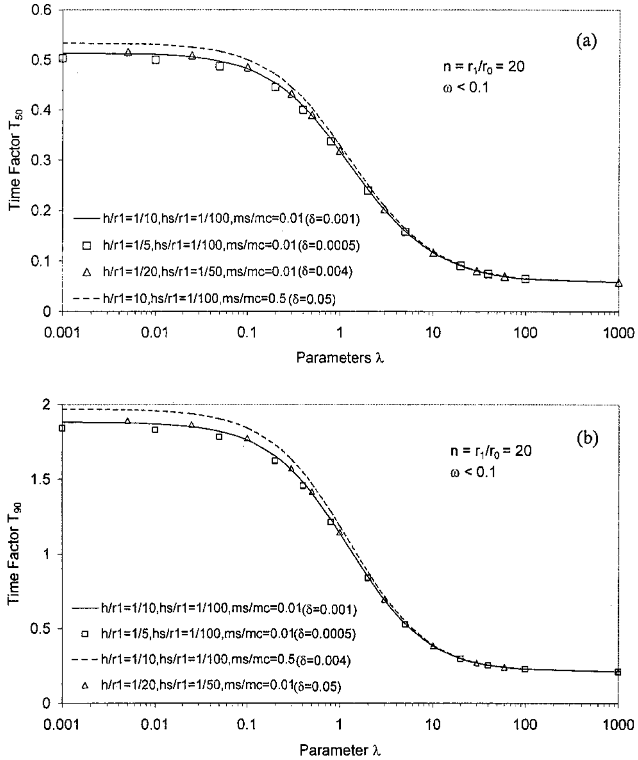

Fig. 5. T_{50} and T_{90} for various values of λ and δ: (a) T_{50} and (b) T_{90}

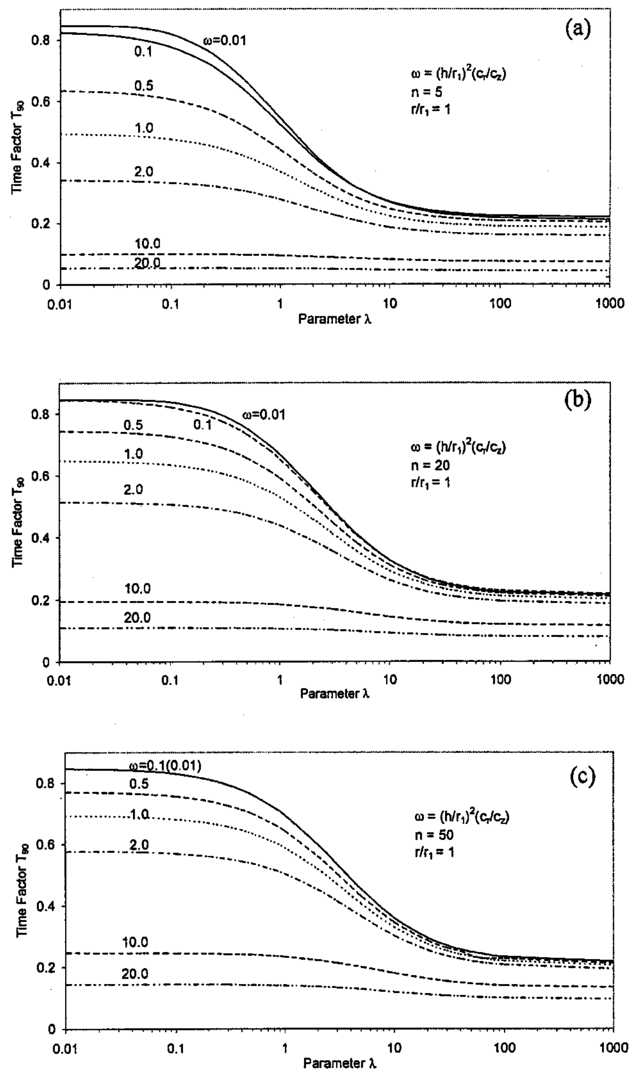

Fig. 6. Variations of time factor with λ for various ω and n: (a) $n=5$; (b) $n=20$; and (c) $n=50$

$$\overline{\alpha}^2=\overline{\beta}^2+\omega\overline{s}^2 \tag{33}$$

where

$$\omega=\left(\frac{c_r}{c_z}\right)\left(\frac{h}{r_1}\right)^2 \tag{34}$$

The above equations indicate that the parameters λ, δ, n, and ω govern the consolidation behavior of clay with drain systems: ω = parameter influencing the lateral flow relative to the vertical flow in clay, λ = major parameter influencing the rate of discharge into the horizontal drain, and δ = parameter expressing the relative compressibility of the thin sand layer. According to the expressions of these parameters, both horizontal and vertical drains influence λ and δ to modify the consolidation behavior of clay, while only vertical drains influence n and ω.

Consolidation Behavior of Clay with Horizontal and Vertical Drains

It is assumed that a homogeneous isotropic clay soil has perfectly pervious surfaces at the top and bottom ends of the clay, and a horizontal thin sand layer is located at the mid-depth in the clay. The nondimensional time to reach the average degree of the consolidation equal to 50 or 90% (T_{50} or T_{90}) are computed for various combinations of k_s/k_z, h/r_1, h_s/r_1, and m_s/m_c to yield various λ and δ values. Fig. 5 shows the variations of the above computed T_{50} or T_{90} with λ under various given δ. As λ decreases, the discharge capacity of the pervious layer becomes smaller compared with the flow of pore water in clay. Thus, when T_{50} or T_{90} increases to reach the flat part of the curve, no or very little pore water is discharged into the pervious layer. In such a case, the pervious layer is least effective for discharging the pore water in clay. As λ increases, on the other hand, the situation is reversed and T_{50} or T_{90} decreases to reach the flat part again. This part corresponds to the condition, in which the excess pore water pressure in the pervious layer becomes zero or nearly zero and, therefore, the pervious layer is fully functional for discharging the pore water. Fig. 5 indicates that the effectiveness of the horizontal pervious layer is affected very little by the difference in δ and rather uniquely governed by λ. This trend with δ is more obvious for larger λ values.

Again, the above clay with a system of vertical drains and a single horizontal drain is considered. Figs. 6 and 7 show the variations of T_{90} with λ for various ω and n, respectively. It is seen that the value λ again appears to rather uniquely govern the effectiveness of horizontal drain regardless of the difference in ω and n, although the curves tend to be shifted to the right slightly more for higher n and smaller ω. When λ is 100 or above, the horizontal drain is most effective for all cases shown. Figs. 6 and 7 indicate that the horizontal drain can be more effective to reduce the consolidation time for higher n and lower ω. This is because, in such conditions with n and ω, the overall consolida-

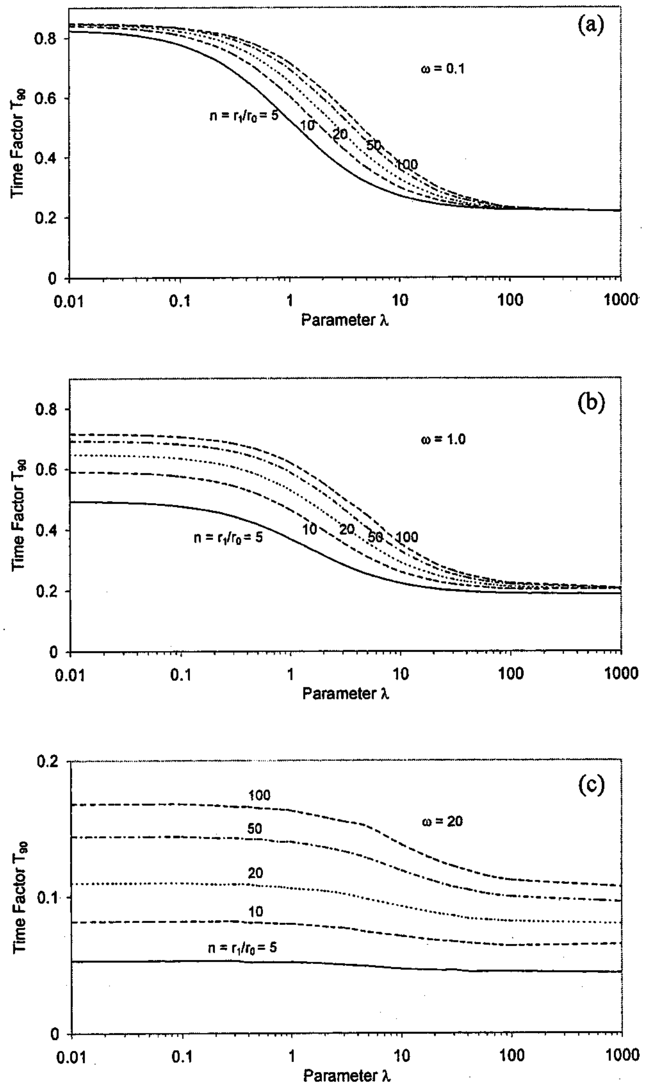

Fig. 7. Variations of time factor with λ for various ω and n: (a) $\omega = 0.1$; (b) $\omega = 1.0$; and (c) $\omega = 20$

tion in clay is more predominantly governed by consolidation in the vertical direction, and thus the horizontal drain to intercept this consolidated pore water can be more effective.

Multiple horizontal drains are now considered. They are assumed to be arranged in such a way that the maximum lengths of vertical drainage paths in clay, to reach the nearest horizontal drains, are identical, as shown in Fig. 8. Then, the system is considered to be made of fundamental units connected in series. When the drains are most effective in such an arrangement (i.e., zero excess pore water pressure in the horizontal drains), the upper and lower ends of an individual fundamental unit are treated as an impervious surface. In this case, consolidation behaviors in fundamental units are mutually uncoupled in the system. Otherwise, they are generally mutually coupled. Fig. 9 shows the ratio of $T_{90(\text{coupled})}$ and $T_{90(\text{uncoupled})}$, computed for clay soil with various numbers of horizontal drains arranged as shown in Fig. 8: in which $T_{90(\text{coupled})}$ is computed for the system as it is, while $T_{90(\text{uncoupled})}$ is computed for an isolated fundamental unit with impervious surfaces at both ends. As expected, the ratio approaches 1 as λ increases for all cases. It is noticed that the parameter λ is rather uniquely related to the effectiveness of horizontal drains regardless of the difference in the number of horizontal drains in the system, and the drains are most effective when λ is equal to 100 or above even for the system with multiple horizontal drains. This can be recognized in Fig. 10, in which the excess pore water pressures in the drains are nearly zero throughout the consolidation time when λ is 100.

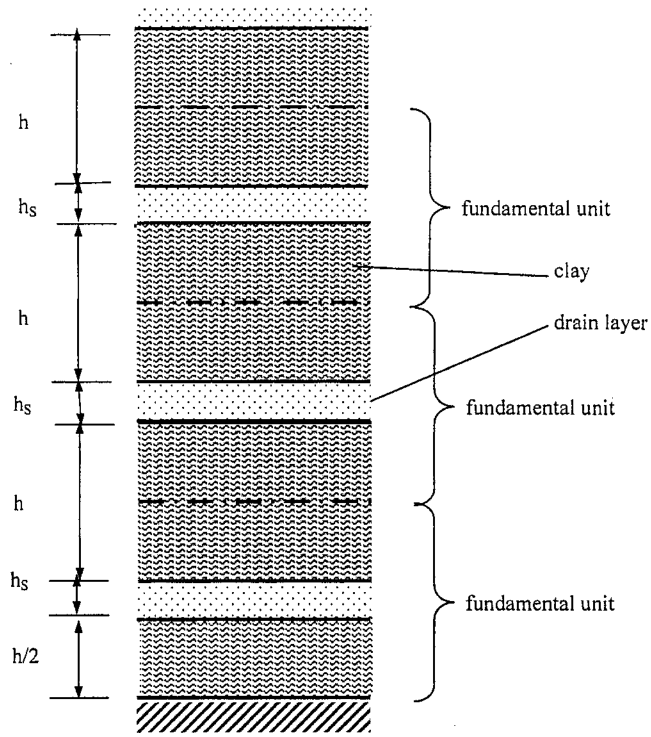

Fig. 8. System of clay and horizontal drain layers

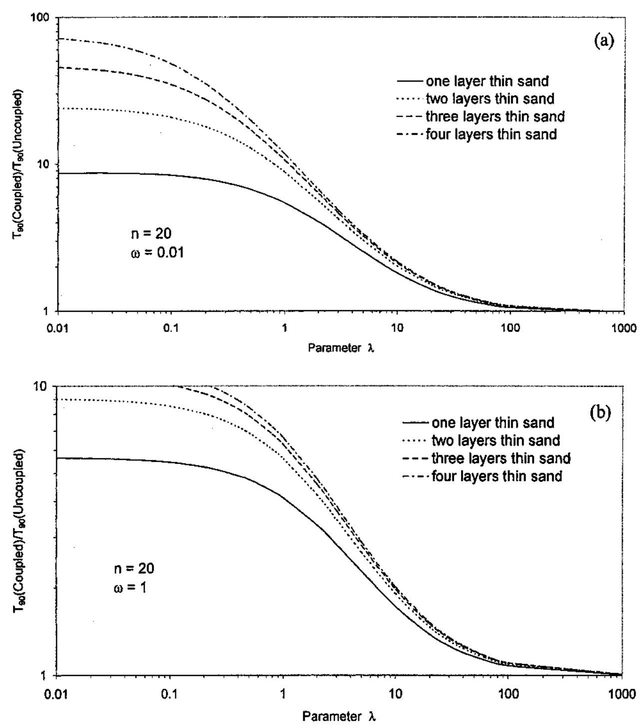

Fig. 9. Variation of time factor (T_{90}) ratio with λ for various number of multiple horizontal drain layers: (a) $\omega = 0.01$ and (b) $\omega = 1$

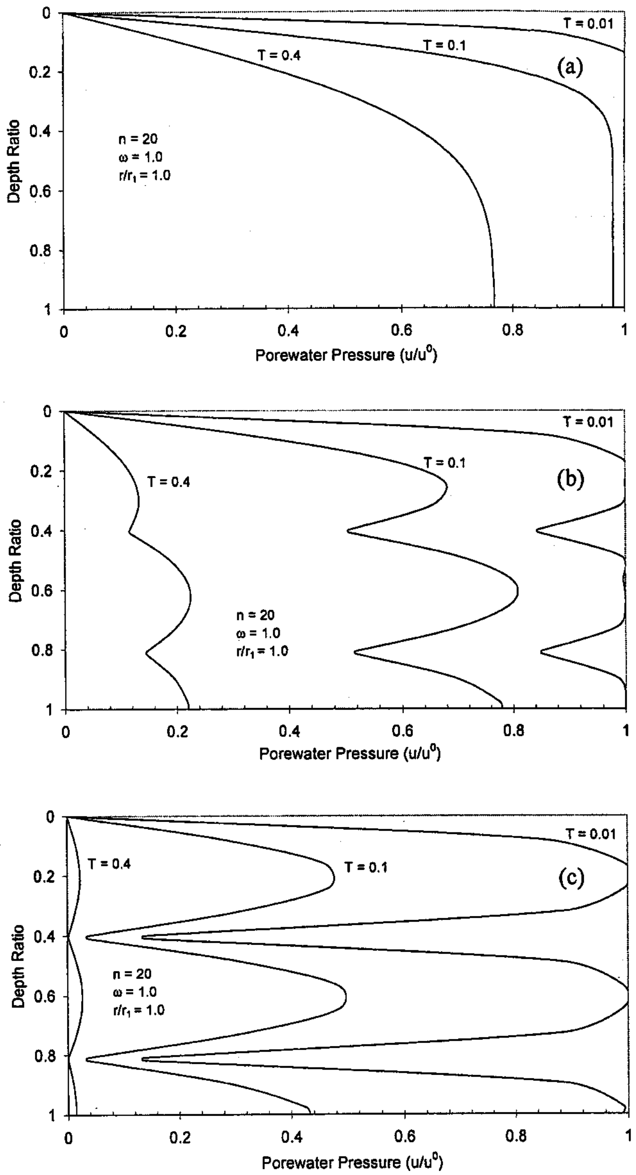

Fig. 10. Excess pore water pressure along depth: (a) $\lambda=0$; (b) $\lambda=5$; and (c) $\lambda=100$

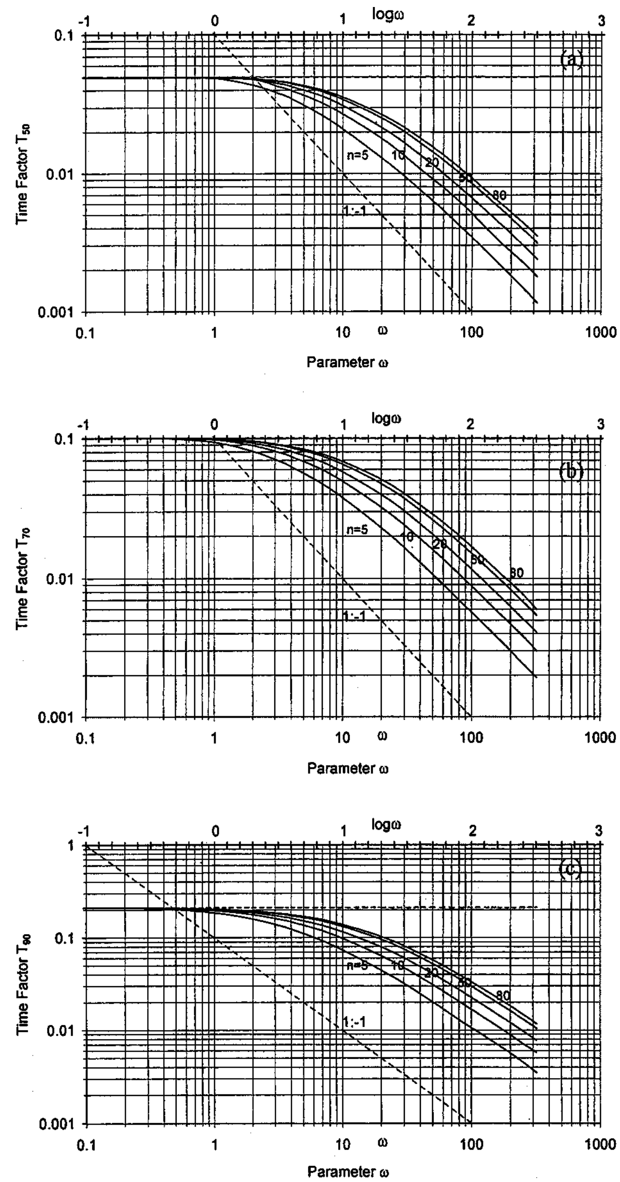

Fig. 11. Design chart for T_{50}, T_{70}, and T_{90}

Design of a System with Horizontal and Vertical Drains

Horizontal, thin, pervious drains such as layers of sand or horizontal sheets of geotextile are used as horizontal drains in combination with vertical cylindrical drains to accelerate the consolidation in clay. Clay is assumed to be homogeneous and isotropic in design consideration. Multiple pervious horizontal layers are arranged as shown in Fig. 8. It is assumed that the following information is given for design: target average degree of consolidation (U_p); time to achieve U_p (t_p); twice the length of vertical drainage path when horizontal drain layers are absent (h'); radius of vertical cylindrical drains (r_0); material properties of clay (k_c and c_c); and permeability of sand (k_s) if the horizontal drains are sand layers.

When the excess pore water pressures in these layers are zero (i.e., $\lambda \geqslant 100$), the horizontal drain works most effectively and the consolidation behaviors of the fundamental units in the system are mutually uncoupled. Therefore, an optimum design is made so that λ for the clay and drain system is equal 100. With $\lambda=100$, the time T to achieve a specific average degree of consolidation at $r=r_1$ depends on ω and n, as shown Fig. 11. Fig. 11 is also applicable to the entire clay ground without horizontal drain layers, when T and ω are expressed, respectively, as

$$T'=\frac{c_z t'}{h'^2} \tag{35a}$$

$$\omega'=\left(\frac{h'}{r_1'}\right)^2 \tag{35b}$$

where h' = entire thickness of clay with both ends pervious or twice the thickness of clay with only one end pervious; notations with prime = those for the ground with no horizontal drains. It is assumed that Fig. 11 is provided for design.

With Eqs. (30a), (34), (35a), and (35b), the following relationship is obtained:

$$T\omega=T'\omega'\frac{t}{t'}\left(\frac{r_1'}{r_1}\right)^2$$

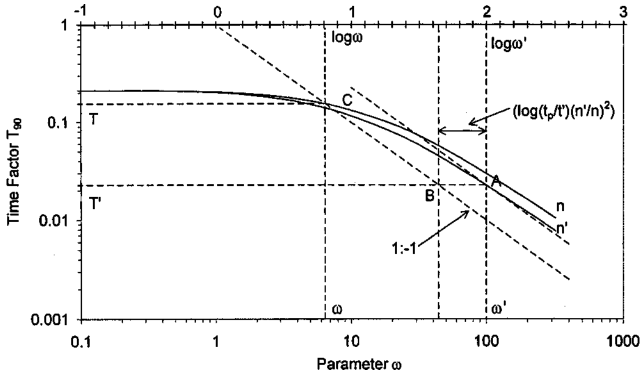

Fig. 12. Geometrical relationship between $T\omega$ and $T'\omega'$

or

$$\log T = -\log \omega + \log(T'\omega') + \log\left[\frac{t}{t'}\left(\frac{r_1'}{r_1}\right)^2\right] \tag{36}$$

Eq. (36) indicates that the $\log T \sim \log \omega$ relationship (including $\log T' \sim \log \omega'$ relationship) is a straight line with a slope of 1 to -1 and can be obtained by shifting the $\log T' \sim \log \omega'$ relationship by $\log[t/t' \cdot (r_1'/r_1)^2]$. T and ω for the most effective condition with horizontal drain must be located in the curve shown in Fig. 11. Therefore, for a give t_p/t' and r_1'/r_1 $(=n'/n)$, such T and ω are uniquely defined as the values at the intercept of the curve in Fig. 11 and the $\log T \sim \log \omega$ straight line. Referring to Fig. 12, this can be done conveniently in the following manner:

1. Compute ω' from Eq. (35*b*) and locate ω' in the design curve for given n' (point A).
2. Find T' corresponding to ω' and n' in the chart and compute t' from T' [Eq. (35*a*)].
3. Shift the point A by $\log[t_p/t' \cdot (n'/n)^2]$ to point B.
4. Draw a straight line with slope of 1 to -1 at point B.
5. Find the intercept of the above straight line and the design curve for given n (point C) to define T.

The value h is defined from the expressions of T [Eq. (30)] with the above-defined T as

$$h = \sqrt{\frac{c_z t_p}{T}} \tag{37}$$

Then, h_s or k_g is defined from the expression of λ [Eq. (32*a*)] with $\lambda = 100$ as

$$k_s h_s \text{ or } k_g = \frac{100 r_1^2 k_c}{h} \tag{38}$$

The above design approach is carried out in the following manner:

1. Define U_p and t_p, and find other inputs (i.e., h', k_s, and k_c).
2. Assume n and n' and compute r_1' $(=n'r_0)$ and r_1 $(=nr_0)$.
3. Find T following the above-explained procedure with the design curves given in Fig. 11.
4. Compute h from Eqs. (37).
5. Compute h_s or k_g from Eq. (38).
6. If any of the computed h, h_s (or k_g), and r_1 are not suitable, repeat the above computation steps with the revised n.
7. Design the drain system based on the above-obtained h, h_s (or k_g), and r_1.

Example 1

The assumed conditions are:

Clay properties: $c_c = 4.0$ m^2/yr, $k_c = 1.0 \times 10^{-9}$ m/s.

Sand properties: $k_s = 1.0 \times 10^{-6}$ m/s.

Geometry: clay thickness $H = 20$ m.

Vertical drain: $r_0 = 0.2$ m.

Boundary conditions: pervious top and impervious bottom surfaces.

The drain system is designed to reach the average degree of consolidation 50% in 0.5 year, with $n = n'$. The following are the computation steps:

1. $U_p = 50\%$ and $t_p = 0.5$ year.
2. Assuming $n = n' = 20$, $r_1 = r_1' = n'\ r_0 = 20 \times 0.2 = 4$ m.
3. $\omega' = (h'/r_1')^2 = (40/4)^2 = 100$.
   $T' = 0.0066$ (point A), from Fig. 11(a) with $\omega' = 100$.
   $$t' = \frac{T'h'^2}{c_c} = \frac{0.0066 \times 40^2}{4} = 2.64 \text{ year}$$
   $$\log\left[\frac{t_p}{t'}\left(\frac{n'}{n}\right)^2\right] = \log\left(\frac{0.5}{2.64}\left(\frac{20}{20}\right)^2\right)$$
   $$= -0.7226 \text{ to locate point B}$$
   $T = 0.047$ (point C) from Fig. 11(a).
4. $h = \sqrt{c_c t_p / T} = \sqrt{4 \times 0.5/0.047} = 6.5$ m; provide three horizontal thin pervious layers with $h = 5.7$ m.
5. Geotexile, $k_g = 100 r_1^2 k_c / h = 100 \times 4^2 \times 10^{-9}/5.7 = 2.8 \times 10^{-7}$ m^2/s.
6. Thin sand layer, $h_s = k_s h_s / k_s = 2.8 \times 10^{-7}/1.0 \times 10^{-6} = 0.28$ m; provide $h_s = 0.3$ m.

Example 2

The assumed conditions are:

Clay properties: $c_c = 4.0$ m^2/year, $k_c = 1.0 \times 10^{-9}$ m/s.

Sand properties: $k_s = 1.0 \times 10^{-6}$ m/s.

Geometry: clay thickness $H = 15$ m.

Vertical drain: $r_0 = 0.2$ m.

Boundary conditions: pervious top and impervious bottom surfaces.

The drain system is designed to reach the average degree of consolidation 50% at $t = t'$ with $n > n'$. The computation steps are as follows:

1. $U_p = 50\%$ and $t_p = t'$.
2. Assuming $n' = 10$ and $n = 20$, $r_1' = n'\ r_0 = 10 \times 0.2 = 2.0$ m and $r_1 = nr_0 = 20 \times 0.2 = 4.0$ m.
3. $\omega' = (h'/r_1')^2 = (30/2)^2 = 225$
   $T' = 0.0026$ (point A), from Fig. 11(a) with $\omega' = 225$
   $$t' = \frac{T'h'^2}{c_c} = \frac{0.0026 \times 30^2}{4} = 0.58 \text{ year}$$
   $$\log\left[\frac{t_p}{t'}\left(\frac{n'}{n}\right)^2\right] = \log\left[\frac{0.58}{0.58}\left(\frac{10}{20}\right)^2\right] = -0.602 \text{ to locate point B}$$
   $T = 0.047$ (point C) from Fig. 11(a).
4. $h = \sqrt{c_c t_p / T} = \sqrt{4 \times 0.58/0.047} = 7.0$ m; provide two horizontal thin pervious layers with $h = 6.0$ m.
5. Geotexile, $k_g = 100 r_1^2 k_c / h = 100 \times 4^2 \times 10^{-9}/6.0 = 2.67 \times 10^{-7}$ m^2/s.
6. Thin sand layer, $h_s = k_s h_s / k_s = 2.67 \times 10^{-7}/1.0 \times 10^{-6} = 0.27$ m; provide $h_s = 0.3$ m.

Conclusions

The consolidation behavior of clay with a system of horizontal drains and vertical cylindrical drains is formulated using the

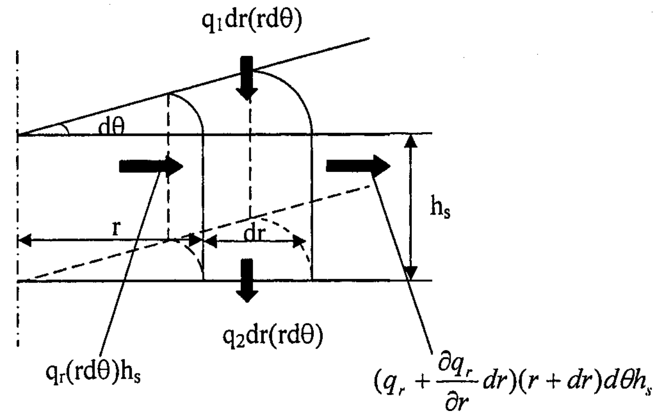

Fig. 13. Flow in thin pervious layer

transfer matrix approach. The developed formulation can handle the inhomogeneous profile in clay and multiple horizontal drains made of either thin sand layers or geotexstile sheets. The number of terms used in series is five terms in the *r* direction, and five to ten terms in the *z* direction depending on the behavior in the series expression. As Terzaghi's consolidation solution, only one or two terms in the expansion in the *z* direction are sufficient to compute the consolidation behavior in the later stage of consolidation but the upper-side number of terms is required in the early stage of consolidation. The formulation is found to be very efficient and convenient for computation.

The consolidation of clay with a system of horizontal and vertical drains is governed by the nondimensional parameters λ, δ, n, and ω. It is found that the important parameter controlling the effectiveness of the horizontal drains is the value λ defined by Eq. (32*a*), which is a function of k_s and h_s of the sand layer (or k_g of geotextile), k_c and h of the clay layer, and r_1 of the vertical drain. When λ is equal to 100 or greater, the excess pore water pressure in the horizontal drains is nearly zero, resulting in the horizontal most effective drains. When ω is smaller and n is larger, installation of horizontal drains can accelerate the consolidation of clay more effectively.

A convenient method is developed for designing an optimum system of multiple horizontal drains and vertical cylindrical drains. The method is based on the idea to design the drain system having $\lambda = 100$ or slightly larger. In order to facilitate the approach, design charts are provided.

Appendix I. Derivation of Eq. (1*b*)

A small width, dr and $rd\theta$, is considered in a thin layer to form an element as illustrated in Fig. 13. Since the excess pore water pressure is uniform within the element, the volumetric change of the element is

$$\Delta V = \varepsilon h_s r dr d\theta \tag{39}$$

where ΔV = volume change of the element; h_s = thickness of the sand layer; and ε = volumetric strains. According to the water flow into and out of the element and Eq. (39), the storage equation is

$$\frac{\partial \varepsilon(r,t)}{\partial t} h_s dr(rd\theta) = q_r(r,t)(rd\theta)h_s - \left(q_r(r,t) + \frac{\partial q_r(r,t)}{\partial r}dr\right) \times (r+dr)d\theta h_s + q_1(r,t)dr(rd\theta) - q_2(r,t)dr(rd\theta)$$

or

$$\frac{\partial \varepsilon(r,t)}{\partial t} = -\frac{q_r(r,t)}{r} - \frac{\partial q_r(r,t)}{\partial r} - \frac{dr}{r}\frac{\partial q_r(r,t)}{\partial r} + \frac{\Delta q(r,t)}{h_s} \tag{40}$$

where $q_r(r,t)$ = lateral flow in the sand layer; $q_1(r,t)$ and $q_2(r,t)$ = vertical flow into and out of the sand layer, respectively; and $\Delta q(r,t) = q_1(r,t) - q_2(r,t)$ = the flow loss due to the lateral drainage in the sand layer while the pore water flows vertically from one clay layer to another.

Introducing Darcy's law, the effective stress principle and the total stress equal to the applied pressure on the ground surface, Eq. (40) can be rewritten as Eq. (2)

Appendix II. Orthogonality of $u_n(r)$

After substitution of Eq. (3) into Eqs. (1) and (2), the equation for $u(r)$ in the nth and mth modes can be written, respectively, as

$$\ddot{u}_n + \frac{1}{r}\dot{u}_n + s_n^2 u_n = 0 \tag{41a}$$

$$\ddot{u}_m + \frac{1}{r}\dot{u}_m + s_m^2 u_m = 0 \tag{41b}$$

where $\dot{u} = du(r)/dr$; and $\ddot{u} = d^2u(r)/dr^2$. Eqs. (41*a*) and (41*b*) are multiplied, respectively, by ru_m and ru_n. Integrating the resultants over r from r_0 to r_1 leads to, respectively,

$$\int_{r_0}^{r_1}(ru_m\ddot{u}_n + u_m\dot{u}_n + s_n^2 ru_m u_n)dr = r\dot{u}_m u_n|_{r_0}^{r_1} - \int_{r_0}^{r_1} r\dot{u}_m\dot{u}_n dr + \int_{r_0}^{r_1} s_n^2 ru_m u_n dr \tag{42a}$$

$$\int_{r_0}^{r_1}(ru_n\ddot{u}_m + u_n\dot{u}_m + s_m^2 ru_n u_m)dr = r\dot{u}_n u_m|_{r_0}^{r_1} - \int_{r_0}^{r_1} r\dot{u}_n\dot{u}_m dr + \int_{r_0}^{r_1} s_m^2 ru_n u_m dr \tag{42b}$$

Subtraction of Eqs. (42*a*) and (42*b*) and substituting the boundary conditions, $u(r_0) = \dot{u}(r_1) = 0$, into the resultant yield

$$(s_m^2 - s_m^2)\int_{r_0}^{r_1} ru_n u_m dr = 0 \tag{43}$$

Since $s_m \neq s_n$, Eq. (43) implies that the orthogonality of u_n exists.

Appendix III. Coefficient in the Expansion by Normal Modes

The function $u(r)$ is expanded by using its eigenvectors such that

$$u(r) = \sum_{n=1} c_n u_n(r) \tag{44}$$

where c_n = coefficient of the nth term. Multiplying Eq. (44) by $ru_m(x)$ and integrating the resultant over r from r_0 to r_1 result in

$$\int_{r_0}^{r_1} r u_m u dx = \sum_{n=1} c_n \int_{r_0}^{r_1} r u_m u_n dx \quad (45)$$

If $u_n(r)$ has orthogonality, the terms with $n \neq m$ are all zero in the right-hand summation. Therefore, c_n is defined as

$$c_n = \frac{\int_{r_0}^{r_1} r u_n u dx}{\int_{r_0}^{x_1} r u_n^2 dx} \quad (46)$$

References

Abid, M. M., and Pyrah, I. C. (1991). "Consolidation behavior of finely laminated clays." *Comput. Geotech.*, 307–323.

Barron, R. A. (1948). "Consolidation of fine grained soils by drain wells." *Trans. Am. Soc. Civ. Eng.*, 113, 718.

Choa, V., Karunaratne, G. P., Ramaswamy, S. D., Vijiaratnam, A., and Lee, S. L. (1979). "Pilot test for soil stabilization at Changi Airport." *Proc., 6th Asian Reg. Conf. Soil Mech. and Found. Eng.*, Singapore, 141–144.

Gibson, R. E., and Shefford, G. C. (1968). "The efficiency of horizontal drainage layers for accelerating consolidation of clay embankments." *Geotechnique*, 18, 327–335.

Gray, H. (1945). "Simultaneous consolidation of contiguous layers of unlike compressible soils." *Trans. Am. Soc. Civ. Eng.*, 110, 1327–1344.

Horne, M. R. (1964). "The consolidation of a stratified soil with vertical and horizontal drainage." *Int. J. Mech. Sci.*, 6, 187–197.

Itoh, M., Shirasawa, M., Itoh, A., Kumagai, K., and Fukuoka, M. (1994). "Well documented case study of a reinforce wall." *Proc., 5th Int. Conf. on Geotextiles Geomembranes, and Related Products*, Singapore, 1.1, 49–52.

Karunarate, G. P., Young, K. Y., Tan, T. S., Tan, S. A., Liang, K. M., Lee, S. L., and Vijiaratnam, A. (1990). "Layered clay–sand scheme reclamation at Changi South Bay." *Proc., 10th Southeast Asian Geotechnical Conf.*, Southeast Asian Society of Soil Mechanics and Foundation Engineering, 1.1, 71–76.

Lee, S. L., Karunaratne, G. P., Young, K. Y., and Ganeshan, V. (1987). "Layered clay–sand scheme of land reclamation." *J. Geotech. Eng.*, 113(9), 984–995.

Loh, S. L. (1998). "Drainage capacity of high strength geotextile in residual soil for reinforcement application." Thesis, National Univ. of Singapore.

Nogami, T., and Paulson, S. K. (1984). "Winkler soil model for axial response analysis of pile groups." *ASCE special technical publication on analysis and design of pile foundations*, Geotechnical Engineering Division, ASCE, San Francisco, October, 278–309.

Nogami, T., and Paulson, S. K. (1985). "Transfer matrix approach for nonlinear pile group response analysis." *Int. J. Numer. Analyt. Meth. Geomech.*, 9, 299–316.

Nogami, T., and Li, M. (2002). "Consolidation of system of clay and thin sand layers." *Soils Foundations, Jpn. Geotech. Soc.*, in press.

Rendulic, L. (1935). "Der hydrodynamische spannungsausgleich in zentral entwässerten entwasserten." *Wasserwirtschaft*, 1.

Rowe, R. K. (1992). "Some challenging applications of geotextiles in filtration and drainage." *Proc., Conf. GEOFAD'92: Goetextiles in Filtration and Drainage*, U.K. Chapter of the International Geotextile Society, Churchill College, Cambridge, U.K., September.

Schiffman, R. L., and Stein, J. R. (1970). "One-dimensional consolidation of layered systems." *J. Soil Mech. Found. Div.*, 96(4), 1499–1504.

Suzuki, S., and Yamada, M. (1990). "Construction of an artificial island for the Kansai Intl. Airport," *Proc., 1989 Sem. on Coastal Dev.*, IEM and Kosai Club, Malaysia, 511–520.

Takai, T., Oikawa, K., and Higuchi, Y. (1989). "Construction of an artificial island for the Kansai Intl. Airport," *Proc., 1989 Sem. on Coastal Dev.*, Bankok, Kosai Club, 511–520.

Tan, S. A., Yong, K. Y., and Lee, S. L. (1992). "Drainage efficiency of sand layer in layered clay–sand reclamation." *J. Geotech. Eng.*, 118(2), 209–228.

Tatsuoka, F., Ando, H. H., Iwaski, K., and Nakamura, K. (1986). "Performance of clay test embankment reinforced with nonwoven geotextile." *Proc., 3rd Int. Conf. on Geotextiles*, 2, Vienna, Austria.

Tominaga, M. M., Sakaki, T., Hashimoto, M., and Paulino, C. C. (1979). "Embankment tests in reclamation area, Manila Bay." *Proc., 6th Asian Reg. Conf. Soil Mech. and Found. Eng.*, Singapore, 195–198.

Watarii, Y. (1984). "Reclamation with clayey soil and method of earth spreading on the surface." *Proc., Seminar on Soil Improvement and Construction Techniques in Soft Ground*, Japanese Society of Soil Mechanics and Foundation Eng., 103–119.

Xie, K. H., Lee, P. K. K., and Cheung, Y. K. (1994). "Consolidation of a two-layer system with vertical ideal drains." *Proc., 8th Int. Conf. on Computer Methods and Advances in Geomechanics*, 1.1, Morgantown, W.V., 789–794.